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Resource Letter MD-1: Maxwell's Demon

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RESOURCE LETTER

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This is one of a series of Resource Letters on different topics intended to guide college physicists, astronomers, and other scientists to some of the literature and other teaching aids that may help improve course content in specified fields. No Resource Letter is meant to be exhaustive and complete; in time there may be more than one letter on some of the main subjects of interest. Comments on these materials as well as suggestions for future topics will be welcomed. Please send such communications to Professor Roger H. Stuewer, Editor, AAPT Resource Letters, School of Physics and Astronomy, 116 Church Street SE, University of Minnesota, Minneapolis, MN 55455.

Resource Letter MD-1: Maxwell's demon

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This Resource Letter provides a comprehensive guide to the voluminous literature that has developed around Maxwell's demon, and offers a perspective on issues for which the hypothetical character Maxwell introduced over 120 years ago has inspired continuing research and debate. The code (E) indicates elementary level or general interest material useful to persons just learning the field; (I) indicates intermediate level or somewhat specialized material; and (A) indicates advanced or highly specialized material. No accompanying AAPT reprint book will be available, because an extensive reprint collection (Ref. 29) edited by the authors will be published separately.

I. INTRODUCTION

James Clerk Maxwell first revealed what has come to be called Maxwell's demon in an 1867 letter to Peter Guthrie Tait. He envisioned an intelligent being who could literally direct molecular flows molecule by molecule. Maxwell thought of this hypothetical controller as a selective valve that could allow passage of certain molecules through a partition separating two halves of a gas, generating a temperature difference across the partition. In his 1871 *Theory of Heat*, Maxwell published his idea, cleverly using it to illustrate the statistical nature of the second law of thermodynamics. In 1874 William Thomson (later Lord Kelvin) coined the term "demon," a name that has stuck with Maxwell's imaginary character for the last 115 years.

Why has Maxwell's temperature demon survived for such a long time? Primarily because it has been a key element in the development and understanding of thermal physics, particularly with respect to the concepts of entropy and irreversibility. Its historical importance and its current interest value make it a natural teaching and learning tool. This can be seen by several examples. First, the tiny demon works on a microscopic scale, in a sea of molecular fluctuations. While its success or failure depends upon how well it can harness this "noise," the demon (and its apparatus) is also subject to thermal noise, which could destroy its effectiveness.

Second, the demon's observations of individual molecules involve quantum measurement phenomena (even though the quantum revolution occurred well after Maxwell's death!). Third, the demon must gather information to reduce the entropy of the gas, and its operation is amenable to an information-theoretic analysis. Leo Szilard realized this in 1929, and Leon Brillouin and Dennis Gabor independently extended Szilard's ideas about 20 years later. Not only were the connections between information and thermodynamic entropy unknown to Maxwell, they are still not entirely agreed upon today. Fourth, the demon sorts molecules on a microscopic level but can affect macroscopic behavior. Maxwell's demon lives at the interface between microscopic and macroscopic physics, and between reversibility and irreversibility. These are rare characteristics.

Fifth, operation of the demon raises the question of how molecular information can be obtained. Until recently, most researchers assumed the use of light signals. This assumption was a key element in the "exorcism" of Maxwell's demon in 1951 by Brillouin and Gabor. But light is not the only way to carry information, and a complete solution of the Maxwell's demon puzzle ought not depend upon its use.

Sixth, the trait of "intelligence" originally ascribed to the demon by Maxwell is somewhat murky, but of interest in the realm of physics. It was addressed by Leo Szilard in his famous 1929 paper, using a pressure demon to operate a one-molecule heat engine. Although Szilard did not specify how the demon acquires information, his treatment suggests that an essential aspect of the demon's intelligence is memory. In 1957, Rothstein pointed out the importance of memory to the informational view of entropy. A statistical mechanical treatment of a demon's memory is evidently given first by Oliver Penrose in 1970. He observed that periodically resetting a demon's memory is essential to its operation, and that such resetting generates entropy in the environment.

In 1982, Charles Bennett, drawing upon Rolf Landauer's landmark 1961 paper on dissipation in computing, independently discovered the importance of memory erasure for a demon, and argued further that information acquisition need not be dissipative. He concluded that information destruction (i.e., memory erasure, which requires use of a work source) is the fundamental act that saves the second law from a Maxwell's demon. This was a surprising, remarkable event in the history of Maxwell's demon, and stimulated new research and debate.

Other examples of the demon's importance can be cited. The original temperature demon was intended to generate a temperature difference in the gas that could then be used to run a heat engine. Kinetic theory, developed by Maxwell but not used by him in this context, enables an estimate of how long it takes to generate a specified temperature difference. Thus the demon can illustrate the utility of kinetic theory. Additionally, Szilard's one-molecule system with a "pressure demon" is amenable to a quantum mechanical analysis, and is a valuable conceptual tool for understanding quantum measurement theory, entropy, and information. For these and other reasons, Maxwell's demon continues to play an important role in our understanding of thermal physics, quantum theory, information theory, analyses of the limits of computing, and the philosophy and history of science.

The considerable span of time since Maxwell's idea emerged and the substantial scope of disciplines it touches give rise to a rich literature that is usefully categorized by the section headings below. The bibliography that follows includes all known primary literature and a significant fraction of the secondary literature on Maxwell's demon.

II. BACKGROUND SOURCES

A. Early contributions

These references cover the period 1867–1914, and constitute fundamental contributions prior to the advent of quantum theory and information theory.

1. **"Maxwell's Account of His Demons,"** J. C. Maxwell, in *Life and Scientific Work of Peter Guthrie Tait*, C. G. Knott (Cambridge U. P., London, 1911), pp. 213–215. Maxwell's 1867 letter to Tait with the seminal idea for Maxwell's demon, and a subsequent undated letter clarifying the concept. (E)
2. **"Letter to J. W. Strutt, 6 December 1870,"** J. C. Maxwell, in *Life of John William Strutt, Third Baron Rayleigh*, edited by R. J. Strutt (Arnold, London, 1924), pp. 47–48. Maxwell's second private announcement of his demon. (E)
3. **Theory of Heat,** J. C. Maxwell (Longmans, Green, London, 1871). Maxwell's public presentation of the demon in his classic book on thermodynamics. His description (Chap. 12) of "a being...who can follow every molecule in its course" is one of the most directly quoted passages in physics. (E)
4. **"The Kinetic Theory of the Dissipation of Energy,"** W. Thomson, Nature **IX,** 441–444 (1874), reprinted in *Lord Kelvin's Mathematical and Physical Papers—V.5*, (Cambridge U. P., London, 1911), and also in S. G. Brush, *Kinetic Theory, Vol. 2—Irreversible Processes* (Pergamon, Oxford, 1966), pp. 176–187. Thomson introduces the term "demon" here and imagines an army of Maxwell's "intelligent demons." (E)
5. **"Uber den Zustand des Wärmegleichgewichtes eines System von Körpern mit Rücksicht auf die Schwerkraft,"** J. Loschmidt, Sitzungsber. Akad. Wiss. Wien **73**, 128–142 (1876). The second law cannot be a mechanical principle because Newtonian mechanics allows the same sequence of motions backward as forward. (A)
6. **"Diffusion,"** J. C. Maxwell, in *Encyclopedia Britannica, 9th Edition* (New York, 1878), pp. 214–221. Maxwell writes "...the idea of dissipation of energy depends on the extent of our knowledge...It is only to a being...who can lay hold of some forms of energy while others elude his grasp, that energy appears to be passing inevitably from the available to the dissipated state." (I, A)
7. **"Tait's Thermodynamics,"** J. C. Maxwell, Nature **17**, 257–259; 278–280 (1878), reprinted in *The Scientific Papers of James Clerk Maxwell, Vol. 2*, edited by J. C. Maxwell (Cambridge U. P., Cambridge, 1890), pp. 660–671. (I)
8. **"The Sorting Demon of Maxwell,"** W. Thomson, R. Inst. Proc. **IX,** 113–114 (1879), reprinted in *Lord Kelvin's Mathematical and Physical Papers—V.5* (Cambridge U. P., London, 1911), pp. 21–23. Detailed description of Maxwell's demon, showing why the term "demon" is appropriate and how the natural dissipation of energy can be reversed. (E)
9. **"Maxwell's Demons,"** H. Whiting, Science **6**, 83 (1885). Analogy between the sorting of a temperature demon and the escape of high-speed molecules from the earth's atmosphere. (E)
10. **"Mechanism and Experience,"** H. Poincaré, Rev. Metaphys. Morale **1**, 534–537 (1893), reprinted in *Kinetic Theory, Vol. II. Irreversible Processes*, edited by S. G. Brush (Pergamon, Oxford, 1966), pp. 203–207. Poincaré writes "...to see heat pass from a cold body to a warm one, it will not be necessary to have the acute vision, the intelligence, and the dexterity of Maxwell's demon; it will suffice to have a little patience." (I)

References 11 and 12 focus on the validity of the second law in view of Brownian motion of the trap door. See also Ref. 74.

11. **"Experimentell nachweisbare der üblichen Thermodynamik widersprechende Molekularphänomene,"** M. Smoluchowski, Phys. Z. **13**, 1069–1080 (1912). (I)
12. **"Gültigkeitsgrenzen des zweiten Hauptsatzes der Wärmtheorie,"** M. Smoluchowski, in M. Planck *et al.*, *Vorträge über die Kinetische Theorie der Materie und der Elektrizität* (Teubner, Leipzig, 1914), pp. 89–121. (I)

B. Historical works

These references underscore the important historical role played by Maxwell's demon.

13. **"Maxwell, His Demon, and the Second Law of Thermodynamics,"** M. J. Klein, Am. Sci. **58**, 84–97 (1970). (E,I)
14. **"Maxwell's Demon,"** E. E. Daub, Stud. Hist. Philos. Sci. **1**, 213–227 (1970). (E,I)
15. **"Molecular Forces, Statistical Representation and Maxwell's Demon,"** P. M. Heimann, Stud. Hist. Philos. Sci. **1**, 189–211 (1970). (E, I)
16. **"James Clerk Maxwell and the Kinetic Theory of Gases: A Review Based on Recent Historical Studies,"** S. G. Brush, Am. J. Phys. **39**, 631–640 (1971). (I)
17. **"Irreversibility and Indeterminism: Fourier to Heisenberg,"** S. G. Brush, J. Hist. Ideas **37**, 603–630 (1976). (I)
18. **The Kind of Motion We Call Heat,** S. G. Brush (North-Holland, New York, 1976). (I)
19. **The Temperature of History. Phases of Science and Culture in the Nineteenth Century,** S. G. Brush (Burt Franklin, New York, 1978). (I)
20. **"Maxwell's Contribution to the Development of Molecular Physics and Statistical Methods,"** M. A. El'yashevich and T. S. Prot'ko, Sov. Phys. Usp. **24**, 876–903 (1981). (A)
21. **"Demons, Angels, and Probability: Some Aspects of British Science in the Nineteenth Century,"** S. Schweber, in *Physics as Natural Philoso-*

phy: Essays in Honor of Lazlo Tisza on His 75th Birthday, edited by A. Shimony and H. Feshbach (MIT, Cambridge, MA, 1982), pp. 319–363. (I, A)

22. **Energy, Force, and Matter: The Conceptual Development of Nineteenth-Century Physics,** P. M. Harman (Cambridge U. P., London, 1982), pp. 8, 139–143. (E, I)
23. **"Comment on the Discovery of the Second Law,"** L. Tansjö, Am. J. Phys. **56,** 179 (1988). (I)

C. Reviews

The references in this section provide a good overview of Maxwell's demon and its connections with thermodynamics, information theory, quantum mechanics, and computation.

24. **"Information, Measurement, and Quantum Mechanics,"** J. Rothstein, Science **114,** 171–175 (1951). Perceptive commentary on the impact of information theory in physics. (I)
25. **"Maxwell's Demon,"** W. Ehrenberg, Sci. Am. **217** (5), 103–110 (1967). Good historical overview through Brillouin's work. (E, I)
26. **"Energy and Information,"** M. Tribus and E. C. McIrvine, Sci. Am. **225** (3), 179–188 (1971). Energy and entropy are scrutinized for phenomena ranging from the hypothetical Maxwell's demon to real computational, audio, and pictorial record activities. (I, E)
27. **"The Fundamental Physical Limits of Computation,"** C. H. Bennett and R. Landauer, Sci. Am. **253** (1), 48–56 (1985). (E, I)
28. **"Demons, Engines and the Second Law,"** C. H. Bennett, Sci. Am. **257** (5), 108–116 (1987). Good survey of the demon's history up to and including recent progress based on the theory of computation. Subsequent letters to the editor and a rejoinder by Bennett are in Sci. Am. **258** (2), 6–9 (1988). (E, I)
29. **Maxwell's Demon: Entropy, Information, Computing,** H. S. Leff and A. F. Rex, to be published by Princeton University Press and Adam Hilger ([British] Institute of Physics). Historical overview of Maxwell's demon, with a collection of important reprints, quotations, and chronological bibliography. (I, A)

D. General background books

The following semipopular and technical books serve as good introductions to Maxwell's demon and related topics.

30. **"Maxwell's Demon,"** in *Mr. Tompkins in Paperback*, G. Gamow (Cambridge U. P., London, 1971). Originally published in *Mr. Tompkins Explores the Atom*, 1944. Delightful introduction to Maxwell's demon, using gambling schemes and fantasy to illustrate statistical physics. (E, I)
31. **What is Life?,** E. Schrödinger (Cambridge U. P., London, 1967; published originally in 1944). Somewhat outdated coverage of the role of thermodynamics in biology. (E)
32. **Life and Energy,** I. Asimov (Bantam, New York, 1965). Elementary discussion (pp. 65–76) showing Maxwell's demon as a tool to dramatize the statistical nature of molecular motion. (E)
33. **Order & Chaos,** S. W. Angrist and L. G. Hepler (Basic Books, New York, 1967). Literate, colorful treatise on thermodynamics for the lay science reader (pp. 193–199) in a chapter, "Demons, Poetry, and Life." (E)
34. **The Macroscope: A New World Scientific System,** J. D. Rosnay (Harper & Row, New York, 1975). Translated by R. Edwards. Treatment of information theory with a unique sociological perspective. (E)
35. **Grammatical Man: Information, Entropy, Language, and Life,** J. Campbell (Simon & Schuster, New York, 1982). Excellent book with a chapter (3), "The Demon Deposed." (E)
36. **Laws of the Game: How the Principles of Nature Govern Chance,** M. Eigen and R. Winkler (Harper & Row, New York, 1983), pp. 157–160. Examination of an imagined cessation of irreversibility in nature, and several hypothetical demons. (E, I)
37. **God and the New Physics,** P. C. W. Davies (Simon & Schuster, New York, 1983), pp. 211–213. Davies asks: If a supreme being existed, constrained to act within the laws of physics, could it prevent the end of the universe? He concludes the answer is negative. (E)
38. **Order Out of Chaos,** I. Prigogine and I. Stengers (Bantam, New York, 1984). Maxwell's demon is used in discussions of biological systems. (E)
39. **The Recursive Universe,** W. Poundstone (Morrow, New York, 1985). Chapter 3 on Maxwell's demon summarizes work from Smoluchowski to Brillouin. (E, I)
40. **The Anthropic Cosmological Principle,** J. D. Barrow and F. J. Tipler (Oxford U. P., New York, 1986). Good overview with reference to existing Maxwell's demon controversies. (E, I)
41. **Seven Ideas That Shook the Universe,** N. Spielberg and B. D. Anderson (Wiley, New York, 1987), pp. 134–135. (E)
42. **Three Scientists and Their Gods,** R. Wright (Times Books, New York, 1988), pp. 91–93. Engaging layman's account of Maxwell's demon. (E)
43. **The World Within the World,** J. D. Barrow (Oxford U. P., Oxford, 1988), pp. 127–130. Good modern treatment covering information acquisition and erasure. (E, I)

III. SZILARD'S ONE-MOLECULE ENGINE

These papers focus on the one-molecule heat engine introduced by Leo Szilard in 1929, quantum mechanical extensions thereof, and related matters. Szilard's simple and elegant model, which entails a Maxwellian "pressure demon," leads to a remarkable range of ideas and results.

44. **"Über die Ausdehnung der phänomenologischen Thermodynamik auf die Schwankungserscheinungen,"** L. Szilard, Z. Phys. **32,** 753–788 (1925), reprinted in *The Collected Works of Leo Szilard: Scientific Papers*, edited by B. T. Feld and G. Weiss Szilard (MIT, Cambridge, MA, 1972), pp. 34–69; English translation ("On the extension of phenomenological thermodynamics to fluctuation phenomena"), pp. 70–102. Szilard's doctoral dissertation, a precursor to his 1929 paper, Ref. 45. (I, A)
45. **"Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen,"** L. Szilard, Z. Phys. **53** 840–856 (1929), reprinted in *The Collected Works of Leo Szilard: Scientific Papers*, edited by B. T. Feld and G. Weiss Szilard (MIT, Cambridge, MA, 1972), pp. 103–119; English translation ("On the decrease of entropy in a thermodynamic system by the intervention of intelligent beings") by A. Rapoport and M. Knoller, pp. 120–129. The English translation was published originally in Behav. Sci. **9,** 301–310 (1964) and is reprinted also in J. A. Wheeler and W. H. Zurek, *Quantum Theory and Measurement* (Princeton U. P., Princeton, NJ, 1983), pp. 539–548. In this famous paper Szilard establishes the basis for relating entropy and information, and recognizes the importance of binary decision processes. (I, A)
46. **"Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen,"** P. Clausing, Z. Phys. **56,** 671–672 (1929). Early criticism of Szilard's 1929 paper, alleging Szilard's analysis is in error. (I, A)
47. **"Light and Information,"** D. Gabor, Prog. Opt. **1,** 111–153 (1964), based on lectures given in 1951. A variant of Szilard's model, using light to determine the molecule's location, is examined. The second law can be violated in classical light theory, but is saved by the quantum theory of radiation. (A)
48. **"Irreversibility; or, Entropy Since 1905,"** K. Popper, Br. J. Philos. Sci. **8,** 151–155 (1957). A sharp criticism of Szilard's logic. (A)
49. **"On the Possibility of a Perpetuum Mobile of the Second Kind,"** P. K. Feyerabend, in *Mind, Matter, and Method: Essays in Philosophy and Science in Honor of Herbert Feigl*, edited by P. K. Feyerabend and G. Maxwell (University of Minnesota Press, Minneapolis, 1966), pp. 409–412. Critical assessment of a variant of Szilard's one-molecule gas, arguing that violation of the second law occurs without the need for information. See also Ref. 122, p. 106 and Ref. 220. (A)
50. **Foundations of Statistical Mechanics,** O. Penrose (Pergamon, Oxford, 1970), pp. 221–238. Apparently the earliest account of irreversible memory erasure for a Maxwell's demon. Penrose shows that resetting (i.e., erasing) a demon's memory is essential, increasing entropy by an amount at least as great as the entropy decrease made possible by the newly available memory capacity. (A)
51. **"Entropy, Information and Szilard's Paradox,"** J. M. Jauch and J. G. Báron, Helv. Phys. Acta **45,** 220–232 (1972). Identification of infor-

mation with the negative physical entropy is found to be unjustified, in conflict with a large body of literature. (A)

52. **"Information Theory and Thermodynamics,"** O. Costa de Beauregard and M. Tribus, Helv. Phys. Acta **47**, 238–247 (1974). Rebuttal to the Jauch–Báron (Ref. 51) attack of Szilard's one-molecule model. (A)

53. **The Philosophy of Karl Popper,** K. Popper (Open Court, LaSalle, IL, 1974), pp. 129–133. Also published under the title, *Unended Quest: An Intellectual Autobiography* (Open Court, LaSalle, IL, 1976). Criticism of Szilard's suggestion that knowledge and entropy are related. See Ref. 220. (A)

54. **"La Logique des Expériences de Pensée et l'Expérience de Szilard,"** G. Rossis, Fundam. Sci. **2**, 151–162 (1981). Reinforcement of Popper's and Feyerabend's criticisms of Szilard's one-molecule heat engine. (I, A)

References 55 and 56 are quantum mechanical analyses of Szilard's one-molecule heat engine that support the view that resetting the demon's memory saves the second law.

55. **"Maxwell's Demon, Szilard's Engine and Quantum Measurements,"** W. H. Zurek, in *Frontiers of Nonequilibrium Statistical Physics,* edited by G. T. Moore and M. O. Scully (Plenum, New York, 1984), pp. 151–161. (A)

56. **"Keeping the Entropy of Measurement: Szilard Revisited,"** E. Lubkin, Int. J. Theor. Phys. **26**, 523–535 (1987). (A)

IV. MAXWELL'S TEMPERATURE DEMON

In contrast with Szilard's one-molecule heat engine, Maxwell's original temperature demon operates in a gas containing many molecules. The following references provide analyses of such systems, focusing on the implications of quantum theory and information theory.

In Refs. 57 and 58, Demers argues that the operation of Maxwell's demon within the limits of the second law of thermodynamics demands blackbody radiation and quantum theory.

57. **"Les Démons de Maxwell et le Second Principe de la Thermodynamique,"** P. Demers, Can. J. Res. **22**, 27–51 (1944). (A)

58. **"Le Second Principe et la Theorie des Quanta,"** P. Demers, Can. J. Res. **23**, 47–55 (1945). (A)

59. **"Maxwell's Demon Cannot Operate: Information and Entropy. I,"** L. Brillouin, J. Appl. Phys. **22**, 334–337 (1951). Influential paper linking Maxwell's demon and the (then) new science of information theory, and introducing the term "negentropy." (I, A)

60. **"The Role of Information Theory in the Inactivation of Maxwell's Demon,"** H. Jacobson, Trans. N.Y. Acad. Sci. **14**, 6–10 (1951). Extension of Brillouin's arguments, exorcising Maxwell's demon using probability arguments. Further details are in Ref. 135. (I)

61. **"Les Relations d'Incertitude d'Heisenberg Empechent-elle le Démon de Maxwell d'Opérer?,"** N. L. Balazs, C. R. **236**, 998–1000 (1953). Maxwell's demon is not restricted by the uncertainty principle if the gas is nondegenerate. See also Refs. 57 and 58. (A)

62. **"L'effet des Statistiques sur le Démon de Maxwell,"** N. L. Balazs, C. R. **236**, 2385–2386 (1953). Restrictions on a Maxwell's demon by the uncertainty principle are discussed for Bose–Einstein and Fermi–Dirac gases. (A)

63. **"Some Comments on Entropy and Information,"** P. Rodd, Am. J. Phys. **32**, 333–335 (1964). Helpful clarification of Brillouin's exorcism of Maxwell's demon. (I)

64. **"A Relation Between the Second Law of Thermodynamics and Quantum Mechanics,"** B. Kivel, Am. J. Phys. **42**, 606–608 (1974). The demon's failure to beat the second law is attributed to energy quantization. (I)

65. **"The Operation of Maxwell's Demon in a Low Entropy System,"** A. F. Rex, Am. J. Phys. **55**, 359–362 (1987). Extension of Brillouin's results to the case in which the two chamber temperatures are initially unequal. (I)

66. **"Available Work from a Finite Source and Sink: How Effective is a Maxwell's Demon?,"** H. S. Leff, Am. J. Phys. **55**, 701–705 (1987). A demon delivering maximum work, using a finite heat source and sink with initial temperatures T_+ and T_-, has thermal efficiency $\approx 1-(T_-/T_+)^{\frac{1}{2}}$. (I)

67. **"Maxwell's Demon, Power, and Time,"** H. S. Leff, Am. J. Phys. **58**, 135 (1990). A demon is found to generate power < 1 nW and to take time > 4 million years to get a 2-K temperature difference in a large roomful of air. (I)

V. RELATED DEMONS

In this section, references are concerned with variants of Maxwell's original temperature demon and Szilard's one-molecule heat engine.

68. **"A Mechanical Maxwell Demon,"** J. L. Costa, H. D. Smyth, and K. T. Compton, Phys. Rev. **30**, 349–353 (1927). Experimental measurement of Maxwell's speed distribution inspired by Maxwell's demon. (I, A)

69. **Thermodynamics for Chemical Engineers,** H. C. Weber (Wiley, New York, 1939), pp. 127–128. Description of a technique attributed to Boltzmann, alleged to avoid the need for intelligence in the demon's operation. (I)

70. **"Can the Rectifier Become a Thermodynamical Demon?,"** L. Brillouin, Phys. Rev. **78**, 627–628 (1950). Illustration that thermal noise in a resistor cannot be rectified to transform heat to electric work. (I)

71. **"The Well-Informed Heat Engine,"** R. C. Raymond, Am. J. Phys. **19**, 109–112 (1951). Examination of a variant of Maxwell's demon that takes advantage of density fluctuations in a gas. (I)

72. **"Physical Demonology,"** J. Rothstein, Methodos **42**, 94–117 (1959). The demon concept is generalized to show that any law can be formulated in terms of the nonexistence of some type of demon. (I, A)

73. **"Well-Informed Heat Engine: Efficiency and Maximum Power,"** C. Finfgeld and S. Machlup, Am. J. Phys. **28**, 324–326 (1960). Raymond's well-informed heat engine is reexamined to get estimates of efficiency and power output. (I)

74. **The Feynman Lectures on Physics - Vol. 1,** R. P. Feynman, R. B. Leighton, and M. Sands (Addison-Wesley, Reading, MA, 1963). Highly recommended discussion (pp. 46.1–46.9) of the ratchet and pawl, discussed a half century earlier by Smoluchowski in connection with Maxwell's demon. It cannot operate as a heat engine that violates the second law. (E, I)

75. **"Seeing Entropy—The Incompleat Thermodynamics of the Maxwell Demon Bottle,"** M. V. Sussman, Chem. Eng. Ed. *Summer*, 149–156 (1974). A demonstration device that illustrates mixing and unmixing, simulating the action of a Maxwell's demon, and helping to understand entropy. (E)

76. **"Maxwell's Demon and Computation,"** R. Laing, Philos. Sci. **41**, 171–178 (1974). Maxwell's demon is viewed as a computing automaton. (E, I)

77. **"Brownian Movement and Microscopic Irreversibility,"** L. G. M. Gordon, Found. Phys. **11**, 103–113 (1981). A microengine model that appears to violate the second law of thermodynamics is proposed. (A)

78. **"The Vortex Tube: A Violation of the Second Law?,"** M. P. Silverman, Eur. J.Phys. **3**, 88–92 (1982). The Hilsch vortex tube is compared with a Maxwell's demon. (I)

79. **Cosmic Code,** H. Pagels (Simon & Schuster, New York, 1982). Discussion (Part I, Sec. 8) of the statistical nature of the second law. (E)

80. **"Maxwell's Demon and Detailed Balancing,"** L. G. M. Gordon, Found. Phys. **13**, 989–997 (1983). Analysis of a gas partitioned by a membrane, each pore of which is controlled by an independent molecular trap door that can exist in either of two states. (A)

References 81–83 deal with the possibility of using Doppler-shifted radiation to allow Maxwell's demon to operate. Denur suggests and supports the idea; Motz and Chardin give arguments against it.

81. **"The Doppler Demon,"** J. Denur, Am. J. Phys. **49**, 352–355 (1981). (I)

82. **"The Doppler Demon Exorcised,"** H. Motz, Am. J. Phys. **51**, 72–73 (1983). (I)

83. **"No Free Lunch for the Doppler Demon,"** G. Chardin, Am. J. Phys. **52**, 252–253 (1984). (I)

84. **The Second Law,** P. W. Atkins (Scientific American Books, New York, 1984). Discussion (pp. 67–79) of the second law using the idea of a "Boltzmann's demon." (E)

85. **"Growth of Order in the Universe,"** D. Layzer, in *Entropy, Information, and Evolution: New Perspectives on Physical and Biological Evolution*, edited by B. H. Weber, *et al.* (MIT, Cambridge, MA, 1988), pp. 23–29. Discussion of what is sometimes referred to as Loschmidt's nondemon or Laplace's demon (see Ref. 72). (I)

VI. INFORMATION AND THERMODYNAMICS

Maxwell's demon has been the most important example that illustrates connections between thermodynamics and information. The following references concentrate on these linkages.

A. General

86. **"Certain Factors Affecting Telegraph Speed,"** H. Nyquist, Bell Syst. Tech. J. **3**, 324–346 (1924). The rate per character at which "intelligence" can be transmitted is taken to be $K \log m$, where m is the number of possible distinct characters and K is a constant. (I, A)

87. **"Transmission of Information,"** R. V. L. Hartley, Bell Syst. Tech. J. **7**, 535–563 (1928). Well-reasoned argument leading to the proposal of a logarithmic measure of information contained in a message. (I, A)

88. **"Entropy and Information,"** N. Wiener, Proc. Symp. Appl. Math. Am. Math. Soc. **2**, 89 (1950), reprinted in *Norbert Wiener: Collected Works with Commentaries*, edited by P. Masani (MIT, Cambridge, MA, 1985), p. 202. Wiener suggests looking for Maxwell demons in photosynthesis. (A)

89. **"Thermodynamics and Information Theory,"** L. Brillouin, Am. Sci. **38**, 594–599. (1950). Discourse on information, thermodynamics, and computation. (I)

90. **"Physical Entropy and Information. II,"** L. Brillouin, J. Appl. Phys. **22**, 338–343 (1951). Discussion of how information can be linked to a decrease in a system's entropy. (A)

91. **"Cybernetics,"** N. Wiener, Scientia (Italy) **87**, 233–235 (1952), in *Norbert Wiener: Collected Works with Commentaries*, edited by P. Masani (MIT, Cambridge, MA, 1985), pp. 203–205. Cybernetics ideas are used to study Maxwell's demon. (A)

92. **"Information and Thermodynamics,"** J. Rothstein, Phys. Rev. **85**, 135 (1952). A useful informational interpretation of the first, second, and third laws of thermodynamics. (I)

93. **"Organization and Entropy,"** J. Rothstein, J. Appl. Phys. **23**, 1281–82 (1952). Organization is characterized as negative entropy in the form of encoded information. Theory is viewed as organization of observation. (A)

94. **"Probabilistic Logics from Unreliable Components,"** J. von Neumann, in *Automata Studies*, Vol. V, No. 10, edited by C. E. Shannon and J. McCarthy (Princeton U. P., Princeton, NJ, 1956), pp. 43–98. Reprinted in *J. von Neumann's Collected Works, Vol. V*, edited by A. H. Taub (Pergamon, New York, 1963), pp. 341–342. Von Neumann points out the similarity between the information theory and statistical mechanical entropy functions. (A)

In Refs. 95–97, statistical mechanics is developed using the maximum entropy principle of information theory. References 95 and 97 focus mainly on equilibrium statistical mechanics, and the anthropomorphic nature of entropy. Reference 96 covers time-dependent phenomena and irreversibility.

95. **"Information Theory and Statistical Mechanics,"** E. T. Jaynes, Phys. Rev. **106**, 620–630 (1957). (A)

96. **"Information Theory and Statistical Mechanics. II,"** E. T. Jaynes, Phys. Rev. **108**, 171–190 (1957). (A)

97. **"Information Theory and Statistical Mechanics,"** E. T. Jaynes, in *Statistical Physics—1962 Brandeis Summer Institute Lectures in Theoretical Physics Vol. 3,* edited by K. Ford (Benjamin, New York, 1963), pp. 181–218. (I, A)

98. **"Nuclear Spin Echo Experiments and the Foundations of Statistical Mechanics,"** J. Rothstein, Am. J. Phys. **25**, 510–518 (1957). Discourse on how thermodynamic irreversibility and mechanical reversibility can be reconciled. Reversibility occurs with perfect memory (storing information), while "forgetting" (discarding information) implies irreversibility. (I)

99. **"Discussion: Information and Organization as the Language of the Operational Viewpoint,"** J. Rothstein, Philos. Sci. **29**, 406–411 (1962). Information theory concepts are related to physical measurement, and organization is related to physics operations and laws. (I, A)

100. **"Entropy, Information, and Quantum Measurements,"** G. Lindblad, Comm. Math. Phys. **33**, 305–322 (1973). Formal treatment of quantum measurements, addressing entropy changes in a system and its measuring apparatus. (A)

101. **"Measurements and Information for Thermodynamic Quantities,"** G. Lindblad, J. Stat. Phys. **11**, 231–255 (1974). Entropy decrease induced by an observer in a system described by a fluctuating thermodynamic parameter is found to be less than the information obtained by the observer. (A)

102. **"Loschmidt's and Zermelo's Paradoxes Do Not Exist,"** J. Rothstein, Found. Phys. **4**, 83–89 (1974). These paradoxes are argued to result from self-contradictory language. Resolution entails an operational, informational analysis. See also Ref. 98. (I, A)

103. **"Where Do We Stand on Maximum Entropy?"** E. T. Jaynes, in *The Maximum Entropy Formalism*, edited by R. D. Levine and M. Tribus (MIT, Cambridge, MA, 1979), pp. 15–118. An examination of the history and potential of the maximum entropy principle. (I)

104. **"Maxwell Demon and the Correspondence Between Information and Entropy,"** R. P. Poplavskii, Sov. Phys. Usp. **22**, 371–380 (1979). Entropic efficiency is defined and used to examine order-producing control processes. (A)

105. **"Foundations of Statistical Mechanics,"** O. Penrose, Rep. Prog. Phys. **42**, 1937–2006 (1979). Review covering topics germane to Maxwell's demon, including the nonunique nature of entropy. (A)

106. **"Thermodynamic Aspects of the Quantum-Mechanical Measuring Process,"** R. Kosloff, Adv. Chem. Phys. **46**, 153–193 (1981). Maxwell's demon and Szilard's and Brillouin's work are starting points for this discussion of quantum measurement theory. (A)

107. **Non-equilibrium Entropy and Irreversibility**, G. Lindblad (Reidel, Dordrecht, The Netherlands, 1983), pp. 113–122. Formal mathematical treatment of information and entropy, with application to Maxwell's demon. (A)

108. **"Wiener on Cybernetics, Information Theory, and Entropy,"** S. Watanabe, in *Norbert Wiener: Collected Works with Commentaries*, edited by P. Masani (MIT, Cambridge, MA, 1985), pp. 215–218. Commentary on Norbert Wiener's papers on the title subject. (A)

109. **"Macroscopic Prediction,"** E. T. Jaynes, in *Complex Systems—Operational Approaches*, edited by H. Haken (Springer-Verlag, Berlin, 1985), pp. 254–269. Jaynes addresses inference based on information rather than on logical deduction via physical law. (A)

110. **"Generalized Scattering,"** E. T. Jaynes, in *Maximum-Entropy and Bayesian Methods in Inverse Problems*, edited by C. Ray Smith and W. T. Grandy, Jr. (Reidel, Dordrecht, The Netherlands, 1985), pp. 377–398. Remaximization of entropy upon constraint changes, with emphasis on understanding why the MAXENT formalism works. (A)

111. **"Entropy and Search Theory,"** E. T. Jaynes, in *Maximum-Entropy and Bayesian Methods in Inverse Problems*, edited by C. Ray Smith and W. T. Grandy, Jr. (Reidel, Dordrecht, The Netherlands, 1985), pp. 443–454. Entropy maximization and optimal strategies are explored for a simple model. (A)

112. **"Entropy and Information: Suggestions for a Common Language,"** J. S. Wicken, Philos. Sci. **54**, 176–193 (1987). Differences between thermodynamic and information theory entropies are clarified. (A)

113. **"Clearing Up Mysteries—The Original Goal,"** E. T. Jaynes, in *Maximum Entropy and Bayesian Methods, Cambridge 1988*, edited by J. Skilling (Kluwer Academic, Dordrecht, The Netherlands, 1989), pp. 1–27. Bayesian inference illuminates diffusion, the EPR paradox, and the second law of thermodynamics in biology. (A)

B. Subjectivity of entropy

Entropy in thermodynamics is defined such that entropy differences can be determined experimentally. In statistical mechanics and information theory, probability assignments are required to calculate entropy and to associate it with missing information. Does this imply that entropy is subjective? This is addressed in the following references. See also Refs. 98 and 102.

114. **"The Symmetry of Time in Physics,"** G. N. Lewis, Science **71**, 569–577 (1930). Maxwell's demon is introduced via three particles in a partitioned cylinder. Entropy increases when "a known distribution goes over into an unknown distribution," supporting a subjective view of entropy. (I)

115. **"Gibbs vs. Boltzmann Entropies,"** E. T. Jaynes, Am. J. Phys. **33**, 391–398 (1965). Gibbs and Boltzmann entropies are compared, and the anthropomorphic nature of entropy is addressed. (I)

116. **"Is the Coarse-Grained Entropy of Classical Statistical Mechanics an Anthropomorphism?,"** A. Grünbaum, in *Modern Developments in Thermodynamics*, edited by B. Gal-or (Wiley, New York, 1974), pp. 413–428. Although entropy depends on human choice of cell size in phase space, it is argued that the answer to the title question is negative. (A)

117. **"Entropy, Information and Maxwell's Demon After Quantum Mechanics,"** R. Bhandari, Pramana **6**, 135–145 (1976). Subjectivity of entropy is examined in light of the quantum measurement problem. (A)

118. **"How Subjective Is Entropy?,"** K. Denbigh, Chem. Br. **17**, 168–185 (1981). A precursor to the more complete Ref. 120. (I, A)

119. **Quantum Theory and the Schism in Physics,** K. R. Popper (Roman & Littlefield, Totowa, NJ, 1982), pp. 109–117. See also Refs. 49 and 53.

120. **Entropy in Relation to Incomplete Knowledge,** K. G. Denbigh and J. S. Denbigh (Cambridge U. P., London, 1985), pp. 1–5; 108–112. Critical, detailed analysis of the notion that entropy is subjective, and rejection of commonly used links between information theory and thermodynamics. It is argued that Brillouin's exorcism of Maxwell's demon can be accomplished without appeals to information theory or negentropy. (A)

C. Contrasts with Brillouin's work

Brillouin's analysis of Maxwell's demon has met with critical responses from some authors. The following references contain criticisms of, or contrasts with, Brillouin's ideas. These are distinct from objections (see Sec. VI B) to subjective interpretations of entropy associated with information theoretic approaches.

121. **Knowing and Guessing: A Quantitative Study of Inference and Information,** S. Watanabe (Wiley, New York, 1969), pp. 245–254. The author finds Brillouin's principle untenable and proposes an alternative approach. (A)

122. **Paradoxes of Physics,** P. Chambadal (Transworld, London, 1971), pp. 80–115. English translation by M. G. Ingham. First published under the title *Les Paradoxes en Physique* (Dunod, Paris, 1971). Critical assessments of Brillouin's solution to the demon puzzle and of Szilard's heat engine. (I)

123. **Entropy: The Devil on the Pillion,** J. Zernike (Kluwer–Deventer, Deventer, The Netherlands, 1972), Chap. 4. It is argued that the demon and its apparatus must be at the same temperature as the gas, which excludes use of a torch. (I)

124. **"Sur la Discussion par Brillouin de l'Expérience de Pensée de Maxwell,"** G. Rossis, Fundam. Sci. **2**, 37–44 (1981). Rossis argues that Brillouin does not demonstrate the connection between entropy and information. (I, A)

125. **Energy and Entropy,** G. N. Alekseev (Mir, Moscow, 1986), pp. 180–183. Qualitative discourse on the statistical nature of the second law. (E)

D. Biological systems

These references address uses of information concepts in biology. See also Refs. 21, 220, and 221.

126. **Thermodynamics and The Free Energy of Chemical Substances,** G. N. Lewis and M. Randall (McGraw-Hill, New York, 1923), pp. 120–121. Brief account that predates Szilard's work and focuses attention on the demon's entropy. (I)

127. **"Life, Thermodynamics, and Cybernetics,"** L. Brillouin, Am. Sci. **37**, 554–568 (1949). Nonmathematical discussion of whether thermodynamics can be applied to biological processes far from equilibrium. Apparently the first published appearance of Brillouin's ideas on Maxwell's demon and the connections between information and entropy. (I)

128. **"Communication, Entropy, and Life,"** R. C. Raymond, Am. Sci. **38**, 273–278 (1950). A living system's entropy is viewed as the sum of the positive thermodynamic entropy the system's constituents would have at thermodynamic equilibrium, and a negative term proportional to the information necessary to build the actual system from its equilibrium state. (I)

129. **The Physical Foundation of Biology,** W. M. Elsasser (Pergamon, New York, 1958), pp. 203–214. Maxwell's demon is viewed as a special case of an idealized "Laplacian spirit" observer. Limitations to the concept of an ideal observer are identified as a central point in the question of compatibility between biological and physical law. (A)

130. **Bioenergetics: The Molecular Basis of Biological Energy Transformation,** A. L. Lehninger (Benjamin, New York, 1965), pp. 225–227. Information storage in living cells is addressed. (I)

131. **Energy Flow in Biology,** H. J. Morowitz (Academic, New York, 1968), pp. 124–133. Brief account of Szilard's model and discussion of the relationship between observer and system in biology. (I)

132. **Entropy for Biologists,** H. J. Morowitz (Academic, New York, 1970), pp. 108–111. A section is devoted to the demon, with emphasis on living cells. (I)

133. **"Thermal Noise and Biological Information,"** H. A. Johnson, Q. Rev. Biol. **62**, 141–152 (1987). Loss of information due to thermal noise in information-handling biological systems is examined. Renal tubules perform a sorting function like that of a Maxwell's demon. (A)

E. Information theory, cybernetics, and related books

The following books each develop the theory of information and/or cybernetics, and most include Maxwell's demon as an example.

134. **The Mathematical Theory of Communication,** C. E. Shannon and W. Weaver (University of Illinois Press, Urbana, 1949). Contains Claude Shannon's celebrated 1948 paper, a cornerstone of information theory. (I, A)

135. **Science and Information Theory,** L. Brillouin (Academic, New York, 1956), Chap. 13. Thorough discussion of Brillouin's contributions to Maxwell's demon. Information is categorized as either free or bound, an idea not present in Brillouin's original exorcism of Maxwell's demon, Ref. 59. (I)

136. **"Discussion,"** A. Rapoport, in *Symposium on Information Theory in Biology*, edited by H. P. Yockey, R. L. Platzman, and H. Quastler (Pergamon, New York, 1958), p. 196. A translator of Szilard's 1929 paper addresses physical laws that are stated in terms of impossibilities. Exorcism of Maxwell's demon is viewed as a means of broadening understanding rather than a defeat. (I)

137. **Cybernetics,** N. Wiener (MIT, Cambridge, MA, 1961). A significant volume in the development of information theory. (I, A)

138. **Symbols, Signals, and Noise,** J. R. Pierce (Harper & Brothers, New York, 1961), pp. 198–207. Survey of the demon problem, covering thermal fluctuations, a variant of the Szilard model, and entropy gain during information acquisition. (I)

139. **Intelligent Machines: An Introduction to Cybernetics,** D. A. Bell (Blaisdell, New York, 1962), pp. 7–10. Maxwell's demon is described along the lines of Brillouin and Raymond (Refs. 59 and 71). (I)

140. **Great Ideas in Information Theory, Language, and Cybernetics,** J. Singh (Dover, New York, 1966), Chap. VII. (I)

141. **Information Theory,** D. A. Bell (Sir Isaac Pitman & Sons, London, 1968), pp. 20–21; 212–219. Critical review of Maxwell's demon. Reversible information transmission is also covered. (I)

142. **Cybernetics A to Z,** V. Pekelis (Mir, Moscow, 1974), pp. 104–110. Maxwell's demon is used to impart an understanding of entropy. (E)

143. **Fundamentals of Cybernetics,** A. Y. Lerner (Plenum, New York, 1975), pp. 256–258. (I)

144. **Optics and Information Theory,** F.T.S. Yu (Wiley, New York, 1976), Chaps. 4–6. Detailed treatment of Maxwell's demon, referring heavily to the work of Gabor and Brillouin (Refs. 47, 59, and 135). (I)

F. Kinetic theory, thermodynamics, and statistical mechanics books

The following thermal physics books deal with aspects of Maxwell's demon. References 145 and 146 contain Jeans' idea that a demon could "effect in a very short time what would probably take a very long time to come about if left to the play of chance." For a contrasting view based on kinetic theory, see Refs. 66 and 67.

145. **Dynamical Theory of Gases,** J. H. Jeans (Dover, New York; reprinted from the 4th ed., 1925). (I)

146. **An Introduction to the Kinetic Theory of Gases,** J. H. Jeans (Macmillan, New York, 1940). (I)

147. **Introduction to Chemical Physics,** J. C. Slater (McGraw-Hill, New York, 1939). Irreversibility is linked to the Heisenberg uncertainty principle, which prevents demons from operating on arbitrarily small scales. See Refs. 61 and 62. (I)

148. **Introduction to Chemical Thermodynamics,** L. E. Steiner (McGraw-Hill, New York, 1941), p. 166. Maxwell's demon is considered in connection with a fluctuation whereby a gas spontaneously collapses to a volume smaller than the container, and a retaining wall is put in place. (I)

149. **Chemical Engineering Thermodynamics,** B. F. Dodge (McGraw-Hill, New York, 1944), pp. 48–49. The author argues that a demon *could* violate the second law. (I)

150. **Elements of Statistical Mechanics,** D. ter Haar (Rinehart, New York, 1958), pp. 160–162. Brief discussion of the work of Maxwell, Szilard, and Wiener. (I)

151. **The Nature of Thermodynamics,** P. W. Bridgman (Harper, New York, 1961), pp. 155–159. Important issues that threaten a demon's ability to operate are addressed. (I)

152. **Principles of General Thermodynamics,** G. N. Hatsopoulos and J. H. Keenan (Wiley, New York, 1965). Maxwell's demon is discussed in connection with thermal physics history and the definition of a thermodynamic system. (I)

153. **Statistical Mechanics,** R. Kubo (North-Holland, Amsterdam, 1965), p. 13. A Maxwell's demon cannot operate continuously because it will heat up, get sick, and lose control. A similar view is in Ref. 74. (I)

154. **The Second Law,** H. A. Bent (Oxford U. P., New York, 1965), pp. 72–76. Discussion of the demon, including an obituary, "Maxwell's Demon (1871-c.1949)," based on Brillouin's resolution of the puzzle. (I)

155. **Entropy and Low Temperature Physics,** J. S. Dugdale (Hutchinson University Library, London, 1966), pp. 151–153. It is emphasized that fluctuations cannot be used systematically to violate the second law. (I)

156. **Thermal Properties of Matter—Vol. II—Thermodynamics and Statistics: WithApplications to Gases,** W. Kauzmann (Benjamin, New York, 1967), pp. 209–211. The view is taken that each demon decision requires conversion of nonthermal energy into heat, keeping the second law intact. (I)

157. **Thermodynamics,** J. P. Holman (McGraw-Hill, New York, 1969), pp. 144–145. (I)

158. **An Introduction to Thermodynamics,** R. S. Silver (Cambridge U. P., London, 1971), pp. 42 and 125. The demon's operation is contrasted with macroscopic engineering processes. (I)

159. **Entropy and Energy Levels,** R. P. H. Gasser and W. G. Richards (Clarendon, Oxford, 1974), pp. 116–119. Maxwell's demon is described, with quantitative connections for a crystal made up of independent two-state molecules. (I)

160. **Two Essays on Entropy,** R. Carnap (University of California Press, Berkeley, 1977), pp. 72–73. Mathematical treatment in which, among other things, a distinction between logical and physical entropy is made. (A)

161. **Heat and Thermodynamics,** M. W. Zemansky and R. H. Dittman (McGraw-Hill, New York, 1981), 6th ed., pp. 297–299. One of few standard textbooks with a good description of the demon's function. (I)

162. **The Theory of Thermodynamics,** J. R. Waldram (Cambridge U. P., London, 1985), pp. 306–308. Provocative discourse on Maxwell's demon, calling entropy a function of how much we know about what the system does, and not of what it actually does. (I)

VII. LIMITS OF COMPUTATION

A fundamental issue in the theory of computation is whether computing processes, in principle, are necessarily dissipative. There is still debate among researchers on this issue. The dominant view is that reading and writing operations can be done with arbitrarily little dissipation—but information erasure must be accompanied by heat transfer to the environment (though not necessarily by irreversibility in the thermodynamic sense; see Ref. 191). References in this section illustrate the progress and spirited debate on dissipation in computing, and address the relevance of this issue to Maxwell's demon.

163. **"Irreversibility and Heat Generation in the Computing Process,"** R. Landauer, IBM J. Res. Dev. **5**, 183–191 (1961). A seminal paper showing that performance of logical functions that do not possess unique inverses is accompanied by heat generation. (I, A)

164. **"Logical Reversibility of Computation,"** C. H. Bennett, IBM J. Res. Dev. **17**, 525–532 (1973). A path-breaking paper showing that a computing automaton can be made logically reversible at every step. (A)

165. **"Classical and Quantum Limitations on Energy Consumption in Computation,"** K. K. Likharev, Int. J. Theor. Phys. **21**, 311–326 (1982). Analysis of a "parametric quantron" model supports the thesis that logical operations can be physically reversible. (A)

166. **"The Thermodynamics of Computation—A Review,"** C. H. Bennett, Int. J. Theor. Phys. **21**, 905–940 (1982). Bennett introduces the analogy between computing and Maxwell's demon in this cogent review of computer models, biological applications, and Szilard's one-molecule heat engine. (A)

167. **"Quantum Mechanical Model of Turing Machines That Dissipate No Energy,"** P. Benioff, Phys. Rev. Lett. **48**, 1581–1585 (1982). (A)

168. **"Computing Without Dissipating Energy,"** A. L. Robinson, Science **223**,1164–1166 (1984). (I)

169. **"Fundamental Physical Limitations of the Computational Process,"** R. Landauer, in *Computer Culture: The Scientific, Intellectual, and Social Impact of the Computer*, Ann. N.Y. Acad. Sci. **426**, 161–171 (1984). (A)

References 170–175 illustrate the spirited debate on dissipation in computation.

170. **"Dissipation in Computation,"** W. Porod, R. O. Grondin, D. K. Ferry, and G. Porod, Phys. Rev. Lett. **52**, 232–235 (1984). A sharp objection to the view that computation with dissipation $< kT \ln 2$ per bit operation is possible, based on the argument that logical reversibility does not imply physical reversibility when $T > 0$. (A)

171. **"Thermodynamically Reversible Computation,"** C. H. Bennett, Phys. Rev. Lett. **53**, 1202 (1984). (A)

172. **"Comment on 'Dissipation in Computation,' "** P. Benioff, Phys. Rev. Lett. **53**, 1203 (1984). (A)

173. **"Dissipation in Computation,"** R. Landauer, Phys. Rev. Lett. **53**, 1205 (1984). (A)

174. **"Comment on 'Dissipation in Computation,' "** R. Toffoli, Phys. Rev. Lett. **53**, 1204 (1984). (A)

175. **"Porod *et al.* Respond,"** W. Porod, R. O. Grondin, D. K. Ferry, and G. Porod, Phys. Rev. Lett. **52**, 1206 (1984). (A)

176. **"Reversibility and Stability of Information Processing Systems,"** W. H. Zurek, Phys. Rev. Lett. **53**, 391–394 (1984). Quantum and classical models of dynamically reversible computers are examined with respect to operational reversibility and stability. (A)

177. **"Low Dissipation Computing in Biological Systems,"** H. M. Hastings and S. Waner, BioSystems **17**, 241–244 (1985). Discourse on lowering energy dissipation for complex computer optimizations. (A)

178. **"Computation and Physics: Wheeler's Meaning Circuit?"** R. Landauer, Found. Phys. **16**, 551–564 (1986). Concise overview of the ultimate physical limitations of computation. (I, A)

179. **"Reversible Computation,"** R. Landauer, in *Der Informationsbegriff in Technik un Wissenschaft*, edited by O. G. Folberth and C. Hackl (Oldenbourg, München, 1986), pp. 139–158. (A)

180. **"Quantum Mechanical Computers,"** R. P. Feynman, Found. Phys. **16**, 507–531 (1986). Supports the thesis that logical operations in principle can be physically reversible and that there is "...no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds dominant sway." (A)

181. **"Multistable Quantum Systems: Information Processing at Microscopic Levels,"** K. Obermayer, G. Mahler, and H. Haken, Phys. Rev. Lett. **58**, 1792–1795 (1987). Proposal to build a nanometer-scale semiconductor device enabling direct measurement of energy dissipation in certain processes. (A)

182. **"Computation: A Fundamental Physical View,"** R. Landauer, Spec. Sci. Technol. **10**, 292–302 (1987). Reprint of an article originally published in IEEE Spectrum **4**, 105–109 (1967). (A)

References 183–189 further illustrate elements of the ongoing debate related to dissipation in information processing.

183. **"Computation: A Fundamental Physical View,"** R. Landauer, Phys. Scr. **35**, 88–95 (1987). The polarization of researchers with respect to the possibility of reversible computing is emphasized. (Distinct from Ref. 182, despite title similarity.) (A)

184. **"Comment on 'Energy Requirements in Communication,'"** W. Porod, Appl. Phys. Lett. **52**, 2191 (1988). (A)

185. **"Response,"** R. Landauer, Appl. Phys. Lett. **52**, 2191–2192 (1988). Rebuttal to Ref. 184. (A)

186. **"Energy Requirements in Communication,"** R. Landauer, Appl. Phys. Lett. **51**, 2056–2058 (1987). (I, A)

187. **"On Ultimate Thermodynamic Limitations in Communication and Computation,"** J. Rothstein, in *NATO ASI Series E. Vol.* 142: *Performance Limits in Communication Theory and Practice*, edited by F. K. Skwirzynski (Kluwer Academic, Dordrecht, The Netherlands, 1988), pp. 43–58. It is argued that reversible measurements are not possible, and "selective" acts of writing and reading have concomitant entropy costs. (A)

188. **"The Computer and the Heat Engine,"** O. Costa de Beauregard, Found. Phys. **19**, 725–727 (1989). (A)

189. **"Response to 'The Computer and the Heat Engine,'"** R. Landauer, Found. Phys. **19**, 729–732 (1989). (A)

190. **"Dissipation and Noise Immunity in Computation and Communication,"** R. Landauer, Nature **335**, 779–784 (1988). (A)

191. **"Notes on the History of Reversible Computation,"** C. H. Bennett, IBM J. Res. Dev. **32**, 16–23 (1988). Bennett covers Szilard's model as an example in which *reversible* memory erasure that transfers heat to the environment saves the second law. (A)

192. **"Computation, Measurement, Communication and Energy Dissipation,"** R. Landauer, in *Signal Processing*, edited by S. Haykin (Prentice-Hall, Englewood Cliffs, NJ, 1989), pp. 18–47. Note that some figures have been printed incorrectly, rotated 90 or 180 deg. (A)

193. **"Thermodynamic Cost of Computation, Algorithmic Complexity and the Information Metric,"** W. H. Zurek, Nature **341** (6238), 119–124 (1989). Algorithmic complexity is adopted as a measure of randomness, and is shown to set limits on the thermodynamic cost of computations and cast a new light on the limitations of Maxwell's demon. (A)

VIII. MISCELLANEOUS

The references in this section address the roles of quantum theory and time in thermodynamics, and the mathematical and physical meanings of entropy.

A. Quantum theory and thermodynamics

References 194 and 195 helped establish the connections between quantum theory and thermodynamics.

194. **"Die Quantenmechanik und der Zweite Hauptsatz der Thermodynamik,"** M. Born, Ann. Phys. **3**, 107 (1948). (A)

195. **"The Kinetic Basis of Thermodynamics,"** M. Born and H. S. Green, Proc. R. Soc. London Ser. A **192**, 166–180 (1948). (A)

196. **"On the Process of Measurement in Quantum Mechanics,"** P. Jordan, Philos. Sci. **16**, 269–278 (1949). Szilard's one-molecule heat engine is examined in the context of quantum mechanical measurement theory. (I, A)

197. **Quantum Theory**, D. Bohm (Prentice-Hall, Englewood Cliffs, NJ, 1951). Brief discussion of the irreversibility of the measurement process in quantum theory. (A)

198. **Mathematical Foundations of Quantum Mechanics,** J. von Neumann (Princeton U. P., Princeton, NJ, 1955). Von Neumann focuses on thermodynamic considerations relating to macroscopic applications of quantum mechanics and measurement theory. (A)

199. **The Philosophy of Quantum Mechanics,** M. Jammer (Wiley, New York, 1974). Smoluchowski's and Szilard's influences on von Neumann's measurement theory (Ref. 198) are discussed. (A)

200. **"Human Perception and the Uncertainty Principle,"** R. C. Harney, Am. J. Phys. **44**, 790–792 (1976). Calculations show the inadequacy of humans to observe directly the Heisenberg uncertainty principle. Although Maxwell's demon is not addressed, extrapolation to a demon's scale is inviting. (I)

201. **The Shaky Game: Einstein, Realism, and the Quantum Theory,** A. Fine (University of Chicago Press, Chicago, 1986), pp. 26–39. The relevant chapter is a reprint of Fine's "Einstein's Critique of Quantum Theory: The Roots and Significance of EPR," originally published in *After Einstein*, edited by P. Barker and C. G. Shugart (Memphis State U. P., Memphis, 1981), pp. 147–159, Chap. 4. A thought experiment of Einstein's (reminiscent of Szilard's one-molecule heat engine) is examined. (A)

202. **"Uncertainty Principle and Minimal Energy Dissipation in the Computer,"** R. Landauer, Int. J. Theor. Phys. **21**, 283–297 (1982). The energy-time uncertainty principle cannot yield information on dissipation in a computer system with a given switching time. (A)

203. **The Ghost in the Atom,** P. C. W. Davies (Cambridge U. P., Cambridge, 1986). Discussion of an electron in a partitioned box has relevance to Szilard's heat engine model and a related thought experiment of Einstein. See also Ref. 201. (E)

204. **"Schrödinger's Immortal Cat,"** A. Peres, Found. Phys. **18**, 57–76 (1988). Review of the quantum measurement problem, with 81 literature citations. (A)

B. Time in thermodynamics

Much of classical thermodynamics is "timeless" in the sense that it deals with time-independent equilibrium states and reversible thermodynamic paths. On the other hand, time is central to the issue of irreversibility. The following references address aspects of time in thermodynamics and information gathering. See also Refs. 66 and 67.

205. **The Physics of Time Asymmetry,** P. C. W. Davies (University of California Press, Berkeley, 1974). Exploration of time, entropy, and information. (A)

206. **"On the Relation Between Information and Energy Systems: A family of Maxwell's demons,"** A. M. Weinberg, Interdiscip. Sci. Rev. **7**, 47–52 (1982). Maxwell's demon is used to make interesting connections between time, energy, and information. (E, I)

207. **The Enigma of Time,** P. T. Landsberg (Adam Hilger, Bristol, 1982). The demon is discussed in relation to energy requirements and the statistical character of the second law. (A)

208. **"The Direction of Time,"** S. Hawking, New Sci. **115** (1568), 46–49 (July 9, 1987). Hawking alludes to the process of writing to a computer memory as dissipative. (E, I)

C. Entropy articles

This section contains articles that provide perspectives on the entropy concept and the relationship between entropy and information. See also Ref. 98.

209. **"The Concept of Entropy,"** K. K. Darrow, Am. J. Phys. **12**, 183–196 (1944). Highly recommended, cogent review. (I)

210. **"Order, Organization, and Entropy,"** M. J. Klein, Br. J. Philos. Sci. **4**, 158–160 (1953). A system's organization and entropy can increase (or decrease) simultaneously. The former relates to the wavefunction; the latter is linked to the number of relevant energy states. (A)

211. **"The Many Faces of Entropy,"** H. Grad, Comm. Pure Appl. Math. **14**, 323–354 (1961). Mathematical review covering the H function, irreversibility, and statistical and classical thermodynamics. (A)

212. **"The Microscopic Interpretation of Entropy,"** D. H. Frisch, Am. J. Phys. **34**, 1171–1173 (1966). Boltzmann's entropy expression is deduced from $dS = dQ/T$ and quantum mechanical expressions of heat and work. (I)

213. **"A Hundred Years of Entropy,"** M. Dutta, Phys. Today **21**, 75–79 (1968). (I)

214. **"Entropy and Disorder,"** P. G. Wright, Contemp. Phys. **11**, 581–588 (1970). Discussion of entropy and intuitive ideas on disorder. (I)

215. **"Informational Generalization of Entropy in Physics,"** J. Rothstein, in *Quantum Theory and Beyond*, edited by T. Bastin (Cambridge U. P., London, 1971). (I)

216. **"Entropy,"** M. Jammer, in *Dictionary of the History of Ideas, Vol.* 2, edited by P. Wiener (Scribner's, New York, 1973). Comprehensive overview of entropy, with a section on Maxwell's demon. (I, A)

217. **"On the Mathematical Definition of Entropy,"** B. Skagarstam, Z. Naturforsch. **29a**, 1239–1243 (1974). Jauch and Báron's entropy is a unique function satisfying extensitivity, positivity, and continuity. (A)

218. **"On the Notions of Entropy and Information,"** B. Skagarstam, J. Stat. Phys. **12**, 449–462 (1975). Historical review of the information-theoretic and thermodynamic entropy concepts. (A)

219. **"General Properties of Entropy,"** A. Wehrl, Rev. Mod. Phys. **50**, 221–260 (1978). Advanced review. (A)

220. **"Generalized Entropy, Boundary Conditions, and Biology,"** J. Rothstein, in *The Maximum Entropy Formalism*, edited by R. D. Levine and M. Tribus (MIT, Cambridge, MA, 1979), pp. 423–468. Generalized entropy in the framework of a generalized thermodynamics is proposed to handle complex "biotonic" laws. Rothstein refutes conclusions in Refs. 49 and 53. (I, A)

221. **"Entropy and the Evolution of Complexity and Individuality,"** J. Rothstein, Ohio State University Report OSU-CISRC-8/88–TR27, pp. 1–51 (1988). To be published in *Conference Proceedings of the Eighth College Park Colloquium on Chemical Evolution: Prebiological Organization*, edited by C. Ponnamperuma and F. Eirich (A. Deepak Publishing, Hampton, VA, 1990). Discussion of the generalized entropy concept and its relevance to quantum theory, thermodynamics, and biology. (A)

222. **"Microscopic and Macroscopic Entropy,"** K. Lindgren, Phys. Rev. A **38**, 4794–4798 (1988). Examination of a microscopic counterpart to entropy, with microstates represented as symbol sequences. (A)

223. **"Algorithmic Randomness and Physical Entropy,"** W. H. Zurek, Phys. Rev. A **40**, 4731–4751 (1989). Physical entropy is defined as the sum of missing information plus algorithmic information content, enabling a formulation of thermodynamics from a Maxwell demon's viewpoint. (A)

Feynman's proof of the Maxwell equations

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Feynman's proof of the Maxwell equations, discovered in 1948 but never published, is here put on record, together with some editorial comments to put the proof into its historical context.

I. THE PROOF

As I mentioned in my talk at the Feynman Memorial Session of the AAAS meeting in San Francisco,[1] Feynman showed me in October 1948 a proof of the Maxwell equations, assuming only Newton's law of motion and the commutation relation between position and velocity for a single nonrelativistic particle. In response to many enquiries, I here publish the proof in a form as close as I can come to Feynman's 1948 exposition. Unfortunately, I preserved neither Feynman's manuscript nor my original notes. What follows is a version reconstructed at some unknown time from notes which I discarded.

Assume a particle exists with position x_j $(j = 1,2,3)$ and velocity $\dot{x}_j$ satisfying Newton's equation

$$m\ddot{x}_j = F_j(x,\dot{x},t), \quad (1)$$

with commutation relations

$$[x_j,x_k] = 0, \quad (2)$$

$$m[x_j,\dot{x}_k] = i\hbar\,\delta_{jk}. \quad (3)$$

Then there exist fields $E(x,t)$ and $H(x,t)$ satisfying the Lorentz force equation

$$F_j = E_j + \epsilon_{jkl}\dot{x}_k H_l, \quad (4)$$

and the Maxwell equations

$$\operatorname{div} H = 0, \quad (5)$$

$$\frac{\partial H}{\partial t} + \operatorname{curl} E = 0. \quad (6)$$

Remark: The other two Maxwell equations,

$$\operatorname{div} E = 4\pi\rho, \quad (7)$$

$$\frac{\partial E}{\partial t} - \operatorname{curl} H = 4\pi j, \quad (8)$$

merely define the external charge and current densities ρ and j.

Proof: Equations (1) and (3) imply

$$[x_j,F_k] + m[\dot{x}_j,\dot{x}_k] = 0. \quad (9)$$

The Jacobi identity

$$[x_l[\dot{x}_j,\dot{x}_k]] + [\dot{x}_j,[\dot{x}_k,x_l]] + [\dot{x}_k,[x_l,\dot{x}_j]] = 0 \quad (10)$$

with (3) and (9) implies

$$[x_l[x_j,F_k]] = 0. \quad (11)$$

Equation (9) also implies

$$[x_j,F_k] = -[x_k,F_j], \quad (12)$$