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Review of: Vector Calculus by Michael Corral

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of the great wealth of work on geometrical applications of minimal models that has been done over the last thirty years or more.

The book begins with a summary of basic results concerning Lie groups, Lie algebras, and homogeneous spaces. The second chapter then gives a thorough introduction to Sullivan's de Rham-type complexes and their minimal models. The applications begin with the third chapter, which discusses the rational homotopy theory of manifolds and includes such interesting results as the necessary and sufficient conditions for a given minimal commutative graded differential algebra to be the minimal model of a simply connected closed manifold. A chapter is then devoted to complex and symplectic manifolds. The fifth chapter studies closed geodesics. These are of great intrinsic interest not only in differential geometry but also in mechanics, where they correspond to periodic solutions. Minimal models are extremely effective tools here: they have led to a very elegant criterion for the existence of infinitely many geometrically distinct closed geodesics in a Riemannian manifold and much else besides. A lot of the results in this chapter concern the free loop space of a manifold (which is also of interest in string theory): it is the growth of its Betti numbers that bears on the number of closed geodesics.

The next chapter gives applications to curvature; and the seventh chapter discusses transformation groups where minimal models have also been very useful. The eighth chapter concerns a variety of topics including complements of submanifolds, symplectic blow-ups, and the Chas–Sullivan loop product on the homology of the free loop space. The final chapter is a compendium of assorted further applications together with a list of some major unsolved problems.

The book is somewhat impressionistic in its style—almost a guide to the literature—rather like Berger's enormously interesting and informative overview of Riemannian geometry [B]. It will be very useful to anyone interested in any of the wide range of topics covered. It would also be a good text for a graduate seminar: there would be plenty of work for students in reading some of the quoted literature (there is an excellent list of references) and filling in some of the details of various topics. There are exercises too.

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Vector Calculus. *By Michael Corral.* **www.mecmath.net, 2008. $11.00. vi+213 pp., softcover (free electronic download available).**

Vector Calculus is a traditional treatment of the topics covered in a third-semester calculus course, often called "Multivariate Calculus" or just "Calc III." While the treatment may be traditional, the distribution and publication terms are a prime example of the newest trends in textbook publication. The author has a standard copyright interest, but has used this government-sanctioned monopoly to grant you additional freedoms in the use of his work through the choice of a GNU Free Documentation License (GFDL). You may download the PDF version from the author's website at no cost, and you may in turn distribute it from your own site. You may make as many printed copies of the book as you like, and you may do this forever. You may modify the content (via the downloadable source files) for your personal use. The only condition imposed by the author is that if you make modifications and distribute the modified versions, then you must apply the same license. By this arrangement, any improvements are made available for the common good.

You may, of course, carefully scrutinize this text in as much detail as you desire by downloading the PDF version. Suffice it to say that the topics chosen are very standard for this course. The first chapter establishes the basics of vector algebra, lines and planes in $\mathbb{R}^3$, and basic calculus of vector-valued functions. The second chapter is centered on partial derivatives of functions of two or more variables, culminating with unconstrained and constrained optimization. The next chapter begins with a concise treatment of double and triple integrals, includes a welcome section on numerical techniques for multiple integrals (Java source code for a Monte Carlo method is included), and finishes with applications to center of mass and probability distributions. The final chapter considers line and surface integrals with the usual discussions of Green's theorem and Stokes' theorem. The only expected topic I found not fully represented is curvature, which appears in the exercises for the first chapter.

The writing style is crisp and concise, while being both informal in tone and accurate in the technical details. Theorems are stated carefully and set off from surrounding text, often with proofs, subtleties are discussed, and pathologies are mentioned but not belabored. In the preface, the author states that on a 1 to 10 scale, with a 1 being "completely informal" and a 10 being "completely rigorous," he rates the book as a 5. When finer points do arise, a footnote will point to one of the more advanced texts described in the excellent annotated bibliography. A refreshing benefit of open-content books is authors striving for the intellectual honesty they feel a topic deserves, without being driven by marketing concerns. This book is a good example of striking the right balance. I would expect a student in a course to find the text very readable in conjunction with lectures, and the independent student should find the explanations sufficient to gain an understanding without assistance.

Many of the crop of new open-content texts are noticeably deficient in exercises, figures, and the use of color. This text has 420 exercises, about 15 per section, graded as "Easy," "Moderate," and "Challenging." As an instructor, after picking and choosing the problems that suit the aims of my course, I think I would find the need to augment the provided exercises with a few of my own, especially to provide some more challenging exercises (there seems to be very good coverage of routine and drill-type problems). Of course, with the source files available, it is nearly trivial to incorporate your own additions to the problem sets. For a student's first exposure to functions of several variables, visualizations of surfaces, tangent planes, vectors, volumes, etc., are crucial, and this text does not disappoint. The excellent graphics are built and rendered with the standard tools: MetaPost, PGF, and Gnuplot. Full color is used effectively in the PDF version, while still printing accurately in grayscale, both in the graphics and in the highlighting of definitions, theorems, and corollaries. An appendix includes a careful tutorial for the student on the use of the open-source plotting package Gnuplot. Other open-content authors could learn something by carefully studying Corral's example.

It is important to note that this text is not an e-book, not an online book, and not a distance-learning resource. It is conceived and designed to be a book. But its promotion and distribution are made possible by computer networks, without an investment of capital in a print run. No need to order an evaluation copy, just download the entire text. Only need a single chapter, a single section, or a single application? Just print the subset you need (while acknowledging the author and the license terms). Should you desire a printed copy, and for a course I think a student would want a printed copy, it may be ordered via the print-on-demand service at lulu.com. The cost for a softcover, grayscale version is merely \$11.00 (plus shipping), which represents the entire production cost at Lulu, with no royalties going to the author. The source files for the book (LaTeX and MetaPost files) may be easily downloaded. It would appear that a couple of additional packages must be added to a standard TeX distribution in order to successfully compile the book through the supplied shell script, but this is a small inconvenience to achieve the high-quality output and graphics. The source appears to be written in a style that would make modifications easy for someone reasonably familiar with TeX.

It is refreshing to see science and mathematics return to a free exchange of ideas, without proprietary interests impeding a rapid and easy interchange. Corral should be applauded for his excellent and unselfish contribution to this movement. An instructor who builds a course around this text will never need to confront the unfortunate decision to take it out of print or the release of a new edition modified simply to frustrate the used-book market. Based on content alone, this text is worth consideration for a traditional multivariate calculus course, is perfect for a quick refresher, is a good choice as a supplemental text for a more advanced course (such as probability), and is a great choice for the independent student on a budget. Additionally, the price is right and the extra inherent freedoms are a welcome novelty.

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Legacy of the Luoshu: The 4,000 Year Search for the Meaning of the Magic Square of Order Three. *By Frank J. Swetz.* AK Peters, Wellesley, MA, 2008. \$39.00. xiv+214 pp., hardcover. ISBN 978-1-56881-427-8.

Mathematics is frequently used to understand the world around us. It has been concerned not just with logical truth, but also with beauty and meaning. In this book Swetz shows us that despite cultural differences and advances in knowledge, for thousands of years its role has been central in understanding more of the world than we perceive directly. Swetz traces the history of one simple mathematical structure, the magic square of order three, across centuries and continents. As promised he reveals a cultural history "that touches on cosmology, mythology, philosophy, religion, occult practices, mathematics, architecture, and even music."

4	9	2
3	5	7
8	1	6

The magic square of order three, or *Luo shu*, is a simple but elegant structure. Adding elements in any row, any column, or the front or back diagonal, one obtains the same sum, 15. Both amateur and professional mathematicians have generalized this "magic" property to larger sizes, more dimensions, and to circles and spirals instead of squares and cubes. Swetz introduces the reader to these many generalizations and provides a starting point for a study of the mathematics of magic squares. But this is not his main focus.

Instead, he takes the reader back to the historical origins of the Luoshu and reveals many interpretations of its meaning. He introduces us to widely held beliefs that the numbers 5 and 9 have a mystical significance. While admitting that many modern readers find such beliefs to be "claptrap," he succeeds admirably in helping the reader to understand them. Indeed the reader appreciates how such a simple object could be seen to be truly magical, a window into understanding the world and the heavens beyond. By developing the context in which the educated people of ancient China interpreted the Luoshu, Swetz lays the foundation for its impact on the practices of the imperial court in China. He traces its evolution within the *yinyang* theory, with *feng shui*, and with Daoism, among others. Its use in forecasting the future and ensuring future success may seem implausible today, but the reader cannot fail to see that these ideas had a remarkable coherence and relevance to the people of the time.

Swetz also leads the reader on a journey following the Luoshu through Southeast