2-2001

Tilted bilayer membranes as simple transmission quarter-wave retardation plates

R. M.A. Azzam
University of New Orleans, razzam@uno.edu

Fadi A. Mahmoud

Tilted bilayer membranes as simple transmission quarter-wave retardation plates

R. M. A. Azzam

Department of Electrical Engineering, University of New Orleans, Lakefront, New Orleans, Louisiana 70148

Fadi A. Mahmoud

Adaptec, Inc., 691 South Milpitas Boulevard, Milpitas, California 95035

Received June 5, 2000; accepted July 26, 2000; revised manuscript received August 22, 2000

A tilted bilayer membrane, which consists of two thin films of transparent optically isotropic materials of different refractive indices, can function as a transmission quarter-wave retarder (QWR) at a high angle of incidence. A specific design using a cryolite-Si membrane in the infrared is presented, and its tolerances to small shifts of wavelength, incidence angle, and film thickness errors are discussed. Some designs provide a dual QWR in transmission and reflection. Such devices provide simple linear-to-circular (and circular-to-linear) polarization transformers. Bilayer eighth-wave retarders without diattenuation are also introduced. © 2001 Optical Society of America

OCIS codes: 120.2130, 230.0230, 240.0310, 260.5430.

1. INTRODUCTION

Wave retarders are versatile optical devices for the control and the analysis of polarized light with numerous applications. The desired differential phase shift between two orthogonal linear polarization components of incident light is often introduced by using linear birefringence in a crystalline plate or total internal reflection.[1]

Coherent multiple-beam interference in a tilted parallel-plane thin dielectric slab (membrane or pellicle) leads to a differential transmission phase shift Δ_t between the parallel (p) and perpendicular (s) polarizations; hence such a slab of an *optically isotropic material* (Fig. 1) acts as the simplest possible wave retardation plate.[2] The retardance Δ_t vanishes at normal incidence, and hence the effect can be considered to arise from form birefringence associated with the tilt of the slab. The upper bound on Δ_t is given by[3]

$$\Delta_{t\ \max} = \arctan\{[N - (1/N)]/2\}, \tag{1}$$

where $N = N_1/N_0$ is the refractive index of the slab relative to that of the surrounding medium (usually air or vacuum, in which case $N_0 = 1$). The maximum retardance is attainable in the limit of grazing incidence, and, according to Eq. (1), its value is $<90°$, so that a single homogeneous slab cannot perform as a quarter-wave retarder (QWR). On the other hand, an eighth-wave retarder (EWR) is readily achieved with a pellicle of refractive index $N > 1.4966$ at an angle of incidence that decreases as N increases.

In this paper it is shown that this basic limitation of the homogeneous thin slab is lifted if the slab is coated with a transparent thin film of a different refractive index. Interference in such a bilayer membrane gives a QWR, albeit at a high angle of incidence. A specific infrared (IR) QWR design using a cryolite (Na_3AlF_6) -Si pellicle is presented. A dual QWR in transmission and reflection is also realizable with certain bilayer membranes. The coated pellicle QWR is a simple (noncrystalline) device that introduces no significant lateral or angular deviation in the transmitted beam. It can be mounted in a rotary stage such that its surface normal precesses in a cone around the incident light beam as an axis. Thus a QWR of variable fast-axis azimuth is obtained. Reflection phase retarders that use periodic or quasiperiodic stacks of many layers have been described by several authors.[4–6]

2. BASIC RELATIONS

The change in the state of polarization of light upon transmission through a bilayer membrane (Fig. 2) is determined by the ratio of complex-amplitude transmission coefficients for the p and s polarizations:

$$\rho_t = T_p/T_s = (\tan\psi_t)\exp(j\Delta_t). \tag{2}$$

Explicit expressions for the transmission (and reflection) coefficients of a bilayer (on a substrate) are available[7]; substitution of these expressions into Eq. (2) gives

$$\rho_t = a(1 + bX_1 + cX_2 + dX_1X_2)/(1 + eX_1 + fX_2 + gX_1X_2), \tag{3}$$

in which

$$a = (t_{01p}/t_{01s})(t_{12p}/t_{12s})(t_{20p}/t_{20s}),$$

$$b = r_{01s}r_{12s}, \qquad e = r_{01p}r_{12p},$$

$$c = r_{12s}r_{20s}, \qquad f = r_{12p}r_{20p},$$

$$d = r_{01s}r_{20s}, \qquad g = r_{01p}r_{20p}. \tag{4}$$

0740-3232/2001/020421-05$15.00

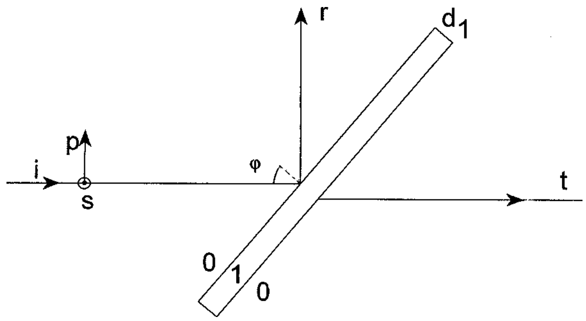

Fig. 1. Reflection and transmission of light by a tilted transparent thin slab of an optically isotropic material (medium 1 of thickness d_1) in a transparent ambient (medium 0). p and s are the linear polarizations parallel and perpendicular to the plane of incidence, respectively, and ϕ is the angle of incidence.

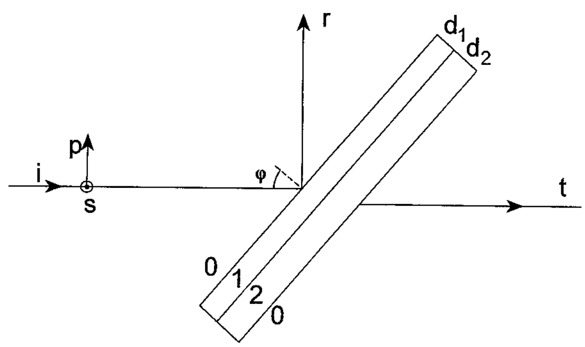

Fig. 2. Reflection and transmission of light by a tilted bilayer membrane of two optically isotropic thin films (media 1 and 2 of thicknesses d_1 and d_2, respectively) in a transparent ambient (medium 0). p and s are the linear polarizations parallel and perpendicular to the plane of incidence, respectively, and ϕ is the angle of incidence.

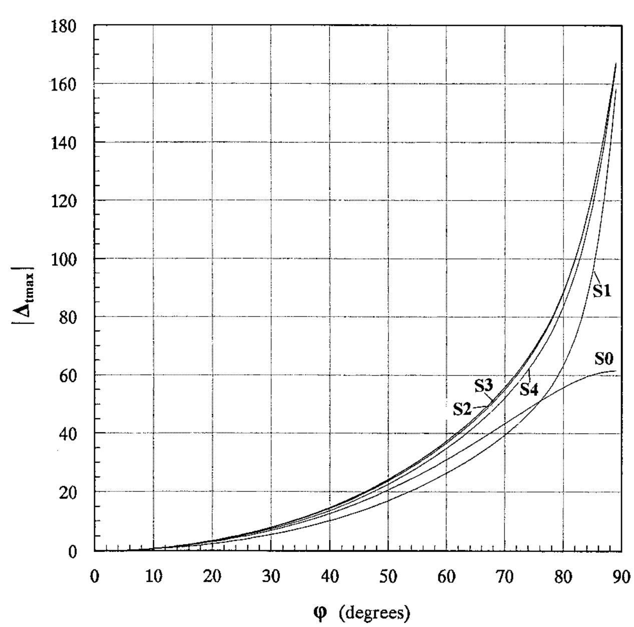

Fig. 3. Maximum differential phase shift in light transmission through a bilayer, $|\Delta_{t\max}|$, versus angle of incidence φ for five material systems S0, S1, S2, S3, and S4, which correspond to $(N_1, N_2) = (4, 4)$, (1.38, 2.35), (1.35, 4), (1.25, 4), and (1.35, 3.42), respectively.

$r_{ij\nu}$ and $t_{ij\nu}$ are the Fresnel reflection and transmission coefficients of the ij interface for the ν polarization ($\nu = p, s$), and

$$X_i = \exp(-j2\pi\zeta_i), \qquad i = 1, 2, \tag{5}$$

where ζ_i is the normalized thickness of the ith film, i.e.,

$$\zeta_i = d_i/D_i, \qquad i = 1, 2, \tag{6}$$

and

$$D_i = (\lambda/2)(N_i^2 - \sin^2\phi)^{-1/2}, \qquad i = 1, 2, \tag{7}$$

is the corresponding film thickness period, λ is the wavelength of the incident light, φ is the angle of incidence, N_i is the refractive index of the ith film, and light is assumed to be incident from air or vacuum ($N_0 = 1$).

3. CONSIDERATION OF SEVERAL MATERIAL SYSTEMS

For given refractive indices (N_1, N_2) of the two films of the bilayer, the normalized film thicknesses (ζ_1, ζ_2) are iterated on in search of the maximum transmission differential phase shift $\Delta_{t\max}$ at a given angle of incidence φ by using Eqs. (2)–(5). Figure 3 is a plot of $|\Delta_{t\max}|$ versus φ for five material systems S0, S1, S2, S3, and S4 which correspond to $(N_1, N_2) = (4, 4)$, (1.38, 2.35), (1.35, 4), (1.25, 4), and (1.35, 3.42), respectively. S0 reduces to the case of a single homogeneous film of Ge. S1 represents a MgF_2-ZnS system in the visible, and S2–S4 represent a low-index (fluoride) film on a high-index Si or Ge layer in the IR.[8] The following conclusions are readily drawn from Fig. 3.

1. The upper bound on $|\Delta_{t\max}|$ for the S0 reference case (the homogeneous Ge layer) is 61.93°, in agreement with Eq. (1).
2. For a bilayer of two films of different refractive indices, the upper bound on $|\Delta_{t\max}|$ is 180° at grazing incidence ($\phi = 90°$).
3. For a bilayer of two films of different refractive indices, $|\Delta_t| = 90°$ is possible at a high angle of incidence ϕ (>80°). Therefore a bilayer membrane can be used as a QWR in transmission. To stay as far from grazing incidence as possible, the refractive-index contrast between the two films of the bilayer should be as high as possible, which is attained in the IR spectral region.

4. CRYOLITE-Si BILAYER TRANSMISSION AND REFLECTION DUAL QUARTER-WAVE RETARDER AT $\phi = 82°$ AND $\lambda = 10.6$ μm

As a specific design we take $(N_1, N_2) = (1.35, 3.42)$, which corresponds to a cryolite-Si bilayer at the 10.6-μm CO_2-laser wavelength, and $\phi = 82°$ angle of incidence.

The normalized thickness ζ_1 is assigned constant values from 0 to 1 in steps of 0.1, and for each value of ζ_1, ζ_2 is scanned over the full range 0–1 in finer steps. Figure 4 shows the resulting family of contours of ρ_t in the complex plane, as calculated from Eq. (3). These contours are circles because ρ_t is a bilinear function of X_2 for constant X_1 according to Eq. (3), and the bilinear transformation maps the unit circle of X_2 onto a circle[9] of ρ_t. Points of intersection of the circles with the imaginary axis in

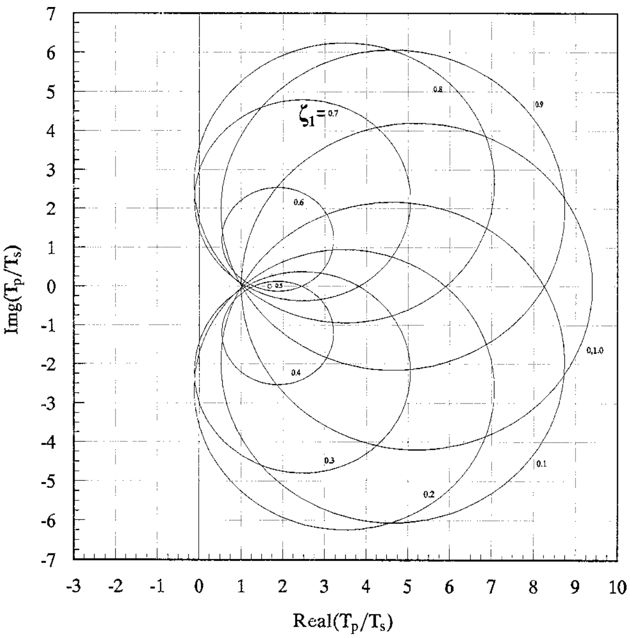

Fig. 4. Family of contours of the ratio of complex-amplitude p and s transmission coefficients ρ_t in the complex plane for a cryolite-Si bilayer with refractive indices (1.35, 3.42), as calculated from Eq. (3), at $\phi = 82°$ angle of incidence. The normalized thickness of the first layer, ζ_1, assumes constant values marked by each curve while the normalized thickness of the second film, ζ_2, is scanned.

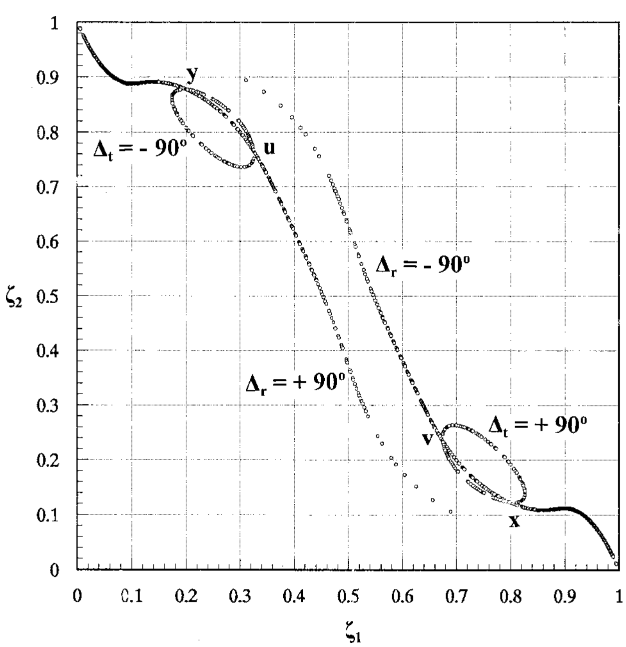

Fig. 5. Multiple solutions (ζ_1, ζ_2) for bilayer transmission quarter-wave retarders (QWR's), for both $\Delta_t = +90°$ and $\Delta_t = -90°$, are presented by the closed contours. Superimposed are the corresponding solution loci for reflection phase shifts $\Delta_r = +90°$ and $\Delta_r = -90°$ for the cryolite-Si bilayer at $\phi = 82°$ angle of incidence. The intersection points x, y, u, and v represent bilayers that function as a dual QWR in transmission and reflection.

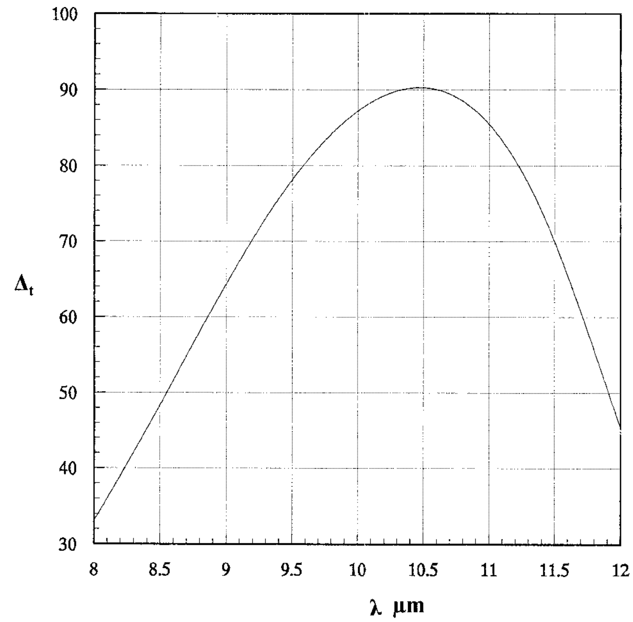

Fig. 6. Δ_t as a function of wavelength around $\lambda = 10.6$ μm for the cryolite-Si pellicle QWR.

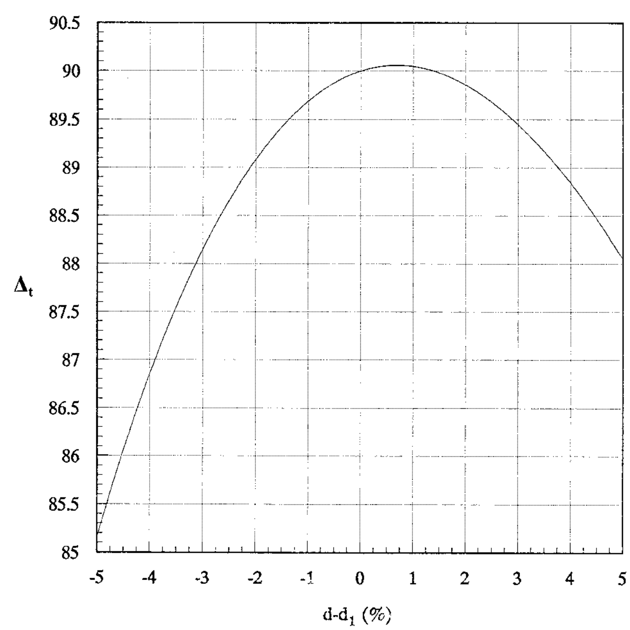

Fig. 7. Effect on Δ_t of errors of ±5% of the cryolite film thickness d_1 in the cryolite-Si pellicle QWR.

Fig. 4 correspond to bilayers that produce a QWR in transmission, i.e., $\Delta_t = \pm 90°$.

Multiple solutions (ζ_1, ζ_2) for a transmission QWR, for both $\Delta_t = +90°$ and $\Delta_t = -90°$, are presented by the closed contours in Fig. 5. Superimposed in Fig. 5 are the corresponding solution loci for reflection phase shifts $\Delta_r = +90°$ and $\Delta_r = -90°$ for the same bilayer. A surprise is the intersection of the loci of transmission and reflection QWR's at the four points x, y, u, and v. These points represent four bilayers that function as a dual QWR ($\Delta_{t,r} = \pm 90°$) in transmission and reflection.

We further consider the design marked x in Fig. 5. The normalized and least metric film thicknesses for this bilayer are (0.7992, 0.1228) and (4.6164, 0.1988) μm, respectively. If the dispersion of the two film materials is neglected, the effect on Δ_t of shifting the wavelength around the design wavelength $\lambda = 10.6$ μm is shown in Fig. 6. The QWR is reasonably achromatic in the neighborhood of the design wavelength ($|\delta\Delta_t| < 1°$ between $\lambda = 10.3$ and 10.7 μm).

Figures 7 and 8 show the effect on Δ_t of errors of $\pm 5\%$ of the thicknesses d_1 and d_2, respectively, at the design wavelength $\lambda = 10.6$ μm and the angle of incidence $\phi = 82°$. It is apparent that the layer thicknesses should be controlled to within $\pm 2\%$ to limit the phase error to $<1°$.

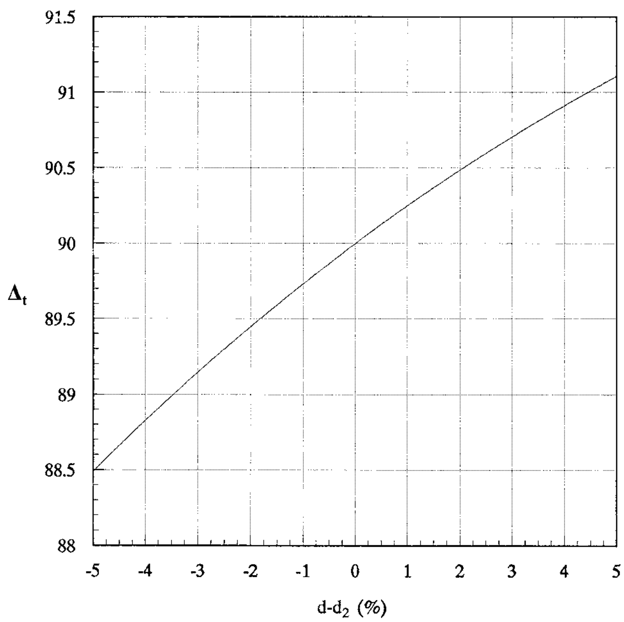

Fig. 8. Effect on Δ_t of errors of $\pm 5\%$ of the Si film thickness d_2 in the cryolite-Si pellicle QWR.

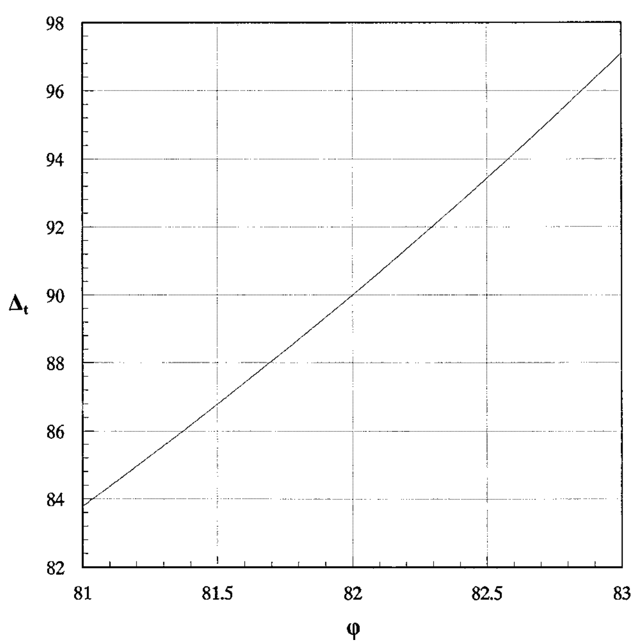

Fig. 9. Effect on Δ_t of shifting the angle of incidence ϕ by ± 1 around $\phi = 82°$ for the cryolite-Si pellicle QWR.

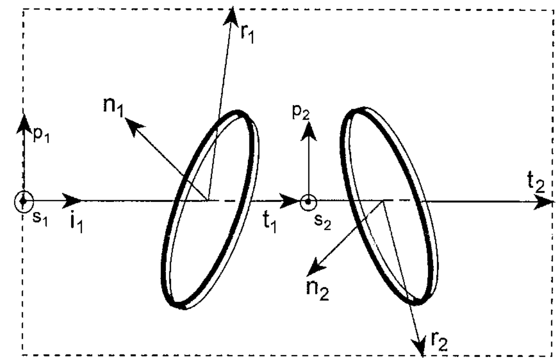

Fig. 10. Two oppositely tilted pellicle eighth-wave retarders in series function as a QWR.

Figure 9 shows the effect on Δ_t of shifting the angle of incidence by $\pm 1°$. As may be expected, operation at the large angle $\phi = 82°$ requires a collimated (e.g., laser) beam with milliradian beam divergence.

Because of the unequal throughput for the p and s polarization (diattenuation[10]) of the bilayer QWR, the electric field vector of incident linearly polarized light must be oriented at the appropriate azimuth θ (measured from the plane of incidence) to produce an output (transmitted) circularly polarized beam. For design x we determine $\theta = 59.862°$, and the insertion loss for this coated-pellicle circular polarizer under this condition is 0.3395.

It is possible to design a bilayer EWR *without diattenuation*. For the same refractive indices $(N_1, N_2) = (1.35, 3.42)$ and the same angle of incidence $\phi = 82°$, an EWR is achieved with normalized thicknesses $(\zeta_1, \zeta_2) = (0.1009, 0.9594)$. In this case $\Delta_t = -45°$, and the throughputs (intensity transmittances) for the p and s polarizations are the same and equal 84.5%. If two such pellicles are used in series and with equal and opposite tilts (Fig. 10), a QWR with improved field of view is obtained. Other two-pellicle designs are possible, at lower angles of incidence, in which the planes of incidence at the two pellicles are made orthogonal to equalize the p and s net transmittances.

5. CONCLUSION

Pellicles are used traditionally as light, yet quite sturdy beam splitters.[11] In this paper it is shown that, by appropriate design, a bilayer pellicle of optically isotropic materials can introduce 90° differential phase shift between the p and s linear polarization components in transmission and hence can function as a simple QWR with negligible lateral or angular shift. To lower the operating angle of incidence and remove diattenuation require multilayer-coated pellicles. This extension of the present work falls outside the scope of this paper.

Address correspondence to R. M. A. Azzam at the location on the title page or by e-mail, razzam@uno.edu.

REFERENCES

1. See, for example, D. Clarke and J. F. Grainger, *Polarized Light and Optical Measurement* (Pergamon, Oxford, UK, 1971).

2. D. A. Holmes, "Wave optics theory of rotatory compensators," J. Opt. Soc. Am. **54**, 1340–1347 (1964).
3. R. M. A. Azzam, "Transmission ellipsometry on transparent unbacked or embedded thin films with application to soap films in air," Appl. Opt. **30**, 2801–2806 (1991).
4. W. H. Southwell, "Multilayer coating design achieving a broadband 90° phase shift," Appl. Opt. **19**, 2688–2692 (1980).
5. J. H. Apfel, "Phase retardance of periodic multilayer mirrors," Appl. Opt. **21**, 733–738 (1982).
6. M. M. K. Howlader and R. M. A. Azzam, "Periodic and quasiperiodic nonquarterwave multilayer coatings for 90-deg reflection phase retardance at 45-deg angle of incidence," Opt. Eng. **34**, 869–875 (1995).
7. R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, Amsterdam, 1987).
8. E. D. Palik, ed., *Handbook of Optical Constants of Solids* (Academic, New York, 1985).
9. A. Kyrala, *Applied Functions of a Complex Variable* (Wiley-Interscience, New York, 1972).
10. R. A. Chipman, "Polarization analysis of optical systems," Opt. Eng. **28**, 90–99 (1989).
11. J. A. Dobrowolski, "Optical properties of films and coatings," in *Handbook of Optics*, M. Bass, editor-in-chief (McGraw-Hill, New York, 1995), Vol. I, Chap. 42.