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## Sonar signal enhancement using fractional Fourier transform

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### ABSTRACT

In this paper we present an approach for signal enhancement of sonar signals. Work reported is based on sonar data collected by the Volume Search Sonar (VSS), as well as VSS synthetic data. The Volume Search Sonar is a beamformed multibeam sonar system with 27 fore and 27 aft beams, covering almost the entire water volume (from above horizontal, through vertical, back to above horizontal). The processing of a data set of measurement in shallow water is performed using the Fractional Fourier Transform algorithm. The proposed technique will allow efficient determination of seafloor bottom characteristics and bottom type using the reverberation signal. A study is carried out to compare the performance of the presented method with conventional methods. Results are shown and future work and recommendations are presented.

**Keywords:** Fractional Fourier transform, impulse response, sonar signal processing, Wigner distribution, volume search sonar.

### 1. INTRODUCTION

The reported data herein is based on sonar data collected by the Volume Search Sonar (VSS), one of the five sonar systems in the AN/AQS-20. The AQS-20 system is an underwater towed body containing a high resolution, side-looking, multibeam sonar system used for mine hunting along the ocean bottom, as well as a forward looking sonar, and the volume search sonar. The system is illustrated in Figure 1. The VSS consists of two separate arrays: the transmit array and the receive array. The VSS is a beamformed multibeam sonar system with 54 beams arranged as 27 fore-aft beam pairs, covering almost the entire water volume (from above horizontal, through vertical, back to above horizontal)<sup>1</sup>. The VSS can be used in two modes: volume mode and SPD mode. The acoustic energy received by the VSS hydrophone array is pre-amplified and conditioned. Conditioning includes dynamic range compensation using time varying gain (TVG), bandshifting to IF (750 KHz), and band pass filtering. After conditioning, analog to digital (A/D) conversion is performed and signals are undersampled at 200 KHz. The beamforming function forms all beams and then quadrature demodulates the beam data to baseband (only the image centered at 50 KHz is retained and basebanded). A hybrid time delay phase shift function is used to beamform by using a Hilbert transform after the element delays. The beam outputs are produced by shading (weighted sum) of array element data, delayed to compensate for cylindrical array geometry. Data from this sonar may be used for bathymetry computation, bottom classification, target detection, and water volume investigations<sup>2</sup>.

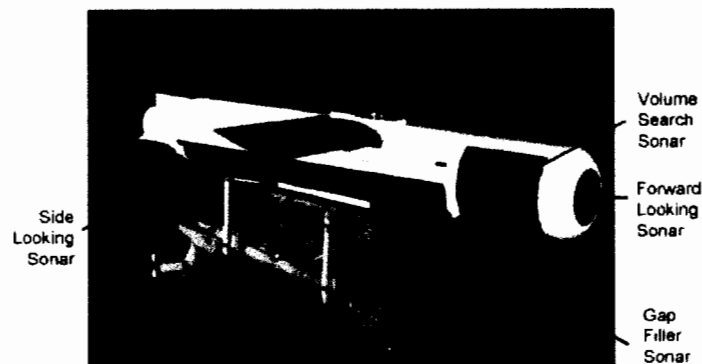


Figure 1: AQS – 20 mine hunting sonar

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Our investigation is focused on the bottom-return signals since we are interested in determination the impulse response of the ocean bottom floor. The bottom-return signal is the convolution between the impulse response of the bottom floor and the transmitted sonar chirp signal. The method developed here is based on Fractional Fourier Transform, a fundamental tool for optical information processing and signal processing. In recent years, interest in and use of time-frequency tools has increased and become more suitable for sonar and radar applications<sup>3,4,5</sup>. Major research directions include the use of time-frequency analysis for target and pattern recognition, noise reduction, beamforming, and optical processing. In this paper we begin by presenting the essential concepts and definitions related to Fractional Fourier transform. The overview is followed by a description of the implementation of the Fractional Fourier transform and the methods proposed for evaluating the impulse response of the ocean bottom. The Fractional Fourier transform requires finding the optimum order of the transform based on the properties of the chirp signal. The bottom impulse response is given by the magnitude of the Fractional Fourier transform applied to the bottom return signal. The technique introduced in this work has been tested both on the synthetic data and real sonar data. A study is carried out to compare the performance of the presented method to conventional methods. Results are shown and future work and recommendations are presented.

## 2. FRACTIONAL FOURIER TRANSFORM OVERVIEW

The Fractional Fourier transform (FrFT) is a generalization of the identity transform and the conventional Fourier transform (FT) into fractional domains. The Fractional Fourier transform can be understood as a Fourier transform to the  $a^{\text{th}}$  power where  $a$  is not required to be an integer. There are several ways to define the FrFT; the most direct and formal one is given by<sup>6</sup>:

$$f_a(u) = \int_{-\infty}^{+\infty} K_a(u, u') f(u') du' \quad (1)$$

where  $K_a(u, u') \equiv A_\alpha \exp[i\pi(\cot \alpha \cdot u^2 - 2 \csc \alpha \cdot u \cdot u' + \cot \alpha \cdot u'^2)]$  and  $A_\alpha = \sqrt{|1 - i \cot \alpha|}$ ,  $\alpha = \frac{a\pi}{2}$  when  $a \neq 2k$

$$K_a(u, u') \equiv \delta(u - u') \text{ when } a = 4k$$

$$K_a(u, u') \equiv \delta(u + u') \text{ when } a = 4k \pm 2$$

where  $k$  is a integer and  $A_\alpha$  is a constant term. The order of the transform is  $a$  and sometimes is referred to as  $\alpha$ . If we set  $a=1$ , that correspond to  $\alpha = \frac{\pi}{2}$  and  $A_\alpha=1$ , so that the FrFT becomes the ordinary Fourier transform of  $f(u)$ :

$$f_1(u) = \int_{-\infty}^{+\infty} e^{-i2\pi uu'} f(u') du' \quad (2)$$

Due to periodic properties, the  $a$  range can be restricted to  $(-2, 2]$  or  $[0, 4)$ , respectively  $\alpha \in (-\pi, \pi]$  or  $\alpha \in [0, 2\pi)$ . The Fractional Fourier transform operator,  $F^\alpha$ , satisfies important properties such as linearity, index additivity  $F^{a_1} F^{a_2} = F^{a_1+a_2}$ , commutativity  $F^{a_1} F^{a_2} = F^{a_2} F^{a_1}$ , and associativity  $(F^{a_1} F^{a_2}) F^{a_3} = F^{a_1} (F^{a_2} F^{a_3})$ . In the operator notation, these identities follow<sup>5</sup>:  $F^0=I$ ;  $F^1=F$ ;  $F^2=P$ ;  $F^3=FP=PF$ ;  $F^4=F^0=I$ ; and  $F^{4k+a}=F^{a+k+a}$ , where  $I$  is a identity operator,  $P$  is a parity operator, and  $k$  and  $k'$  are arbitrary integers. According to the above definition (1), the zero-order transform of a function is the same as the function itself  $f(u)$ , the first order transform is the Fourier transform of  $f(u)$ , and the  $\pm 2^{\text{nd}}$  order transform is equal to  $f(-u)$ .

One of the most important properties of the FrFT states that the Wigner distribution of the FrFT of a function is a rotated version of the Wigner distribution of the original function<sup>6</sup>:

$$W_{f_a}(u, \mu) = W_f(u \cos \alpha - \mu \sin \alpha, u \sin \alpha + \mu \cos \alpha). \quad (3)$$

The Wigner distribution of a signal  $f$  is defined as<sup>6</sup>:

$$W_f(u, \mu) \equiv \int f(u + u'/2) f^*(u - u'/2) e^{-i2\pi \mu u'} du' \quad (4)$$

and can be interpreted as a function that indicates the distribution of the signal energy over the time-frequency space.

The most significant properties of the Wigner distribution are stated in the following equations:

$$1. \int W_f(u, \mu) d\mu = |f(u)|^2 \quad (5)$$

$$2. \int W_f(u, \mu) du = |F(u)|^2 \quad (6)$$

$$3. \iint W_f(u, \mu) dud\mu = \|f\|^2 = En[f] \quad (7)$$

$$4. g = h(u) * f(u) \text{ has Wigner distribution } Wg(u, \mu) = \int W_h(u - u', \mu) W_f(u', \mu) du' \quad (8)$$

$$5. g = h(u)f(u) \text{ has Wigner distribution } Wg(u, \mu) = \int W_h(u, \mu - \mu') W_f(u, \mu') d\mu' \quad (9)$$

6. Wigner distribution of the Fourier transform is the Wigner Distribution of the original function rotated clockwise by the right angle.

The Wigner distribution is completely symmetric with respect to time-frequency domains, it is everywhere real but not always positive. The Wigner distribution exhibits advantages over the spectrogram (short-time Fourier transform): the conditional averages are exactly the instantaneous frequency and the group delay, whereas the spectrogram fails to achieve this result, no matter what window is chosen. The Wigner distribution is not a linear transformation, a fact that complicates the use of the Wigner distribution for time-frequency filtering.

The ambiguity function has a correlative interpretation and it is defined as <sup>6</sup>:

$$A_f(\bar{u}, \bar{\mu}) \equiv \int f(u' + \bar{u}/2) f^*(u' + \bar{u}/2) e^{-i2\pi\bar{\mu}u'} du' \quad (10)$$

This ambiguity function is related to the Wigner distribution as a two-dimensional Fourier transform

$$A_f(\bar{u}, \bar{\mu}) = \iint W_f(u, \mu) e^{-i2\pi(\bar{\mu}u - \bar{u}\mu)} dud\mu \quad (11)$$

Another relationship between the Wigner distribution and the Fractional Fourier transform is given by the Radon transform operation,  $RDN_\alpha$ , which maps a two-dimensional function to its integral projection onto an axis making angle  $\alpha$  with the  $u$  axis <sup>6</sup>:

$$\int W_{f_\alpha}(u, \mu) d\mu = RDN_\alpha[W_f(u, \mu)] \quad (12)$$

Using equation (12), equations (7) and (8) can be generalized and expressed in terms of the Radon transform as:

$$RDN_\alpha[W_f(u, \mu)] = |f_\alpha(u)|^2 \quad (13)$$

Equation (13) is a powerful relation that can be applied to determine the relationship between the magnitude of the  $\alpha$  order of the Fractional Fourier transform  $f_\alpha(u)$  and the Wigner distribution  $W_f(u, \mu)$ .

### 3. APPLICATION OF FRACTIONAL FOURIER TRANSFORM

In order to evaluate Fractional Fourier transform techniques, several methods have been proposed<sup>7</sup>. Fast computation of the Fractional Fourier transform implies different decompositions that lead to different algorithms. Successive steps of simple operations such as chirp multiplication followed by chirp convolution followed by another chirp multiplication yield the fast convolution algorithm<sup>7</sup>. Optimization of the main interval of the fractional order increases calculation accuracy<sup>8</sup>.

In this paper we use a Fractional Fourier transform Matlab routine available from the Mathworks website<sup>10</sup>. First, a chirp with the specific parameters characterizing the system in questions —such as a bandwidth of 10400 Hz and a chirp duration of 4.32 ms for VSS— are generated. In order to corroborate our techniques we generate a synthetic impulse response of the seafloor, a Green function is utilized. The synthetic sonar return signal is generated by the convolution between the Green function and the transmitted VSS chirp. This synthetic data were used for testing both methods: classical frequency-domain deconvolution and our proposed deconvolution using Fractional Fourier transform.

The classical method applies the inverse Fourier transform to equation (14) to obtain the impulse response.

$$H(\omega) = H_I(\omega) + j \cdot H_Q(\omega) \quad (14)$$

$$H_I(\omega) = \frac{R_I(\omega)P_I(\omega) + R_Q(\omega)P_Q(\omega)}{P_I^2(\omega) + P_Q^2(\omega)} \quad (15)$$

$$H_Q(\omega) = \frac{-R_I(\omega)P_Q(\omega) + R_Q(\omega)P_I(\omega)}{P_I^2(\omega) + P_Q^2(\omega)} \quad (16)$$

In the above equations  $R(\omega)$  and  $P(\omega)$  are the Fourier transforms of complex baseband received signal and of transmitted pulse, respectively. The subscripts I and Q denote, respectively, the real (in-phase) and imaginary (quadrature-phase).

The second method consists of using the Fractional Fourier transform that is applied to the sonar return data. The order of the transform is determined by the chirp properties: the rate of change  $\lambda$ , sampling rate  $f_s$ , and the length of the data segment  $N$ ,<sup>3</sup>:

$$a = (2/\pi) \cdot \tan^{-1}(f_s^2/N/2\lambda) \quad (17)$$

Wigner distribution has been used to visually determine the correct order of the transform. The optimum transfer order is achieved when the representation of the chirp in the Wigner distribution is a delta function. If the properties of the chirp are not known,  $a$  can be optimized visually. The impulse response is given by the absolute value of the correct order of the Fractional Fourier transform of the function that represents the return data:

$$|f_a(u)| \approx h(t) \quad (18)$$

#### 4. EXPERIMENTAL RESULTS

In this section we present the experimental results for synthetic data as well as for real sonar data. The synthetic transmitted chirp pulse presented in Figure 2 consists of a synthesized version of the actual VSS transmitted pulse. The synthetic Green function has been simulated using an exponential function and three impulses. The synthetic data has been generated by convolving a chirp signal with the previously mentioned characteristics and a Green function as illustrated in Figures 3 and 4. The impulse response of the seafloor can be obtained in two different ways. The first method uses the classical deconvolution between the return data and the chirp signal producing the results presented in Figure 4 (b).

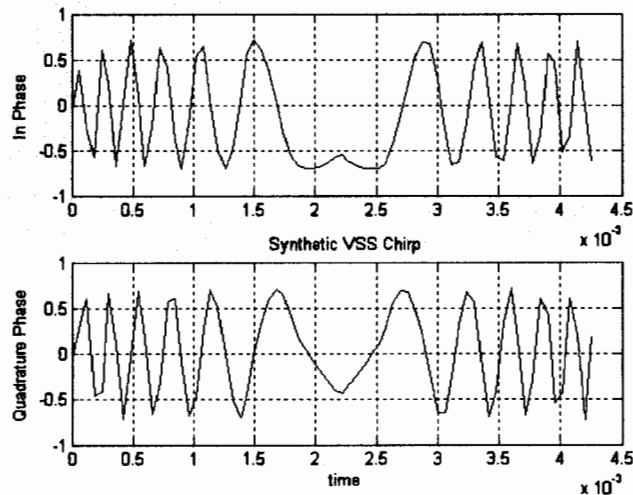


Figure 2: In-phase and Quadrature-phase components of the synthetic VSS pulse vs. time

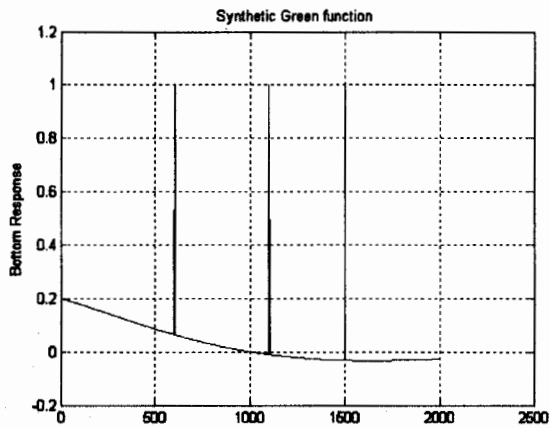


Figure 3: Synthetic Green function

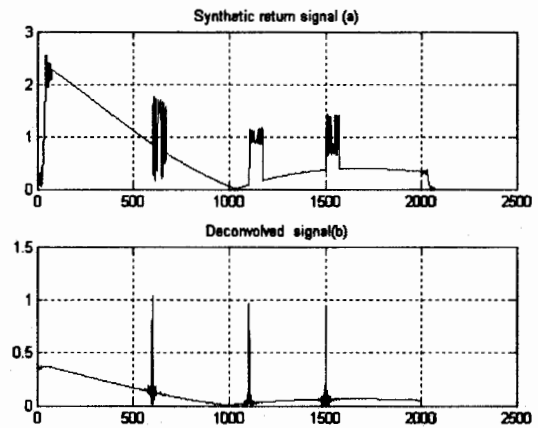


Figure 4: (a) Synthetic return signal (convolution of VSS Synthetic Source with the simulated Green function)  
(b) Deconvolved signal (using classical method from ref 2)

Given the deconvolved signal (Figure 4 (b)) it is simple to find the time (or sample number) of the returns. The return signal location in time is found to be equal to the original location in the Green function. The impulses in the original Green's function occur at sample numbers 600, 1100, and 1500. The peaks of the deconvolved return (once shifted by the length of the source), also occur at 600, 1100, and 1500.

The second method investigated in this paper employs Fractional Fourier Transform applied to the return data. In order to determine the order of the transform we used equation (17) and we validated its value by examination of the chirp's Wigner distribution as shown in Figures 5 and 6. In general the Wigner distribution of the chirp function is found to be concentrated along the line giving the instantaneous frequency of the chirp<sup>6</sup>. Taking the  $\alpha^{\text{th}}$  Fractional Fourier Transform of a signal is equivalent to rotating the Wigner distribution by an angle  $\alpha = \frac{a\pi}{2}$  in the clockwise direction<sup>5</sup>. The Wigner Distribution of the synthetic return data corresponds to the convolution of the Wigner distribution of the chirp signal with the Wigner distribution of the Green function.

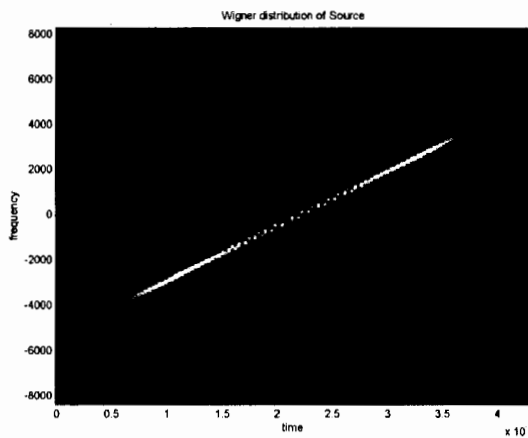


Figure 5: Wigner Distribution of the source with BW=10400 Hz

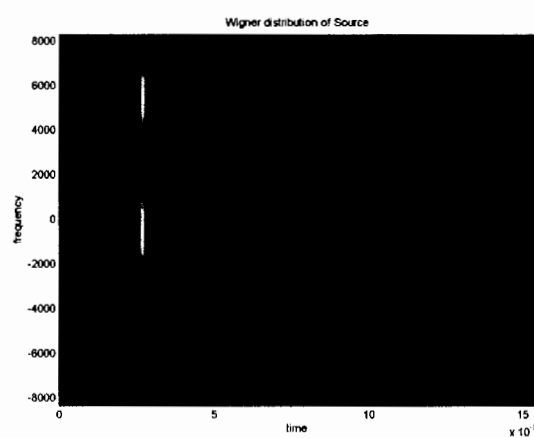


Figure 6: Wigner Distribution of the source  $a=0.035$

After choosing the optimum order (0.035) the Fractional Fourier transform was applied to the bottom synthetic data return. The Wigner distribution of the chirp's Fractional Fourier transform at optimum order is a delta function as illustrated in Figure 6. The synthetic bottom impulse response (synthetic Green function) was obtained by taking the magnitude of the Fractional Fourier transform of the bottom synthetic data return as shown in Figure 7. Although a slight shift occurred in determining the bottom impulse response, a good match between it and the original Green function has been achieved. The Fractional Fourier transform is represented as a function of sample number, hence the x-axis is a-dimensional.

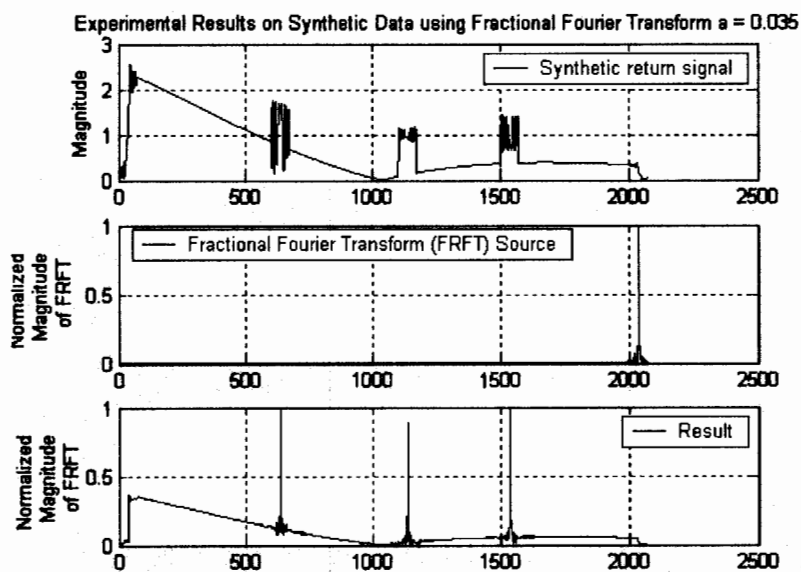


Figure 7: Experimental Results on Synthetic Data using Fractional Fourier transform

We applied the same techniques to actual VSS sonar data and the results are shown in Figures 8 thru 11. The data available consist of a small number of pings. A typical nadir beam amplitude return in its raw received form is illustrated in Figure 8. The "transmitted pulse" (as seen by the receiver array) and the main return are clear on this plot, where the bottom return occurs at 0.132 seconds from transmission. The total length of the signal is 0.9648 seconds. As expected the nadir beam raw data shows a clear bottom return with high amplitude and little spreading. The received signal shown is normalized to a maximum amplitude of 1.

The deconvolution method in ref 2 and the Fractional Fourier transform method presented here have been applied to the same beams and pings and their respective results are presented in Figures 9 and 10. We used the same window of 256 samples for both methods. The optimum order of the Fractional Fourier transform corresponds to the highest pulse compression and it was found to be 0.269 for this specific chirp. The amplitude of the Fractional Fourier transform applied to the bottom return data for the optimum order represents the bottom impulse response (Figure 9).

Figure 11 illustrates the bottom impulse response using both methods discussed in this paper. The plots are shifted so that they can be compared easily by visual inspection.

The energy levels, mean and standard deviation of the bottom impulse response for 100 samples corresponding to a range 10% - 90% have been computed for both methods and they are presented in Table 1.

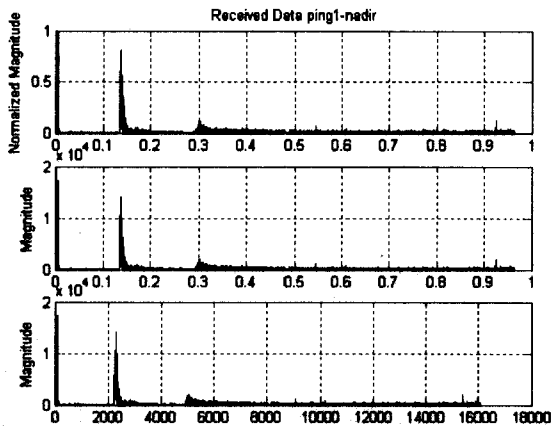


Figure 8: Received Ping data

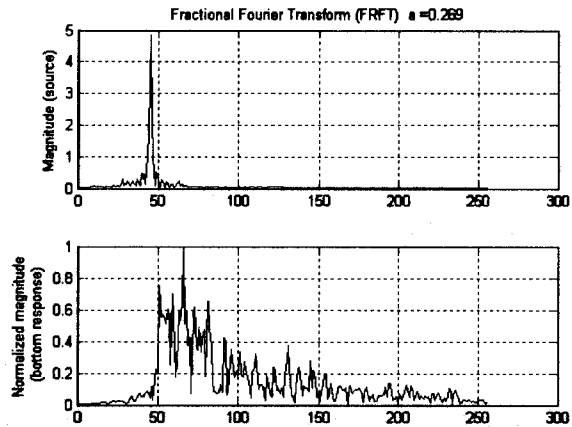


Figure 9: Bottom impulse response

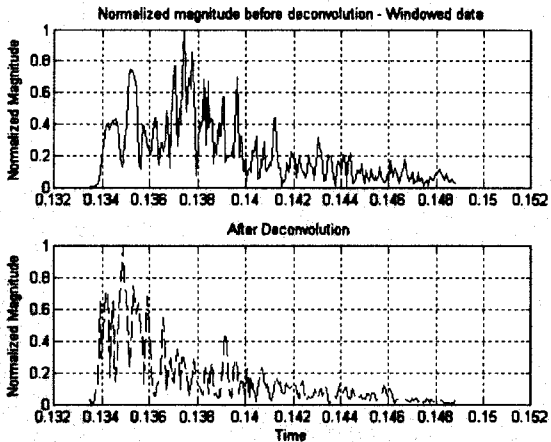


Figure 10: Normalized Magnitude of the windowed data before and after deconvolution

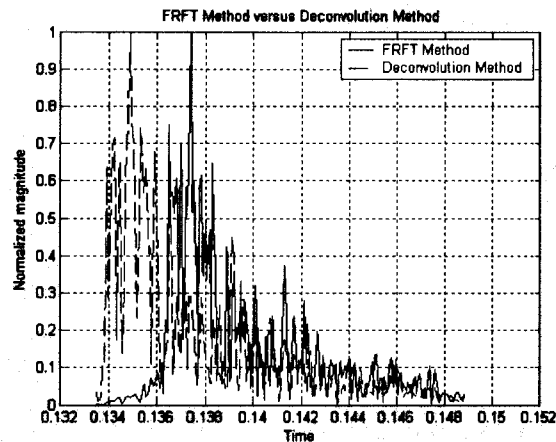


Figure 11: Comparison between Fractional Fourier Transform method (solid line) and Deconvolution method (dash line)

	Mean	Standard Deviation	Energy
Fractional Fourier Transform Method	0.2681	0.2025	10.1385
Deconvolution Method from ref 2	0.3039	0.2316	12.6609
Percent Difference [%]	11.77	12.56	19.92

Table 1: Comparison between Fractional Fourier Transform method and Deconvolution method

## 5. SUMMARY AND CONCLUSIONS

In this paper we proposed a technique for determining the bottom impulse response by using the Fractional Fourier transform that has great potential in sonar signal processing. We also presented a classical method for determining the bottom impulse response based on frequency domain deconvolution. The two methods have been tested and compared



on synthetic as well as on real sonar data. The experimental results shown demonstrate a good agreement between the two methods. Future work includes a complete statistical analysis of the obtained impulse responses for further sediment classification.

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