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Optimal beam splitters for the division-of-amplitude photopolarimeter

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Optimal optical parameters of the beam splitter that is used in the division-of-amplitude photopolarimeter are determined. These are (1) 50%–50% split ratio of the all-dielectric beam splitter, (2) differential phase shifts in reflection and transmission Δ_r and Δ_t that differ by $\pm\pi/2$, and (3) ellipsometric parameters $(\psi_r, \psi_t) = (27.368°, 62.632°)$ or $(62.632°, 27.368°)$. It is also shown that for any nonabsorbing beam splitter that splits incident unpolarized light equally, the relationship $\psi_r + \psi_t = \pi/2$ is always satisfied. © 2003 Optical Society of America

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The most general state of (partial elliptical) polarization of quasi-monochromatic light is described by the four Stokes parameters S_0, S_1, S_2, and S_3. These parameters are often lumped in a 4×1 Stokes vector

$$\mathbf{S} = [S_0 \ \ S_1 \ \ S_2 \ \ S_3]^{\mathrm{T}}, \tag{1}$$

where T indicates the matrix transpose. Numerous instruments (optical polarimeters) have been devised[1–3] for measuring some or all of the components of **S**.

In the division-of-amplitude photopolarimeter[4,5] (DOAP), shown in Fig. 1, the incident light beam (i) whose Stokes parameters are to be measured is split into four separate beams by use of a beam splitter BS and two Wollaston prisms WP1 and WP2 or equivalent polarizing beam splitters. Linear detection of the light fluxes of the four component beams by photodetectors D_0, D_1, D_2, and D_3 yields an electrical output signal vector,

$$\mathbf{I} = [i_0 \ \ i_1 \ \ i_2 \ \ i_3]^{\mathrm{T}}, \tag{2}$$

which is linearly related to the Stokes vector **S** of incident light by

$$\mathbf{I} = \mathbf{AS}. \tag{3}$$

From Eq. (3) the unknown polarization vector **S** is completely recovered from the measured signal vector **I** by

$$\mathbf{S} = \mathbf{A}^{-1}\mathbf{I} \tag{4}$$

provided that the instrument matrix **A** (which is usually determined by calibration) is nonsingular, so that its inverse $\mathbf{A}^{-1}$ exists. Several DOAP instruments have been constructed and applied to reflection and scattering Mueller-matrix ellipsometry.[6–10] Because the DOAP generates four linearly independent projections of the unknown Stokes vector of incident light simultaneously (via four independent parallel channels), it is capable of complete and fast (time-resolved) measurement of light polarization or scattering matrices under dynamic conditions.

When the Wollaston prisms are oriented to filter the orthogonal linear polarization components of the reflected (r) and transmitted (t) light along the bisectors of the p and s reference coordinate axes, parallel and perpendicular to the plane of incidence at the beam splitter, respectively, the determinant of the instrument matrix becomes[5]

$$\det \mathbf{A} = (RT)^2 \sin 2\psi_r \sin 2\psi_t (\cos 2\psi_r - \cos 2\psi_t) \sin(\Delta_r - \Delta_t). \tag{5}$$

In Eq. (5), R and T are the intensity reflectance and transmittance of the BS for incident unpolarized light, and (ψ_r, Δ_r) and (ψ_t, Δ_t) are the ellipsometric parameters[11] of the beam splitter in reflection and transmission, respectively. A constant premultiplier, proportional to the product of the photoelectric sensitivities of the four detectors, has been dropped from the right-hand side of Eq. (5).

Det $\mathbf{A} \neq 0$, and the instrument matrix is nonsingular, if all of the following conditions

$$\psi_r, \ \ \psi_t \neq 0 \ \ \text{or} \ \ \pi/2, \tag{6a}$$

$$\psi_r \neq \psi_t, \tag{6b}$$

$$\Delta_r - \Delta_t \neq 0 \ \ \text{or} \ \ \pi, \tag{6c}$$

are satisfied simultaneously. Condition (6a) indicates that the beam splitter should not be polarizing in reflection or transmission; condition (6b) is identically satisfied if the throughputs of the beam splitter for the p and s polarizations are unequal; and condition (6c) embodies the one essential polarimetric requirement concerning the phase shifts introduced by the beam splitter in reflection and transmission.[12]

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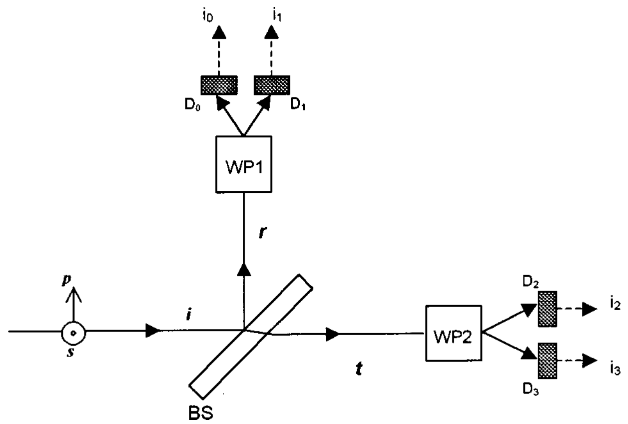

Fig. 1. Schematic depiction of the DOAP. BS, the beam splitter to be optimized; WP1 and WP2, Wollaston prisms; D_0, D_1, D_2, and D_3, linear photodetectors that produce output electrical signals i_0, i_1, i_2, and i_3, respectively. p and s, the linear polarization directions parallel and perpendicular, respectively, to the plane of incidence at the beam splitter.

Whereas the singularity conditions[5] follow readily from Eq. (5), beam-splitter parameters for optimum performance of the DOAP are not apparent. In this communication it is our objective to determine a set of optimum conditions.

Optimum optical parameters for the beam splitter are defined as those that make the instrument matrix as far from singular as possible,[13] by maximizing the absolute value of every term in the determinant of Eq. (5).

First, the $\sin(\Delta_r - \Delta_t)$ term in Eq. (5) has maximum absolute value of 1 when

$$\Delta_r - \Delta_t = \pm\pi/2. \tag{7}$$

Next, for a nonabsorbing, all-dielectric beam splitter,

$$T = 1 - R, \tag{8}$$

and the first term on the right-hand side of Eq. (5),

$$P = (RT)^2 = R^2(1 - R)^2, \tag{9}$$

is maximum when the derivative $\mathrm{d}P/\mathrm{d}R = 2R(1 - 3R + 2R^2) = 0$, which gives

$$R = T = 0.5. \tag{10}$$

(The other two roots, $R = 0$ and $R = 1$, are obviously unacceptable.) This proves that a 50%–50% beam splitter is optimal from a polarimetric point of view, as one might intuitively expect. The maximum value of $(RT)^2 = 1/16$.

Third, we consider the psi factor in Eq. (5), namely,

$$\begin{aligned} Q &= \sin 2\psi_r \sin 2\psi_t(\cos 2\psi_r - \cos 2\psi_t) \\ &= \tfrac{1}{2}(\sin 4\psi_r \sin 2\psi_t - \sin 2\psi_r \sin 4\psi_t). \end{aligned} \tag{11}$$

Figure 2(a) shows a three-dimensional plot of Q as a function of ψ_r and ψ_t over the entire range of values of these two parameters. Figure 2(b) shows a family of constant-Q contours in the ψ_r, ψ_t plane. This figure illustrates clearly the $Q = 0$ singularity conditions (6a) and (6b). Optimum values of ψ_r and ψ_t correspond to the location of the positive peak and negative trough of Q. The exact optimum values are determined by solving the following simultaneous equations:

$$\partial Q/\partial\psi_r = 0 = 2\cos 4\psi_r \sin 2\psi_t - \cos 2\psi_r \sin 4\psi_t \tag{12a}$$

$$\partial Q/\partial\psi_t = 0 = \sin 4\psi_r \cos 2\psi_t - 2\sin 2\psi_r \cos 4\psi_t. \tag{12b}$$

Equations (12a) and (12b) are replaced by the following set of equivalent conditions:

$$\cos 4\psi_r = \cos 4\psi_t, \tag{13a}$$

$$\cos 4\psi_r = \cos 2\psi_t \cos 2\psi_r, \tag{13b}$$

as is obtained by factoring out, and dropping, $2\sin 2\psi_t$ and $2\sin 2\psi_r$ [which are nonzero according to condition (6a)] from the right-hand sides of Eqs. (12a) and (12b), respectively.

From Eq. (13a) we get[14]

$$\psi_r + \psi_t = \pi/2. \tag{14}$$

And from Eqs. (13b) and (14) we find that

$$\cos^2 2\psi_r = 1/3. \tag{15}$$

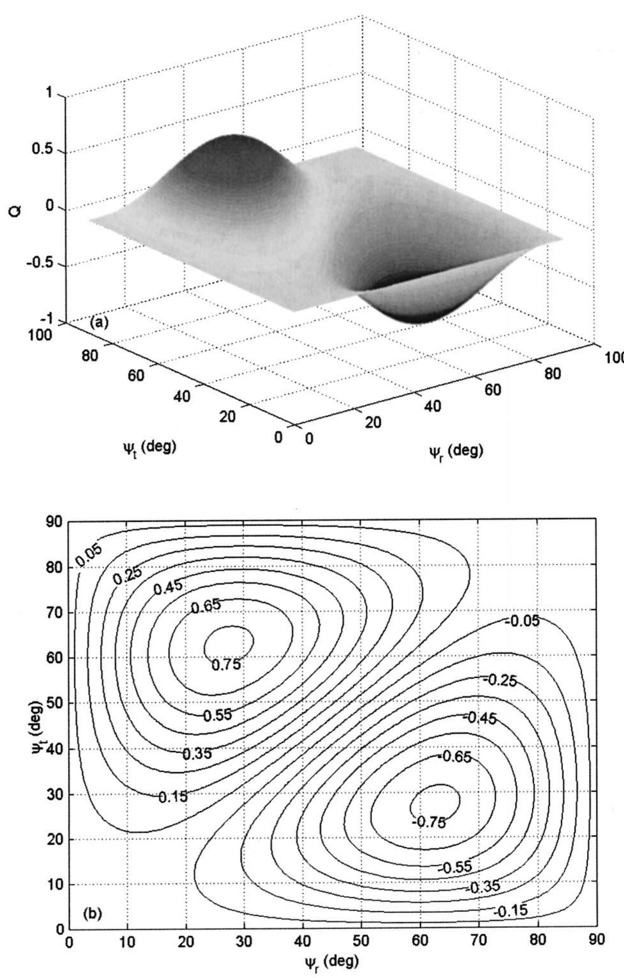

Fig. 2. (a) Three-dimensional plot of Q [Eq. (11)] as a function of ψ_r and ψ_t. (b) Family of constant-Q contours in the ψ_r ψ_t plane. Note that $Q(\psi_r, \psi_t) = -Q(\psi_t, \psi_r)$, a condition that follows directly from Eq. (11).

Finally, Eqs. (14) and (15) yield the desired optimum parameters

$$(\psi_r, \psi_t) = (27.368°, 62.632°) \text{ and } (62.632°, 27.368°), \quad (16)$$

in agreement with Fig. (2b). From Eqs. (11) and (16), the maximum value of Q is given by

$$Q_{\max} = 4/3\sqrt{3}) = 0.7698, \quad (17)$$

and the minimum $= -Q_{\max}$. Figure 3 shows a close-up view of the constant-Q contours in the immediate neighborhood of the maximum. This completes the determination of the optimal parameters of the beam splitter to be used in the DOAP.

In the course of this work we arrived at the following interesting and somewhat general result: For any nonabsorbing, all-dielectric, beam splitter that splits incident unpolarized light equally, the relationship $\psi_r + \psi_t = \pi/2$ [Eq. (14)] is always satisfied. To prove this, write

$$\tan^2 \psi_t = T_p/T_s, \quad (18a)$$

$$\tan^2 \psi_r = R_p/R_s = (1 - T_p)/(1 - T_s), \quad (18b)$$

where T_p, T_s and R_p, R_s are the intensity transmittances and reflectances of the beam splitter for incident p- and s-polarized light, respectively. Equations (18a) and (18b) are solved for T_p and T_s in terms of $\tan^2 \psi_r$ and $\tan^2 \psi_t$, and subsequently we set

$$T = (T_p + T_s)/2 = 1/2. \quad (19)$$

This gives

$$\tan^2 \psi_r \tan^2 \psi_t = 1. \quad (20)$$

Because ψ_r and ψ_t are nonnegative (by definition[11]), Eq. (14) immediately follows from Eq. (20).

The DOAP optimization analysis presented here is similar to the approach that was applied previously to optimize the four-detector polarimeter[15–17] (FDP). (The FDP uses partially reflective photodetectors at oblique incidence and a nonplanar light path to measure the Stokes vector **S**.) Hence it is not surprising that the quarter-wave differential phase shift [Eq. (7) here and Eq. (27) of Ref. 16] is found to be optimum in both cases. Similarly, the optimum ψ_r and ψ_t of Eq. (15) for the DOAP beam splitter are identical to the optimum ψ_0 values of the first detector (D_0) of the FDP.[18]

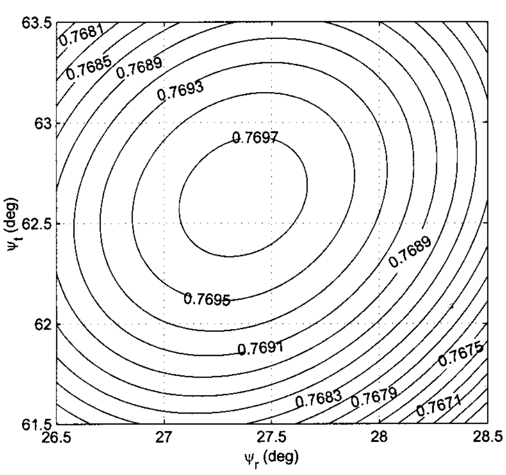

Fig. 3. Close-up view of the constant-Q contours in the immediate neighborhood of the maximum.

Earlier polarimeter optimization studies[6,9,19,20] employed other criteria for optimality; e.g., the minimization of variously defined condition numbers of the instrument matrix.[21] Our analytical approach, which is based on the maximum absolute value of the normalized determinant of the polarimeter instrument matrix, is simple, transparent, and less computationally intensive than those previous techniques.

In summary, we have determined optimal optical parameters of the beam splitter used for the DOAP. Specific thin-film beam splitters that achieve all of the optimum conditions discussed above simultaneously were presented recently.[22]

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12. For a thick uncoated dielectric slab BS, Δ_r and Δ_t assume the trivial values of 0 or $\pm\pi$, and condition (6c) is violated; hence $\det \mathbf{A} = 0$. Therefore thin-film coatings are required for the BS of DOAP.
13. According to Eq. (4), a given error $\delta\mathbf{I}$ in the measured signal vector $\mathbf{I}$ results in a corresponding error $\delta\mathbf{S}$ of the derived Stokes vector $\mathbf{S}$, which is given by $\delta\mathbf{S} = \mathbf{A}^{-1}\delta\mathbf{I}$. Because $\mathbf{A}^{-1}$ is proportional to $1/\det \mathbf{A}$, maximization of $|\det \mathbf{A}|$ is consistent with reduction of the error $\delta\mathbf{S}$.
14. Equation (13a) has an infinite number of solutions, and it is by convention[11] that the ellipsometric angle ψ is confined to the first quadrant, $0 \leq \psi \leq \pi/2$.
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