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Acoustic Effect of Holes on a Brass Disk

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Introduction

Recently it has become popular for music companies to manufacture cymbals with large symmetrically placed holes . The physics of how these perturbations effect the sound is unknown. This summer the effect of a single hole, and then multiple holes, on a brass disk was studied, motivated by the Sabian Ozone cymbal.

Figure 1. Three of the instruments used in this study. From left to right is the Sabian AAX Ozone cymbal , the Sabian AAX crash cymbal, and the 10 inch brass disk with a 3" inch diameter hole cut in the Puget Sound shop.

Background

Musical instruments produce sound through vibrations. If driven acoustically at a natural (resonant) frequency the instrument will vibrate at a normal mode, meaning that all parts of the disk are vibrating with the same frequency. At each vibrational mode there are nodes (regions of no motion) and anti nodes (regions of greatest motion). The patterns consist of nodal diameters and circles represented by integers *m* and *n* respectively.

For a disk, every mode with diameters is doubly degenerate, meaning that there are two modes with the same shape at the same frequency but oriented differently. If alterations are made to the disk which change the symmetry, the degenerate modes can split and occur at different frequencies.

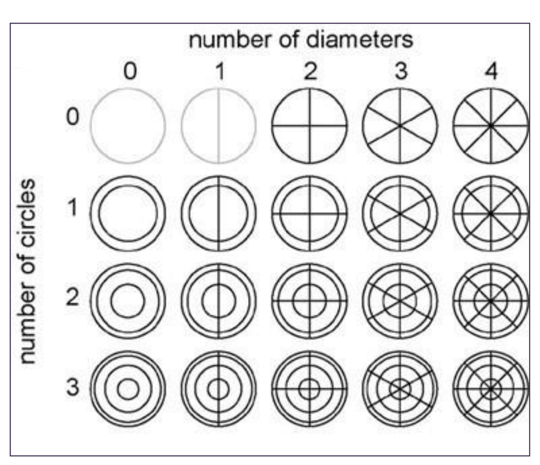

Figure 2. This shows the shape of modes for a circular plate, where the outside circle is the edge of the disk and the lines or circles drawn inside are nodes. The number of diameters and circles are labeled in the (*m*, *n*) notation.

Theory

The frequencies of the plate depend on *m* and *n*, but are also proportional to $\sqrt{D/\rho}$ where *D* is the stiffness and *ρ* is the density. This means that decreasing the stiffness can only lower the frequency, and decreasing the density can only increase the frequency. The stiffness is $D \sim Eh^3$, where *E* is Young's Modulus, and *h* is the thickness of the plate. A hole can be thought of as a region of both lower stiffness and density. The effect of stiffness is greatest where the mode has the most curvature. For density the effect is greatest where the mode has the largest amplitude. Because of these competing effects modal frequencies may increase or decrease depending on the position and size of the holes.

References

T. D. Rossing (Ed.), *Springer Handbook of Acoustics* (ed.), (New York, 2007).

T. D. Rossing , *Science of Percussion Instruments,* (River Edge, New Jersey, 2000).

Data and Analysis

1 Hole

It was found that the location and size of the hole in a mode pattern determines whether the frequency increases or decreases. The following general patterns were observed:

1. For modes with nodal diameters only the frequency decreases, and the smaller *m* the greater the change in frequency
2. For modes with nodal circles only the frequency decreases up to a certain hole size and then increases as the hole is made larger
3. Modes with both nodal diameters and circles tend to split
   - With one diameter both frequencies increase
   - With more than one diameter one frequency increases and one decreases

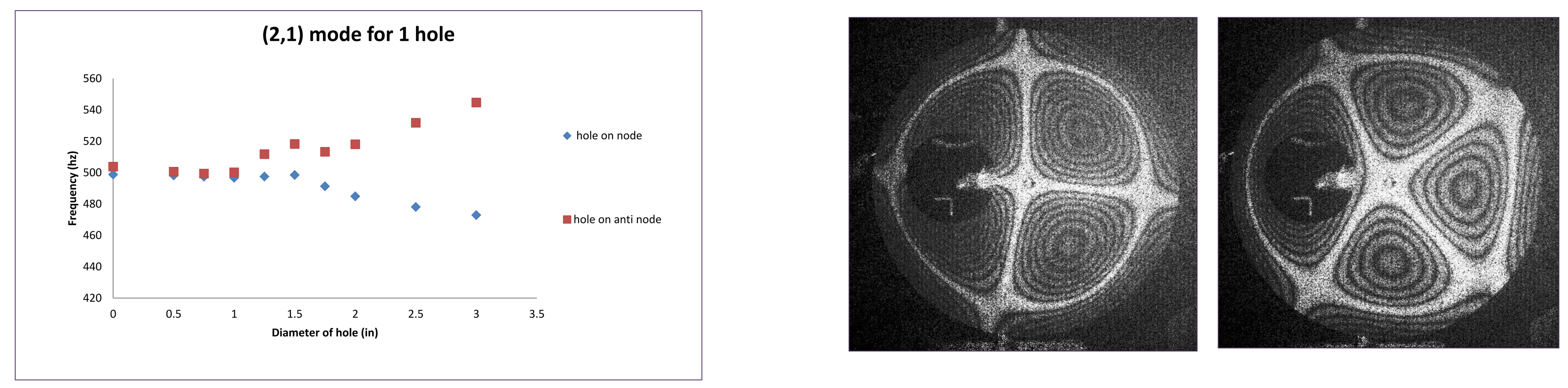

Figure 5. Above left is a graph of frequency versus holes size for the (2,1) mode pair. It can be seen that the degenerate pair splits once the hole is about 1 inch in diameter. Above right are the ESPI image s of the (2,1) mode with a 3 inch hole. The image with the hole on the node corresponds to the blue data, while the hole on the anti node corresponds to the red data. (This is a representative example of the second part of pattern 3 above.)

6 Holes

We find that generally the frequencies decrease when six holes are cut into a disk. However the degenerate modes do not split unless they have a multiple of 3 nodal diameters. This can be explained through a group theory result that relates the symmetry of the perturbation to the symmetry of a mode. The degenerate modes can only split if all of the perturbations (holes) can lie on a node in one case and on an anti node in the other. This rule can be represented by the following equation

$$\frac{2m}{a} = p, \qquad p = 1, 2, 3, \ldots, \tag{1}$$

where *m* is the number of nodal diameters and *a* is the number of perturbations (holes). For *a*=6, this means that m must be a multiple of three for splitting to occur.

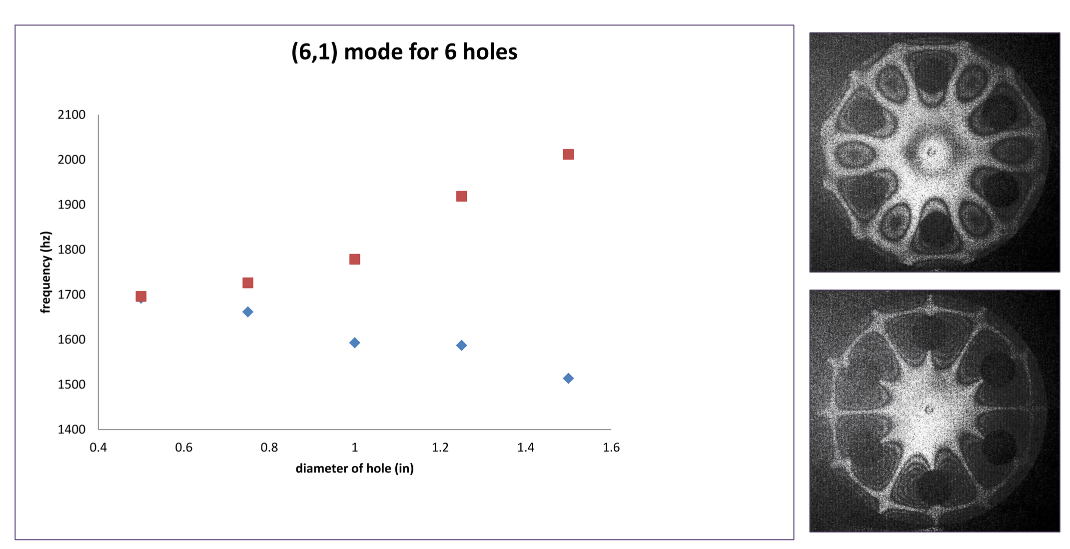

Figure 6. This graph shows the splitting of the (6,1) degenerate pair as the holes are made larger. The top image on the right is of the mode that increases in frequency and the bottom one decreases.

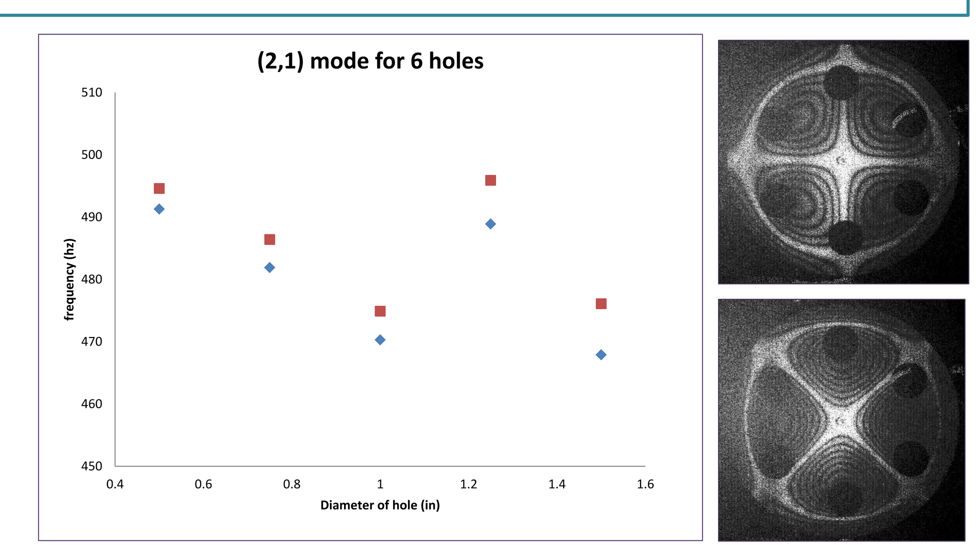

Figure 7. The graph of the (2,1) mode shows the frequency generally decreasing as the hole is made larger. Note that with *m*=2, there is virtually no frequency splitting. The top ESPI image corresponds to the blue data and the bottom image to the red data.

Frequency changes for 20 modes comparing largest hole size to unperturbed disk for the single and six hole cases

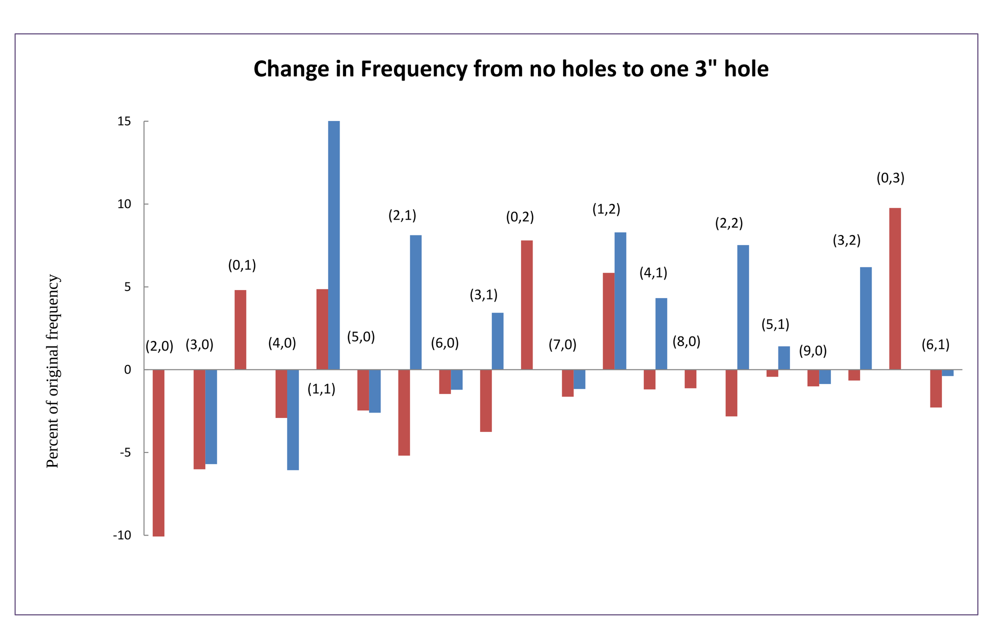

Figure 6. This shows the change in frequency for the first 20 modes for a plate with no holes compared to a plate with a 3 inch hole. Degenerate mode pairs are shown in red and blue. It can be seen that a single hole can cause frequencies to increase, decrease, or remain relatively unchanged.

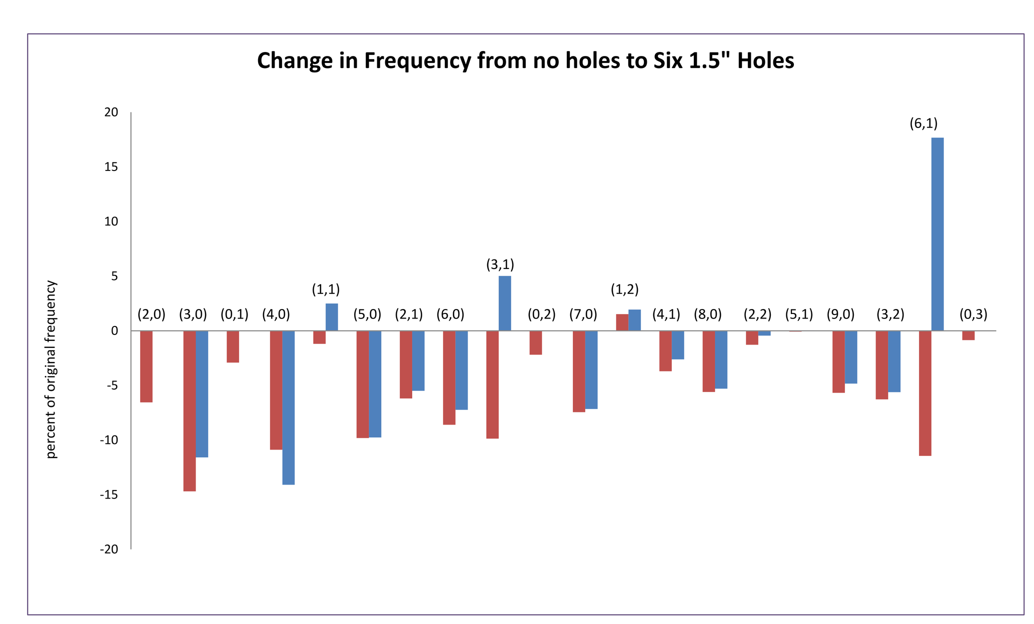

Figure 7. This shows the change in frequency for the first 20 modes for a disk with no holes compared to a disk with six 1.5 inch holes. In general the frequencies decrease. The modes where the frequency increases tend to satisfy equation 1 and have at least one circle.

Apparatus and Procedure

Brass disks were used for measuring the effect of a hole. The disks, though they have a different profile shape and are made from a slightly different alloy, provide a simpler model than the cymbal for studying the effect of a hole on vibrations. The disks were cut from a large sheet of metal in the Puget Sound shop. The lowest 30 vibration modes were measured for each hole size. This was repeated for a disk with six holes.

To record the images of the vibrational modes an electronic speckle pattern interferometer (ESPI) was used (Figure 3). Light from a laser was split into two beams, one of which reflected off of the acoustically driven disk. The other beam served as a reference and did not strike the disk. The speckle patterns from both beams were recombined creating an interference pattern that varied depending on the disk vibration. A CCD camera recorded the interference patterns and a computer with LabVIEW software was used to process the images. The output was an image where nodal regions were white and the contours of equal amplitude motion were grey.

In addition, finite element software NEi Fusion was used to numerically model each object (Figure 4).

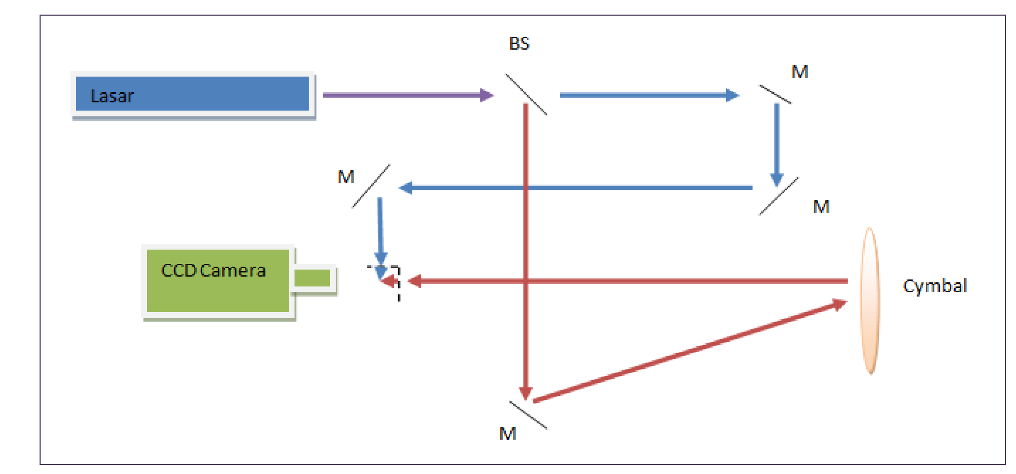

Figure 3. This is a drawing of the path the laser beam takes for the ESPI. BS is beam splitter and M is mirror, the two colors represent the different paths after the beam is split; the blue is the reference beam and the red the object beam.

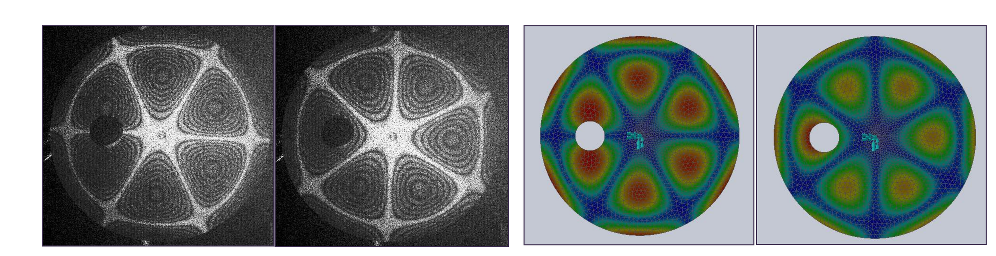

Figure 4. On the left are images of the two degenerate components of the (3,1) mode taken using the ESPI system. The right are corresponding images from the finite element software. Note the alignment of the degenerate pair relative to the 1.5 inch hole, with either the hole on the node or on the anti node.

Conclusion

In general it is possible for the frequency of a given mode to increase, decrease, or remain about the same depending on the size, location, and number of holes. It was found that the splitting of degenerate modes was consistent with the symmetry relation expressed in equation 1. When a degenerate pair splits the modes align with the perturbation such that nodal diameters of the lower frequency mode fall on the hole(s). The higher frequency mode is aligned with the holes halfway between nodal diameters. The numerical model agrees with the experimental data in nearly all cases.

Future Work

We will try to develop a model to predict frequency changes in the flat disk. To better investigate the effect of holes on the cymbal we will cut six small symmetrically placed holes into a Sabian AAX cymbal and record the modes as the holes are made larger.

Acknowledgements

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