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Design of film-substrate single-reflection retarders*

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The design steps for film-substrate single-reflection retarders are briefly stated and applied to the SiO₂-Si film-substrate system at wavelength 6328 Å. The criterion of minimum-maximum error of the ellipsometric angle ψ is used to choose angle-of-incidence-tunable designs. Use is made of the $(\phi-d)$ plane (angle of incidence versus thickness) to determine whether a given film-substrate system with known optical properties and film thickness can operate as a reflection retarder and to determine the associated angles of incidence and retardation angles. This leads to the concept of permissible-thickness bands and forbidden gaps for operation of a film-substrate system as a reflection retarder. Experimental measurements on one of the proposed designs proved the validity of the method.

Index Headings: Films; Reflection; Optical devices; Silicon.

Retarders are commonly constructed by use of the anisotropic property of birefringence exhibited by certain crystals (e.g., quartz) or are based on total internal reflection (the Fresnel rhomb).¹ In this paper, we consider the design of a different class of retarders, based on external reflection from a film-substrate system. Although these retarders attenuate the incident light to a certain extent, it is expected that such attenuation can be tolerated in many situations.² The film-substrate external-reflection retarder has been suggested recently³; here, we elaborate on this interesting device.⁴ These retarders should prove to be particularly useful in wavelength regions (e.g., vacuum uv) where materials of good optical quality necessary for the construction of conventional retarders are not readily available. Although we examine one film-substrate system (SiO₂-Si) as an example and assume a single reflection, the considerable amount of control provided by the choice of materials and the number of reflections will make possible designs of greatly improved performance. This subject is currently under investigation.

Operation of a given film-substrate system with known optical properties and film thickness as a reflection retarder is discussed. A simple method to determine the angles of incidence (if any) at which this mode of operation is possible, and the associated retardation angles, is given. The concept of permissible-thickness bands and forbidden gaps for operation of a film-substrate system as a reflection retarder is also presented.

The experimental results obtained on one of the proposed designs are reported. The results confirm the proposed design.

I. DESIGN PROCEDURE FOR FILM-SUBSTRATE SINGLE-REFLECTION RETARDERS

A specified value of the ellipsometric function ρ , the ratio of the complex amplitude-reflection coefficients R_p and R_s for light polarized parallel (p) and perpendicular (s) to the plane of incidence, can be realized at a given wavelength by the design of a film-substrate system with known optical constants (see Fig. 1). The design is aimed at finding the least film thickness d' and angle of incidence ϕ' at which the given system has the prespecified value of ρ , leading to a particular reflection-type optical device.³

In this section, we will outline the design steps of reflection retarders; we will discuss the choice of designs for angle-of-incidence-tunable retarders in Sec. III. We take SiO₂-Si as an example of a film-substrate system and choose the wavelength of operation 6328 Å for its wide availability. The refractive indices of SiO₂ and Si at 6328 Å are $N_1=1.46$ and $N_2=3.85-j0.02$, respectively.⁵ The design procedure is general and can be applied at other wavelengths and to other materials; it is also applicable, with minor modification,⁶ to the design of any reflection-type optical device.

We summarize the design steps for a transparent film on an absorbing-substrate single-reflection retarder:

- (1) From the desired retardation angle Δ calculate

$$\rho = e^{j\Delta} \tag{1}$$

- (2) Plot the quantity $1 - |X|$ as a function of the angle of incidence ϕ , by use of the value of ρ obtained in step 1, where X is the thickness complex-exponential function⁷

$$X = \frac{-(B - \rho E) \pm [(B - \rho E)^2 - 4(C - \rho F)(A - \rho D)]^{1/2}}{2(C - \rho F)} \tag{2}$$

$$A = r_{01p}, \quad B = r_{12p} + r_{01p} r_{01s} r_{12s}, \quad C = r_{12p} r_{01s} r_{12s},$$

$$D = r_{01s}, \quad E = r_{12s} + r_{01p} r_{12p} r_{01s}, \quad F = r_{01p} r_{12p} r_{12s},$$

and $r_{ij\nu}$ ($\nu = p, s$) are the air-film ($ij = 01$) and film-substrate ($ij = 12$) interface Fresnel reflection coefficients. Figure 2 shows such a design curve, $1 - |X|$ versus ϕ , for a chosen retardation angle $\Delta = 30^\circ$.

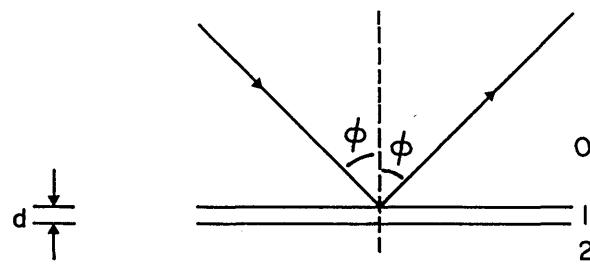


FIG. 1. The film-substrate system: ambient (0), film (1) of thickness d , and substrate (2).

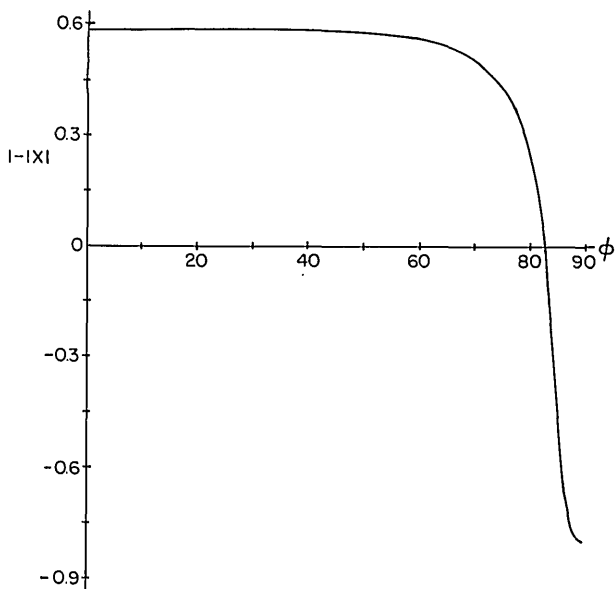


FIG. 2. Design curve, $1 - |X|$ vs ϕ , for a SiO_2 -Si film-substrate single-reflection retarder with retardation angle $\Delta = 30^\circ$ at 6328 \AA .

(3) Find the angle of incidence ϕ' at which the obtained curve intersects the ϕ axis.⁸ This angle is the required angle of incidence. For the chosen example, we have $\phi' = 82.53^\circ$.

(4) Calculate the angle (argument) α of the thickness complex-exponential function X at ϕ' , using Eq. (2). Also, check that the magnitude of X is unity.

(5) Calculate the film thickness d' necessary for the operation of the film-substrate system as an exact reflection retarder, by use of⁷

$$d' = (\alpha/2\pi) D_{\phi'} + m D_{\phi'}, \quad (3)$$

where

$$D_{\phi'} = \frac{1}{2} \lambda [N_1^2 - N_0^2 \sin^2 \phi']^{-1/2}.$$

$D_{\phi'}$ is the film-thickness period evaluated at ϕ' , and m is an integer that can be chosen to obtain the film thickness in the required range. For the retardation angle $\Delta = 30^\circ$, we found that the least film thickness equals 1084.5 \AA , and the film thickness period $D_{\phi'} = 2952.4 \text{ \AA}$.

II. DESIGN RESULTS FOR SI-SiO₂ SYSTEM AT 6328 Å

The design procedure of Sec. I has been carried out for different values of the retardation angle Δ , to cover the whole range $-180^\circ < \Delta < +180^\circ$. The angle of incidence ϕ' and the film thickness d' were obtained for each value of Δ . In addition, for each retardation angle Δ , the film-thickness period $D_{\phi'}$ and the reflectances R_p and R_s were calculated. Table I lists the results. The values of $\Delta = 0^\circ$ and $\Delta = \pm 180^\circ$ were excluded from the calculations, and are not given in Table I, because $\Delta = 0^\circ$ occurs at all film thicknesses at grazing incidence ($\phi = 90^\circ$) and $\Delta = \pm 180^\circ$ occurs at all film thicknesses at perpendicular incidence ($\phi = 0^\circ$).

Figures 3-5 represent the design results. Figure 3 gives the angle of incidence ϕ' for each retardation angle Δ . It shows two branches, one for ϕ' values associated with positive retardation angles $B_{\phi'}^+$, and the other for ϕ' values associated with negative retardation angles $B_{\phi'}^-$. From Fig. 3, it is clear that the two branches, $B_{\phi'}^+$ and $B_{\phi'}^-$, are almost mirror images of one another.⁹ Also, Fig. 3 shows that the angle of incidence ϕ' approaches 90° (0°) as the retardation angle Δ approaches 0° ($\pm 180^\circ$), as expected.

Figure 4 also shows two branches for the d' - Δ curve, namely $B_{d'}^+$ and $B_{d'}^-$, corresponding to film thicknesses d' that give positive and negative retardations Δ in the ranges $0^\circ < \Delta < +180^\circ$ and $-180^\circ < \Delta < 0^\circ$, respectively. The discontinuity of the d' - Δ curve at $\Delta = 0^\circ$ is not unexpected, because the condition, $\Delta = 0$, can be realized at any value of the film thickness. It is important to note that the two branches, $B_{d'}^+$ and $B_{d'}^-$, exist in two isolated

TABLE I. This table gives, for each value of the retardation angle Δ , the angle of incident ϕ' and the least film thickness d' necessary for operation of the SiO_2 -Si film-substrate system as an exact reflection retarder at 6328 \AA and the associated film-thickness period $D_{\phi'}$ and reflectances R_p and R_s (note that $R_p = R_s$).^a

Δ (deg)	ϕ' (deg)	d' (Å)	$D_{\phi'}$ (Å)	R_p	R_s
10	87.506	1087.9	2971.9	0.8191	0.8191
20	85.016	1086.6	2964.5	0.6728	0.6728
30	82.530	1084.5	2952.4	0.5555	0.5555
40	80.048	1081.6	2935.8	0.4621	0.4621
50	77.562	1077.9	2915.2	0.3880	0.3880
60	75.063	1073.5	2890.7	0.3291	0.3291
70	72.534	1068.5	2862.6	0.2822	0.2822
80	69.953	1062.7	2831.0	0.2448	0.2448
90	67.290	1056.3	2795.9	0.2148	0.2148
100	64.505	1049.1	2757.2	0.1907	0.1907
110	61.547	1041.1	2714.5	0.1711	0.1711
120	58.341	1032.1	2667.3	0.1551	0.1551
130	54.779	1022.0	2614.8	0.1420	0.1420
140	50.690	1010.5	2555.5	0.1311	0.1311
150	45.779	997.1	2487.4	0.1220	0.1220
160	39.441	981.0	2406.9	0.1141	0.1141
170	30.035	960.9	2306.9	0.1069	0.1069
-10	87.512	1880.1	2971.9	0.8192	0.8192
-20	85.028	1874.1	2964.5	0.6730	0.6730
-30	82.547	1864.2	2952.5	0.5558	0.5558
-40	80.070	1850.7	2936.0	0.4624	0.4624
-50	77.590	1833.8	2915.4	0.3882	0.3882
-60	75.097	1813.8	2891.0	0.3293	0.3293
-70	72.573	1790.9	2863.0	0.2825	0.2825
-80	69.997	1765.1	2831.5	0.2451	0.2451
-90	67.340	1736.6	2796.6	0.2151	0.2151
-100	64.561	1705.3	2758.0	0.1909	0.1909
-110	61.608	1670.7	2715.4	0.1713	0.1713
-120	58.407	1632.7	2668.3	0.1553	0.1553
-130	54.850	1590.3	2615.8	0.1422	0.1422
-140	50.768	1542.7	2556.6	0.1314	0.1314
-150	45.862	1488.2	2488.5	0.1222	0.1222
-160	39.528	1423.8	2408.0	0.1143	0.1143
-170	30.122	1343.9	2307.7	0.1071	0.1071

^aBy combining a number of reflections (e.g., two or three), we can use the results of this table to generate new multiple-reflection retarders that maintain the parallelism or collinearity of the incident and emergent beams.

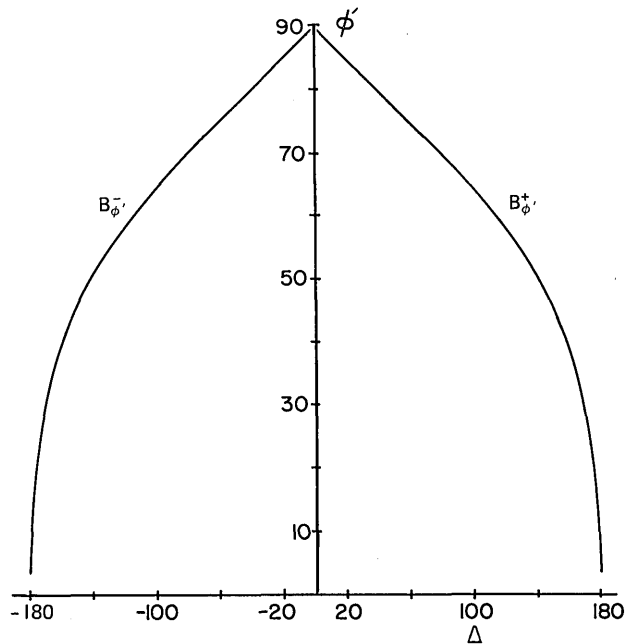


FIG. 3. Angle of incidence ϕ' (in degrees) vs retardation angle Δ (in degrees) for exact operation as reflection retarders, for Si-SiO₂ system at 6328 Å.

(nonoverlapping) thickness ranges $933 \leq d' \leq 1088$ Å and $1232 \leq d' \leq 1883$ Å, respectively.

In Fig. 5, the two branches B_{α}^+ and B_{α}^- give the reflectances $\mathcal{R}_p = \mathcal{R}_s = \mathcal{R}$ for the positive and negative ranges of Δ , respectively. Each branch is almost a mirror

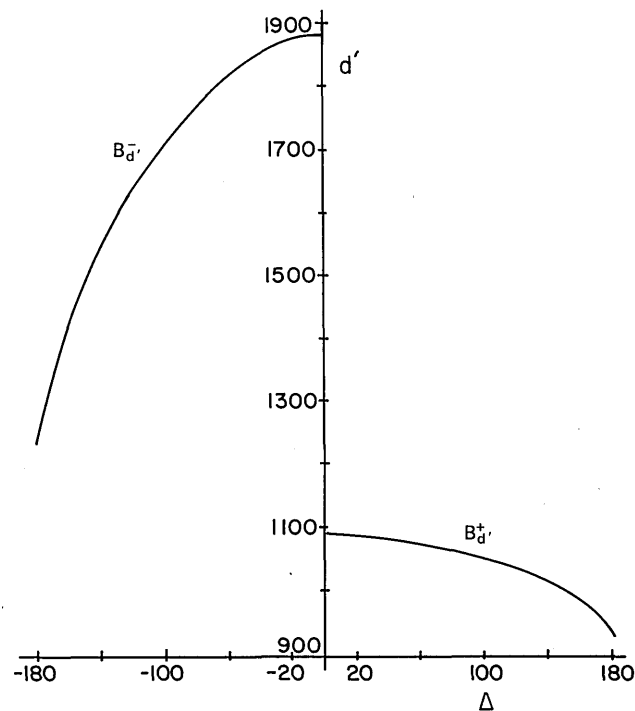


FIG. 4. Least film thickness d' (in angstroms) vs retardation angle Δ (in degrees) for exact operation as reflection retarders for Si-SiO₂ system at 6328 Å.

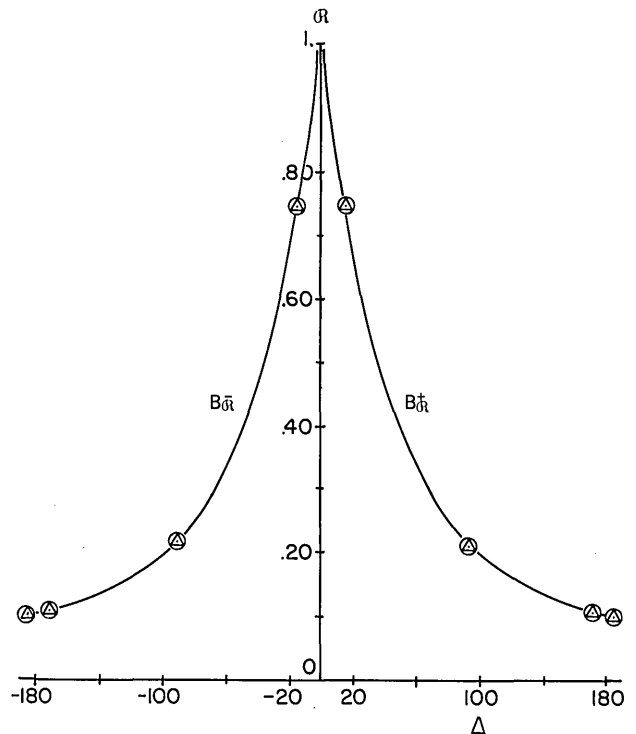


FIG. 5. The p and s reflectances, $\mathcal{R}_p(\Delta)$ and $\mathcal{R}_s(\circ)$ vs retardation angle Δ (in degrees) for exact SiO₂-Si film-substrate single-reflection retarders at 6328 Å.

image of the other. The reflectances asymptotically approach unity at $\Delta = 0^\circ$, which should take place at grazing incidence.

III. ANGLE-OF-INCIDENCE-TUNABLE RETARDERS

The difference between the smallest and largest film thicknesses required for operation of the Si-SiO₂ system as an exact reflection retarder over the range $0^\circ < \Delta < 180^\circ$ at 6328 Å is approximately 150 Å (see Fig. 4). This suggests that an angle-of-incidence-tunable retarder is feasible, by choice of one of the designs for positive retardations Δ in Table I. (Because this difference of film thickness is relatively small, an error in oxidizing the silicon wafer to the desired thickness of the SiO₂ film will only change the

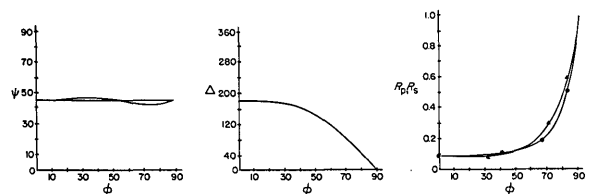


FIG. 6. Performance curves for SiO₂-Si film-substrate single-reflection retarder of thickness $d = 1010$ Å at 6328 Å. Left: ellipsometric angle ψ (in degrees) vs angle of incidence ϕ (in degrees). Middle: ellipsometric angle Δ (in degrees) vs angle of incidence ϕ (in degrees). This is the retarder's tuning curve. Right: p and s reflectances $\mathcal{R}_p(\Delta)$ and $\mathcal{R}_s(\circ)$ vs angle of incidence ϕ (in degrees), respectively.

angle of incidence ϕ' at which the system operates as an exact retarder.) In fact, any of the designs in Table I will operate as an angle-of-incidence-tunable reflection retarder with an acceptable error over a wide range of ϕ . It seems logical to consider a film whose thickness is the average ($d=1010 \text{ \AA}$) between the upper ($d=1088 \text{ \AA}$) and lower ($d=933 \text{ \AA}$) thickness limits for an angle-of-incidence-tunable film-substrate retarder. The performance curves of this design are shown in Fig. 6. The angle of incidence at which this design operates as an exact retarder is $\phi'=50.69^\circ$, with retardation angle $\Delta=140.0^\circ$.

As can be appreciated from Table I, the difference between the smallest and largest film thicknesses required for the system to operate as exact reflection retarders over the range $-180^\circ < \Delta < 0^\circ$ is relatively large, 650 \AA . Therefore, there is no suitable film thickness at which the system can operate as an angle-of-incidence-tunable retarder, over a wide range of ϕ , with a reasonable error.

By study of the behavior of all of the designs, we can select some that are more suitable than others as angle-of-incidence-tunable retarders.¹⁰ The ψ - ϕ curves for each of the designs of Table I, with $0^\circ < \Delta < +180^\circ$, are shown, collectively, in Fig. 7. In Fig. 8, only the envelope of the corresponding Δ - ϕ tuning curves is shown, because the difference of Δ between any two successive tuning curves is small. All of the curves meet at $\phi=0^\circ$ and $\phi=90^\circ$. The closeness of these tuning curves, Fig. 8, means that almost the same angle-

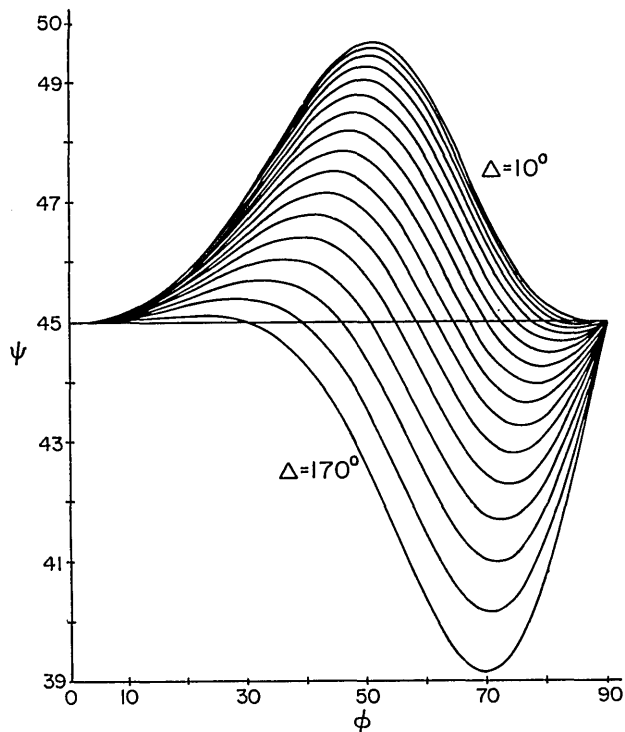


FIG. 7. Ellipsometric angle ψ (in degrees) versus the angle of incidence ϕ (in degrees) for Si-SiO₂ exact reflection retarder designs with retardation angles $\Delta=10^\circ$ - 170° with a step of 10° at 6328 \AA .

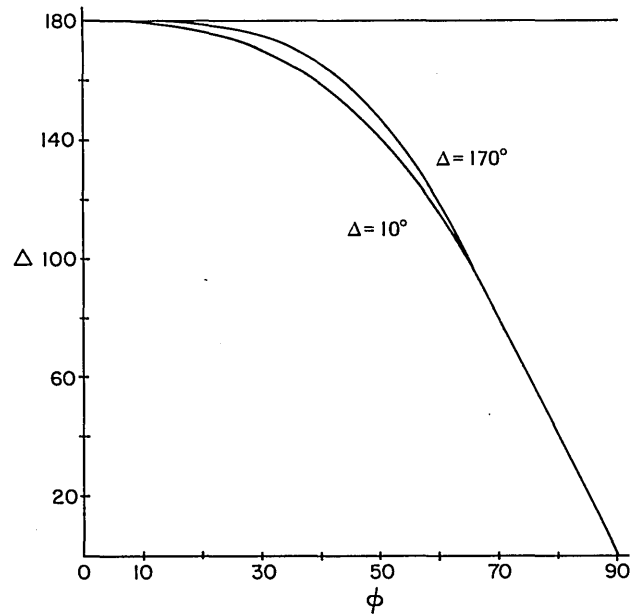


FIG. 8. The retardation angle Δ (in degrees) vs the angle of incidence ϕ (in degrees) for Si-SiO₂ exact reflection retarder designs with retardation angles $\Delta=10^\circ$ and 170° at 6328 \AA .

of-incidence-tunable retardation will be obtained for the designs listed in Table I with $0^\circ < \Delta < +180^\circ$. Therefore, a small error of film thickness can be tolerated.

The choice of an angle-of-incidence-tunable design, for a certain range of angle of incidence, can be based on the criterion of minimum-maximum error (MME) of the ellipsometric angle ψ . This error ($\psi - 45^\circ$) should not exceed a prespecified value. [ψ is related to the ratio of the p and s reflectances R_p and R_s by $\psi = \tan^{-1}(R_p/R_s)^{1/2}$]. We determined the film thickness values that lead to ψ -MME over the angle of incidence ranges 0° - 45° , 45° - 90° , and 0° - 90° . The results are summarized in Table II. The performance curves of the designs of Table II are shown in Figs. 9 and 10. Figure 9 shows, from left to right, the ψ - ϕ curve, the Δ - ϕ tuning curve, and the reflectances R_p - and R_s - ϕ curves for $d=981.01 \text{ \AA}$, respectively. Figure 10 shows the corresponding curves for $d=1041.1 \text{ \AA}$.

Other criteria for screening the angle-of-incidence-tunable retarder designs such as the rate of change of the ellipsometric angles ψ and Δ with respect to the film thickness d and angle of incidence ϕ , $\partial\psi/\partial d$, $\partial\Delta/\partial d$;

TABLE II. Angle-of-incidence-tunable Si-SiO₂ retarders at 6328 \AA that achieve minimum-maximum error of ψ over selected angle-of-incidence ranges 0° - 45° , 45° - 90° , and 0° - 90° .

range of ϕ (degrees)	d' (angstroms)	maximum error in ψ ($\psi - 45^\circ$) (degrees)	ϕ' (degrees)
0-45	981.01	0.61055	39.441
45-90	1041.1	2.13	61.547
0-90	1041.1	2.16	61.547

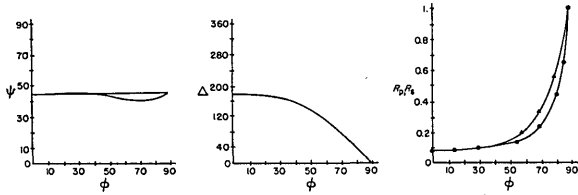


FIG. 9. Performance curves, as in Fig. 6, but for $d=981.01 \text{ \AA}$.

$\partial\psi/\partial\phi$, and $\partial\Delta/\partial\phi$, respectively, are being investigated. Also under investigation is the effect of other choices of film and substrate materials as well as the use of two or three reflections to maintain the parallelism or collinearity between the light beams incident on and emergent from the retarder.

IV. OPERATION OF A GIVEN FILM-SUBSTRATE SYSTEM WITH KNOWN OPTICAL CONSTANTS AND FILM THICKNESS AS AN EXACT REFLECTION RETARDER

The unit circle U in the complex ρ plane is the locus of all possible reflection retarders for any film-substrate system. If the constant-thickness contour (CTC) for a given value of film thickness intersects the unit circle U , then at the angles of incidence at the intersection points the given film-substrate system will operate as an exact reflection retarder with different retardations. Figure 11(right) shows the unit circle U and the CTC's A and B for film thicknesses $d=18700 \text{ \AA}$ and $d=3000 \text{ \AA}$, respectively, for the Si-SiO₂ system at 6328 \AA .

Instead of plotting the CTC in the complex- ρ plane and determining its intersection points with the unit circle U , we can easily obtain the possible angles of incidence ϕ for operation as an exact reflection retarder and the associated retardation angles Δ , by use of the ϕ - d plane, Fig. 11(left). First, we draw the images of both the unit circle U and the CTC in the ϕ - d plane. From their intersection points, we find the values of ϕ . By straightforward calculations, we get the values of Δ . If the two images do not intersect one another, the exact reflection-retardation mode can not be associated with this film thickness.

To get the image of the unit circle U in the ϕ - d plane, U' , we calculate the values of ϕ' and d' over the ranges $0^\circ < \Delta < +180^\circ$ and $-180^\circ < \Delta < 0^\circ$, and plot the results in the ϕ - d plane. (Such calculations have already been made for the Si-SiO₂ system in Sec. II at 6328 \AA .) In Fig. 11(left) the image of the upper half of the unit

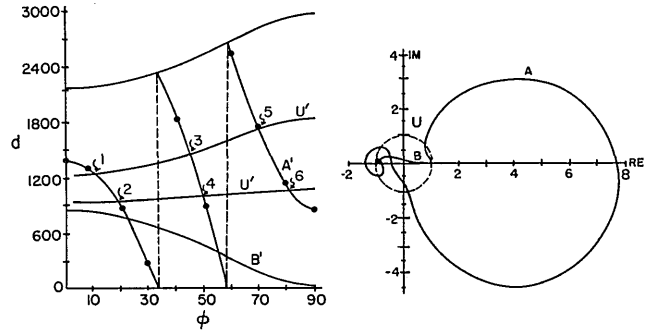


FIG. 11. (Right) The CTC's A and B for film thicknesses $d=18700 \text{ \AA}$ and $d=3000 \text{ \AA}$, respectively, in the complex- ρ plane for Si-SiO₂ system at 6328 \AA . Curve U , the unit circle, is the locus of all possible reflection retarders. (Left) The images A' , B' , and U' in the ϕ - d plane of the CTC's A and B , and the unit circle U of the complex- ρ plane, respectively, for Si-SiO₂ system at 6328 \AA .

circle U is the bottom branch of U' , and the image of the lower half is the upper branch. Because the vertical lines at $\phi=0^\circ$ and $\phi=90^\circ$ in the ϕ - d plane are the images of only two points on the unit circle U , $\rho=-1$ and $\rho=+1$, respectively, the image of the entire unit circle

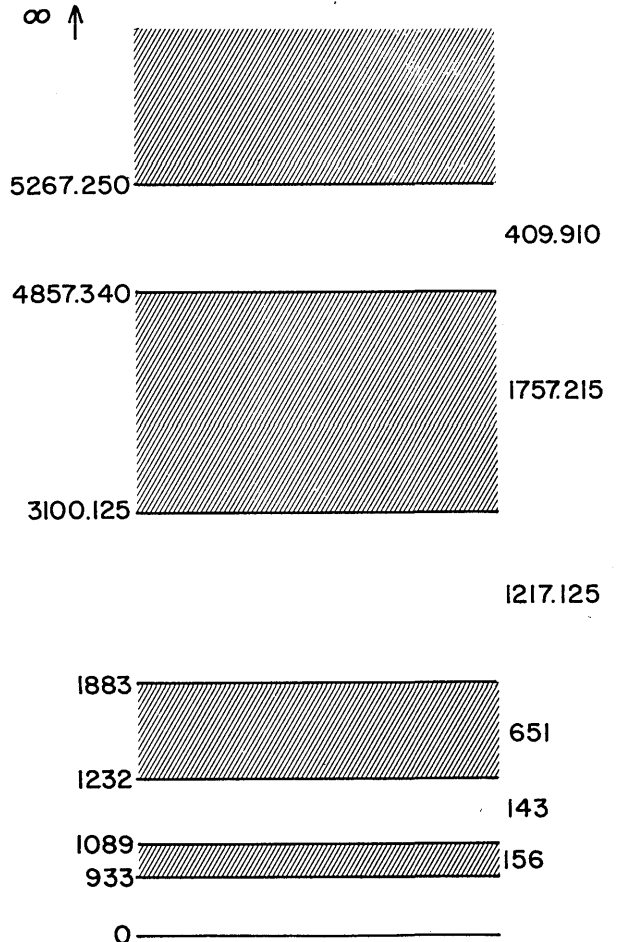


FIG. 12. Permissible-thickness bands and forbidden-thickness gaps for operation of the Si-SiO₂ system as an exact reflection retarder at 6328 \AA .

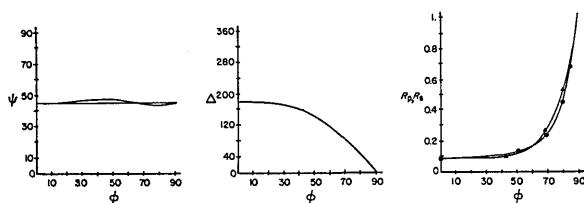


FIG. 10. Performance curves, as in Fig. 6, but for $d=1041.1 \text{ \AA}$.

in the ρ plane is a closed curve U' in the ϕ - d plane.

The image of any CTC in the reduced thickness range $0 \leq d < D_0$ is obtained very simply by subtracting from the given film thickness $d = \text{const}$ (assumed greater than D_0) the appropriate multiple of the film-thickness period D_0 , at each angle of incidence ϕ in the range $0^\circ \leq \phi \leq 90^\circ$. Curve A' in Fig. 11(left) is the image of the CTC A in Fig. 11(right), $d = 18700 \text{ \AA}$. The images U' and A' of both the unit circle U and the CTC A intersect one another at the points U_1, U_2, U_3, U_4, U_5 , and U_6 . The corresponding angles of incidence¹¹ are $\phi'_1 = 9.813^\circ$, $\phi'_2 = 18.375^\circ$, $\phi'_3 = 43.875^\circ$, $\phi'_4 = 48.75^\circ$, $\phi'_5 = 69.375^\circ$, and $\phi'_6 = 80.125^\circ$. From these angles of incidence and the given film thickness ($d = 18700 \text{ \AA}$) we can get the associated retardation angle and the p and s reflectances by direct calculation. This gives the retardation angles $\Delta_1 = -179.15^\circ$, $\Delta_2 = 176.78^\circ$, $\Delta_3 = -153.59^\circ$, $\Delta_4 = 144.13^\circ$, $\Delta_5 = -82.37^\circ$, and $\Delta_6 = 39.69^\circ$ and the reflectances \mathcal{R} ($\mathcal{R}_p = \mathcal{R}_s = \mathcal{R}$) are $\mathcal{R}_1 = 0.100$, $\mathcal{R}_2 = 0.102$, $\mathcal{R}_3 = 0.119$, $\mathcal{R}_4 = 0.127$, $\mathcal{R}_5 = 0.237$, and $\mathcal{R}_6 = 0.465$.

The CTC for $d = 3000 \text{ \AA}$ and its image, curves B and B' in Fig. 11, represent the case where the CTC and the unit circle (B and U in the complex ρ plane) and consequently their images (B' and U' in the ϕ - d plane) do not intersect one another.¹² Therefore, at this film thickness, the system cannot operate as an exact reflection retarder at any angle of incidence. This implies the existence of ranges of film thickness for which the mode of operation as an exact reflection retarder is possible, separated by gaps for which it is not. For the Si-SiO₂ system at 6328 \AA the permissible-thickness bands are shown in Table III in addition to the sign of the possible retardation angles Δ in each band. Table IV gives the forbidden-thickness gaps and their widths. These tables are obtained by adding the film thickness periods $D_{0^0} = 2167.125 \text{ \AA}$ and $D_{90^0} = 2974.340 \text{ \AA}$ to the corresponding minimum $d_{\text{min}}^- = 1232 \text{ \AA}$ and $d_{\text{min}}^+ = 933 \text{ \AA}$ (at $\phi = 0^\circ$) and maximum $d_{\text{max}}^- = 1883 \text{ \AA}$ and $d_{\text{max}}^+ = 1088 \text{ \AA}$ (at $\phi = 90^\circ$) least thickness for exact retarder operation with negative and positive retardations, respectively.¹³ By use of Table III or IV, we can immediately determine if at a certain film thickness the film-substrate system can operate as an exact reflection retarder, or not. Figure 12 shows a schematic diagram of the permissible-thickness bands and the forbidden-thickness gaps.

TABLE III. Permissible-thickness bands and their widths (in angstroms) and the retardation sign for operation of the Si-SiO₂ system as an exact reflection retarder at 6328 \AA .

d_{min} (Å)	d_{max} (Å)	bandwidth (Å)	retardation sign
933	1089	156	+
1232	1883	651	-
3100.125	3399.125	299	+
3399.125	4063.340	664.215	±
4063.340	4857.340	794	-
5267.250	5566.250	299	+
5566.250	7037.680	1471.430	±
7037.680	7434.375	396.695	-
7434.375	∞	∞	±

TABLE IV. Forbidden-thickness gaps and their bandwidths (in angstroms) for operation of the Si-SiO₂ system as an exact reflection retarder at 6328 \AA .

d_{min} (Å)	d_{max} (Å)	bandwidth (Å)
0	933	933
1089.000	1232.000	143.000
1883.000	3100.125	1217.125
4857.340	5267.250	409.910

V. EXPERIMENTAL

To verify the operation of one design of a film-substrate reflection retarder, the ellipsometric angles ψ and Δ were measured with a conventional ellipsometer, by use of a commercial oxidized silicon wafer of a nominal film thickness $d = 1000 \text{ \AA}$ at wavelength 6328 \AA . On the basis of the published data⁵ for the optical constants of Si and SiO₂, and the nominal film thickness, the system was expected to work as a reflection retarder with retardation angle $\Delta \approx 150^\circ$ at $\phi \approx 45.78^\circ$ (for exact operation the film thickness should be $d = 997 \text{ \AA}$), see Table I. From our measurements on this Si-SiO₂ system at $\lambda = 6328 \text{ \AA}$, we found, at $\phi = 46.5^\circ$, that $\psi = 46.916^\circ$ and $\Delta = 149.974^\circ$. The difference between $\tan\psi$ and unity is 0.068, which is of the same order of magnitude as the relative transmittance of a typical compensator used in ellipsometers. The error of Δ (0.026°) is also reasonable. The measurement thus confirms the suggested design.

VI. COMPUTER PROGRAMS

Computer programs written in Fortran IV Language for carrying out the design procedure and obtaining the performance data are available from the authors.

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¹J. M. Bennett and H. E. Bennett, in *Handbook of Optics*, edited by W. G. Driscoll and W. Vaughan (McGraw-Hill, New York, 1975).

²The magnitude of the attenuation is independent of the incident polarization.

³R. M. A. Azzam, A.-R. M. Zaghloul, and N. M. Bashara, *J. Opt. Soc. Am.* 65, 252 (1975).

⁴Note that, whereas the use of a film-substrate system as a reflection retarder is new, its use as a reflection polarizer is well known. See, for example, M. Ruiz-Urbieta and E. M. Sparrow, *J. Opt. Soc. Am.* 62, 1188 (1972) and other papers in this series.

⁵*Ellipsometric Tables of the Si-SiO₂ System for Mercury and He-Ne Laser Spectral Lines*, edited by G. Gergely (Akademical Kiado, Budapest, 1971).

⁶In step 1, instead of using $\rho = e^{j\Delta}$, we use the general form $\rho = \tan\psi e^{j\Delta}$; steps 2-5 remain unchanged.

⁷The derivation of Eqs. (2) and (3) is given in Ref. 3.

⁸Alternatively, the angle of incidence ϕ' can be obtained by any numerical method (e.g., successive bisection) to find the root of the equation $|X| = 1$.

⁹Our investigation of the effect of substrate absorption on the

results of the film-substrate single-reflection retarder designs has shown that the mirror symmetry of the two branches B^+ and B^- occurs exactly only when the substrate is totally transparent. Deviation from exact symmetry increases with substrate absorption.

¹⁰Angle-of-incidence-tunable retarders are those with the least thickness. By adding multiples of D_0 , the performance of the retarder gets worse, refer to Fig. 11(left).

¹¹An alternative procedure would be to add the appropriate mul-

tiples of D_0 , at each ϕ , to the image of the unit circle and to obtain the intersection points of this vertically translated image with the straight line $d = \text{const}$. In general, the first method is more convenient.

¹²The existence of exact reflection-retardation modes depends, for a particular system at a given wavelength, on the film thickness only.

¹³ d_{\min} and d_{\max} are obtained by extrapolation to $\Delta = 0^\circ$ and $\Delta = \pm 180^\circ$ of the branches shown in Fig. 4.

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