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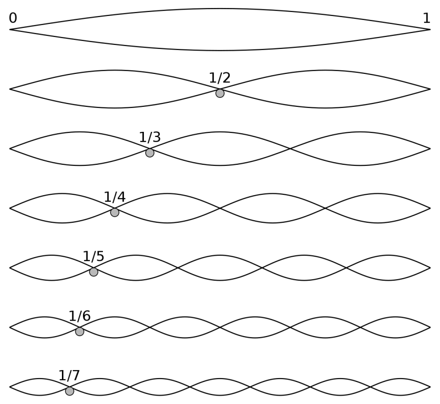
Bowing Behavior of Strings With Non-Uniform Mass Density

Andrew J. Brandt
abrandt@pugetsound.edu



Bowing Behavior of Strings with Non-Uniform Mass Density

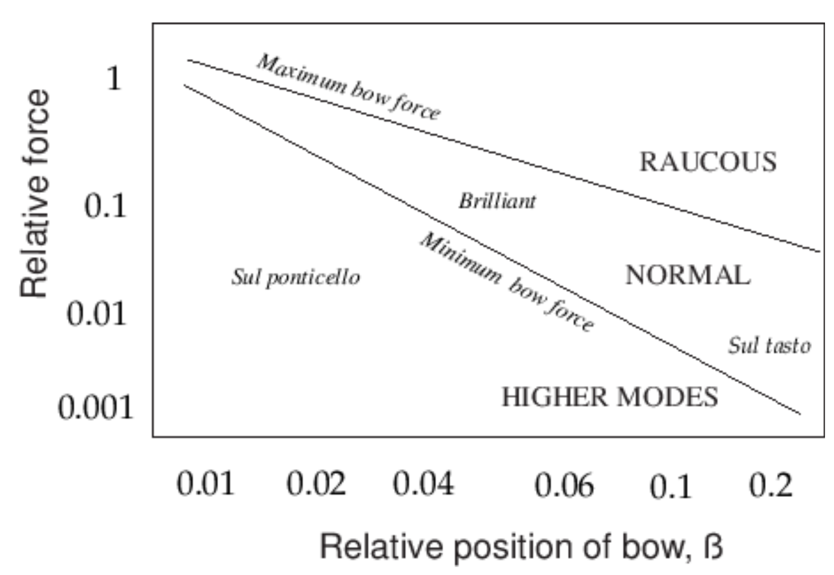
Andrew Brandt - Mathematics ’18, Dr. Greg Elliott

University of Puget Sound Physics Department

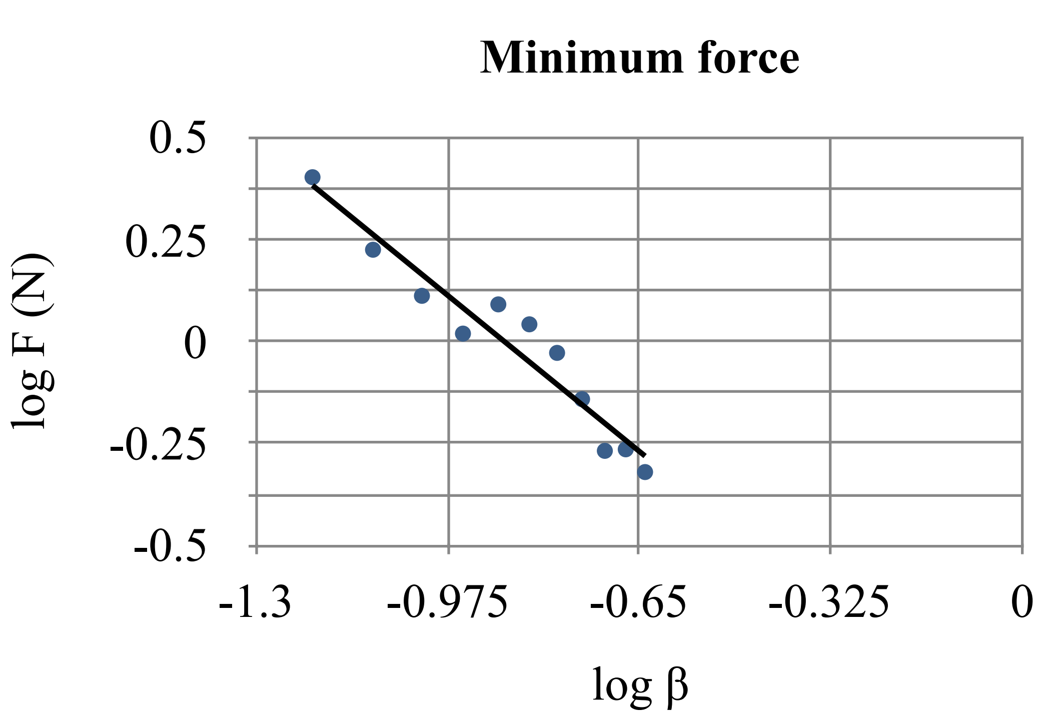
Introduction

The bowing behavior of strings with uniform mass density is well-documented; when bowed or plucked, they produce the fundamental tone and harmonic overtones. When a non-uniform string is plucked, it creates the fundamental tone and non-harmonic overtones. Most interestingly, bowing a non-uniform string produces the fundamental tone and harmonic overtones. This phenomenon, known as mode-locking, inspired us to this research, an exploration of how non-uniform we can make a string with regards to its mass density and still experience harmonic overtones.

Schelleng Diagram



A Schelleng diagram of a bowing system categorizes the different sounds produced and maps the regions of sound quality on a two dimensional diagram of bowing pressure versus the relative distance of the bowing from the bridge. Above is a sample Schelleng diagram on a logarithmic scale showing the minimum and maximum force curves along with the sounds produced. Below is the Schelleng minimum force curve for a uniform string that we tested which displays the standard shape of a Schelleng curve on a logarithmic scale. The goal of this project was to determine how the shape of the Schelleng minimum force curve changes for a string with a non-uniform mass density.



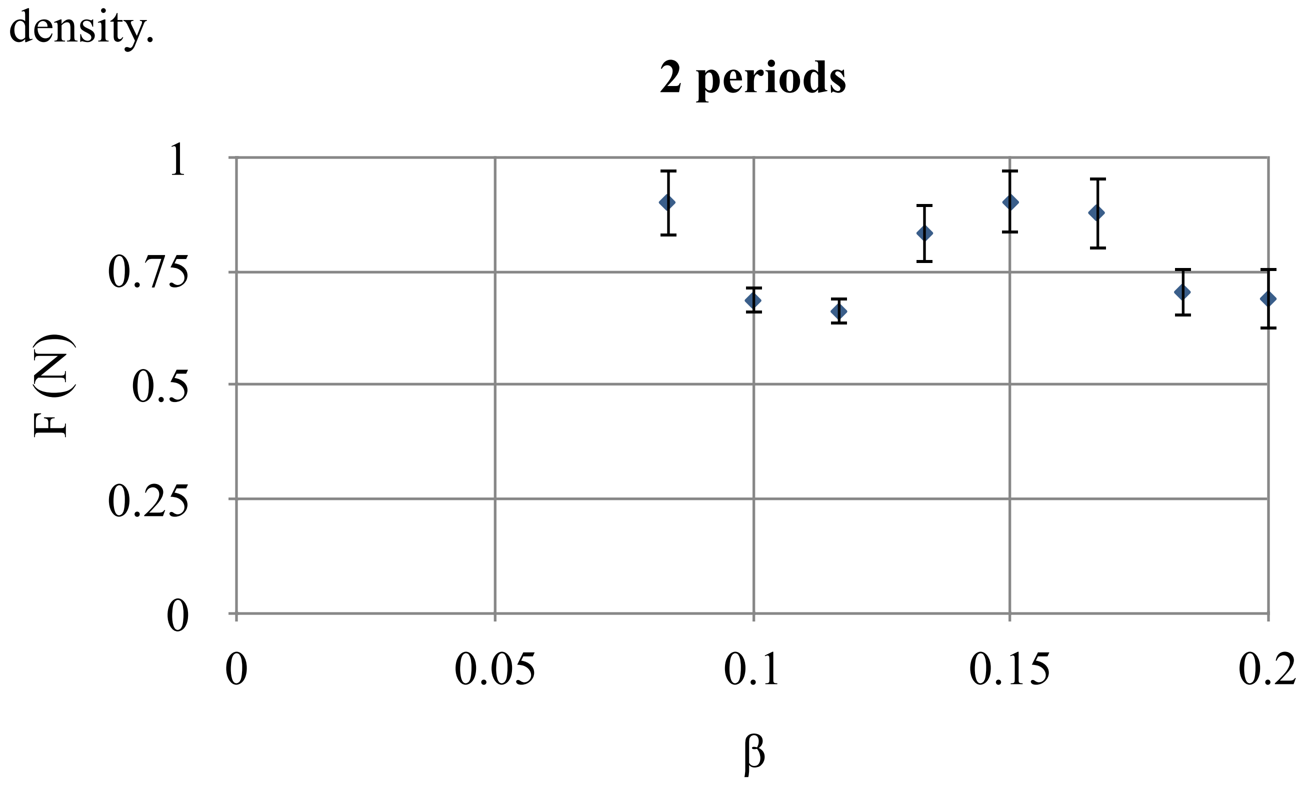
Our Work

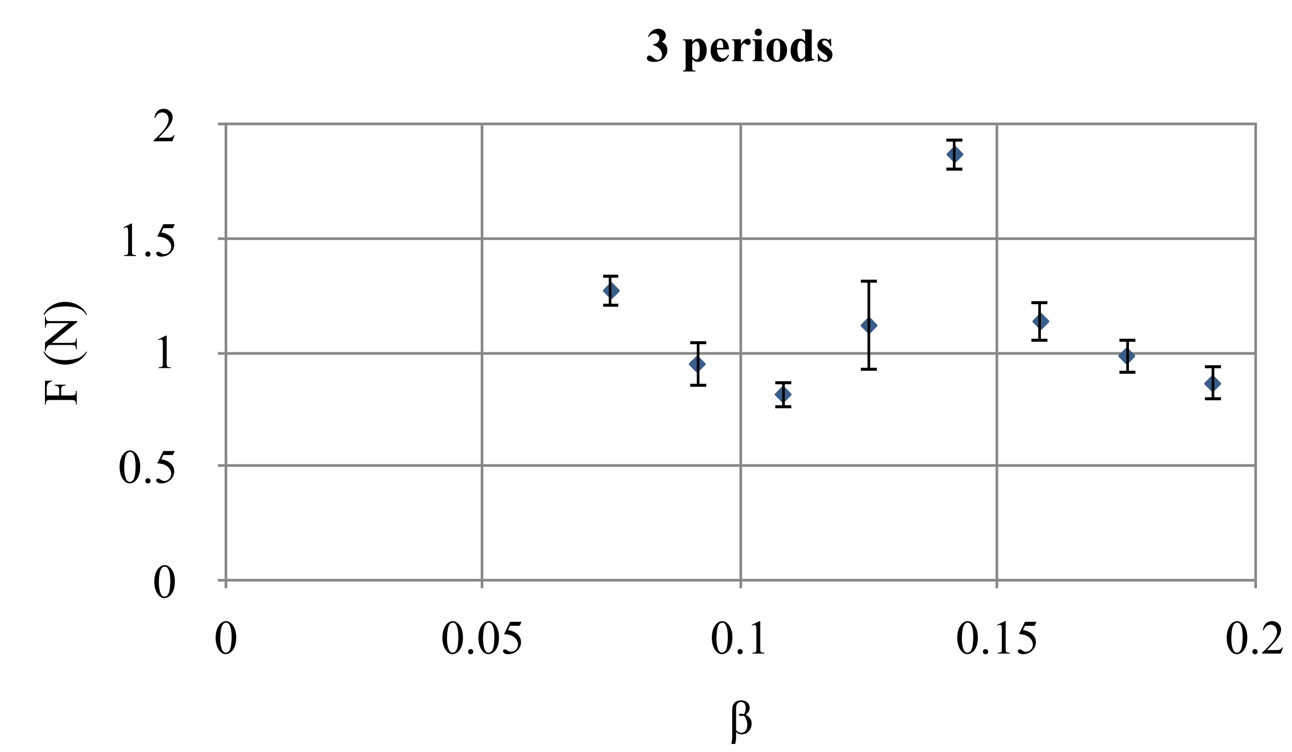
- Built and verified the reliability of an experimental setup in the lab to measure applied force with our unconventional hurdy-gurdy wheel on uniform and non-uniform strings.
- Created five strings with non-uniform mass density.
- Tested non-uniform strings with our apparatus to determine shape of their Schelleng curves.

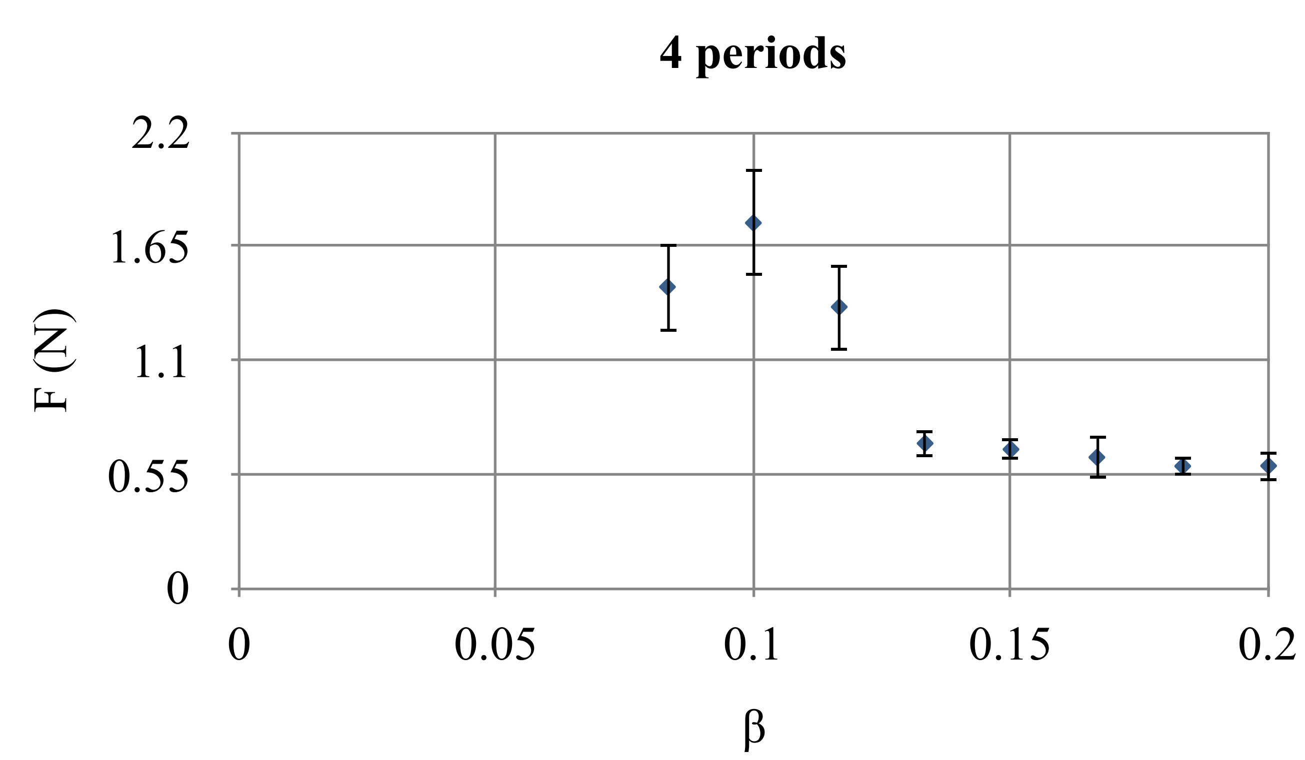
Cosinusoidal Variation

Our non-uniform strings had two different styles of mass distribution. The first three followed a cosinusoidal pattern of mass density, varied ± 40% in amplitude of density from our standard uniform string, and had either two, three, or four periods. The plucking of these strings has been studied by Dr Elliott previously and their behavior is well-understood.

The first graph below displays the Schelleng diagram for the cosinusoidal string with two periods over its entire length on a non-logarithmic scale. This string starts off acting similar to the power function of the uniform string but experiences a marked increase in necessary force near the point where mass density is highest (the peak of the cosine graph). The second and third graphs show cosinusoidal strings with three and four periods, respectively. As the number of cosine periods increases among these strings, the spike in the force measurement moves closer to the bridge. Considering this result achieved using strings with gradual mass density changes, we next investigated the behavior of strings with stark discontinuities in mass density.

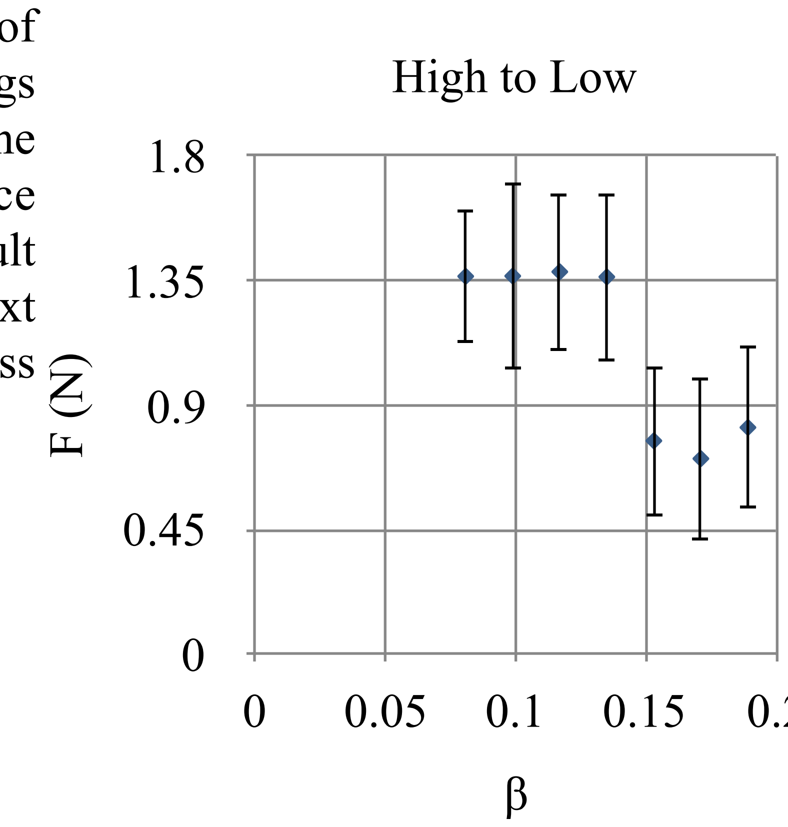


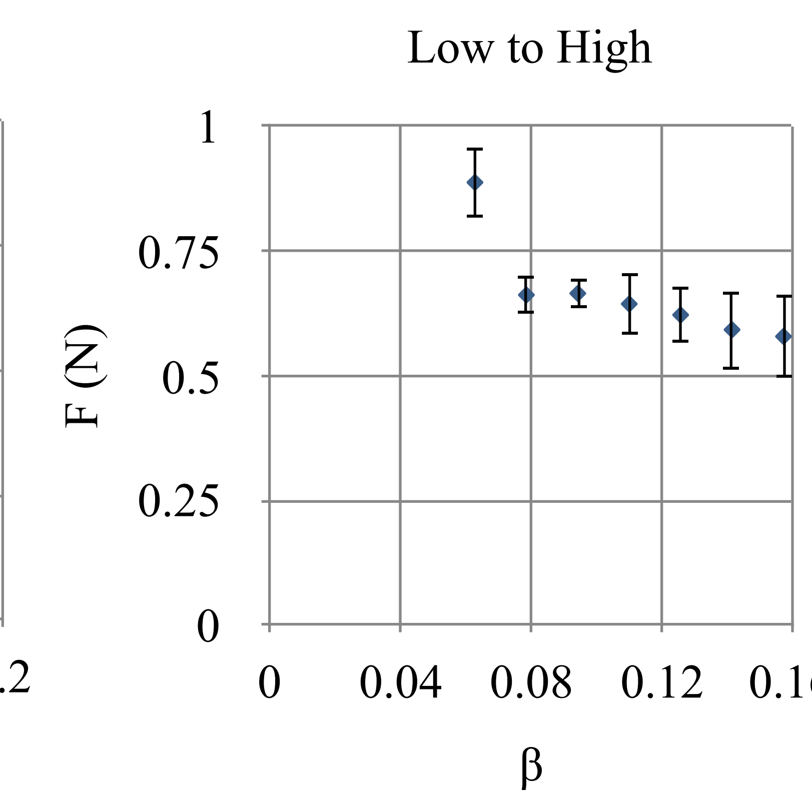




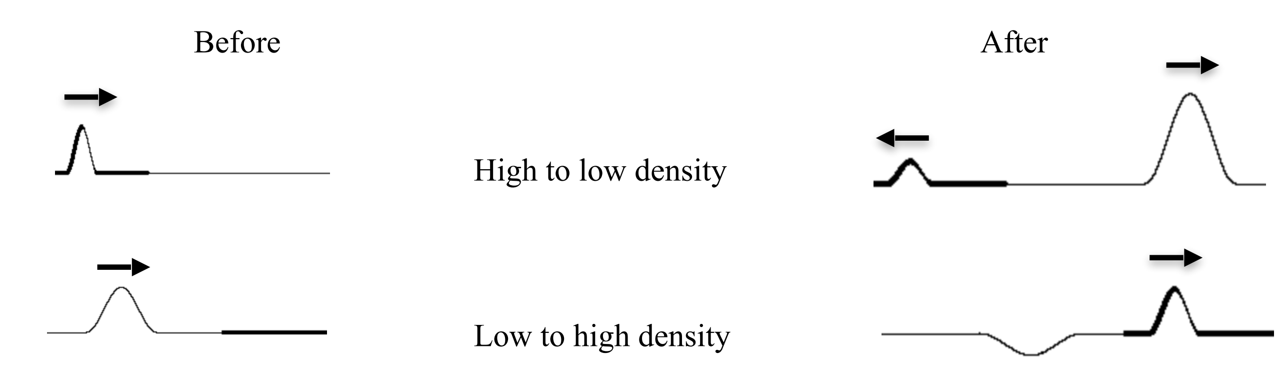
Impedance Discontinuities

The other two strings we made had two distinct sections, each with constant density, that met at a discontinuity inside the tested region. One had a small section with high density while the rest of the string was low density and vice versa for the other string. The discontinuities were located at β = .11 for the first and β = .1 for the second. Examining the Schelleng graphs of these strings below reveals that the high to low string has a sharp force plateau whereas the low to high string has a gentle, downward slope after a slight drop-off. Our hypothesis is that the proximity of the discontinuities to force drop-offs is related to the behavior of wave pulses at impedance discontinuities.





As seen below, the result of a wave pulse encountering an impedance depends on whether the new medium is more or less dense than the original. We hypothesize that the smaller force drop-off for the low to high string is due to the inverted nature of the reflected wave seen below.



Future Work

Later this fall, as part of my thesis work, I plan to make and test more strings with impedance discontinuities to further our understanding of how these mass density changes affect the shape of the Schelleng curves. Furthermore, we plan to explore the possibility of developing a mathematical model to describe the relationship between a string’s density function and the minimum bow force curve.

Acknowledgements

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