Summer Research

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Quantitative Three-Dimensional Basal Ice Roughness from Scanning Electron Microscopy (SEM)

Katie Gray
University of Puget Sound, kgray@pugetsound.edu

Shreeti Patel
University of Puget Sound, spatel@pugetsound.edu

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Katie Gray*, Shreeti Patel, and Steven Neshyba

1. Crystals comparable to those found in Cirrus clouds can be grown in and imaged using an SEM.

Using a backscatter detector in the chamber, both the temperature and water vapor pressure can be adjusted to create environments in which the crystals can grow and ablate. It's important to note an image brightness as detected by the SEM response is determined by *facet orientation* and *detector positions (A-D)* in a reproduceable process.
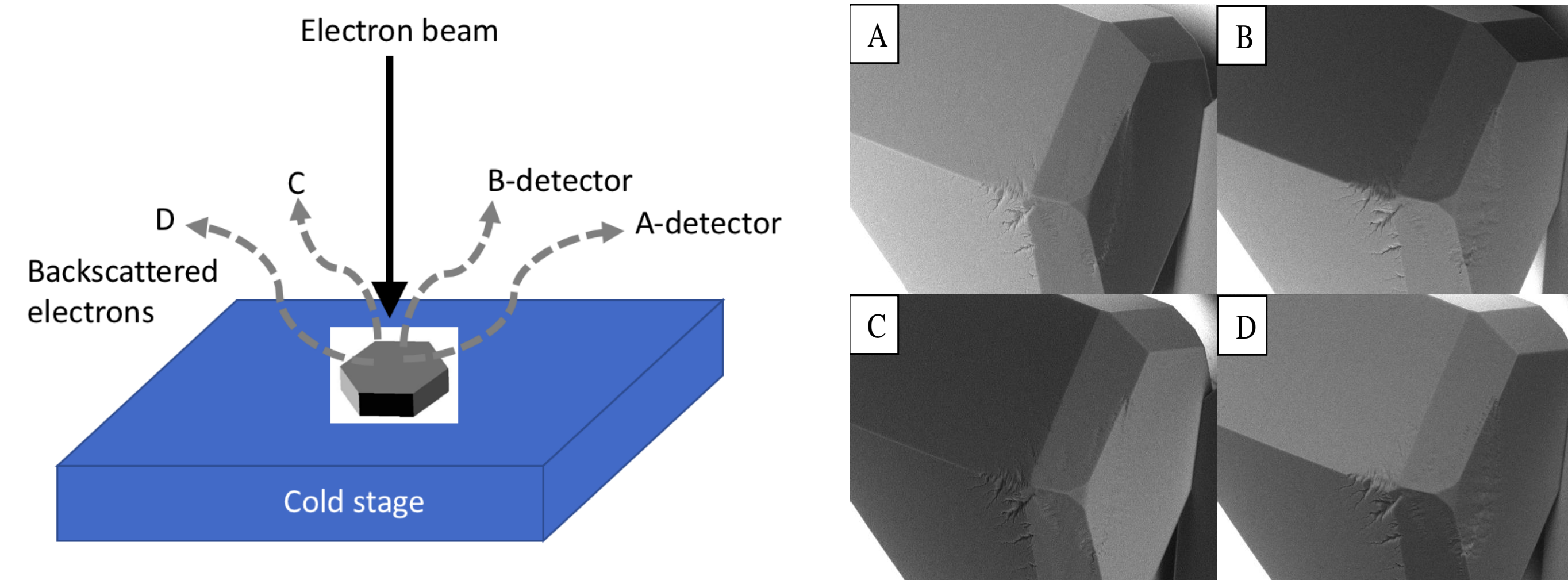
2. Calibrating the SEM image response to known facet orientations allows us to predict the response for any other variation of the same crystal.

The key variable to SEM response is the orientation parameter,

$$s \equiv \vec{n} \cdot (\vec{d}_l - \vec{b}) / |\vec{n}|$$

where $\vec{n}$ is the normal vector to the crystal facet at a given point, $\vec{d}_l$ identifies the orientation of the lth SEM detector, and $\vec{b}$ indicates the orientation of the electron beam. As shown in the figure at right, observed SEM intensity satisfies,

$$c_{obs} \approx ms + b \equiv c_{model}$$

where a linear best fit yields the parameters m_l (sensitivity) and b_l (background brightness). $ms + b$ defines the *forward model.*
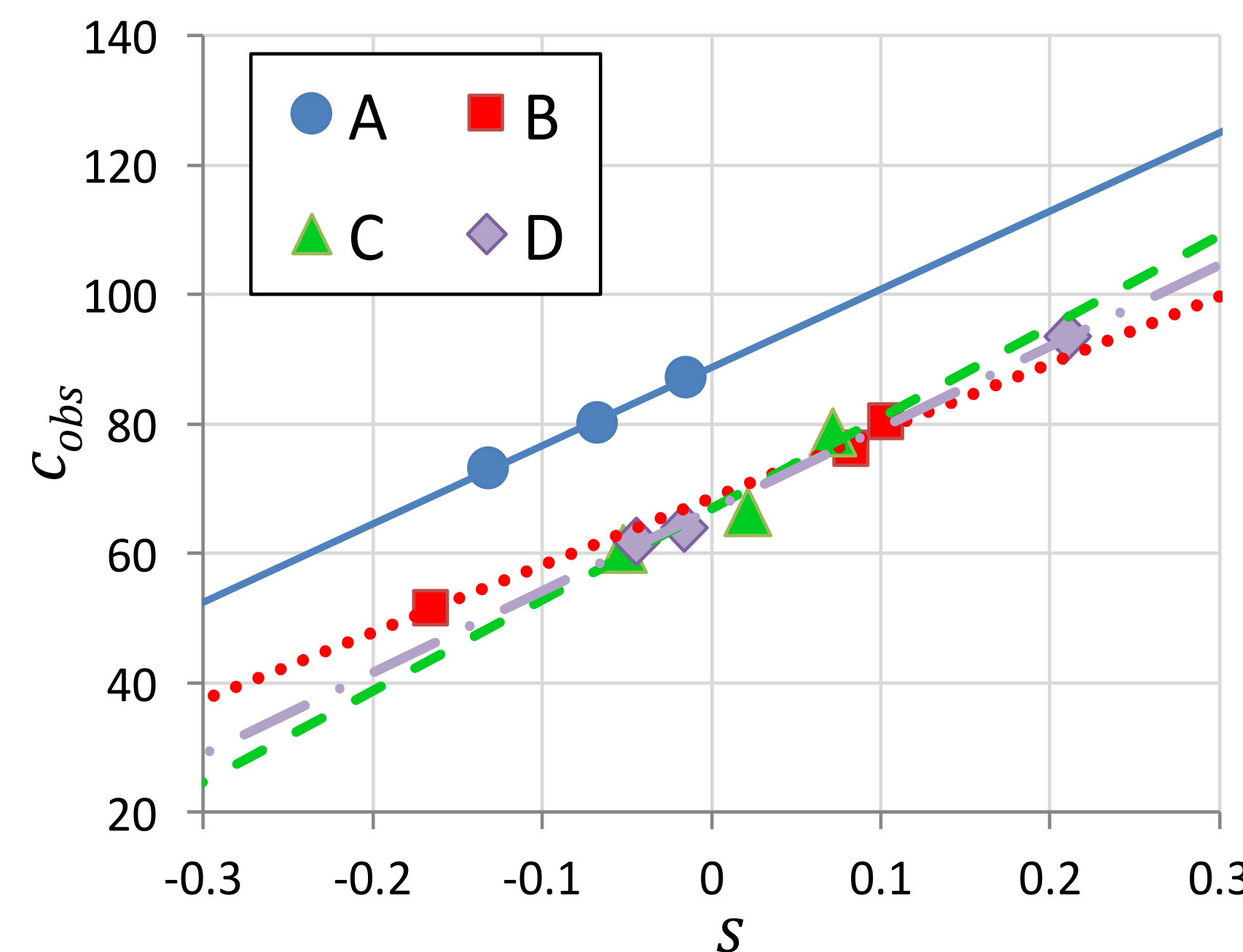
3. Gauss-Newton inversion in a Bayesian Framework (GNBF) permits robust, iterative construction of $h(x,y)$ from observed SEM images.

Providing a global solution to $Z(x,y)$ while minimizing noise. The iterative formula has the form

$$h_{n+1} = h_a + (S_a^{-1} + K_n^T S_e^{-1} K_n)^{-1} [K_n^T S_e^{-1} (c_{obs} - c_n + K_n(h_n - h_a))]$$

where c_n is the result of the forward model at iteration n, S_e is the variance in c_{obs}, and S_a is the *a priori* model variance. Satisfactory convergence usually occurs after 3-4 iterations.

The resulting data set of $h(x,y)$ is extremely large, each SEM image is made of thousands of pixels which are retrieved using the GNBF method, a high resolution probability density function is created, $\rho(Z^2)$, where Z^2 is the square of the gradient of $h(x,y)$.

## 4. Theory. One theoretical representation of a probability density is given by,

$$\rho(Z^2) = \frac{2}{\pi^{1/2}\sigma} \exp(-\frac{Z^2}{\sigma^2})$$

where σ is a statistical measure of the roughness on the surface of ice crystals, which can be obtained by least-squared fitting.

The appearance of a straight line in probability density functions of the ice crystals demonstrates a growth mechanism that is consistent with *scaling* and *intermittency. Scaling* contains the idea that growth mechanisms spans multiple spatial scales. A hypothetical binary scaling growth mechanism is shown at right,
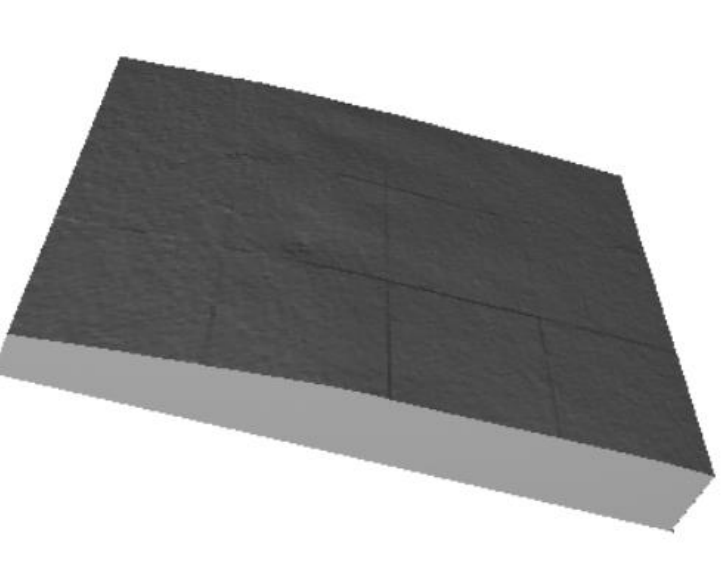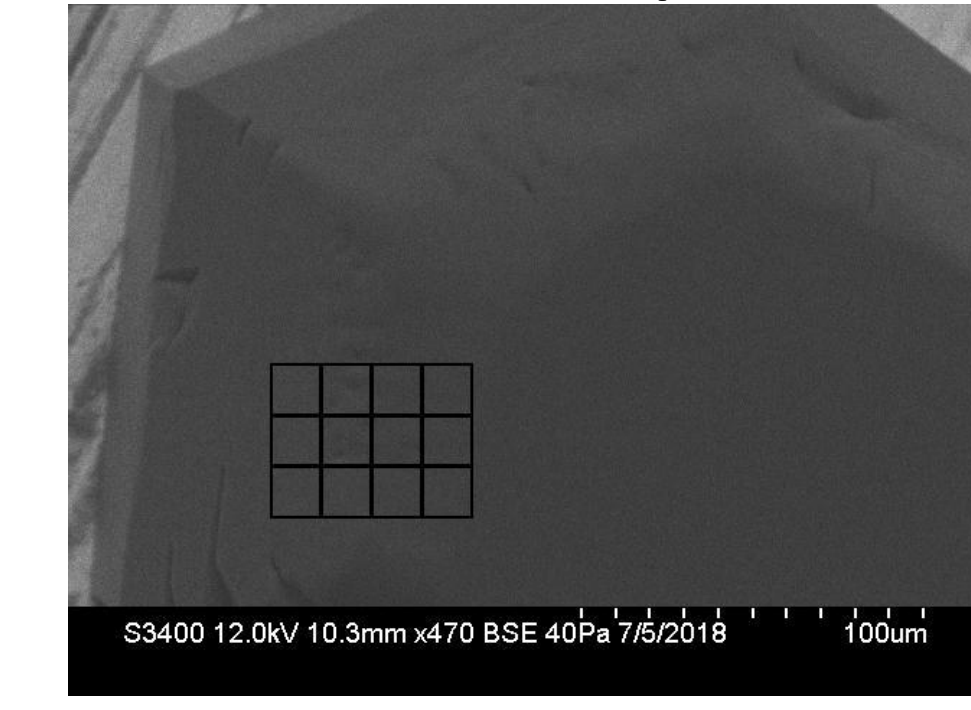

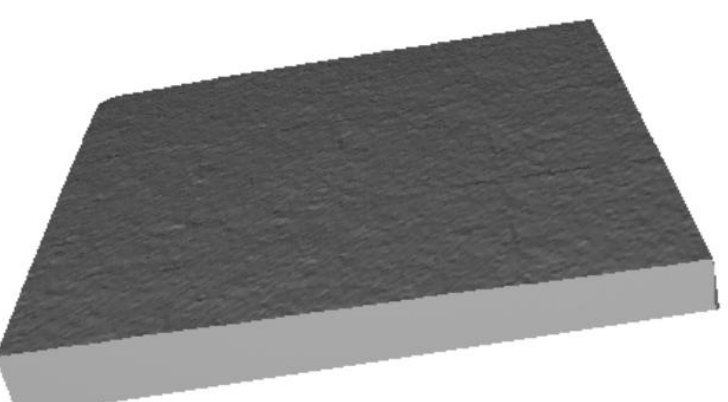## 5. A Sampling of Results

2018-07-05
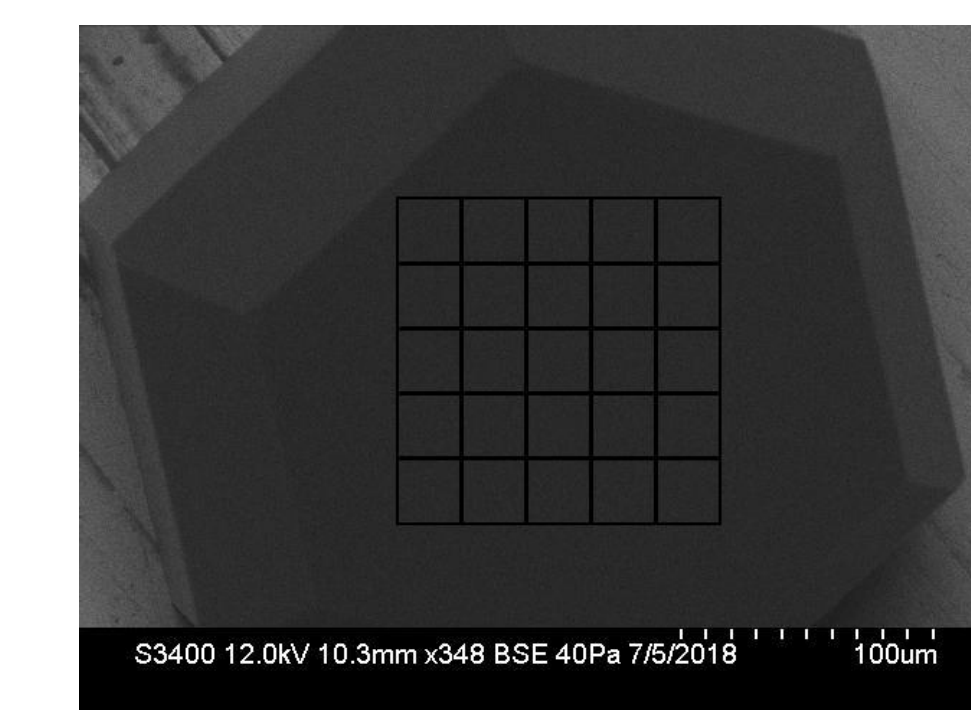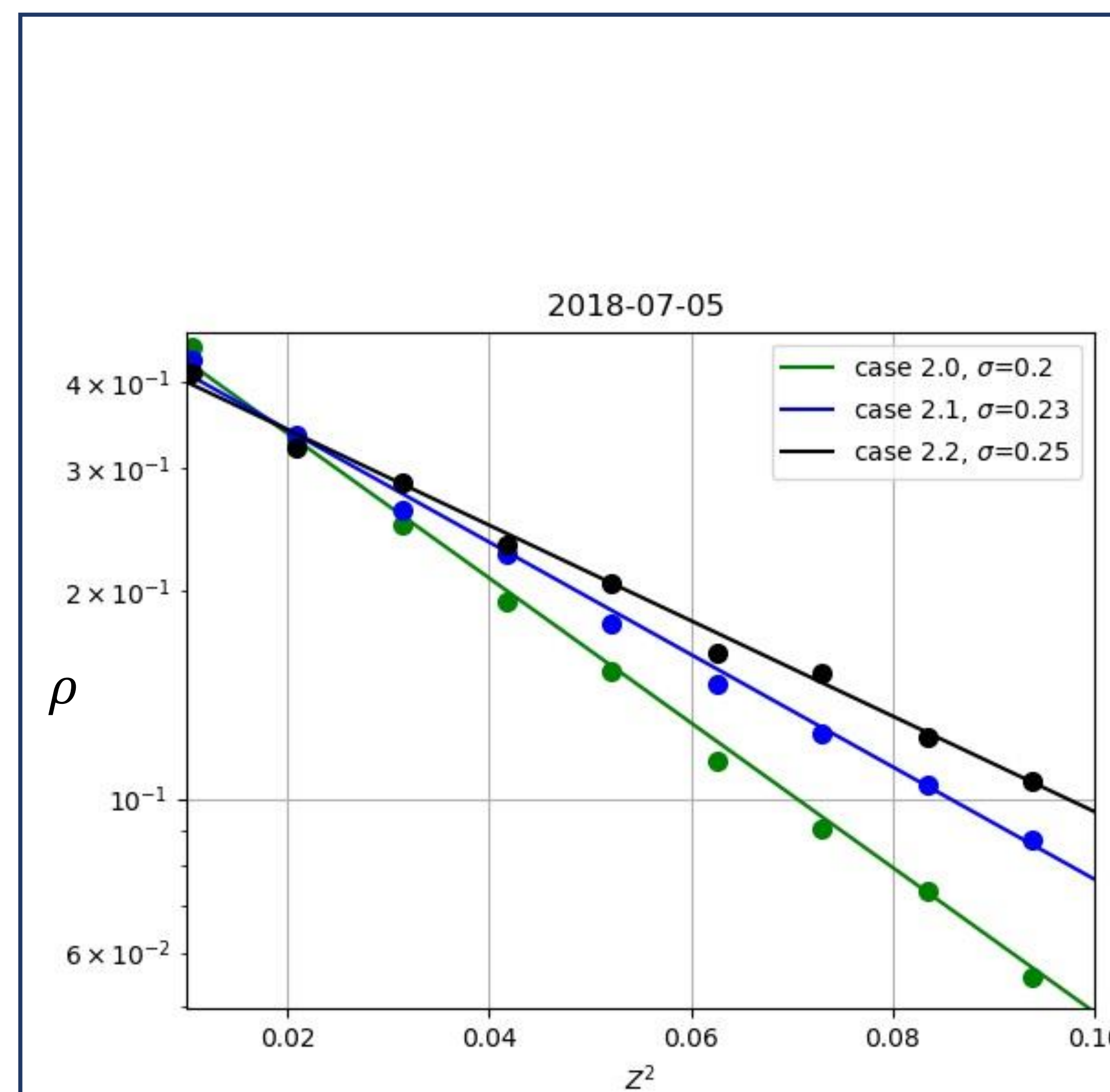
case 1.0, σ=0.21
case 1.4, σ=0.24
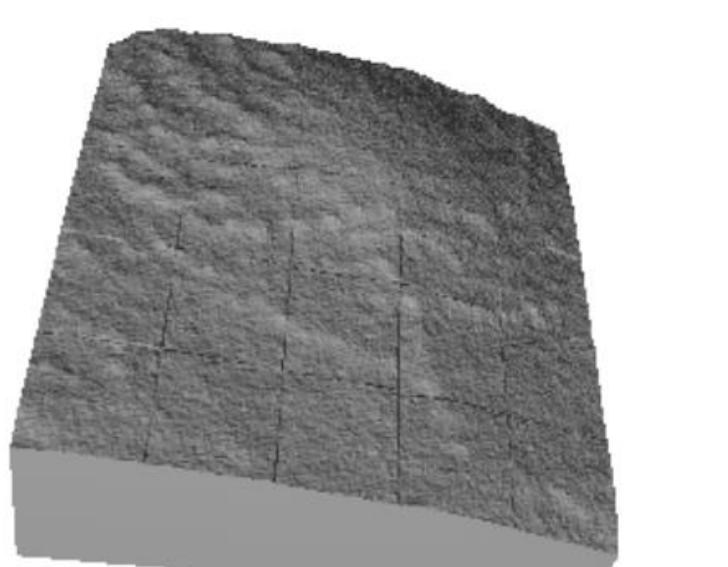case 1.6, σ=0.2
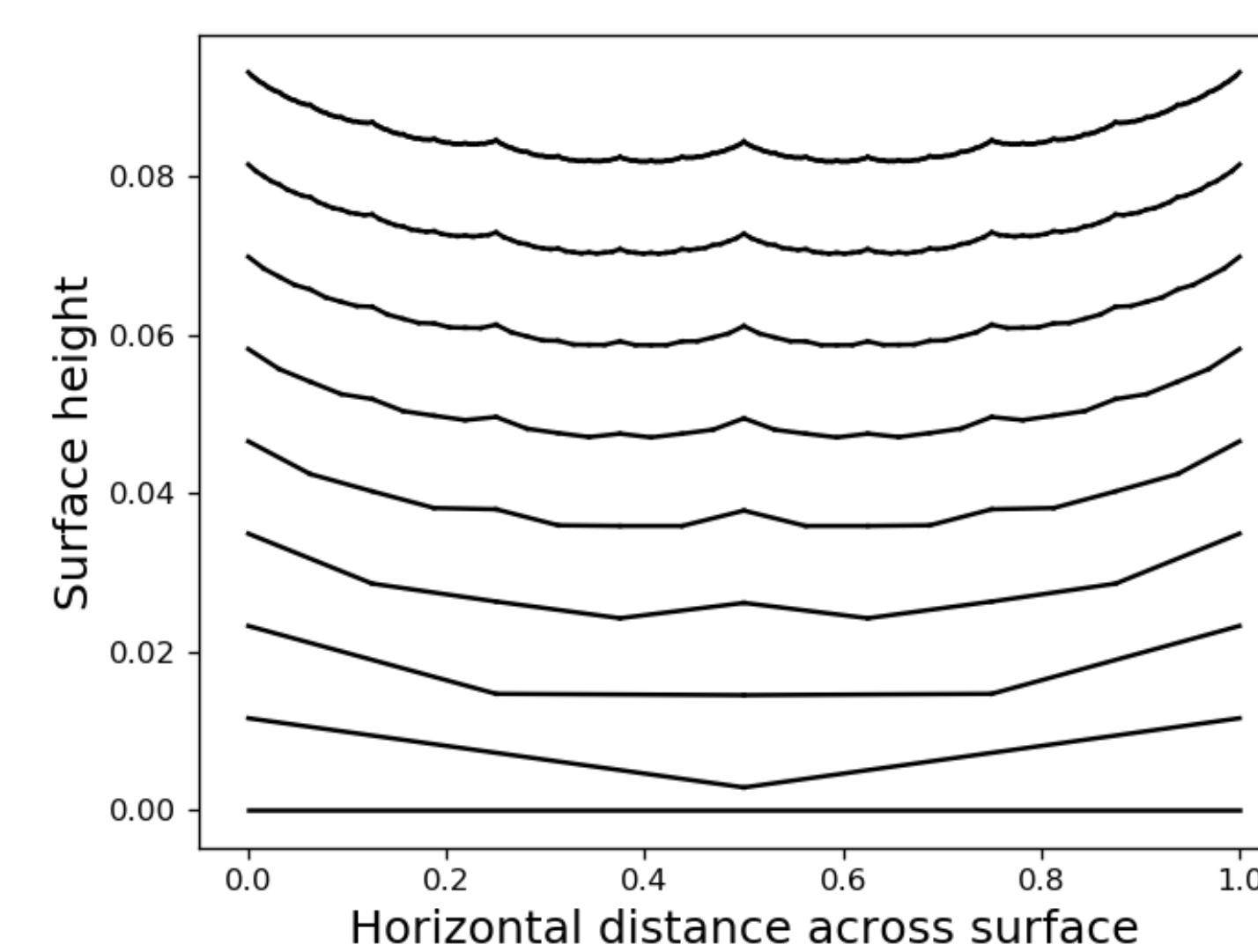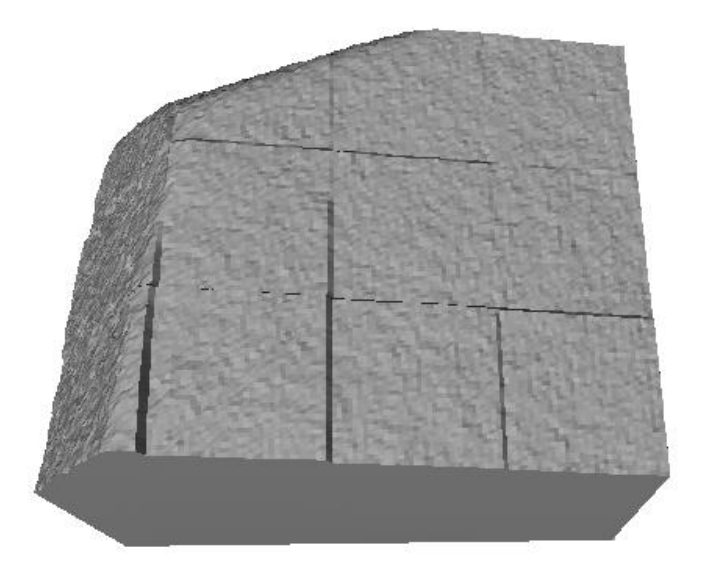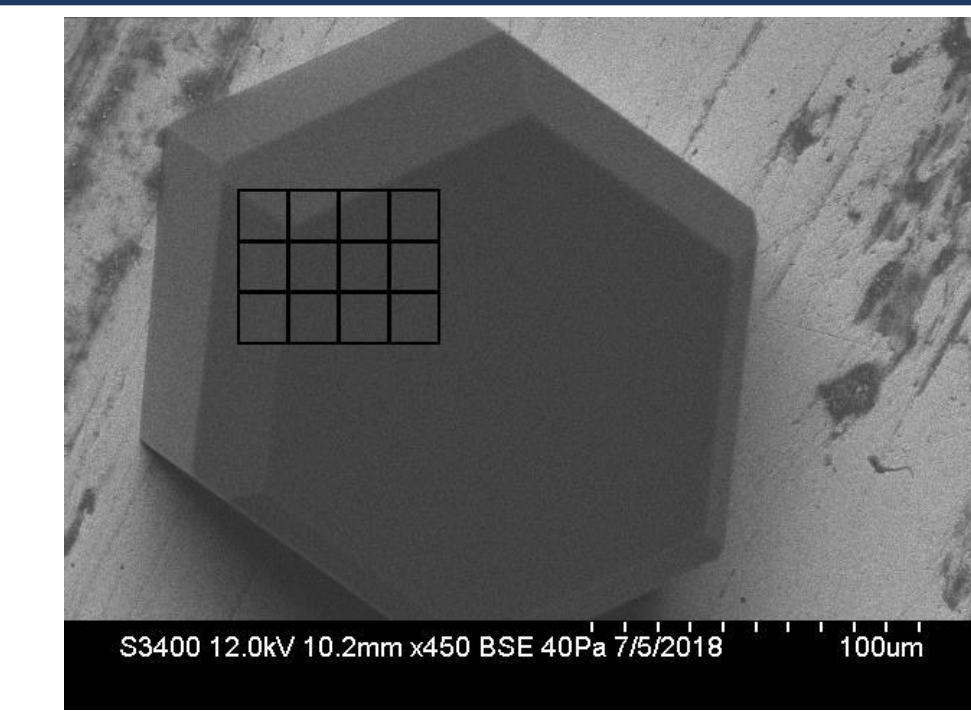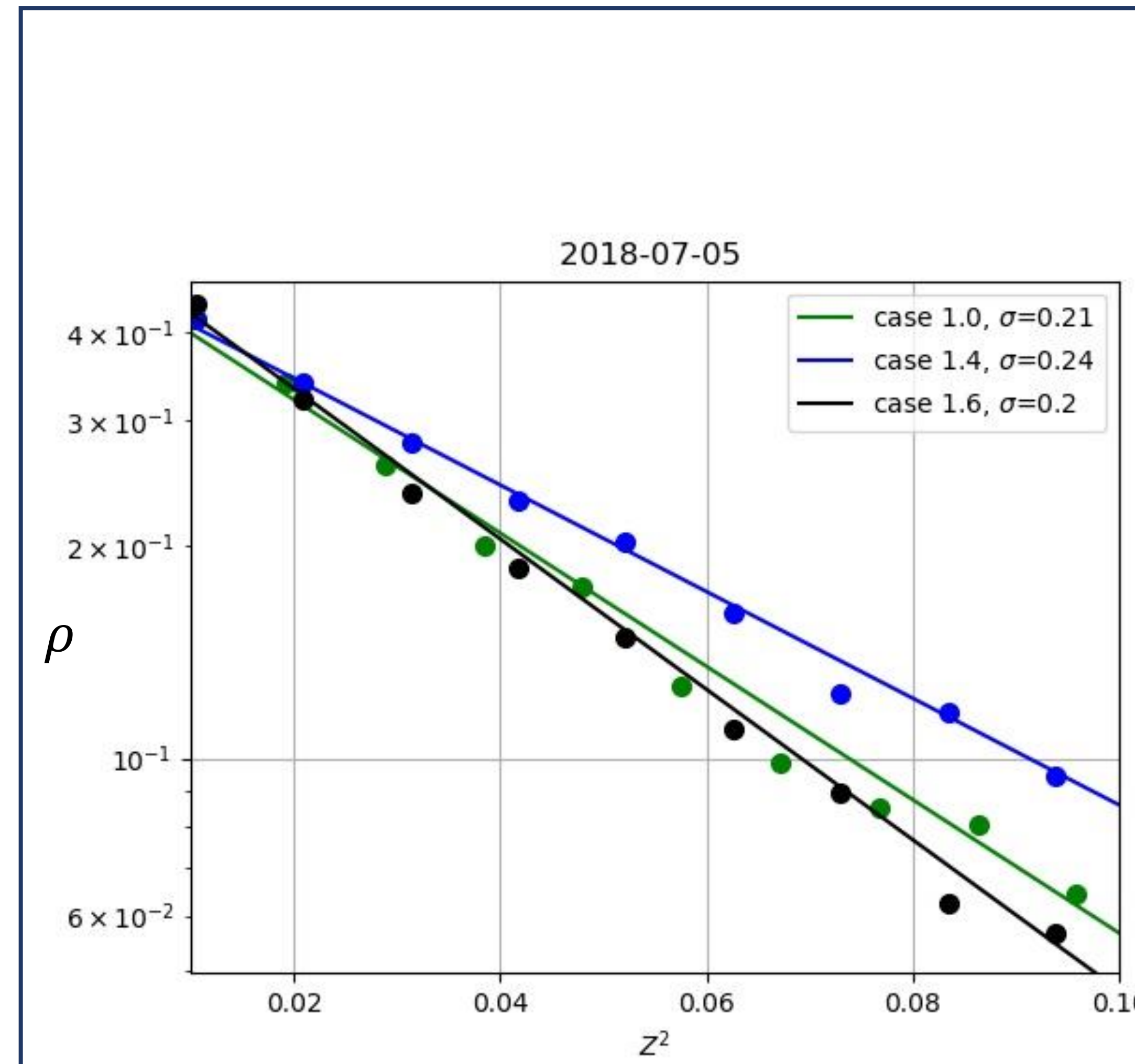
Case 1.0 Temperature -35°C at 40 Pa
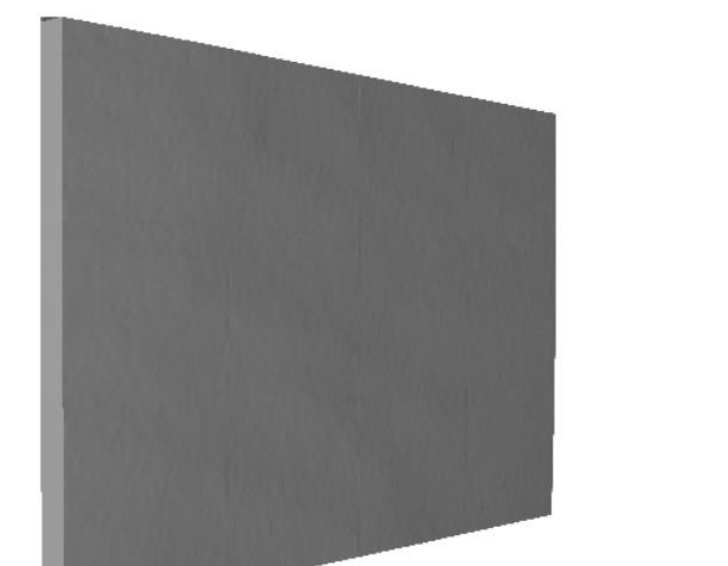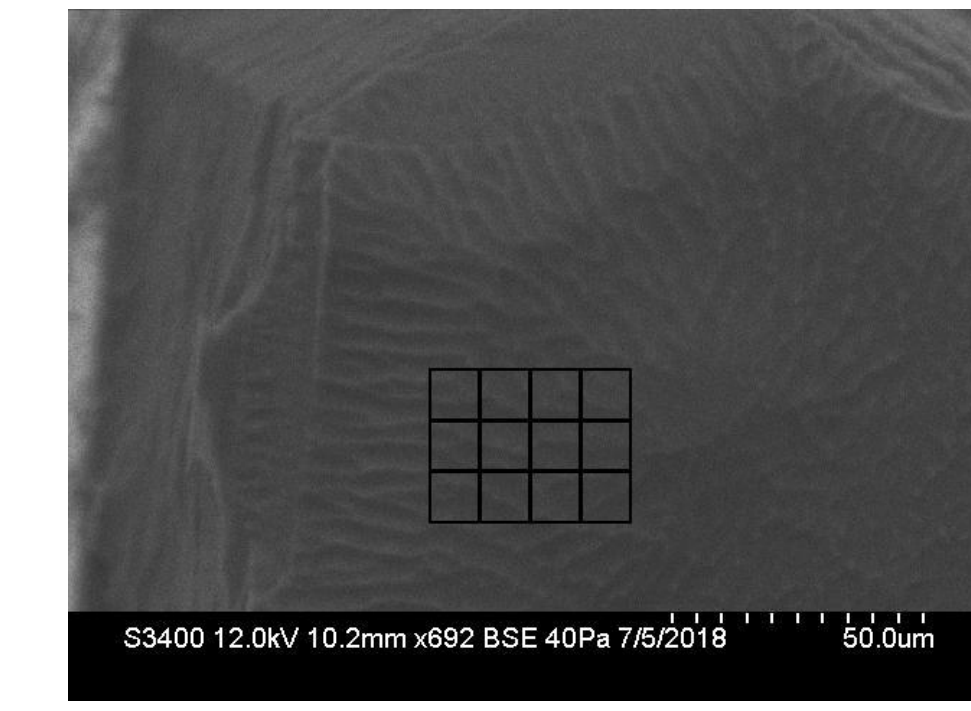
Case 1.4 Temperature -30 °C at 40 Pa

Case 1.6 Temperature -40 °C at 40 Pa

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case 2.0, σ=0.2
case 2.1, σ=0.23
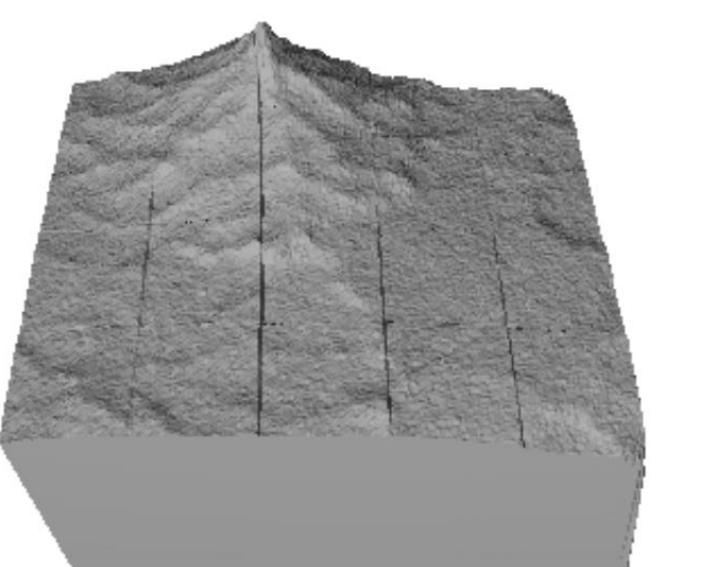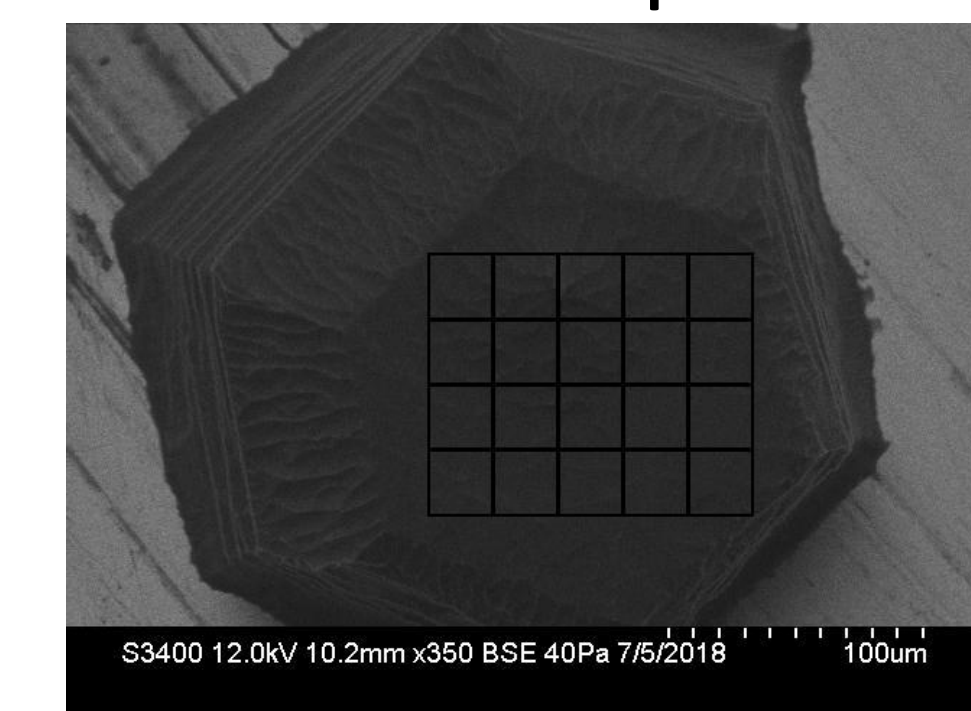case 2.2, σ=0.25

Case 2.0 Temperature -37°C at 40 Pa
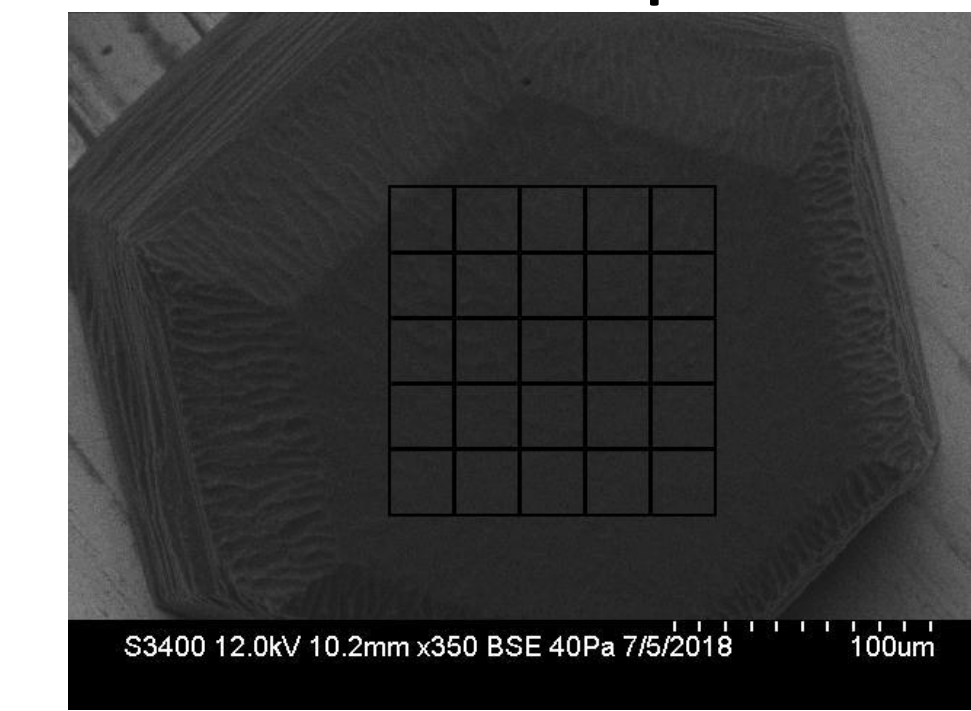
Case 2.1 Temperature -33°C at 40 Pa

Case 2.2 Temperature -29°C at 40 Pa

6. Conclusions

- Roughness on the surface of the basal facet of ice crystals has been quantitatively measured for the first time. Quantitative measure of roughness (sigma) does occur in a straight line, with σ ranging as large as 0.25 which lies in the range of remote sensing instruments.
- A straight line in the space $\log(\rho^2)$ provides compelling evidence that growth occurs by a **scaling mechanism**. We have proposed a binary mechanism consistent with this picture.
- The growth mechanism also appears to be influenced by **crystalline symmetry**.

7. References and Acknowledgements

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