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Angular range for reflection of p-polarized light at the surface of an absorbing medium with reflectance below that at normal incidence

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The range of incidence angle, $0 < \varphi < \varphi_e$, over which *p*-polarized light is reflected at interfaces between transparent and absorbing media with reflectance below that at normal incidence is determined. Contours of constant φ_e in the complex plane of the relative dielectric constant ε are presented. A method for determining the real and imaginary parts of the complex refractive index, $\varepsilon^{1/2} = n + jk$, which is based on measuring φ_e and the pseudo-Brewster angle φ_{pB}, is viable in the domain of fractional optical constants, $n, k < 1$. © 2002 Optical Society of America

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1. INTRODUCTION

The reflection of collimated monochromatic *p*-polarized light at the planar interface between a transparent medium of incidence of real dielectric constant ε_1 and an absorbing medium of refraction of complex dielectric constant ε_2 is governed by the Fresnel reflection coefficient[1]

$$r_p = [\varepsilon \cos \varphi - (\varepsilon - \sin^2 \varphi)^{1/2}]/[\varepsilon \cos \varphi + (\varepsilon - \sin^2 \varphi)^{1/2}], \tag{1}$$

where

$$\varepsilon = \varepsilon_2/\varepsilon_1 \tag{2}$$

and φ is the angle of incidence; see Fig. 1. For a given complex ε, the absolute reflectance $r_p r_p^*$ initially decreases as φ increases from 0, reaches a minimum at the pseudo-Brewster angle φ_{pB}, and then increases monotonically from minimum reflectance to 1 as φ increases from φ_{pB} to 90°. Explicit solutions for φ_{pB} for a given complex ε have been derived by several authors.[2–4] Azzam and Ugbo[5] also determined analytically the contours of constant φ_{pB} in the complex ε plane.

In this paper we are interested in the angular range $0 < \varphi < \varphi_e$ over which the reflectance for *p*-polarized light at oblique incidence is less than that at normal incidence. The upper limit φ_e, which lies between φ_{pB} and 90°, is determined by equating the oblique and normal-incidence reflectances, i.e.,

$$r_p(\varphi) r_p^*(\varphi) = r_p(0) r_p^*(0). \tag{3}$$

For the special case of an interface between two transparent media (ε real and >0), the minimum reflectance is zero, φ_{pB} reverts to the usual Brewster angle $\varphi_B = \arctan \varepsilon^{1/2}$, and Eq. (3) has an explicit solution $\varphi = \varphi_e$ given by[6]

$$\tan \varphi_e = (\varepsilon^2 + \varepsilon)^{1/2}. \tag{4}$$

Another interesting conclusion from Ref. 6 is that the difference $\varphi_e - \varphi_B$ reaches a maximum of 13.9852° when $\varepsilon = 3.6135$, and $\varphi_B = 62.2528°$.

For the general case of an absorbing medium of refraction (complex ε), no analytical solution exists for Eqs. (1) and (3), and φ_e must be determined numerically. Our approach in this paper is to determine all possible values of $\varphi_e - \varphi_{pB}$ that are consistent with a given φ_{pB} (Section 2). The maximum difference $(\varphi_e - \varphi_{pB})_{max} = 20.447°$ occurs in the limit when ε is real negative, and $\varphi_{pB} \approx 44°$. We also determine the constant-φ_e contours in the complex planes of ε and $\varepsilon^{1/2}$ ($= n + jk$, the relative complex refractive index) in Section 3. In Section 4, we propose a technique for determining n and k, which is based on measuring the two angles φ_{pB} and φ_e.

2. ANGULAR RANGE $\varphi_e - \varphi_{pB}$ FOR SPECIFIED PSEUDO-BREWSTER ANGLE φ_{pB}

All possible values of complex $\varepsilon = |\varepsilon| \exp(j\theta)$, that are consistent with a given φ_{pB} are determined by[5]

$$|\varepsilon| = \iota \cos(\zeta/3), \tag{5}$$

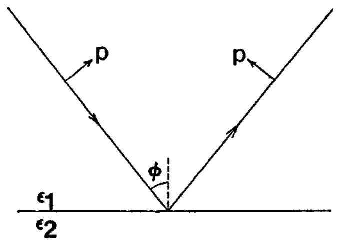

Fig. 1. Reflection of p-polarized light at an angle φ by the planar interface of two media with dielectric constants ε_1 and ε_2.

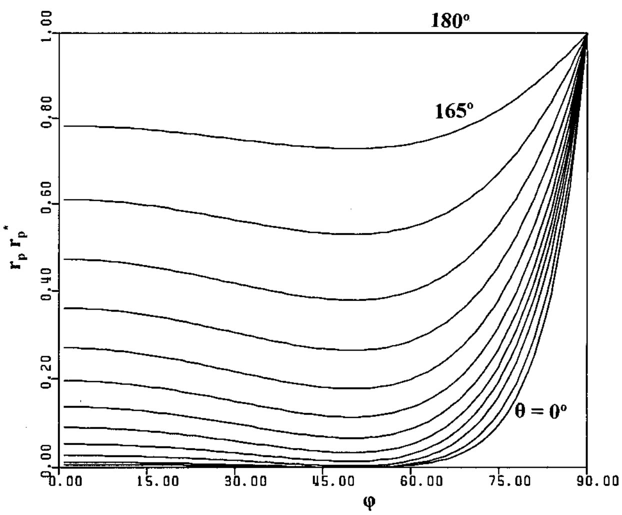

Fig. 2. Family of reflectance-versus-angle ($r_p r_p^*$-versus-φ) curves that share the same pseudo-Brewster angle $\varphi_{pB} = 50°$. The associated values of complex $\varepsilon = |\varepsilon|\exp(j\theta)$ are obtained from Eqs. (5)–(7) by allowing θ to assume values from 0 to 180° in steps of 15°.

where

$$\iota = 2\tan^2\varphi_{pB}(1 - \tfrac{2}{3}\sin^2\varphi_{pB})^{1/2}, \quad (6)$$

$$\zeta = \arccos[-\cos\theta\cos^2\varphi_{pB}(1 - \tfrac{2}{3}\sin^2\varphi_{pB})^{-3/2}], \quad (7)$$

by scanning θ from 0 to 180°. Constant-φ_{pB} contours in the complex ε plane were presented in Ref. 5 based on Eqs. (5)–(7).

Figure 2 shows a family of reflectance-versus-angle ($r_p r_p^*$-versus-φ) curves for 13 values of complex ε that share the same pseudo-Brewster angle $\varphi_{pB} = 50°$, as obtained by allowing θ to assume values from 0 to 180° in steps of 15°. Both the normal-incidence reflectance and the minimum reflectance at $\varphi_{pB} = 50°$ increase monotonically with θ. In the limit of $\theta = 180°$ (i.e., ε is real negative), the reflectance is total (=1) at all angles.

Figure 3 shows the minimum reflectance, $(r_p r_p^*)_{\min}$, as a function of θ for constant values of φ_{pB} from 5° to 80° in steps of 5°. For small pseudo-Brewster angles (5° to 15°), an initial steep rise of the minimum reflectance with θ is followed by a more gradual increase toward 1. For $\varphi_{pB} > 30°$, the increase of minimum reflectance with θ appears parabolic and is nearly independent of φ_{pB}.

Figure 4 shows $\varphi_e - \varphi_{pB}$ as a function of θ for constant φ_{pB} from 5° to 80° in steps of 5°. The maximum difference $(\varphi_e - \varphi_{pB})_{\max} = 20.447°$ occurs when $\theta = 180°$ and $\varphi_{pB} \approx 44°$. For large values of φ_{pB}, $(\varphi_e - \varphi_{pB})$ is nearly constant (e.g., at $\varphi_{pB} = 80°$, $\varphi_e - \varphi_{pB}$ increases from 8.270° to 8.351° as θ increases from 0 to 180°).

3. CONSTANT-φ_e CONTOURS IN THE COMPLEX PLANES OF ε AND $\varepsilon^{1/2}$

Over the range of incidence angles $0 < \varphi < \varphi_e$ the p reflectance at oblique incidence is less than that at normal incidence. It is of interest to consider the constant-φ_e contours in the complex ε plane. Figure 5 shows a family of such contours for φ_e from 45 to 80° in steps of 5° and φ_e from 80 to 85° in steps of 1°. These results are obtained by solving Eqs. (1) and (3) numerically. The curves resemble a family of semicircles centered at the origin. (However, each contour is *not* a semicircle.) Figure 6 shows the corresponding family of contours in the complex-refractive-index plane, $\varepsilon^{1/2} = n + jk$. The de-

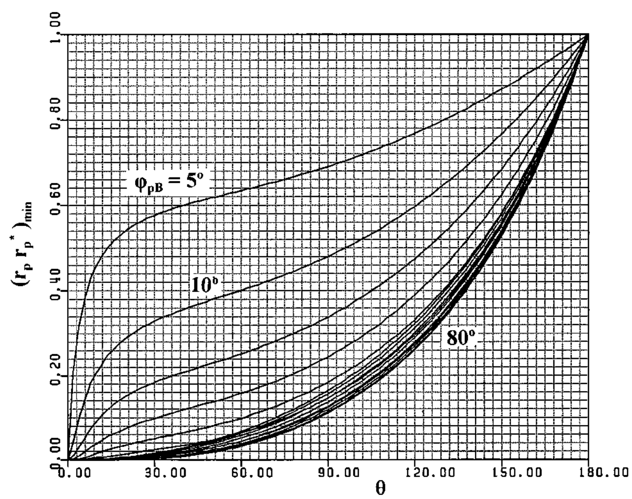

Fig. 3. Minimum reflectance at the pseudo-Brewster angle φ_{pB}, $(r_p r_p^*)_{\min}$, as a function of θ for constant values of φ_{pB} from 5 to 80° in steps of 5°.

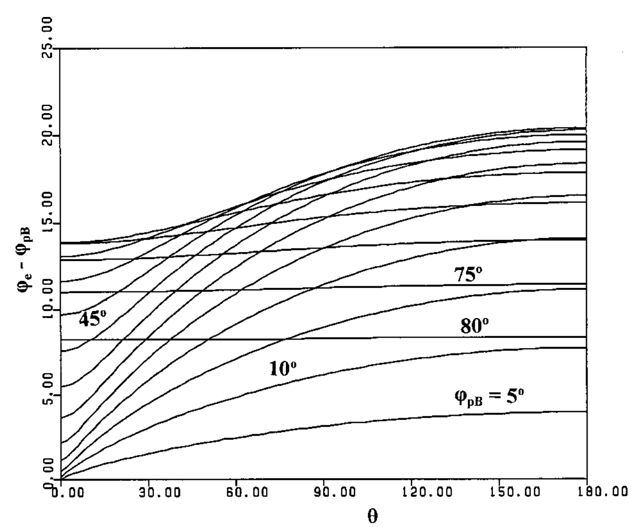

Fig. 4. Angle difference $\varphi_e - \varphi_{pB}$ as a function of θ, for constant values of φ_{pB} from 5 to 80° in steps of 5°. φ_e defines the upper limit of the range of incidence angle for which the p reflectance at oblique incidence is less than that at normal incidence.

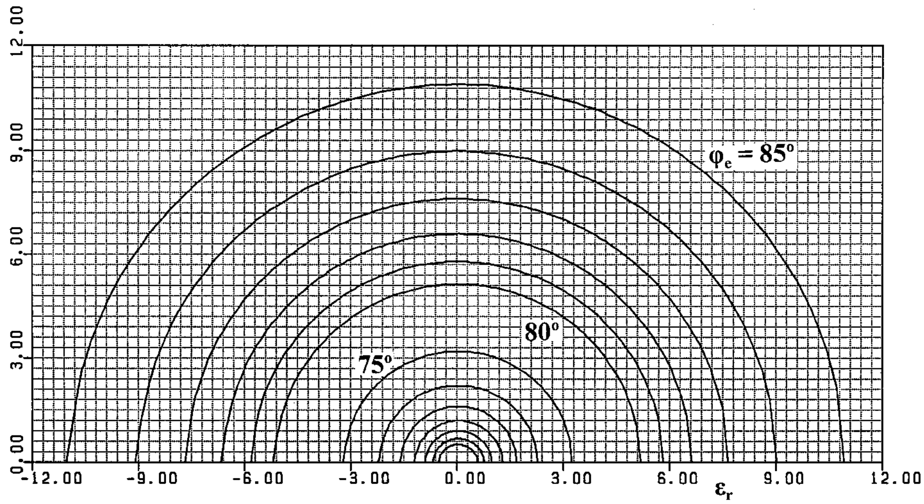

Fig. 5. Family of contours of constant $\varphi_e = 45$ to $80°$ in steps of $5°$, and $\varphi_e = 80$ to $85°$ in steps of $1°$. φ_e defines the upper limit of the range of incidence angle for which the p reflectance at oblique incidence is less than that at normal incidence.

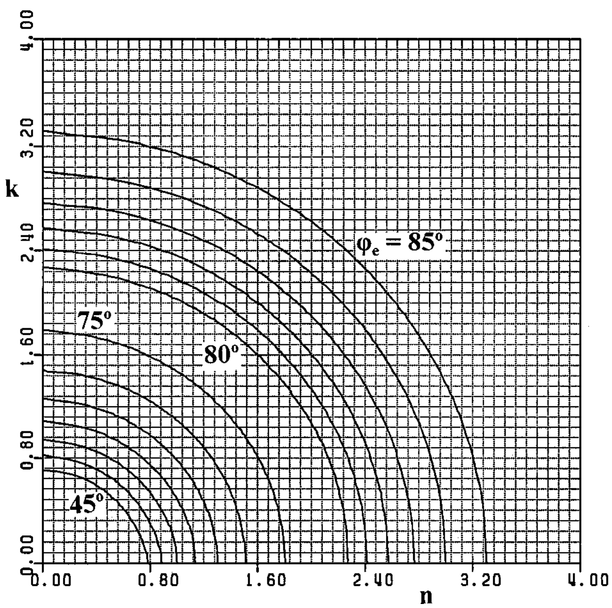

Fig. 6. Family of constant-φ_e contours in the nk complex refractive index plane for the same values of φ_e as in Fig. 5.

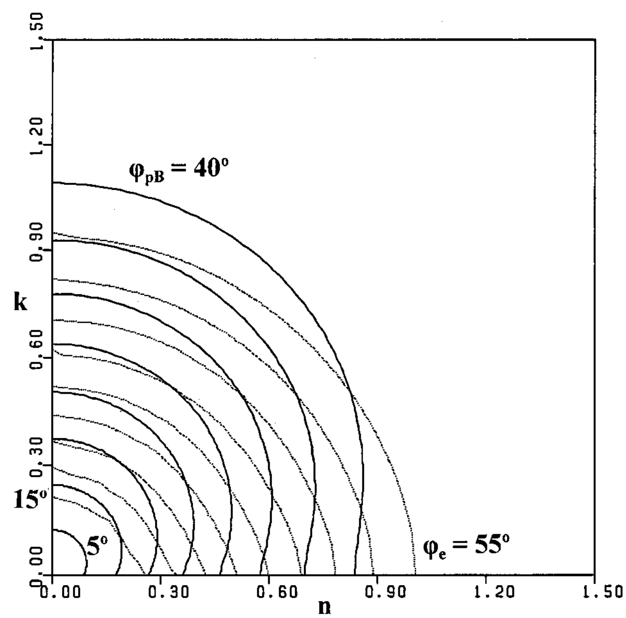

Fig. 7. Families of constant-φ_{pB} and constant-φ_e contours in the $n-k$ plane in the domain of fractional optical constants ($n, k < 1$).

viation of each contour from a quadrant of a circle is more apparent at lower angles (e.g., at $\varphi_e = 45°$).

4. TECHNIQUE FOR DETERMINING *n* AND *k* FROM THE MEASURED ANGLES φ_{pB} AND φ_e

Azzam described an analytical technique for determining the optical constants n and k of an absorbing medium from two pseudo-Brewster angles measured in two transparent incidence media.[7] It is of interest to consider whether n and k can be determined from the two angles φ_{pB} and φ_e measured in the same medium of incidence. In general, angular measurements are attractive, because no absolute reflectance measurements are required. (For a review of numerous reflectance-based techniques, the reader may consult papers by Humphreys-Owen[2] and Hunter.[8])

Figure 7 shows two superimposed families of constant-φ_{pB} and constant-φ_e contours in the $n-k$ plane in the domain of fractional optical constants. This domain is important in that it relates to attenuated total internal reflection when light is incident from a dense medium. The contours are shown for $\varphi_{pB} = 5$ to $40°$ in steps of $5°$ and for $\varphi_e = 15$ to $55°$ in steps of $5°$. The angles of intersection of curves of one family with curves of the other provide a measure of the precision with which n and k can be determined. It is apparent from Fig. 7 that n and k can be reasonably well determined when $k < n$.

Figure 8 is similar to Fig. 7, except that values of $n, k > 1$ are now considered. In Fig. 8 the families of

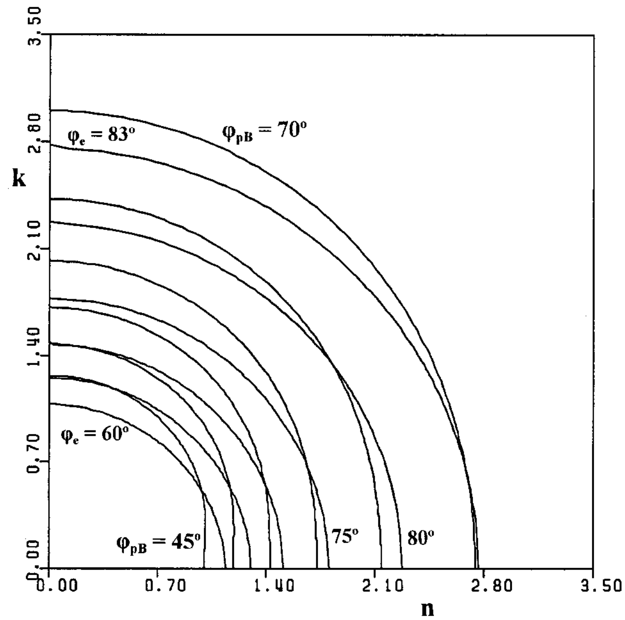

Fig. 8. Families of constant-φ_{pB} and constant-φ_e contours in the $n-k$ plane for $n, k > 1$.

constant-φ_{pB} and constant-φ_e contours are generated for $\varphi_{pB} = 45$ to 70° in steps of 5°, and for $\varphi_e = 60$ to 80° in steps of 5°, and $\varphi_e = 83°$. Because of the smaller intersection angles, the present two-angle method would not provide an accurate method of determining n and k

5. SUMMARY

We have determined the range of incidence angles, $0 < \varphi < \varphi_e$, over which the reflectance of p-polarized light at oblique incidence is less than that at normal incidence, for any transparent medium/absorbing medium interface. Constant-φ_e contours in the complex planes of the dielectric constant ε and refractive index $\varepsilon^{1/2} = n + jk$ are obtained. Finally, it is shown that fractional optical constants n and k can be determined if the pseudo-Brewster angle and the angle φ_e [which satisfies Eq. (3)] are measured.

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