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Reflection of an electromagnetic plane wave with 0 or π phase shift at the surface of an absorbing medium

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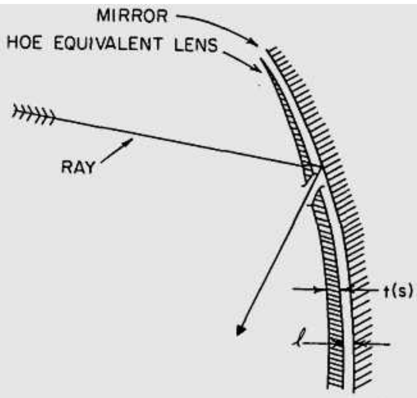

FIG. 2. Equivalent lens for a HOE on a mirror. Note: $t(s) \to 0$ and $l \to 0$.

For $\theta < \pi/2$ and very large n ($n > 10{,}000$)

$$n - \cos\theta \simeq n - \cos\theta' \simeq n - 1,$$

so Eq. (6) can be approximated by

$$(n-1)(\delta - \delta') = sin\theta - sin\theta' \qquad (7)$$

to an accuracy of 0.01%.

The surface angles δ and δ' are very small. Therefore, the relationship can be written in terms of the change in thickness t with respect to the surface coordinate s:

$$(n-1)\, dt/ds = \sin\theta - \sin\theta'. \qquad (8)$$

This is the relationship that has been compared with the grating equation, Eq. (2). Note that all of these approximations improve as n increases. In the limit ($n \to \infty$) the model is exact.

GRATING ON A MIRROR

A holographic grating on a reflecting substrate can also be represented by the equivalent lens model. For ray tracing, the grating is represented as a lens infinitesimally close to the mirror. The ray is refracted by the equivalent lens and then reflected by the mirror to the image plane (Fig. 2). Note that the lens analog is a purely mathematical construction. As such, the ray need not be refracted again after the reflection. In Fig. 2 the lens is shown with some thickness t and there is a space l between the elements. Both lengths approach zero. The drawing shows finite thickness only for clarity.

SUMMARY

Rays traced through the HOE lens equivalent can be made arbitrarily close to those traced using diffraction theory. One must simply choose a large enough index of refraction n. The lens model can also be used to represent a grating on a mirror.

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[1] W. C. Sweatt, J. Opt. Soc. Am. 67, 804 (1977).

Reflection of an electromagnetic plane wave with 0 or π phase shift at the surface of an absorbing medium

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An electromagnetic plane wave incident obliquely from a transparent medium onto the surface of an absorbing medium can be reflected with 0 or π phase shift if (i) the wave is p (TM) polarized, and (ii) the complex relative dielectric function ϵ is such that $0 \leqslant |\epsilon|^2/2\mathrm{Re}(\epsilon) \leqslant 1$. Furthermore, the locus of ϵ such that the reflection coefficient for the p polarization is real at the same angle of incidence, is a circle, and that of $\epsilon^{1/2}$ (the complex relative refractive index) is Bernoulli's lemniscate.

Except under conditions of total internal reflection, a plane electromagnetic wave experiences a "trivial" phase shift of either 0 or π upon reflection at the interface between two transparent (homogeneous and isotropic) media at all angles of incidence and for both the s(TE) and p(TM) polarizations. However, when the medium of refraction (or incidence) becomes absorbing, reflection induces a phase shift δ that is generally neither 0 or π. In fact, if surface layers (films) can be ruled out, $\delta \neq 0$ or π may be considered as a manifestation of the existence of absorption. The question arises: Can δ revert to one of its trivial values of 0 or π even though absorption is present? The answer is yes. In this Letter we determine when this happens.

The interface Fresnel reflection coefficient for the p polarization (with electric vibration parallel to the plane of incidence) can be written as[1]

$$r = (1-x)/(1+x), \qquad (1)$$

$$x = (\epsilon - \sin^2\phi)^{1/2}/\epsilon \cos\phi, \qquad (2)$$

where ϕ is the angle of incidence and ϵ is the relative dielectric function of the two media, that is, the ratio of the dielectric

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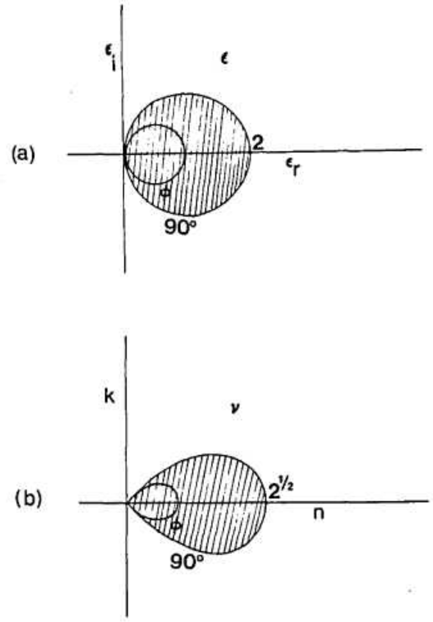

FIG. 1. (a) Domain of the relative dielectric function ϵ (hatched) in which the reflection coefficient for the p polarization r_p can be real. The locus of ϵ such that r_p is real at the same fixed angle of incidence ϕ is a circle with center ($\sin^2\phi$, 0) and radius $\sin^2\phi$. (b) Domain of the complex relative refractive index ν (hatched) in which r_p can be real. The locus of ν such that r_p is real at the same fixed angle of incidence ϕ is Bernoulli's lemniscate.

function of the medium of refraction to that of the medium of incidence.[2] For r to become real, Eq. (1) shows that x must be real. Consequently, x^2 must be real and positive, or, from Eq. (2),

$$(\epsilon - \sin^2\phi)/\epsilon^2 \cos^2\phi = \eta, \tag{3}$$

where η is a positive real number. Equation (3) can be rewritten as a quadratic in ϵ whose roots are

$$\epsilon = [1 \pm (1 - \eta \sin^2 2\phi)^{1/2}]/2\eta \cos^2\phi. \tag{4}$$

In Eq. (4) when $\eta \sin^2 2\phi \leq 1$, ϵ is a positive real number. This is the case of interfaces between transparent media. More interesting is the case when $\eta \sin^2 2\phi > 1$. Equation (4) then indicates that ϵ becomes complex, $\epsilon = (\epsilon_r, \epsilon_i)$, with real and imaginary parts given by

$$\epsilon_r = 1/2, \eta \cos^2\phi, \tag{5a}$$

$$\epsilon_i = \pm(-1 + \eta \sin^2 2\phi)^{1/2}/2\eta \cos^2\phi. \tag{5b}$$

Elimination of η between Eqs. (5a) and (5b) gives

$$\epsilon_r^2 + \epsilon_i^2 - 2\epsilon_r \sin^2\phi = 0. \tag{6}$$

Equation (6) indicates that the locus of $\epsilon = (\epsilon_r, \epsilon_i)$, such that the reflection coefficient for the p polarization at a given angle of incidence ϕ is real,[3] is a circle in the complex ϵ plane with center on the real axis, $\epsilon = (\sin^2\phi, 0)$, and radius of $\sin^2\phi$, Fig. 1(a). Varying ϕ generates coaxial circles that touch the imaginary axis at the origin, with centers on the segment of the real axis $0 \leq \epsilon \leq 1$. $\phi = 0$ (normal incidence) gives a null circle coincident with the origin ($\epsilon = 0$), while $\phi = 90°$ (grazing incidence) yields a circle with unit radius with center at (1,0). The interior and boundary of the latter circle define the domain of the ϵ plane where the reflection coefficient for the p polarization is real at a specific angle of incidence ϕ for each ϵ.

It is often convenient, particularly in the optical region of the electromagnetic spectrum, to work with refractive indices as well as dielectric functions. If we denote the relative refractive index by $\nu = (n,k)$, then $\epsilon = \nu^2$ and

$$\begin{aligned} \epsilon_r &= n^2 - k^2, \\ \epsilon_i &= 2nk. \end{aligned} \tag{7}$$

If we substitute Eqs. (7) into Eq. (6), we obtain

$$(n^2 + k^2)^2 = 2 \sin^2\phi(n^2 - k^2), \tag{8}$$

which, interestingly enough, is the standard equation of the lemniscate of Bernoulli[4] in the ν plane, Fig. 1(b). Varying ϕ between 0 and 90° generates a family of these curves whose symmetry axes are coincident with the n and k coordinate axes and with maximum dimension, $n(k = 0) = \sqrt{2} \sin\phi$, that increases from 0 to $\sqrt{2}$, respectively. The interior and boundary of the largest lemniscate (that corresponds to ϕ = 90°) defines the domain of the ν plane in which r_p can become real. Notice that in this domain $n \leq \sqrt{2}$ and $|k| < n$.

So far we have considered the p polarization only. For the s polarization, the reflection coefficient r_s is also given by Eq. (1) but with $x = (\epsilon - \sin^2\phi)^{1/2}/ \cos\phi$. In this case r_s is real when x is real, which, in turn, is satisfied only if ϵ is real. This is the uninteresting case of interfaces between transparent media.

To conclude, when the relative dielectric function ϵ, or relative refractive index ν, is complex, only the reflection coefficient for the p polarization (r_p) can become real. If ζ is defined by

$$\zeta = (\epsilon_r^2 + \epsilon_i^2)/2\epsilon_r = (n^2 + k^2)^2/2(n^2 - k^2), \tag{9}$$

then r_p is real when

$$0 \leq \zeta \leq 1, \tag{10}$$

and the angle of incidence at which this happens is given by $\phi = \sin^{-1}\zeta^{1/2}$. For a given ϕ, the locus of $\epsilon = (\epsilon_r, \epsilon_i)$ is a circle, and that of $\nu = (n,k)$ is the lemniscate of Bernoulli.

[1] See, for example, M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1975), 5th edition, p. 40.

[2] For passive or absorbing media, ϵ (and the relative refractive index ν) is limited to the first or fourth quadrant of the complex plane depending upon whether the $e^{-j\omega t}$ or $e^{j\omega t}$ time dependence of the harmonic fields is selected, respectively. By allowing ϵ (and ν) to assume values in the entire right-half plane, both choices of the time dependence are simultaneously represented.

[3] The reflection phase shift δ (arg r) is either 0 or π dependent upon the sign conventions used. If the $e^{j\omega t}$ time dependence is chosen, and the p directions in the incident and reflected beams are selected such that they are antiparallel at normal incidence (in other words, if we assume the Nebraska (Muller) conventions), then we have $\delta = \pi$.

[4] S. M. Selby, Editor, *Standard Mathematical Tables* (Chemical Rubber Company, Cleveland, 1972), 20th edition, p. 377.