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Abstract

In this study we examined the responses of 16 low-achieving students to reform-based mathematics instruction in 5 elementary classrooms for 1 year. We used qualitative methods at 2 schools to identify the needs of low achievers in these classrooms, which were using an innovative curriculum. Through classroom observations and interviews with teachers, we studied the involvement of low achievers in whole-class discussions and pair work. Results suggested that both the organization and task demands of the reform classrooms presented verbal and social challenges to low achievers that need to be addressed if these students are to benefit from reform-based mathematics instruction.

The elementary mathematics classroom was once a place of clear goals and familiar routines. Students were expected to learn the basic operations so that they could solve computational problems quickly and correctly. During math lessons, students had to listen carefully as the teacher explained the desired way to solve various types of problems and then work independently to practice the teacher's method until it was automatic. Students who had difficulties were eligible for extra help from a range of service providers, such as Title 1 and resource specialists, that typically consisted of additional practice to increase the speed and accuracy of their computations. To do well in mathematics, students needed to listen to the teacher, memorize important procedures, and write rapidly.

Current reform in mathematics calls for sweeping changes, changes that dramatically alter the goals and routines of elementary mathematics classrooms (National Council of Teachers of Mathematics, 1989).

Students are to continue to learn basic computational skills, but more of their time and energy is to be devoted to solving challenging problems that are open-ended or that can be solved using different strategies. Students are to explain their mathematical reasoning to others and be able to follow the explanations of their peers in an attempt to construct personally meaningful understandings of mathematical concepts (Cobb, Wood, Yackel, & McNeal, 1992; Fraivillig, Murphy, & Fuson, 1999). Students are to contribute actively to the shared understanding in the classroom because the teacher is no longer to be the only source of knowledge (Williams & Baxter, 1996). To do well in reform-based mathematics, students need to listen to their teacher as well as their peers, be able to explain their mathematical reasoning to others, and build their own understanding of mathematical concepts.

But students who have difficulties learning mathematics are curiously absent from the reform documents and classroom-based research. The NCTM Standards (National Council of Teachers of Mathematics, 1989) offer few, if any, guidelines for how the standards might be modified for students who are at risk of academic failure or have a learning disability in mathematics. Mathematics researchers have rarely focused on the effects of reform-based pedagogy and curricula on low achievers, offering primarily anecdotal reports (e.g., Fennema, Franke, Carpenter, & Carey, 1993). In some cases researchers seem to imply that new mathematics pedagogy and materials are effective for all students without special adaptations to curriculum, instructional techniques, or classroom organization (e.g., Resnick, Bill, Lesgold, & Leer, 1991). An underlying assumption of the reform is that the new mathematics pedagogy and curricula are effective for all students, including low achievers.

As one might expect, concerns about the effects of reform mathematics on low achievers can be found in the special education literature. Some researchers in special education doubt that the proposed methods and materials associated with reform mathematics are appropriate for students with learning disabilities or those at risk for special education (Carnine, Dixon, & Silbert, 1998; Carnine, Jones, & Dixon, 1994; Hofmeister, 1993). For example, special educators have long recommended the use of a clear set of procedures when teaching mathematics to reduce ambiguity (Carnine et al., 1994). These researchers view the discussion of alternative strategies and invented algorithms, a common approach in reform-based mathematics instruction, as problematic for low achievers because they believe multiple approaches to solving problems or even computing can only lead to confusion. These researchers see one simple set of rules as the best approach to teaching these students.

In addition, research on attempts at mainstreaming special education students, particularly students with learning disabilities, suggests that general education teachers have a difficult time accommodating the needs of these students (Baker & Zigmond, 1990; Schumm et al., 1995; Scruggs & Mastropieri, 1996). It is important to note that this mainstreaming research has typically been conducted in settings where general education teachers have been using traditional pedagogy and curricular materials. There are two dramatically different interpretations of the mainstreaming work. One view is that traditional pedagogy is to blame for the difficulties low achievers experience, and that with reform-based curricula and pedagogy, many students who formerly struggled in traditional mathematics instruction will thrive. A contrasting view is that students who have difficulties in a traditional mathematics setting will have even greater problems with the advanced topics and problem-solving activities in reform-based mathematics. Clearly, additional classroom-based research is needed to inform such debates. The lack of research coupled with the concerns of the special education community highlight the need for studies on the effects of reform mathematics instruction on low achievers.

Our investigation focused on the classroom dynamics of reform-based mathematics instruction with special attention to low achievers. For example, we worked to identify the mathematical tasks that students were expected to complete, as well as students' responses to these tasks. Using classroom observations and interviews with teachers, we looked for emergent patterns and differences in five third-grade classrooms that we studied for 1 year.

It is important to note that our present analysis of five classrooms using an innovative curriculum (described below) was informed by our earlier quantitative analysis of the students' mathematical achievement (Woodward & Baxter, 1997). In the earlier study we compared the mathematics achievement of students in these five classrooms with the achievement of students in four classrooms using a traditional curriculum. We collected student achievement data using both a standardized measure, the Iowa Test of Basic Skills (ITBS), and an alternative measure of problem solving, the Individual Mathematics Assessment (IMA). The IMA is an individual interview that probes students' thinking as they solve problems that can be approached in different ways (see Woodward & Baxter, 1997, for a more detailed description of the IMA). Our results from this quasi-experimental comparison study indicated that the innovative curriculum was effective for average- and high-ability students. For example, high-ability students' mean scores improved from the seventy-fourth percentile to the eighty-third percentile, and average-ability students' scores improved from the forty-seventh to the fifty-fourth percentile in concepts on the ITBS. Both increases were statistically significant; however, for students in the lowest third of the academic distribution, progress was marginal. For example, low-ability students' mean scores on the ITBS were at the twenty-fourth percentile in the fall and the twenty-sixth percentile in the spring. Although our earlier study suggested that the low achievers were having difficulties with the innovative mathematics program, it did not document the nature of these difficulties. The present analysis of the observational and interview data from the five innovative classrooms is intended to identify the challenges low achievers faced in these classrooms.

Method

We used a constant comparative method to focus our data collection and analysis (Glaser, 1978; Glaser & Strauss, 1967). A distinguishing feature of this research design is that formal analysis begins early in the study and thus controls the scope of the data collection while increasing the theoretical relevance of multiple-site studies. Because most classrooms in these two schools have few low-achieving students, we decided to study five classrooms to increase the number of students under study. We treated each classroom as a unit of analysis, slowly building a picture of the routines and dynamics in the classroom that comprised reform-based mathematics instruction. We focused on the responses of low achievers to mathematics instruction. Our work in each classroom informed data collection in the other classrooms, as we looked for patterns as well as differences across the five classrooms.

Setting

We received permission to work in two elementary schools located in the Pacific Northwest. The two schools were selected because they were using Everyday Mathematics (Bell, Bell, & Hartfield, 1993), a curriculum closely aligned with the 1989 NCTM Standards. The two schools were comparable along many variables. Both were middle class with similar socioeconomic status (determined by the very low number of students on free or reduced-price lunch). One school was in a suburban setting and the other was in a medium-sized city.

Staff members at both schools held comparable beliefs regarding mathematics instruction. First- through fifth-grade teachers at each school completed the Mathematics Belief Scale (Fennema, Carpenter, & Loef, 1990), an updated version of the Teacher Belief Scale (Peterson, Fennema, Carpenter, & Loef, 1991). This measure has been used in a number of studies investigating the effects of innovative mathematics instruction. The teachers' responses on the measure's three subscales indicated that they generally agreed with views associated with cognitively guided instruction. Specifically, the teachers tended to believe that children learn mathematics by constructing knowledge, that there is an integrated relation among skills, understanding, and problem solving, and that mathematics instruction should be more facilitative and exploratory rather than primarily teacher directed. On each of the three Mathematics Belief subscales, the teachers averaged from 3.75 to 3.88 on individual items (1 = strongly disagree to 5 = strongly agree).

Participants

Teachers. We invited all five third-grade teachers at the two schools to participate in the study. All agreed to be observed and interviewed throughout the school year. The teachers were experienced, with an average of 21 years (ranging from 18 to 24 years) of teaching. In addition, two of the five teachers were certified in special education and had taught in resource room settings early in their careers. As stated above, the five teachers also held similar views of teaching mathematics, as measured by the Mathematics Belief Scale.

Students. A total of 104 third-grade students at the two schools participated in the study. Seven of the students were classified as learning disabled on their Individual Education Plans (IEPs), and they were receiving special education services for mathematics in mainstreamed settings. However, we found that this number did not reflect the probable number of students with learning disabilities in mathematics. In fact, a clear theme in our preliminary teacher interviews at both schools was that more students could have been referred for special education services in mathematics but were not for a variety of reasons.

Some of the teachers mentioned that the special education teacher primarily served low-incidence students (e.g., those with autism or physical disabilities) and/or students who had reading problems, leaving "little room" for students who needed help in mathematics. In addition, three of the five teachers chose not to refer students. Two of these teachers retained identified special education students in their classrooms for mathematics instruction because they did not want to contend with the logistical problems of sending students out of the class for mathematics at important or inconvenient times in the day. These teachers also expressed skepticism about the quality of mathematics instruction that their students would receive in the special education classroom. They thought that the traditional direct instruction approach that the special education teacher used would do little to help their students learn the mathematics they needed for success in future grades.

Because of this apparent discrepancy between those who could have been referred and the number of students actually referred for special education, we expanded the pool of students who would serve as the focus of this study. In October, we administered the mathematics portion of the ITBS to all third-grade students in both schools and used their scores as a basis for further identifying students who could be considered at risk for special education services in mathematics. A total test score at or below the thirty-fourth percentile was used as a criterion for identifying the target at-risk students. We also asked the teachers to divide their students into three groups: high, medium, or low. Students who were ranked in the low group by their teacher and scored at or below the thirty-fourth percentile on the ITBS were placed in the target group.

Consequently, in addition to the seven students with learning disabilities, we identified nine other students who did not have special education IEPs to be part of the target group.

Materials

As mentioned earlier, the two schools in this study were using a reform-oriented curriculum, the Everyday Mathematics program. This program reflects over 6 years of development efforts by mathematics educators at the University of Chicago School Mathematics Project (UCSMP). This program deemphasizes computation and differs from many traditional elementary math curricula in the way concepts are introduced and then reintroduced within and across grade levels. In this "spiraling" curriculum, major concepts are presented initially and then reappear later in the year and in the next grade level, where they are addressed in greater depth. The program also stresses mathematics vocabulary, and it is not uncommon for a day's lesson to have three to five new terms that the teacher introduces and discusses.

Further, there is a significant emphasis on innovative forms of problem solving. Unlike word problems in traditional math curricula that often lend themselves to a key word approach, the problems or "number stories" in Everyday Mathematics often derive from the students' everyday world or from life science, geography, or other school subjects. The developers agree with other mathematics educators (e.g., Carpenter, 1985) that students come to school with informal and intuitive problem-solving abilities. The lessons draw on this knowledge as a basis for math problem-solving exercises. Students are encouraged to use or develop a variety of number models to display relevant quantities (e.g., total and parts; start, change, end; quantity, quantity, difference) that can be manipulated in solving these problems. The third-grade level of Everyday Mathematics is rich in problem-solving activities that rarely involve the one-step problems common to traditional commercial curricula for general and special education students.

Everyday Mathematics, then, incorporates and emphasizes many of the NCTM standards. Students spend considerable time identifying patterns, estimating, and developing number sense. The curriculum encourages teachers to conduct whole-class discussions in which students describe their problem-solving strategies. An array of math tools and manipulatives—calculators, scales, measuring devices, unifix cubes—are an important part of the daily lessons. Finally, students work for a small portion of each day's lesson in their math workbooks or "journals." The journals generally contain five to 12 items that are intended to reinforce the main concepts of the day's lesson.

Procedures

Each classroom served as a unit of analysis as we worked to understand the relationships among the teacher, low achievers, their classmates, and the mathematics instruction. Throughout the year we looked for similarities across classrooms as well as differences.

Data collection. We (the authors) collected observational and interview data throughout the academic year. During classroom observations, which totaled 34 for the five classrooms, observers focused on 16 target students who had either been identified as low achieving or were at risk for special education services in mathematics. There were two to four target students in each class. We noted these students' interactions with teachers and with other students, as well as their involvement in the lessons. For each observation we wrote a narrative summary of the lesson and completed a one-page summary that highlighted key features of the lesson, such as (*a*) organizational structure (e.g., small groups, individual work); (*b*) teacher's role (e.g., direct teaching, facilitating); and (*c*) pedagogical methods (e.g., discussions, writing, manipulatives).

In addition, conversations between teachers and observers occurred weekly. These conversations were especially important because teachers discussed their concerns about particular students, plans for the lesson, and thoughts on the mathematics instruction. We included summaries of these conversations in our field notes. Finally, a 1-hour interview was conducted with each teacher at the end of the school year (see appendix). These interviews followed a semistructured, thematic format recommended by Seidman (1991) and were audiotaped and transcribed. The interviews focused on the progress of the target students, the teachers' experiences in implementing the innovative curriculum, and the teachers' thoughts on how to address the needs of low achievers.

Data analysis. As we observed in the classrooms, we communicated weekly, exploring themes through conversations, memos, and the exchange of written thematic analyses. Miles and Huberman (1984) recommend memos to summarize field notes prior to the conclusion of a study, because ongoing memos can be a useful way to frame and reframe the focus of inquiry as a study evolves. The thematic analyses build on ideas identified in the memos (Glaser & Strauss, 1967). A thematic analysis might describe an intriguing pattern illustrated with examples from classroom observations or interview transcripts.

Results

Our findings are presented in three sections. In the first section we describe a typical math lesson that illustrates the teachers' use of instructional time and the expectations for students. We present a lesson that focuses on the entire class to establish the context for our observations of the target students. In the second section we examine similarities across the five classrooms, focusing on two situations—whole-class discussions and pair work—in which the target students faced difficulties. In the third section we present important differences among classrooms by contrasting the teachers' use of manipulatives and small groups. The differences highlight the complexity of implementing mathematics reforms as well as the effects on low achievers.

Whole-Class Lesson 53

As we observed in the classrooms it soon became apparent that all five teachers organized their math lessons into similar sections that followed the suggestions of the reform-based curriculum. Most lessons included a whole-class discussion and pair work. Relatively little class time was given to independent practice. Table 1 shows how one teacher conducted Lesson 53, which was typical of how teachers used their time during this lesson.

The lesson presented in Table 1 began with an ongoing investigation that included collecting, recording, graphing, and analyzing data (see activity 1, Table 1). On many days students simply needed to record sunrise, sunset, and the high and low temperatures for the day. Other days the students looked for patterns or graphed information. Data-recording activities went quickly, as the 2 minutes for this activity reflect, whereas more time was spent—up to 20 minutes—when graphs were constructed. The problem-solving task in activity 2 could also be called "homework correction," but the open-ended nature of the assignment—write an original word problem using multiplication and ask someone at home to solve it—turned the homework discussion into a celebration of creativity and alternative solutions.

Activity 3 was a result of the teacher's realization that there was not enough time for all of the eager students to present their homework problems to the whole class. The teacher directed the students to read their problems to their partners who would then try to solve them. An interaction between two average-ability partners, Anna and Kathy, captures an interesting mathemati-

TABLE 1. Description of Whole-Class Lesson 53

Time	Activity	Organization	Task/Content
11:08–11:10	1	Whole group	Data collection and analysis: Record sunrise/sunset time and high/low temperatures for the day
11:10–11:21	2	Whole group	Problem solving: Read and solve multiplication word problems that students had written at home
11:21–11:31	3	Pairs	Problem solving: Share and solve partner's word problem using multiplication or division
11:31–11:42	4	Whole group	Journal page on place value in decimals

cal discovery that was typical of many lessons. Anna read her problem to Kathy: Five cats were sitting. Each cat saw five other cats. How many cats were there?

> Anna started to answer the problem, when an aide suggested that Kathy give it a try. Kathy promptly answered "25 cats." The aide suggested that Kathy draw a picture or use blocks or something else to show how she got her answer. Kathy drew five circles in a column on her paper. Then she drew a row of five circles next to each of the five initial circles. Her finished drawing was five rows of six circles. Kathy looked at her picture and then very excitedly counted the circles. Anna looked at Kathy's picture in disbelief at first. The aide asked Anna to read her problem again. The two girls excitedly talked over how they had gotten 30 instead of 25. Anna erased her original answer and wrote 30 on her paper (obs G:2.2:3)

Anna and Kathy were both surprised when Kathy's drawing led them to a new understanding of Anna's problem. Working together, they were able to explore a simple multiplication problem and, with the help of a picture, convince themselves that a different answer was correct.

The final activity for the lesson was a worksheet to practice place value in decimals. The teacher worked through the problems with the whole class, using student-constructed place-value books—flip books with digits from 0 to 9 for five places (tens, ones, tenths, hundredths, and thousandths).

This lesson illustrates two important features of the lessons we observed: group work, including whole-class and pair work, and student talk. Typically, the teachers allotted the majority of class time for whole-group and pair work, and the activities were designed to be carried out in these contexts. Even the final journal page on decimal place value was offered as a pair activity rather than an individual assignment. The lesson also highlights the importance of students' comments: half the class period was devoted to students talking about the problems that they had written. Many lessons included lengthy discussions of students' solutions to problems where the teacher primarily called on students.

Our initial analysis of how time was used in the five classrooms focused on whole-group discussions and pair work because these appeared to be the primary contexts for mathematical learning. We reviewed all of our observations that included whole-class discussions and/or pair work to see how the target students were involved in each type of activity.

Patterns across the Five Classrooms

It is not surprising that whole-class discussions and pair work figured prominently in all five classrooms; both are integral parts of reform mathematics. Ideally, discussions facilitate conjecture and argumentation in an environment where students create and develop ideas that "matter mathematically" (Ball, 1993). Discussions should allow students to construct meaning rather than simply memorize strategies or algorithms (Cobb, Wood, & Yackel, 1993). Pair work should be an opportunity for students to learn from the questions and suggestions of a peer. Students are able to formulate their thoughts and try them out in a much less public arena than the whole-class discussions. Both whole-class discussions and pair work can allow students to engage in the questioning and explanation that lead to doing mathematics.

Whole-class discussions. The reform-based curriculum highly recommended using open-ended discussions where the intent was to validate student-derived solutions to problems, including algorithmic procedures. The discussions that we observed occurred at various times during the lessons. Occasionally, they happened at the beginning of a lesson as the teacher reviewed the previous day's work or set the stage for that day's work. On other occasions, whole-class discussions followed periods of small-group, paired, or independent activities. Regardless of when they occurred, these discussions offered opportunities for students to articulate their thinking and examine the thinking of their peers.

During our 34 observations we noted only three occasions when low achievers volunteered to speak during class discussions. Moreover, when they did volunteer, they offered one-word answers or remained silent while a peer spoke. A poignant example of this was when Thomas, a target student with emotional problems and learning disabilities, and his average-ability partner, Claudette, were asked to explain their solution to a problem.

> Ms. Hoyle, the teacher, asked the class if anyone had a different strategy. Claudette and her partner Thomas raised their hands. After being called on, Thomas asked Claudette if he could go up to the overhead, too. Thomas anxiously stood beside her at the front of the room as she wrote [on the overhead projector] and explained their solution to the problem. (obs Na 12.2)

Although it was clear that Thomas wanted to be in front of the class, he also remained silent throughout his partner's explanation. Other students directed questions to Claudette rather than Thomas. In contrast, when other pairs of students in the class explained their problem solutions at the overhead projector, both partners spoke, at times interrupting each other or passing the all-important overhead projector pen back and forth, as they drew pictures to illustrate their thinking. In this instance, even when a target student volunteered to join in a class discussion, he was silent.

All five teachers tried to involve the target students in discussions; however, even when teachers were able to do so, these students generally gave only one- or two-word answers. For example, during one class discussion three target students were called on nine times and all nine responses were one word in length. These brief answers were most often a memorized math fact. One class was discussing how to figure the difference in the amount of daylight between the present day and the previous day.

> Mr. Jackson drew a name from the "I can Can" and read off Lily [a target student]. Lily replied that she didn't get it. Mr. Jackson asked if she knew the answer if he told her 11:08-3. Lily shook her head no. Mr. Jackson replied calmly, "8 minus 3." Lily paused a few moments and then replied, "5." Mr. Jackson looked around the room and asked, "Thumbs up if you agree." Most of the students thrust their hands high up into the air, their thumbs extended. (obs F:10.14)

In this example, the teacher made the problem simpler and simpler until Lily only had to perform a familiar computation to obtain the correct answer. Lily did not state her mathematical thinking; she simply stated a memorized math fact. Thus, the target students did not enter into class discussions as speakers who questioned the thinking of others or tried to justify their own reasoning. Across all five classrooms, there was not one whole-class discussion in which a target student spoke more than two words. The target students were primarily the audience during class discussions.

In reform mathematics, whole-class discussions provide opportunities for students who speak to ask questions and articulate their thinking. Students who listen have the opportunity to compare their thinking with the comments of others. Both speaker and listener are potentially productive roles for students. Unfortunately, as listeners in class discussions, the target students were often "off-task." In every one of our observations we found patterns of nonengagement by the target students. For example, during a class discussion on the U.S. census that focused on large numbers, the teacher asked for estimates of the U.S. population in 1790. A lively conversation followed.

> Most of the students leaned forward to hear Mr. Jackson and follow the discussion. The two target students, Mandy and Norika, did not. Mandy was doodling on her calendar, while Norika rested her head on her desk. (obs G:1.19)

In all of our observations of whole-class discussions, the target students were often observed playing quietly with a small object in their laps, staring out the window, writing on a piece of paper, or avoiding eye contact with the teacher. The teachers were not unaware of the students' behavior; however, their efforts to redirect the target students' attention usually failed. For example, Jack, a large, physically aggressive boy, poked and punched students sitting nearby. Darcy, a quiet girl, often read a book during class discussions. Robert, a soft-spoken, easily distracted boy, shuffled the papers in the many bulging folders he kept in his desk. The teachers and instructional aides dealt directly with these behaviors. Jack's teacher placed him on a tightly structured behavior management system, and an aide spent a recess helping Robert organize his desk. In response, Jack became less physical during discussions, but he squirmed in his seat, and Robert played quietly with a set of keys. Despite the teachers' efforts to focus and include the target students in discussions, they remained, for the most part, aloof and uninvolved.

Our observations and interviews revealed two features of the classroom discussions that made them especially challenging for the target students and may have contributed to their low participation. First, the class discussions were often difficult to follow. The spoken thoughts of third graders are not always complete and well organized. The following example illustrates the opacity of students' comments during a discussion on strategies to calculate the amount of daylight using sunrise and sunset times. The two students, Beth and Nathan, who speak during this discussion are both of average ability in mathematics.

Ms. Hoyle (the teacher):	Who can explain their solution? (Half the students raise their hands. Ms. Hoyle calls on Beth.)
Beth:	9:07.
Ms. Hoyle:	How did you find the answer?
Beth:	(shrugging) I just figured it out.
Ms. Hoyle:	Tell me what you did in your head.
Beth:	(hesitantly) I minused 7:28 from 7:29 and added one. (Note: 7:28 was today's sunrise time, and 7:29 was the previous day's sunrise time. The length of day for the previous day was 9:06.)
Ms. Hoyle:	(smiling) Good strategy, Beth. Anyone have a different explanation? (Several students raise their hands. Ms. Hoyle calls on Nathan.)
Nathan:	(walking to the overhead projector) I found the morning (writes 12:00 − 7:28 = 4:32) and then the afternoon (writes 4:35) and then added them to get 8:67 (writes 4:32 + 4:35 = 8:67).
Ms. Hoyle:	Well, we've got two different answers here. What's going

on? (A few students raise their hands, as Ms. Hoyle waits over a minute until 11 students, including Nathan, have raised their hands. Ms. Hoyle calls on Nathan.)

Nathan: I've got another hour (pointing to the 67 minutes), so it should be 9:07. (Ms. Hoyle then tells the students to graph the answer and asks them what might happen on the graph next.)

Ms. Hoyle: Might it rise again and then drop? Or what? (When students offer predictions, Ms. Hoyle asks "Why?") (obs Na:12.2)

This discussion continued for 20 minutes while two other students, both of high ability in mathematics, described their strategies for solving problems. Other students were quiet, and many appeared to understand the speakers, nodding their heads in agreement. Many students were eager to go to the overhead projector to draw a picture as a way of explaining their ideas. During this discussion students listened to their peers' explanations and paused to consider the difference in the two students' answers. In this respect, the discussion represented the kind of "doing mathematics" that commonly appears in the reform literature.

This vignette also illustrates the lack of clarity in many students' comments during class discussions. Beth's first response to the teacher's request for a solution strategy was to simply state the correct answer. When the teacher asked how she worked the problem, her initial response was, "I just figured it out." It took another prompt from the teacher, "Tell me what you did in your head," and a great deal of patience, to get Beth to put her thinking into words. When she finally did explain her solution, it was quite telegraphic and algorithmic: "I subtracted 7:28 from 7:29 and added one." Her answer makes sense if one realizes that the sun rose 1 minute *earlier* than it had the previous day, so she simply needed to add 1 minute to the total daylight for the previous day to find the total daylight for the current day.

The important point here is that students' explanations of their solutions were often difficult to follow, and too often students ignored the comments of their peers as they waited for their turn to speak. One teacher described this problem, stressing the importance of dealing with

> the sharpest kids saying, "I just thought it up in my head." The kids really need to be taught, "Well, what did your mind do to think it? What was your first step?" Then, walk them through so they can get a format. Then they can describe it more eloquently next time and other kids would start picking it up too. The hardest thing for me to do was to have the other kids listening and observing. Very often they would just wait for their turn and not listen. (int M:5.31:4)

These classroom discussions placed high verbal and cognitive demands on all students, who had to be able to understand and respond quickly to questions and comments by peers as well as their teachers. The rapid exchanges and the confidence required to present a detailed explanation might be daunting to low achievers. In addition, unraveling the comments of peers might also prove to be extremely difficult for target students. Ball (1993) has written thoughtfully of mathematical discussions with her third-grade class, questioning her own ability to always understand what her students were trying to explain. If a university researcher and experienced teacher, such as Ball, has problems understanding statements during class discussions, it is likely that the target students were struggling as well.

A second feature of the discussions that worked against the target students was the relatively small number of students who were able to speak during a 15-minute discussion. The time needed for any kind of extended explanation precluded more students from discussing their ideas or even being called on by the teacher. Many students, particularly low achievers, appeared to avoid active participation because their more capable and highly verbal peers were likely to volunteer their solutions. The time taken by the more academically capable students who did volunteer usually consumed the entire portion of the lesson allotted to discussion. In fact, there was usually a surplus of volunteers who did not have an opportunity to present their ideas or solutions. As a consequence, it was relatively easy for the low achievers to remain quiet during whole-class discussions.

In summary, the target students were seldom involved in whole-class discussions. They rarely spoke, and when they did it was to pass or to give an answer to a computational problem. As listeners, the target students did not fare well either. While their peers were speaking, they tended to be easily distracted. Two features of the whole-class discussions appeared to challenge the target students: the confusing nature of students' comments and the limited opportunities for students to speak.

Pair work. The pair and small-group activities that took place during the lessons were generally informal in structure. The teachers allowed students to select their partners, only rarely intervening when management problems arose. The purpose of these activities was to give students opportunities to construct representations, play math games, or solve problems using mathematical tools such as scales, calculators, or rulers. Throughout the year, observers noted that in all five classrooms the majority of students were actively involved for most of the time when working in pairs. In fact, teachers occasionally had to interrupt students' pair work when it was time to move on to another part of the lesson.

In contrast to their behavior during whole-class discussions, the target students appeared much more engaged when they worked with a partner. Across the five classrooms we observed 28 occasions, five to six per class, when students worked in pairs. On three of these occasions target students were unengaged and did not complete the assigned task. For the majority of the pair work the target students talked with their partners and shared in handling the manipulatives. They also seemed attentive, making eye contact and nodding in agreement, as other students solved problems or offered explanations.

However, closer examination of these interactions raised questions about the kind of mathematics in which target students were engaged. In 24 of the 28 pair-work observations, the target students primarily copied their partner's work or organized materials. For example, during one lesson on ordering fractions from smallest to largest, a target student, Ginger, worked with an average-ability peer, Jennifer. The observer noted:

> Ginger was quietly finding all of the fraction bars that equaled zero. She collected a green, yellow, blue, white, purple, and red fraction bar that each showed no shaded parts (i.e., each represented zero). Jennifer picked up Ginger's pile of zero equivalents and reordered them from zero halves through zero twelfths. Ginger watched as Jennifer worked. Jennifer next laid out the following fraction bars in a row: 1/12, 1/10, 1/6, 1/5, 1/4, 1/3, 1/2. Again, Ginger watched silently and then suggested to Jennifer, "You put them in my hand and I'll put them there." (obs G:3.2)

The two girls continued to work together with Ginger carefully lining up the bars and then handing them to Jennifer upon request. Ginger never suggested how to arrange the fraction bars; she simply responded to her partner's directions. The two girls referred to the fraction bars by color rather than mathematical name, which further reduced the mathematical thinking needed to complete the task.

During this episode, Ginger primarily managed the materials. She kept the fraction bars in neat piles and willingly filled Jennifer's requests. Although both girls were handling the materials and focused on the activity, each made a very different contribution to the task. One of the advantages of group or pair work is that contributions from different students can move the group forward to complete its task. From that perspective, both girls had something to offer and most likely felt positive about their work together. It is important to note, however, that a task that is designed to help students understand a concept or relationship can lose its mathematical meaning for some students when students divide the work into mathematical and nonmathematical subtasks.

At other times, the students played games in pairs. The games served an important function in the EM curriculum because they allowed students to practice skills and become facile in certain procedures. During one lesson the students were to play a math fact game with a partner. Each pair of students needed a deck of cards and a tally sheet for keeping score. Most students quickly organized their materials and began to play the game, but two target girls, Ginger and Laura, had difficulties.

> At 11:28 Mr. Jackson told the students to get a partner and play a game to find the biggest difference between two decimal numbers. Most of the students settled into the game, drawing cards and trying to win by drawing two cards with the biggest difference. Laura wandered around the room for 5 minutes. Mr. Jackson spotted her and asked two girls to include her. The two girls hesitated, as they had a special procedure all worked out, but they modified their game to include Laura.
>
> Ginger moved around the room for 10 minutes. Mr. Jackson finally sat her down with Molly, another target student, but they talked rather than played the game. (obs G:3.9)

After about 10 minutes the majority of students had completed 18–20 cycles of the game. During the same time one pair of target students had completed one cycle of the game; they had spent most of their time adjusting their work space and looking out the window. Even with the teacher's help they still had minimal practice before the period was over.

This episode suggests that management procedures that worked for the majority of the class did not necessarily lead the target students to focus on the task. While 22 students in the class found a partner, organized materials, and began to play the game within a few minutes, the four target students in the class needed the teacher's help. These students often could not begin work until the teacher had repeated the directions. They also needed assistance and feedback to solve the first two or three problems.

In summary, the target students were actively engaged in pair work, especially when working with an average- or high-ability peer. While participating in the pair work, however, they usually were involved in nonmathematical or low-level, functional tasks: The target students organized materials. They often needed additional guidance from the teacher or an aide before they could begin work. Although the target students were involved in pair work, their contributions tended to be supportive (e.g., organizing materials) rather than substantive.

The patterns across the five classrooms revealed challenges that confronted the target students during math lessons. The whole-class discussions were often difficult to follow and offered relatively few opportunities for students to speak. Thus, target students usually listened rather than spoke in class discussions. As audience members, their involvement was problematic. Often they stared out a window, played with objects, or read. In contrast, during pair work the target students were actively involved, but typically they acted as materials managers. When target students were paired with another target student, they tended to have a difficult time beginning work and staying involved.

Differences among Classrooms

In addition to the patterns that we discussed in the previous section, we also noted differences among classrooms. Although our primary goal was to better understand the difficulties low achievers face when using an innovative mathematics curriculum, we also noted instances where the teachers used strategies that seemed to benefit the target students. Both the ways that teachers grouped students and the ways that the teachers used manipulatives appeared to increase the target students' involvement in lessons.

Four of the teachers used primarily whole-class and pairs approaches when grouping students for mathematics lessons. When an instructional aide was present, the aide typically moved among the target students, reading instructions to them, helping them find materials, and reviewing explanations. The aides were especially important during pair work, when they assisted many students who were raising their hands for help or distracting other students. For example, during one class discussion the aide walked quietly around the groups of student desks handing out rewards (a paper coupon good for a treat) to students who were listening and raising their hands to speak. In these four classes the aides provided individual help to the target students and helped monitor the behavior of the entire class.

In the fifth class, the teacher, Ms. Monroe, often formed "ad hoc" groups to focus on a particular problem or skill. These groups were composed of eight to 11 students, all low achievers. Ms. Monroe's ad hoc groups were fluid in that different students joined the groups based on the teacher's goals for the lesson. Sometimes she reviewed a topic for students who were struggling; at other times she provided additional practice of skills to build automaticity. Her instructional aide worked with the rest of the class while she taught the ad hoc group. The striking feature of Ms. Monroe's ad hoc groups was the high involvement of all students.

For example, in one geometry lesson Ms. Monroe was reviewing terms such as parallel lines and ray. She began the lesson by asking for a definition of parallel lines. She then asked the students to show parallel lines with their arms. The students, including a target student, Jack, pointed straight up. "Can you do it a different way?" the teacher asked. Two students pointed straight out. Using the board and students' bodies, the class reviewed key terms such as ray and intersection. Next Ms. Monroe asked two students, Jennifer and Kirby, to take the ends of a piece of yarn that then became the Jennifer-Kirby line segment. Two other students, Polly and Douglas, were given a piece of yarn and asked to make a parallel line segment. The class agreed that the Polly-Douglas line segment was parallel to the Jennifer-Kirby line segment. The teacher then passed out yarn and asked the students to make line segments that intersected the Jennifer-Kirby and Polly-Douglas line segments. One student noted, "Here, we can make these form right angles." Ms. Monroe then drew the symbols for right angles on the board and defined the term for the students.

At this point the students spontaneously started counting the degrees, noting that one of the right angles in the intersecting yarn was 90 degrees, that there was another one that was 90 degrees, and another and another. Ms. Monroe posed the question, "I wonder how many degrees we have?" One student suggested, "We could get a calculator." Another student rounded off to 100 and said, "100 and 100 is 200, and 100 more is 300, and another 100 is 400. Ms. Monroe reinforced the students' efforts and said, "Oh, yes, you're rounding off." And another student said, "Well, that's not quite right." The teacher helped students add 90 four times to get 360.

Next Ms. Monroe directed students to create parallel line segments on their geoboards, then intersecting lines and right angles. Students worked quickly and independently with the geoboards, forming the line segments. Jack, the target student, worked quickly and was able to build all of the required shapes on his geoboard.

It is important to note that this was much more than a vocabulary lesson. The students worked with a wide array of geometric terms, building conceptual understandings of important mathematical ideas, such as parallel, rather than memorizing a list of definitions generated by the teacher.

Through the ad hoc groups, Ms. Monroe was able to involve all students in the lesson. Jack, an easily distracted target student, responded to her requests and was eager to answer questions. He joined in the group activities and was able to complete the geoboard tasks independently.

Ms. Monroe's lesson also illustrates another difference among the teachers, their use of manipulatives, a key feature of the reform-based curriculum. Manipulatives were used regularly in all five of the classes; however, the effects of the manipulatives on target students' participation varied considerably across classrooms. In some classes the target students appeared to work ineffectively with manipulatives, whereas in two classrooms target students met with more success. All five teachers devoted considerable time to organizing routines for passing out and collecting the cubes, blocks, and other materials that were part of the reform-based program. The management of the materials was not a problem for the majority of the class. The difference among the classes was the mathematical role that the materials played: in some cases they were a distracter, in others a conceptual scaffold.

In three classrooms manipulatives became the focus rather than a means to think about mathematical ideas. In the earlier example of the two girls working with the fraction bars, Ginger, the target student, kept the materials in neat piles. Her partner worked on the mathematical task and simply asked Ginger for particular bars. Both girls focused on an irrelevant feature of the bars (i.e., color) to complete their work. The fraction bars served a positive role in that Ginger was involved in the task; however, the bars did not appear to further her understanding of relationships among fractions.

In contrast, two teachers used manipulatives in ways that engaged target students in mathematical thinking. A distinctive feature of their instruction was the use of many different representations of a concept prior to the use of the manipulative specified in the curriculum. Ms. Monroe's geometry lesson illustrates how she used manipulatives and offers many contrasts to the other classes. Perhaps most striking is the involvement of every student in the building of line segments and rays. The teacher reviewed the terms in a way that engaged students, as they all pointed with their arms to form parallel lines. She also used different representations (e.g., arms and yarn) to review mathematical terms, so that by the time the geoboards were passed out, each student had practiced forming parallel and intersecting lines in two contexts. The geoboards became a third setting for the students to think about parallel and intersecting lines.

The spontaneous exploration of right angles in the middle of the episode was a good example of student-initiated inquiry. The students suggested different strategies for adding the right angles, and the teacher helped as needed and allowed the time necessary to reach a conclusion.

In summary, as we looked across the five classrooms, we found that the class discussions and pair work were especially challenging for target students. Limited opportunities and the verbal inability to process and contribute to class discussions kept their role to one of audience. As audience members, they often appeared distracted. Pair work engaged the target students but often at superficial levels. They also had a tendency to be unfocused and not use their time well, especially when paired with another target student. We also noted important differences among teachers in the ways they grouped students for instruction and in their use of manipulatives. One teacher's formation of ad hoc groups seemed to increase the involvement of target students. Two teachers used sequences of manipulatives to engage students and provide many different ways to think about mathematical concepts.

Discussion

The purpose of our study is to understand the difficulties that low achievers face when working with reform-based mathematics curricula. We studied these students in classrooms where two-thirds of students showed significant gains in their problem solving and mathematical computation (Woodward & Baxter, 1997). As we observed in these classrooms, however, low achievers were only minimally involved in lessons. During whole-class discussions, these students rarely spoke and often appeared distracted as other students explained their thinking. In contrast, when working with a partner, the low-achieving students were engaged in the task, touching materials and talking with their partners, but often the roles of the partners were quite different. While the low-achieving students usually assumed a nonmathematical task, such as managing materials, their average- or high-achieving partners made mathematical decisions. When two low achievers worked together, they were slow to begin work and were easily distracted. They also tended to need additional help from the teacher or an aide to understand the task and begin work.

This rather bleak picture of the minimal involvement of low achievers is not without glimmers of hope. The experienced teachers in our study used a variety of strategies to increase the participation of low achievers in mathematics lessons. One teacher used ad hoc groups to focus on tasks. Low achievers tended to be active participants in these groups. In contrast, two other teachers used manipulatives in innovative ways to engage the low achievers. The instructional strategies of experienced teachers certainly merit further study, but it is equally important that researchers try to understand the underlying causes of these students' difficulties before guidelines are developed for teachers who are working with reform-based mathematics curricula. Our work indicates that two features of reform-based mathematics—the formation of a community of learners and the increased cognitive load of the curricula—are especially important to consider in relation to low achievers.

Community of Learners

In reform-based lessons, low achievers face the challenge of becoming part of a community of learners in which students are to construct their own understanding of mathematical concepts through conversations with peers and the teacher. An underlying assumption is that students can exchange ideas and learn from each other. Ideally, small-group practices should reflect whole-class norms, particularly where students actively explore and "argue out" solutions to problems (Cobb et al., 1993). However, a considerable body of research suggests that low achievers tend to remain passive in small groups (King, 1993; Mulryan, 1995) and that low-quality interactions occur as a function of ability or status differences within groups (Battistich, Solomon, & Delucchi, 1993; Good, Mulryan, & McCaslin, 1992). Thus, although it is critical that students be part of the classroom community of learners, low achievers are often marginal members, remaining silent or distracted during whole-group discussions.

One possible explanation for the low achievers' minimal involvement during whole-class discussions is offered by Good's (1981) passivity model, which suggests that subtle teacher behaviors (e.g., criticizing low achievers for inadequate answers, offering less praise) may sustain passive involvement by these students. According to Good, these students eventually develop an array of coping mechanisms for "getting by" during whole-class instruction. We repeatedly saw students gazing out the window, playing with small toys, and otherwise not responding to the teacher's and peers' comments during whole-class discussions, yet our observational data also suggested an important divergence from Good's model.

Instead of reflecting negative teacher behaviors as Good describes, the teachers we observed generally were positive and supportive with all of their students. They dealt with behavior problems calmly and quickly. All five teachers were experienced and effective classroom managers. For teachers who struggle with classroom management, one might expect to see more of the negative behaviors Good described; however, our data do not fit the passivity model in this respect.

A pragmatic explanation for the silence of low achievers during class discussions is the lack of opportunities to speak. In a class of 25 students only a few students were able to describe their thinking during a typical discussion. Not even all of the volunteers were able to speak. The time taken by the more academically capable students who did volunteer usually consumed the entire portion of the lesson allotted to discussion. As a consequence, it was relatively easy for the low achievers to remain quiet during whole-class discussion.

Lack of opportunities to speak was only part of the problem, for when the low achievers did speak during whole-class discussions or small-group work, their contributions were most often low level. A subtle but critical variable that hampers the quality of small-group activities for these students is metacognition. The problems that low achievers and students with learning disabilities have in regard to metacognitive behavior around academic tasks are well documented in the special education literature (Montague, 1992, 1995; Wong, 1993). For example, the kind of idle movement that follows a teacher's directions to form small groups and work on specific problems and low achievers' propensity to adopt passive roles in problem-solving activities may be explained, in part, by their difficulties with metacognition. Detailed research on small-group problem solving (Artzt & Armour-Thomas, 1992) suggests that an ongoing interplay of cognitive and metacognitive processes is essential for successful problem solving. The challenge is to help these students engage in mathematical conversations, a difficult challenge that requires students to follow the comments of their peers and to express their own mathematical ideas. These conversations, whether in small groups or among the whole class, tax low-achieving students' listening and thinking skills.

Cognitive Demands of the Curriculum

Part of the burden for the low achievers is the mental demands or cognitive load of the reform-based mathematics curriculum. As noted earlier, the curriculum that teachers used in this study follows a spiral model in which major concepts are introduced and then reintroduced over time. The purpose of this model is to provide increasing depth for the concepts presented. The program also stresses daily mathematics vocabulary related to the concept strands. In addition, like many commercial curricula, the Everyday Mathematics program presents several concepts within a single lesson. For example, in one third-grade lesson, students calculate the area of rectangles and draw line segments as well as work on addition and subtraction problems as a review activity. The two lessons immediately following this lesson contain a review of estimation skills and a numeric conversion exercise (e.g., write another name for 300 tens). Furthermore, the small-group and independent activities in these three lessons involve a variety of representations such as manipulatives, pictures, and models. Most lessons include hands-on activities (e.g., students measure, count objects, construct place value books). Again, these are strong features of the program that are consistent with current reform and, for most students, provide useful ways to engage their thinking about mathematical ideas.

However, this structure can create problems for low achievers. The complex array of concepts and vocabulary introduced throughout the year creates information-processing problems for these students. In a series of related studies, Chandler and Sweller explored the consequences of materials that demand a high level of integration and problem solving (Chandler & Sweller, 1991; Sweller & Chandler, 1994). As one might expect, they found that materials that required students to integrate concepts from multiple sources at one time were difficult to learn.

In addition, cognitive load becomes a significant issue for highly structured curricula, like the one used in this study, because of the developer's attempt to build conceptual understanding systematically over many years. These kinds of curricula are typically designed for the average student in that they present materials in accordance with what is expected at certain stages of child development. These structural assumptions interfere with learning opportunities for low achievers for several reasons. For example, these students often lack prerequisite knowledge and usually need additional time to review when concepts from previous years are introduced. Teachers pressed to move through curriculum at a reasonable pace do not have natural avenues for giving the kind of detailed attention low achievers may require. The structure of the reform-based curriculum plus the demands of speaking and listening in class create a cognitive load that is challenging for all students but especially for low achievers.

Conclusion

We designed our study to better understand the experiences of low achievers in reform-based mathematics classrooms. We found that both the form and substance of mathematics instruction present tremendous challenges to these students, as they are to articulate their mathematical thinking and, through conjecture, argumentation, and verification, develop a shared understanding of mathematical concepts. The bar has been raised dramatically in mathematics: all students are to work toward a higher level of mathematical literacy.

It is critical to consider how low achievers can become actively involved in the present reform. The assumption that all students will flourish with the challenging mathematics curricula and pedagogy that comprise reform needs to be questioned. Reform is a complex process that involves many factors at many levels (Elmore, 1996). The current mathematics reform is, for the most part, being implemented in traditionally configured schools: One teacher works with 28 students. In our observations we saw how low achievers seemed to disappear during whole-class discussions. The organization of schools creates structural constraints that impede teachers' abilities to reach low achievers.

In addition, the ambitious and laudable goals of current reform require more than a reorganization of existing resources if educators are to include low-achieving students. This is one context where "less is more" does not make sense. Slavin (1989) argues that low achievers need more resources. To create greater opportunities for low achievers, politicians, administrators, and educators must provide more time, more attention, and more structured learning experiences. These students need more time to talk and more scaffolding to develop their verbal skills.

We strongly believe that it is unwarranted to conclude from our work that reform-based mathematics should be abandoned when teaching low achievers; however, our work does suggest that many of these students may be struggling and need additional support. The task for teachers, administrators, and math educators is to identify the instructional and structural changes that will make low achievers active participants in reform-based math instruction. As Clay (1996) notes in her writing on early literacy learning, "Classes are instructed, but classes do not learn; only individuals learn" (p. 202).

Appendix

Teacher Interview

Teacher Interview

1. Goals for mathematics instruction
   a. What do you want your students to know about mathematics as they complete the third grade?
   Probes
   Content: Which topics are important?
   Process: What should they be able to do (what types of problems should they be able to solve?)
2. Curriculum design
   a. Do you think that the Chicago program contains the mathematical content that is necessary for third graders?
   b. What are the features of the program that you see as helping students to develop their understanding of mathematics? What are the features that hinder students in their understanding of mathematics?
3. Schedules
   a. How much time each day/week do you devote to math instruction? How did you decide on this schedule?
   b. What do you see as the trade-offs with this schedule?
   Probes
   Pros/cons for math learning
   Pros/cons for other instructional goals
   c. Ideally, how much time and how often do you need to teach math?
4. Pacing
   a. How much of the text will you complete this year?
   b. How long do you usually spend on each lesson? Is this enough time, need more time, too much time?
   c. Which lessons/sections of the text have you skipped/modified? Why?
   d. Which lessons/materials have you added? Why?
   e. When do you decide to move on?
5. Individual differences
   a. How do you know what mathematics different students are learning?
   b. What information is most helpful in tracking the progress of individual students?
   c. What are the techniques/strategies that you use to help students who are struggling with the mathematics?

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