Statistical Mechanics and the Past Hypothesis

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Abstract:

Statistical mechanics is a time invariant explanation of thermodynamic phenomena at a microphysical level. However, given that the laws of thermodynamics are not time-reversal symmetric, it is unclear whether to introduce the asymmetry through boundary conditions (through the past hypothesis) or through the dynamic laws themselves. In this paper, I defend the need of a boundary condition for statistical mechanics against two main objections: that there is no independent knowledge of the past hypothesis, and that the dynamic laws in statistical mechanics should be time-reversal asymmetric. I first introduce core notions of statistical mechanics, explain the past hypothesis and its motivation. Then, I bring up the two main objections against the past hypothesis and my subsequent defense against them.

Statistical Mechanics is a time invariant, microphysical explanation of thermodynamics. However, to make correct retroactive predictions (retrodictions) through statistical mechanics, the past hypothesis must be included. In this essay, I will defend the past hypothesis and statistical mechanics explanation of thermodynamics against two objections: that there is no independent knowledge of the past hypothesis, and that time-invariance should be incorporated into the laws governing dynamics instead. Over the course of this paper, I will first give an overview of statistical mechanics, before motivating the past hypothesis. Then, I will present the two objections and answer them.

Statistical mechanics is a theory that is supposed to explain thermodynamic laws in microphysical terms. At the time, scientists wanted to explain the Second Law of Thermodynamics, that the entropy of the universe (as an isolated system) never decreases, in terms of the particles that constitute the system. So, they devised statistical mechanics, which can be broken down into three components: first, 6N phase space, second, the statistical postulate, and third, Boltzmann's theorem. First, to explain the phenomena that we observe, like, for example, ice melting in a cup of water, we need to consider a higher dimensional space that encodes all the information of each particle in a system. This space is called phase space, and has 6N dimensions for N particles. Three dimensions are used to describe the position of the particle in the x, y, and z direction, and three dimensions are used to describe the velocity of the particle in the x, y, and z direction. In this 6N dimensional space, all the information for each particle can be represented. Here, one point in the 6N phase space corresponds to a single combination of all position and velocity values for each particle, also known as a microstate. Individual microstates can also be grouped together based on the macroscopic properties of the system they define. These properties include temperature, pressure, and volume, to name a few. These groupings of microstates are called macrostates and can be expressed in phase space by the volume that the number of microstates corresponding to one particular macrostate takes up. This leads to the statistical postulate, which states that the volume of a macrostate in phase space corresponds to the probability that that macrostate will be observed. Therefore, the bigger the volume of a macrostate in phase space, the greater the likelihood that we will observe the system in that state. Lastly, Boltzmann's theorem describes the evolution of a system over time. It states that the vast majority of microstates in a given macrostate are going to evolve into a microstate that is a part of a macrostate with lower entropy, also represented as a macrostate with a larger volume in phase space.

With this conception of thermodynamics, scientists can make accurate predictions of the future. For example, consider an ice cube in a cup of water at room temperature. In this system, there are more possible orientations for the water molecules if they were in liquid phase than if they were in the solid phase, where they need to be in an ordered structure. Therefore, the volume of phase space that corresponds to all the water molecules being in liquid phase is larger than the volume of phase space that corresponds to some of the water molecules being in the solid phase, or being an ice cube. So, by statistical mechanics, and Boltzmann's theorem in particular, we should see the ice turn into water in the future. This prediction matches our observations, where we always see ice melt into water at room temperature.

Now I will go onto the main problem with statistical mechanics. Here, our microphysical explanation of thermodynamics is time-reversal symmetric. This means that, with only statistical mechanics, we would predict that if we go back in time, our system would also be in a macrostate with higher entropy. This diverges from our knowledge of the past. So, either statistical mechanics is wrong, or our knowledge and records of the past are incorrect. So, statistical mechanics gives us bad retrodictions. This argument is called the reversibility objection. Because Boltzmann's theorem is derived from fundamental Newtonian mechanical laws, all of which are time-reversal symmetric, Boltzmann's theorem is also time-reversal symmetric. But, the processes we see are not. For example, we do not see a cube of ice in a glass of water melt if we were to go back in time. We would see the ice cube grow. But, based on statistical mechanics, it is very unlikely for that cube of ice to get bigger, even if we were to go back in time, simply because the macrostate that corresponds to a larger ice cube takes up less volume in phase space. So, because statistical mechanics cannot make accurate retrodictions, it must be an incorrect theory of thermodynamics.

Because of this problem, scientists added on the past hypothesis. The past hypothesis states that the universe was at a low entropy state at the beginning of time. This effectively acts as a boundary condition for statistical mechanics. With the past hypothesis, we can use statistical mechanics to make accurate retrodictions. For example, if we were to retroactively predict what an ice cube in a glass of water at room temperature would be like at an earlier time with the past hypothesis in mind, we would be able to say that it is very likely for the ice cube to have been a bigger ice cube in the glass of water. This lines up with our memory of the ice cube in the glass of water, and therefore allows statistical mechanics to make accurate retrodictions.

There are two objections to the combined theory of statistical mechanics and the past hypothesis that I will go over in this paper. The first objection is that there is no independent verification or motivation for the past hypothesis. It seems, at least at this point, that the past hypothesis is just tacked onto statistical mechanics without any justification. Therefore, in order to justify why adding the past hypothesis to statistical mechanics is the right way to correct the theory, we need some independent justification of the past hypothesis.

Jill North seems to answer this objection in her paper "Time and Thermodynamics." There, she suggests that there is cosmological evidence that, at the very least, can independently support the past hypothesis. This cosmological data indicates that right after the big bang at the beginning of time, the universe was uniformly distributed in thermal equilibrium. At this point, the universe existed in an extremely low entropy state due to gravity. This cosmological evidence provides independent support of the past hypothesis. Therefore, adding on the past hypothesis to statistical mechanics creates a correct theory that can accurately explain our observations.

This response, however, seems to assume that our observations and records of the past are correct. In order to take measurements and interpret cosmological data, we rely on the fact that our records of those measurements and data are accurate. But, our goal is to prove that the conclusion drawn from just the statistical mechanics theory of thermodynamics is wrong. That is to say, we are trying to prove that our records of the past are correct. So, assuming that our records of the past are correct to make conclusions from the cosmological data is not independent verification of the past hypothesis.

At this point, it seems like the kind of independent verification that North is trying to give in her cosmological data argument is different from the independent verification that the objection might be referring to. For North, the cosmological data seems to be answering an objector who may want justification for the past hypothesis without using thermodynamic laws like statistical mechanics or the second law of thermodynamics. That is to say, we want to show that the world exists at low entropy by utilizing physical laws other than thermodynamic laws. To this extent, her response seems to hold. However, it cannot answer an objector who is looking for independent verification of the past hypothesis such that the assumptions used for justifying the past hypothesis are not the same as those used in statistical mechanics. So, for this objector, another answer is needed.

Unfortunately, we cannot answer this objector by providing justification of the past hypothesis without using the assumption that our past records are correct. However, there may be another reason to believe that the past hypothesis is correct, and in turn, that our records of the past are correct. To do science, we need to believe that our perceptive faculties, like memories, and our records of them, are correct. The ramifications of our records being wrong extend beyond just the status of statistical mechanics and the past hypothesis. If the past was so completely different that we cannot trust our records for cosmological support for the past hypothesis, it would also mean that we could not trust our records for anything else. Other laws that many scientists take to be correct, like Newtonian mechanics, were developed based off data and observations, and more importantly, our memories and records of those observations. Therefore, believing that our records are flawed would mean that we would not have reliable evidence for any of our physics. If we were to reject all our current knowledge of physics by saying that our records are inaccurate, then it would make sense that statistical mechanics and the past hypothesis should also be rejected. But, if we do not want to reject our current knowledge of physics or the way in which we currently conduct science, we must assume that our perceptive faculties and the data we gather from these faculties are correct. So, we cannot believe that our records are incorrect. Therefore, we should trust the cosmological data that upholds the past hypothesis. So, statistical mechanics and past hypothesis theory of thermodynamics is a good theory of thermodynamics.

Now, I will address the second objection against the statistical mechanics and past hypothesis theory of thermodynamics. It seems like statistical mechanics was shown to be a bad theory when it could not make proper retrodictions. Adding the past hypothesis to statistical mechanics, however, does not seem like the right way to fix this problem. The problem with statistical mechanics may be the fact that the dynamics are time-reversal invariant. After all, there is no way for us to use time-reversal invariant dynamic laws to explain a time reversal asymmetric process. Another way to say it is this: there is no reason we need to have a time-reversal symmetric account of thermodynamics, like we do in statistical mechanics. It could be the case that the fundamental laws that govern thermodynamics should be time reversal asymmetric. Adding asymmetry into statistical mechanics by changing the boundary conditions is not the right way to get a good microscopic theory of thermodynamics.

First, it may be a good idea to go over some possible time-reversal asymmetric theories of thermodynamics. I will introduce two leading propositions brought up by North, namely ergodic theory and the GRW theory. According to Boltzmann, a system is ergodic if "for almost all initial conditions, its trajectory passes through every point in the available phase space" (North 26). So, if a system is ergodic, we can use the mathematical theorems from ergodic theory to explain the entropy increase through the dynamics. GRW theory, on the other hand, is a collapse theory that is used to explain phenomena seen in quantum mechanics. It adds a probabilistic collapse law that, along with the Schrödinger equation, explains how wavefunctions collapse for particles. This collapse law can be multiplied by a gaussian to give the probability that a wavefunction will collapse into other possible wavefunctions. The notable feature about the collapse law in GRW theory is that it does not give chances for different past wavefunctions, only future possible wavefunctions that a given wavefunction can collapse into. Therefore, it is a fundamental, time-reversal asymmetric law. This can be used to explain thermodynamic phenomena because the wavefunction is localized to a certain location in phase space, predicting which microstate, and therefore which macrostate, the system collapses to.

In her paper, North gives some specific replies for the aforementioned time-reversal asymmetric theories and for incorporating time asymmetry into the dynamics of thermodynamics. First, the two theories mentioned before may not even be true. So far, normal systems have not been shown to be ergodic, which means that the theorems derived from ergodic theory may not apply. Furthermore, GRW theory may not be the right theory of quantum mechanics, as other theories, like Bohm’s theory, are still contenders for explaining quantum mechanical phenomena. Second, time-reversal asymmetric theories generally seem to still require asymmetric boundary conditions to make correct thermodynamic predictions and retrodictions. So, it seems like the theories that do incorporate time-reversal asymmetric laws cannot avoid incorporating a boundary condition.

This answer, however, seems to only address particular instances of time-reversal asymmetric theories of thermodynamics. It does not answer why we should accept a thermodynamic theory that has time-reversal invariant dynamic laws, or why time-reversal invariant dynamic laws should be preferred over time-reversal asymmetric dynamic laws. For this, we must consider how thermodynamic laws are derived and what things they govern. Thermodynamic laws govern particles that make up a system, which are subject to Newtonian mechanics. This means that the laws which govern the individual movement of different particles in a system are time-reversal invariant. For the most part, thermodynamic laws that govern the evolution of particles over time, like Boltzmann’s theorem, for example, are also derived from the individual movements of the particles in the system. Therefore, they must also be time-reversal invariant. If these laws were time-reversal asymmetric, there would need to be some explanation as to why some laws that govern particle dynamics in a system are time-reversal symmetric while other laws, which also seem to govern the same particle dynamics in a system, are not. Furthermore, if these time-reversal asymmetric laws were derived from Newtonian mechanics, it would be difficult to pinpoint where asymmetry would come into play. Having time-reversal invariant laws that govern the movement of particles in our theory of thermodynamics avoids this problem, and therefore, should be preferred over time-reversal asymmetric laws of thermodynamics.

In conclusion, the current thermodynamic theory of statistical mechanics paired with the past hypothesis seems to hold against two prominent objections that I raise in this paper. But, it should be acknowledged that this theory of thermodynamics may not be the most intuitive. This unease, however, is likely because the processes we observe are themselves time-reversal asymmetric and not because the theory itself is incorrect. Ultimately, statistical mechanics and the past hypothesis, considered together, give the best explanation of the thermodynamic processes that we observe.

References

North, Jill. "Time in Thermodynamics." *Oxford Handbooks Online*, 2011.