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Photopolarimeter based on planar grating diffraction

R. M.A. Azzam
University of New Orleans, razzam@uno.edu

K. A. Giardina

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R. M. A. Azzam and K. A. Giardina

Department of Electrical Engineering, University of New Orleans, Lakefront, New Orleans, Louisiana 70148

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A division-of-amplitude photopolarimeter (DOAP) is described that employs a diffraction grating in the conventional spectrometer orientation with the grating grooves normal to the plane of incidence. Four coplanar diffracted orders are used for polarimetric analysis to determine all four Stokes parameters of incident light simultaneously and virtually instantaneously (with the speed being determined solely by the photodetectors and their associated electronics); a fifth order is used for alignment by autocollimation or by use of a position-sensing quadrant detector. To sensitize the instrument for the +45° and −45° azimuths of incident linearly polarized light and for the handedness of incident circular polarization (i.e., for the third and fourth Stokes parameters), we insert two linear polarizers in two diffracted orders with their transmission axes inclined at appropriate angles with respect to the plane of incidence. The calibration and testing of an instrument of this type that uses an Al-coated 600-groove/mm holographic grating at 632.8-nm wavelength are reported as an example.

1. INTRODUCTION

Diffraction gratings have had a major impact on spectroscopy since their invention in 1821 by Joseph Fraunhofer, who used them to measure the absorption lines of the solar spectrum.[1] Gratings have also proved to be some of the most versatile optical elements, with numerous other applications in modern optical science and engineering,[2] e.g., as waveguide couplers; wavelength division multiplexers and demultiplexers; distributed feedback elements in lasers; and holographic beam deflectors, combiners, and interconnects for optical computers.

Polarization effects that accompany grating diffraction have been known since they were first noted by Fraunhofer.[1-4] However, these effects have generally been thought of as spurious and have not been put to practical use. A notable exception is that of the nondiffracting wire-grid polarizer,[4] which is used to linearly polarize infrared radiation (and other longer-wavelength electromagnetic waves) by the differential absorption of the component waves of incident light whose electric vectors oscillate parallel and perpendicular to the wires. More recently, form birefringence associated with high-spatial-frequency gratings has been used to make new wave retarders.[5-7]

Recently[8,9] we have proposed that multiple-beam grating diffraction be used for polarization analysis or optical polarimetry. In particular, when a grating is set for conical diffraction[10] (with the grating vector not in the plane of incidence) and linear photodetectors intercept four or more diffracted orders, the beam-splitting and polarization-altering properties of the grating permit time-resolved Stokes-parameter photopolarimetry, without any moving optical elements or polarization modulators. Furthermore, when the single-element photodetectors are replaced with detector arrays, the instrument becomes spectroscopic. In a related development Todorov and Nikolova[11] described a spectrophotopolarimeter that uses two transmission gratings in series. The first of these gratings is presumed not to change the state of polarization of light, and the second is a nonconventional holographic grating that is fabricated with the use of the superposition of orthogonally circularly polarized waves in a photodichroic material. Our grating photopolarimeter differs from that of Todorov and Nikolova in two important respects: (1) only one grating is used instead of two, and (2) this grating is of conventional construction, e.g., a reflective metallic ruled or holographic grating, which is readily available commercially as an off-the-shelf item. Furthermore, conical diffraction offers the possibility of eliminating the need for any additional linear polarizers.

Optical polarimetry[12-16] is a field of vast scope, and many photopolarimeters have been devised to measure light polarization.[17,18] A typical instrument uses a single linear photodetector, in front of which are placed polarizing optical elements one or more of which is subjected to some form of modulation (e.g., synchronous rotation or phase modulation). Fourier analysis of the output signal of the photodetector is used to extract the four Stokes parameters of incident light.[19]

The simultaneous and nearly instantaneous measurement of all four Stokes parameters of light with no moving parts or modulators requires a multichannel scheme with four (or more) detectors. An elegant scheme uses a spatial arrangement of four photodetectors[20] (of which three are reflective) and no other optical components. This four-detector photopolarimeter (FDP) has been implemented and studied extensively.[21-25] The FDP is spectroscopic only in that it can be operated over a range of wavelengths but with light of only one wavelength incident upon the FDP at a time.[22] Therefore, kinetic polarization analysis as a function of wavelength of incident broadband radiation is possible only if a sufficiently fast (and polarization-independent) wavelength-scanning element, e.g., a tunable Fabry–Perot étalon, is added in front of the FDP. The grating spectrophotopolarimeter overcomes this limitation by using the inherent dispersive power of the grating and available high-resolution de-

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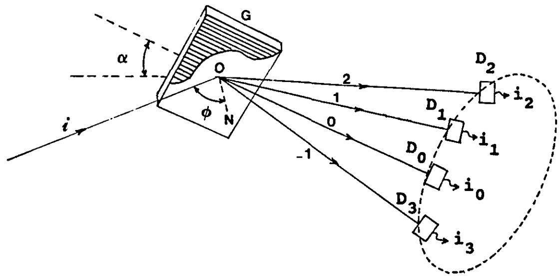


Fig. 1. Photopolarimeter using conical grating diffraction. G is the grating in the conical diffraction mount with the grooves inclined at an angle, α, with respect to the plane of incidence. The incident light beam i (whose Stokes parameters are to be measured) strikes the grating at an angle of incidence ϕ measured from the grating normal *ON*. At least four diffracted beams, which are indicated by their order numbers, are intercepted by linear photodetectors D_m to produce corresponding output electrical signals i_m. Linear polarizers (not shown) may be added in front of the photodetectors to improve the overall polarimetric sensitivity of the system.

tector arrays. This is important, for example, in spectroscopic ellipsometry of fast-changing physicochemical reactions on surfaces.

Other multichannel schemes employ the division of wave front[26,27] and division of amplitude.[28–32] The diffraction-grating photopolarimeter uses the division-of-amplitude principle.

2. PHOTOPOLARIMETER BASED ON CONICAL GRATING DIFFRACTION

In the division-of-amplitude photopolarimeter[28] (DOAP) an incident light beam whose four Stokes parameters are to be measured is split into four beams by using an appropriately coated[29] beam splitter and two Wollaston prisms (or equivalent polarizing beam splitters). Linear detection of the light fluxes of the four component beams produces four output electrical signals that determine the four Stokes parameters of incident light by means of an instrument matrix that is obtained by calibration. The DOAP permits time-resolved measurement of the most general state of partial elliptical polarization of light, as it uses no moving parts or modulators. Several DOAP instruments have been constructed recently.[30–33]

In the grating-based division-of-amplitude photopolarimeter (G-DOAP)[8] a diffraction grating replaces all three beam splitters of the DOAP. Conical diffraction is used (with the grating grooves neither parallel nor perpendicular to the plane of incidence) (Fig. 1) to sensitize the grating diffraction efficiency with respect to the handedness of incident circularly polarized light and also with respect to the switching of the incident linear-polarization direction between the +45° and −45° azimuths as measured from the plane of incidence. Consequently, when the mth diffracted beam is intercepted by a linear photodetector, D_m, the output electrical signal i_m is a linear combination of all four Stokes parameters S_i ($i = 0, 1, 2, 3$) of the incident light; i.e.,

$$i_m = \sum_{n=0}^{3} a_{mn} S_n, \qquad m = 0, 1, 2, \ldots. \tag{1}$$

The mth projection vector $\mathbf{a}_m = [a_{m0} \quad a_{m1} \quad a_{m2} \quad a_{m3}]$ is equal to the first row of the Mueller matrix of diffraction of the mth order multiplied by a scale factor that represents the sensitivity of the photodetector, D_m.

When four orders are detected, the corresponding set of four signals defines a current vector, $\mathbf{I} = [i_0 \quad i_1 \quad i_2 \quad i_3]^t$, which is linearly related to the input Stokes vector $\mathbf{S} = [S_0 \quad S_1 \quad S_2 \quad S_3]^t$ (t denotes the transpose) by

$$\mathbf{I} = \mathbf{AS}. \tag{2}$$

The 4 × 4 instrument matrix $\mathbf{A} = (a_{mn})$ is characteristic of the G-DOAP at a given wavelength and is a function of the chosen grating, the angle of incidence and the angle of inclination of the grooves with respect to the plane of incidence, and the selected diffracted orders (if more than four are available). Under conditions of conical diffraction,[10] $\mathbf{A}$ is expected to be in general nonsingular. From Eq. (2), $\mathbf{S}$ is then given by

$$\mathbf{S} = \mathbf{A}^{-1}\mathbf{I}. \tag{3}$$

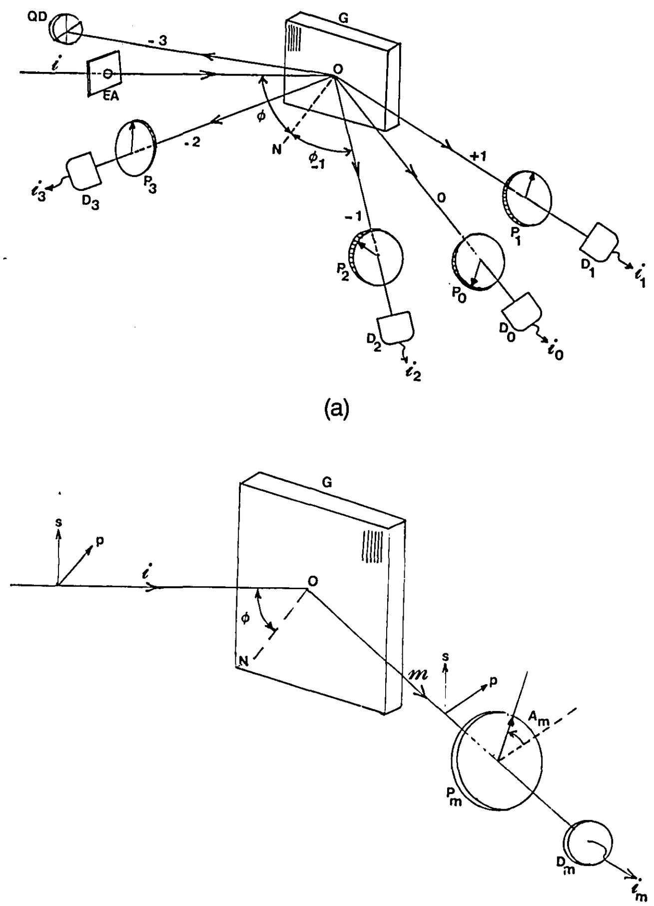
The instrument matrix $\mathbf{A}$ of the G-DOAP is determined by calibration with the same procedures established for the FDP.[22,23]

The principle of the G-DOAP's using conical diffraction and no additional polarizers was tested recently.[8] An Al-coated 600-groove/mm holographic grating set at a 65° angle of incidence was used, with the grooves inclined at 45° with respect to the plane of incidence, as well as a 632.8-nm He–Ne laser and small-area P-I-N photodiodes. The associated instrument matrix $\mathbf{A}$ is not so far from singular as desired, and the overall polarization sensitivity (or polarimetric resolving power[25]) is limited. This situation does not improve substantially with other choices of the incidence or groove inclination angle. Operation with other types of gratings has not been tried.

The polarization sensitivity of the G-DOAP is substantially increased by the insertion of linear (e.g., dichroic sheet) polarizers in front of the photodetectors. This increases the length of each normalized projection vector[24] to nearly the maximum value of 1 and increases the determinant of the instrument matrix. However, if linear polarizers are introduced, the conical diffraction geometry is no longer essential for the operation of the G-DOAP. Planar diffraction (with the grating in the conventional spectrometer orientation with the grooves normal to the plane of incidence) offers a more attractive alternative. Planar diffraction makes the instrument more compact (in one plane) and allows one-dimensional linear detector arrays to replace two-dimensional area arrays in spectroscopic work. The theory, implementation, and testing of the G-DOAP with the use of planar diffraction are examined in the remainder of this paper.

3. PHOTOPOLARIMETER BASED ON PLANAR GRATING DIFFRACTION

Figure 2(a) is a schematic diagram of the G-DOAP in the planar diffraction configuration, and Fig. 2(b) introduces the parameters and angles involved in the detection of the mth diffracted order. [For negative orders, the order number is changed to match the signal number, as indicated in Fig. 2(a).] In Fig. 2(b) the directions marked p and s are parallel and perpendicular, respectively, to the plane of incidence, which is also the common plane of



(b)

Fig. 2. Photopolarimeter using planar grating diffraction. (a) G is the grating in the normal spectrometer orientation with the grooves perpendicular to the plane of incidence. The incident light beam i (whose Stokes parameters are to be measured) passes through an entrance aperture, EA, and strikes the grating at an angle of incidence ϕ measured from the grating normal ON. At least four diffracted beams, which are indicated by their order numbers, are intercepted by linear photodetectors D_m, to produce corresponding output electrical signals i_m. A fifth diffracted beam intercepts a quadrant detector, QD, which is used for alignment. P_m are linear polarizers placed in front of photodetectors D_m with their transmission axes oriented at appropriate angles with respect to the common plane of incidence and diffraction. Of the four polarizers shown, only two are essential for polarimetric analysis. (b) Parameters and angles involved in the detection of the mth diffracted order.

diffraction for all orders; p and s also identify the linear eigenpolarizations of reflection or diffraction. The diffraction of the mth order is completely described by three parameters: r_m, the power reflectance or diffraction efficiency for incident unpolarized or circularly polarized light, and the ellipsometric angles[14] ψ_m and Δ_m, that determine the ratio of the far-field complex p and s reflection (or diffraction) coefficients for the same order. The linear polarizer (or analyzer) P_m is oriented with its transmission axis at an azimuth A_m measured from the plane of incidence. The output signal i_m of the mth photodetector D_m is still given by Eq. (1), where the mth projection vector $\mathbf{a}_m$ is now given by

$$\mathbf{a}_m = (k_m r_m/2)\{[1 - \cos(2\psi_m)\cos(2A_m)]\ \ [\cos(2A_m) - \cos(2\psi_m)]\ \ [\sin(2A_m)\sin(2\psi_m)\cos(\Delta_m)]\ \ [\sin(2A_m)\sin(2\psi_m)\sin(\Delta_m)]\}, \tag{4}$$

where k_m is the responsivity of the mth detector and an ideal polarizer is assumed. (The assumption of an ideal polarizer is for analytical convenience only. The operation of the G-DOAP, however, is not significantly dependent on the actual properties of the polarizers used; such properties are absorbed into the instrument matrix, which is determined by calibration and not by calculation.)

Let us assume that four diffracted orders are detected. The instrument matrix $\mathbf{A}$ then consists of four rows each of the form given by Eq. (4). For the general case the determinant of the instrument matrix det($\mathbf{A}$) is too involved to be presented here. For the purpose of illustration, we assume that the polarizers are oriented at the uniformly distributed azimuths $A_0 = 0$, $A_1 = 45°$, $A_2 = 90°$, and $A_3 = 135°$. This simplifies the instrument matrix considerably, and its determinant reduces to

$$\det(\mathbf{A}) = (1/8)\left(\prod_{q=0}^{3} k_q r_q\right)[1 - \cos(2\psi_0)][1 + \cos(2\psi_2)] \times [\sin(2\psi_1)][\sin(2\psi_3)]\sin(\Delta_1 - \Delta_3). \tag{5}$$

In general, det($\mathbf{A}$) $\neq 0$ and the instrument matrix is nonsingular. Singularities occur when any of the multiplied factors in Eq. (5) is accidentally zero. These singularities are conveniently grouped as follows:

1. The diffraction efficiency r_m of any order or the responsivity of any detector k_m is zero. This obviously leads to the loss of one signal, and the full Stokes vector cannot be determined.
2. The zeroth order is purely s polarized ($\psi_0 = 0$), or the second order is purely p polarized ($\psi_2 = 90°$).
3. The p or s polarization is supressed in the first or third order, i.e., ψ_1 or $\psi_3 = 0$ or $90°$. This means that the diffraction grating functions as a linear polarizer in one of these orders.
4. The differential reflection phase shifts Δ_1 and Δ_3 of the first and third orders happen to be equal to or to differ by $\pm 180°$.

For a monochromatic or quasi-monochromatic photopolarimeter operating at a given wavelength, all the singularities listed above are readily avoidable. However, when operation over a range of wavelengths is anticipated (with linear detector arrays), one or the other of the above singularities may occur at discrete wavelengths. Even those can be avoided by appropriate design.

4. REDUCING THE NUMBER OF POLARIZERS IN THE PLANAR G-DOAP

If a polarizer is removed from one of the diffracted orders, the associated projection vector is reduced from that given by Eq. (4) to the following form:

$$\mathbf{a}_m = (k_m r_m/2)[1 \quad -\cos(2\psi_m) \quad 0 \quad 0]. \tag{6}$$

The corresponding instrument matrix simplifies accordingly. To avoid rendering the matrix singular, one can remove at most two polarizers. Put differently, the minimum number of polarizers that can be used in the planar

G-DOAP (to be placed in distinct orders) is two. For specificity, let the two polarizers be placed in the zeroth and second diffracted orders at azimuths A_0 and A_2, respectively, and let no polarizers be present in the first and third diffracted orders. The determinant of the instrument matrix in this case is given by

$$\begin{aligned}\det(\mathbf{A}) = (1/8)\left(\prod_{q=0}^{3} k_q r_q\right)[\sin(2A_0)\sin(2A_2)] \\ \times [\sin(2\psi_0)\sin(2\psi_2)][\cos(2\psi_1) - \cos(2\psi_3)] \\ \times [\sin(\Delta_0 - \Delta_2)]. \qquad (7)\end{aligned}$$

In general, $\det(\mathbf{A}) \neq 0$ and the instrument matrix is nonsingular. Singularities occur if any multiplied factor in Eq. (7) is accidentally zero. Besides the obvious singularities that arise from zero diffraction efficiency or zero detector responsivity, the other singularities occur when

1. The transmission axis of either polarizer is in or normal to the plane of incidence ($A_m = 0$ or 90°, $m = 0$ or 2).
2. The diffraction of the p or s polarization is suppressed in the zeroth or second order (ψ_0 or $\psi_2 = 0$ or 90°).
3. The (ellipsometric) parameter ψ is the same for the first and third orders (the two beams without polarizers), i.e., $\psi_1 = \psi_3$.
4. The differential reflection phase shifts Δ_0 and Δ_2 (for the two beams with polarizers) are equal or differ by ±180°.

Again, the above listed singularities can assuredly be avoided in a monochromatic design. The essential ψ's and Δ's can be measured by ellipsometry, and the instrument matrix, as determined by calibration, is checked to be nonsingular. (For a grating with well-defined characteristics, the instrument matrix can also be calculated from electromagnetic theory.[34,35]) For operation over a spectral range, one or the other of singularities 2–4 listed above may occur at discrete wavelengths.

5. EXPERIMENTAL RESULTS

A G-DOAP has been constructed and operated in the planar diffraction regime with the use of the same 600-groove/mm Al-coated holographic grating at wavelength $\lambda = 632.8$ nm. The angle of incidence ϕ was adjusted to 49.41° to produce autocollimation (exact backreflection) for the fourth diffracted order, as is predicted by the grating equation

$$\begin{aligned}\sin(\phi_m) &= \sin(\phi) + m(\lambda/\Lambda) \\ &= 0.7594 + 0.3797m, \qquad (8)\end{aligned}$$

where Λ is the grating period and m is the order number. Five diffracted beams for $m = 0, -1, -2, -3, -4$ appear at angles of 49.41°, 22.32°, 0°, −22.32°, and −49.41°, respectively, measured from the normal to the grating surface. A negative angle ϕ_m indicates that the corresponding diffracted order propagates on the same side of the normal as the incident beam. The first four of these orders are used for polarimetric analysis and are numbered without the minus sign. The fifth beam ($m = -4$) is used for alignment by autocollimation. Sheet polarizers, P_0 and P_3, are placed in the zeroth- and third-order beams only. Polarizer P_0 is oriented so as to maximize the differential response of photodetector D_0 to the incident right- and left-handed circular polarization states. Polarizer P_3 is oriented to maximize the differential response of photodetector D_3 to the incident orthogonal linear polarizations at the +45° and −45° azimuths. P-I-N photodiodes are used as the detectors, and their outputs are amplified and then processed using a 12-bit analog-to-digital converter and an on-line desktop 286 computer. The experimental setup for the calibration and testing of the G-DOAP is shown in Fig. 3 and is similar to that used with the FDP.[21,23]

The instrument is calibrated with the "equator–poles" method of Ref. 23, and the resulting instrument matrix **A** and its inverse **C** are

$$\mathbf{A} = \begin{bmatrix} 0.629 & 0.361 & 0.371 & 0.355 \\ 0.927 & 0.353 & -0.001 & 0.002 \\ 1.484 & -0.770 & 0.005 & -0.008 \\ 0.441 & -0.289 & -0.325 & 0.055 \end{bmatrix}, \qquad (9)$$

$$\mathbf{C} = \begin{bmatrix} 0.002 & 0.621 & 0.284 & 0.004 \\ -0.018 & 1.211 & -0.738 & -0.036 \\ 0.420 & -0.534 & 0.922 & -2.582 \\ 2.390 & -1.771 & -0.715 & 2.724 \end{bmatrix}. \qquad (10)$$

$\det(\mathbf{A}) = -0.169$, and the matrix is far from singular.[36] The lengths of the normalized projection vectors[24] are 0.998, 0.381, 0.519, and 0.994 for the zeroth, first, second, and third orders, respectively. The data that produced the first three columns of **A** are shown in Fig. 4. I_m are the output signals of the four photodetectors normalized with respect to the output signal of the reference detector, D_r, and P is the azimuth of the incident linear polarization. The diamonds are the experimental points, and the continuous curves are the best-fit theoretical curves, which include the dominant $\cos(2P)$ and $\sin(2P)$ terms plus much smaller $\cos(4P)$ and $\sin(4P)$ terms that represent mostly a signal-level-dependent quantization error. (Inclusion of the fourth harmonics does not affect the determination of the matrix elements in the first three columns.) The residual rms error is $\lesssim 0.001$.

The instrument's ability to determine a broad range of elliptical polarization states was checked with the fixed-polarizer rotating quarter-wave-retarder (QWR) test (also called the figure-of-8 test because of the shape of the locus

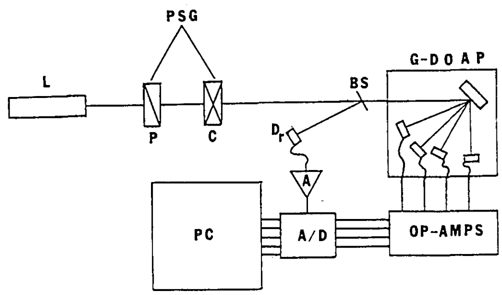


Fig. 3. Experimental setup for the calibration and testing of the planar G-DOAP, the grating-based division-of-amplitude photopolarimeter. L is a 632.8-nm laser source, PSG is a polarization-state generator that consists of crystalline polarizer P and quarter-wave plate (or compensator) C, BS is a beam splitter, and D_r is a reference detector that is followed by an amplifier, A. OP-AMPS are operational amplifiers, A/D are analog-to-digital converters, and PC is a desktop personal computer.

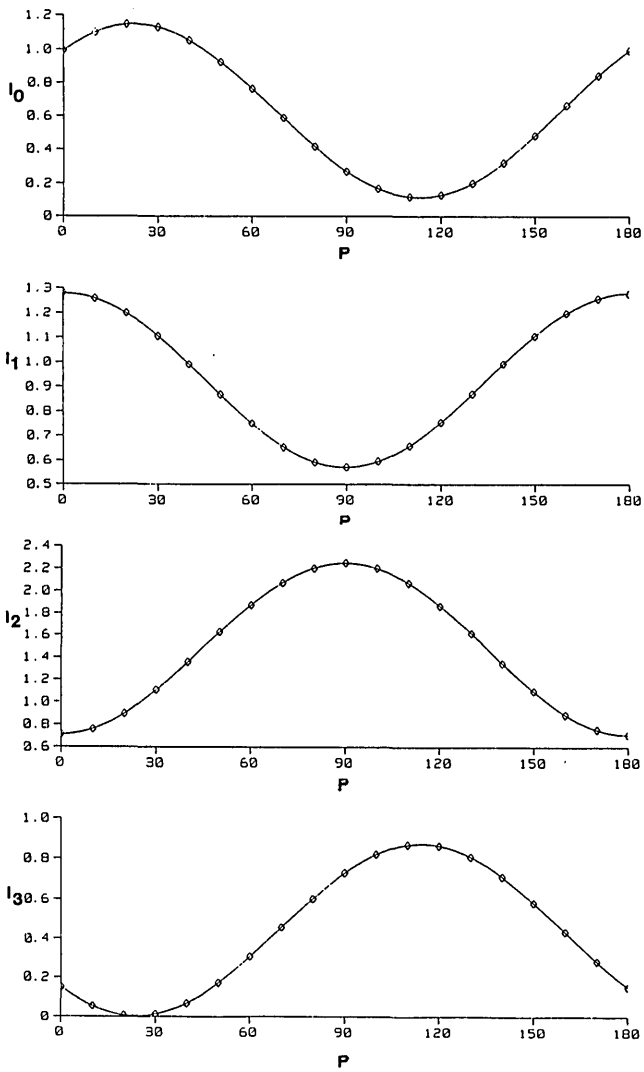


Fig. 4. Experimental data and best-fit curves that are involved in the determination of the first three columns of the instrument matrix of Eq. (9). See text.

of the test states on the Poincaré sphere[21]). The deviations of the G-DOAP-determined normalized Stokes parameters from those produced by an ideal polarization-state generator with an assumed perfect QWR are shown in Fig. 5. The diamonds are the experimental data points, and the continuous curves are generated by means of the imperfection model of the QWR discussed in Ref. 23. The residual rms error is ≈0.001 for all three normalized Stokes parameters, after a correction is made for the dc offset in the third Stokes parameter [Fig. 5(c)]. We therefore conclude that our prototype G-DOAP in the planar diffraction configuration can measure the normalized Stokes parameters with a precision of the order of 0.001, which is determined primarily by the digitization error associated with the 12-bit analog-to-digital converter.

6. SELECTION OF THE INCIDENCE ANGLE AND OF DIFFRACTED ORDERS FOR THE PLACEMENT OF POLARIZERS

In operating the G-DOAP in the planar configuration, we set the grating to produce autocollimation for the fourth

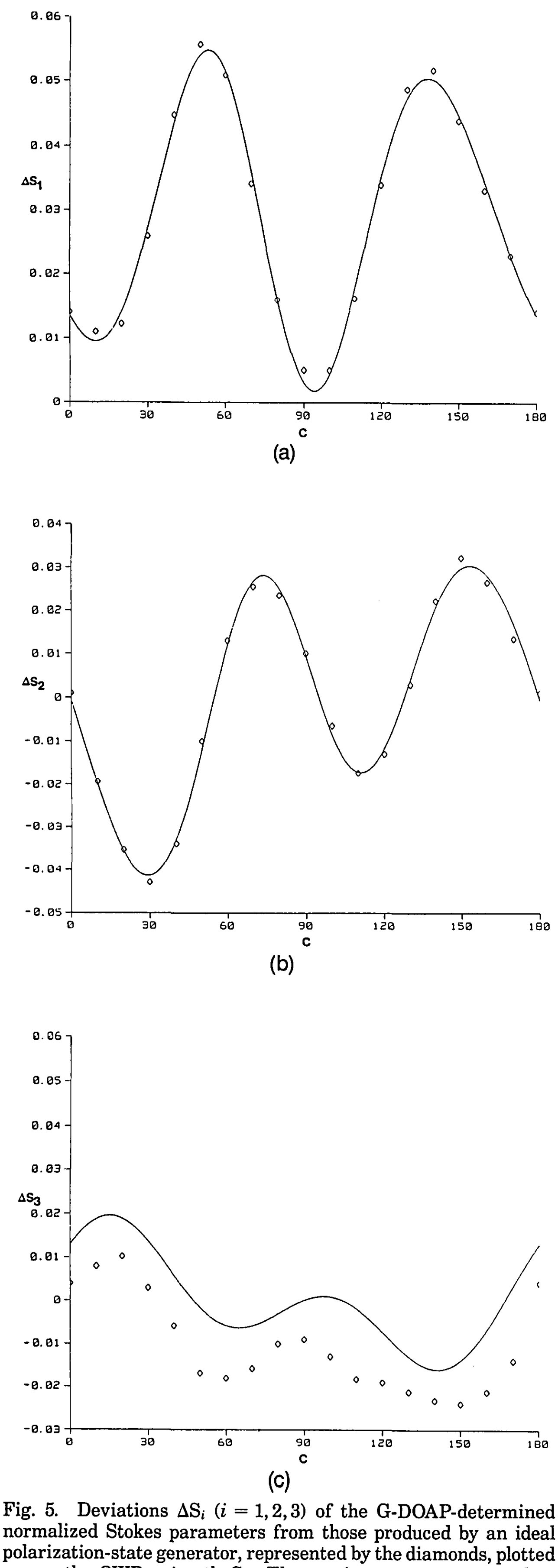


Fig. 5. Deviations ΔS_i ($i = 1, 2, 3$) of the G-DOAP-determined normalized Stokes parameters from those produced by an ideal polarization-state generator, represented by the diamonds, plotted versus the QWR azimuth C. The continuous curves are calculated using the imperfect-QWR model of Ref. 23.

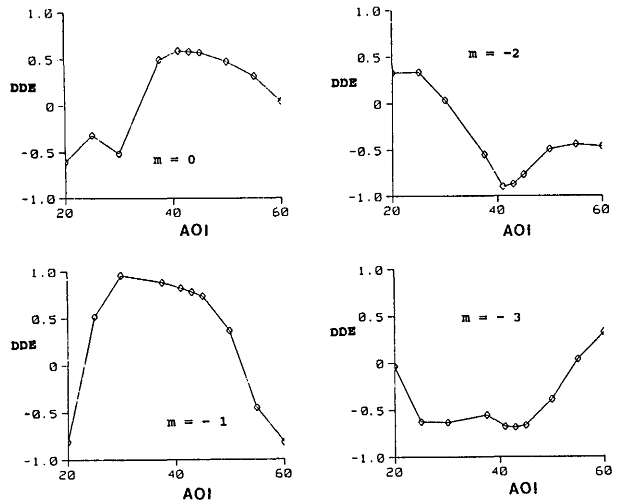


Fig. 6. The measured differential diffraction efficiency, DDE, for p- and s-polarized light for four diffracted orders ($m = 0, -1, -2, -3$) plotted versus the angle of incidence (AOI).

diffracted order as a means of alignment. Inasmuch as alignment can also be achieved by a position-sensing quadrant detector placed in one of the diffracted orders [see Fig. 2(a)], other choices of the incidence angle are possible. For the grating that we have selected, another good choice of incidence angle is 41°. At this angle the instrument matrix elements are insensitive to small angle-of-incidence errors. This is illustrated in Fig. 6, which shows the differential diffraction efficiency[37] for p- and s-polarized light for the zeroth, first, second, and third orders plotted versus the angle of incidence. Figure 6 also indicates that, at 41°, the first and second orders are nearly orthogonally linearly polarized. Obviously, these are the two orders in which no polarizers need to be placed. Another factor that should be considered in selecting orders for the placement of polarizers is the partition of energy among the diffracted orders for unpolarized incident light. Clearly, polarizers may be placed in those more intense orders of higher average diffraction efficiency.[37]

7. OUTLOOK

Grating diffraction has been central to spectroscopy and its many applications in virtually every branch of science and technology. Our research is intended to broaden the domain of the use of diffraction gratings to include photopolarimetry and spectrophotopolarimetry. Specifically, the multiple-beam-splitting, polarization-altering, and dispersive properties of diffraction gratings, when combined with currently available high-resolution detector arrays, make possible the realization of a new Stokes-parameter spectrophotopolarimeter. This instrument is capable of time-resolved and spectral measurements of the state of polarization of light without the use of any moving parts or modulators. The existence of extra diffracted orders (>the required minimum of 4) increases the accuracy with which the Stokes vector is determined from redundant projections.[8] Furthermore, self-alignment is facilitated by autocollimation or by the use of a position-sensing (quadrant) detector to intercept one of the diffracted beams. Thus the positioning of the instrument relative to an incoming beam can be reproduced accurately between calibration and measurement. The use of planar diffraction as demonstrated in this paper simplifies the geometry and reduces the size of the instrument. Furthermore, the flexibility and control that are available in the production of gratings of the desired spatial frequencies, groove shapes, and coating materials can be used to optimize the performance of the grating photopolarimeter. Also, the control of the grating substrate curvature adds an imaging feature to the polarimeter that is not found in other polarimeter designs.

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