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Achieving a given reflectance for unpolarized light by controlling the incidence angle and the thickness of a transparent thin film on an absorbing substrate: application to energy equipartition in the four-detector photopolarimeter

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Achieving a given reflectance for unpolarized light by controlling the incidence angle and the thickness of a transparent thin film on an absorbing substrate: application to energy equipartition in the four-detector photopolarimeter

R. M. A. Azzam and Kurt A. Giardina

At a given wavelength λ we determine all possible solution pairs (ϕ, ζ) of the incidence angle ϕ and the thickness ζ of a transparent thin film on an absorbing substrate that achieve a given unpolarized light reflectance $\mathscr{R}_u$. The trajectory of the point that represents a solution pair in the ζ, ϕ plane depends on the optical properties of the film and substrate and on whether $\mathscr{R}_u$ is greater than or less than the normal-incidence reflectance $\overline{\mathscr{R}}_0$ of the bare substrate. When $\mathscr{R}_u > \overline{\mathscr{R}}_0$, the specified reflectance is achieved over a limited range of ϕ. At the least possible incidence angle, the film thickness is $\approx$ ⅛th wave. As an application we consider SiO_2 films on Si detectors that produce $\mathscr{R}_u$ = 0.75, 0.6667, and 0.50 at λ = 337 and 633 nm. If the first three detectors of the four-detector photopolarimeter (FDP) are coated to have these reflectance levels, with the reflectance diminishing in the direction of propagation of the light beam, and the last detector is antireflection coated (e.g., with a quarter-wave Si_3N_4 layer), equipartition of energy among the four detectors is accomplished for incident unpolarized light. Such a condition is desirable in the operation of the FDP. The ellipsometric parameters of the coated surfaces and the FDP instrument matrix are also calculated.

I. Introduction

Consider the external reflection in air of unpolarized quasi-monochromatic light[1] at an angle of incidence ϕ by the plane surface of a transparent (dielectric) or absorbing (semiconductor or metallic) substrate, Fig. 1(a), which may be coated by a transparent thin film[2] of uniform thickness d, Fig. 1(b). Suppose that it is required to achieve a specified or given reflectance level $\mathscr{R}_g > 0$. For a bare substrate, the desired reflectance can be attained at a particular ϕ if $\mathscr{R}_g > \overline{\mathscr{R}}_0$, where $\overline{\mathscr{R}}_0$ is the bare-substrate reflectance at normal incidence and ϕ is obtained by solving the equation

$$\mathscr{R}_u(\phi) = \mathscr{R}_g. \tag{1}$$

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Reflectances in the $0 < \mathscr{R}_g < \overline{\mathscr{R}}_0$ range may or may not be realized by a bare substrate. This depends on the substrate complex refractive index $N_2 = n_2 - jk_2$, which determines whether $\mathscr{R}_u(\phi)$ is a monotonic or nonmonotonic function[3] of ϕ.

In this paper we deal with the more general case of a substrate that is coated by a dielectric thin film, Fig. 1(b). Instead of the metric film thickness d, we use the normalized thickness

$$\zeta = d/D_\phi, \tag{2}$$

where

$$D_\phi = (\lambda/2)(N_1^2 - \sin^2\phi)^{-1/2} \tag{3}$$

is the film thickness period, λ is the vacuum wavelength of light, and N_1 (real) is the film refractive index. For the coated surface, Eq. (1) is replaced by

$$\mathscr{R}_u(\zeta, \phi) = \mathscr{R}_g. \tag{4}$$

The unpolarized light reflectance $\mathscr{R}_u$ is itself deter-

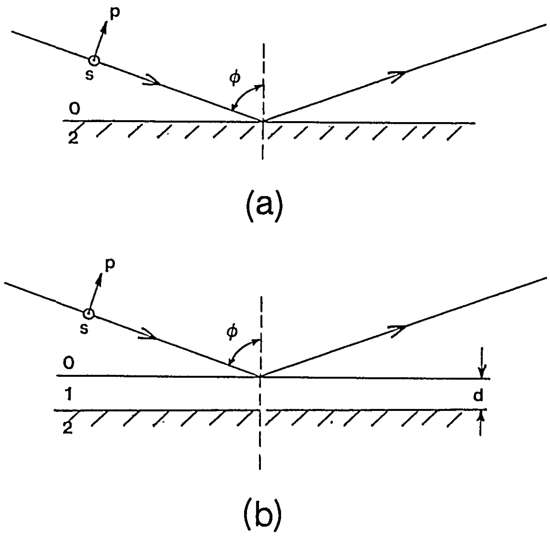

Fig. 1. Reflection of light by (a) a bare substrate, (b) a substrate coated by a uniform layer of thickness d.

mined by

$$\mathscr{R}_u = (\mathscr{R}_p + \mathscr{R}_s)/2 = (R_p R_p^* + R_s R_s^*)/2, \qquad (5)$$

where R_p and R_s are the complex reflection coefficients for the p and s linear polarizations, parallel and perpendicular to the plane of incidence, respectively. The mathematical expressions for R_p and R_s of a bare or coated substrate can be found elsewhere[4] and are not repeated here.

For a given film–substrate system with known optical properties (N_1, N_2) at a given wavelength λ, all possible solutions of Eq. (4) are found for set values of $\mathscr{R}_g$. The other reflection characteristics of the coated surfaces that satisfy Eq. (4), namely, the p and s reflectances $\mathscr{R}_p$ and $\mathscr{R}_s$ and the ellipsometric angles ψ and Δ, where $\tan \psi \exp(j\Delta) = R_p/R_s$, are also calculated.

Our interest in this problem originates in a requirement that the first, second, and third detectors of the four-detector photopolarimeter (FDP), Fig. 2, have

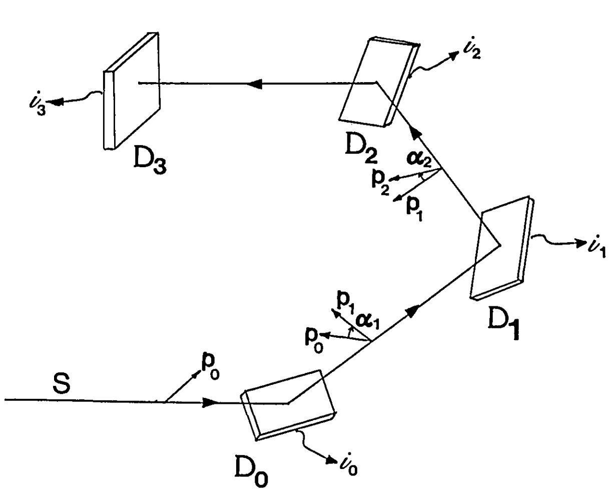

Fig. 2. Four-detector photopolarimeter.

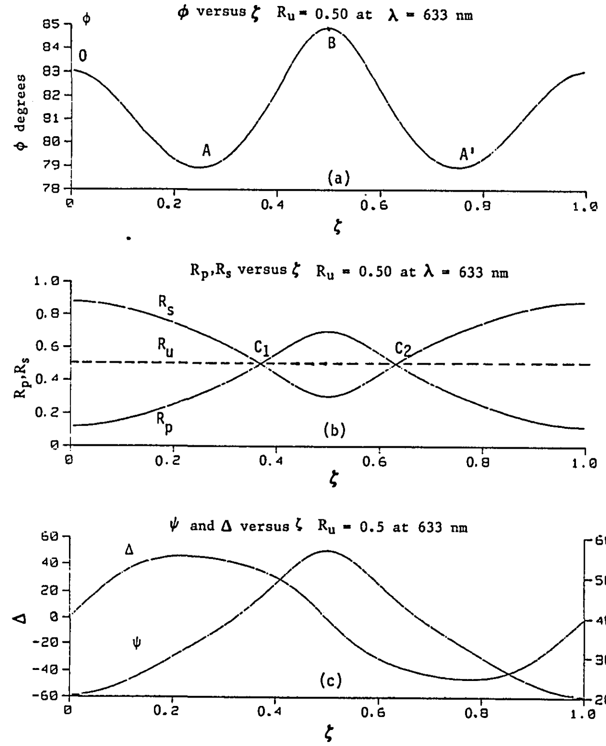

Fig. 3. (a) Constraint on the normalized thickness ζ of an SiO_2 film on a Si substrate and the angle of incidence ϕ such that the unpolarized light reflectance $\mathscr{R}_u = 0.50$ at $\lambda = 633$ nm. (b) Reflectances $\mathscr{R}_p$ and $\mathscr{R}_s$ for the p and s polarizations versus the normalized film thickness ζ of the SiO_2 coatings on Si that achieve $\mathscr{R}_u = 0.50$ at $\lambda = 633$ nm. The average reflectance $(\mathscr{R}_p + \mathscr{R}_s)/2$ = constant = 0.5 is as expected. (c) Ellipsometric angles ψ and Δ versus ζ for the same SiO_2 films on Si for which $\mathscr{R}_u = 0.5$.

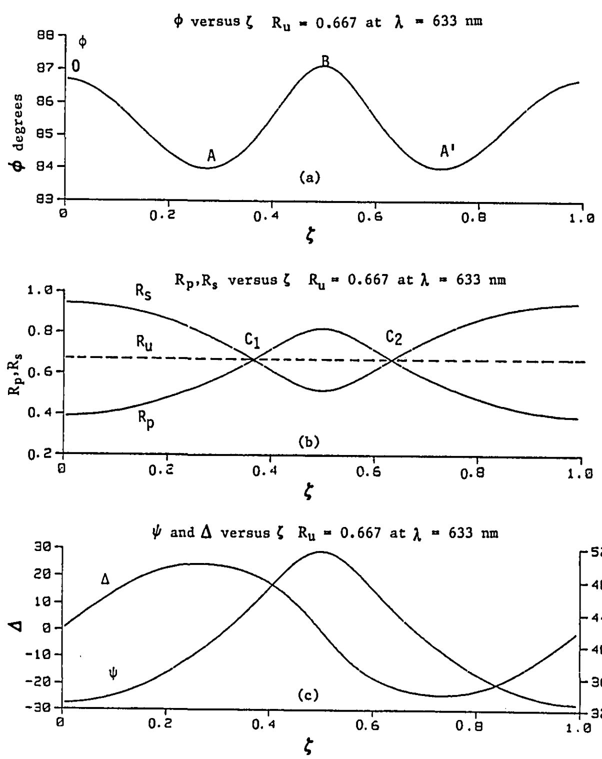

Fig. 4. Same as Fig. 3 except that $\mathscr{R}_u = 0.667$.

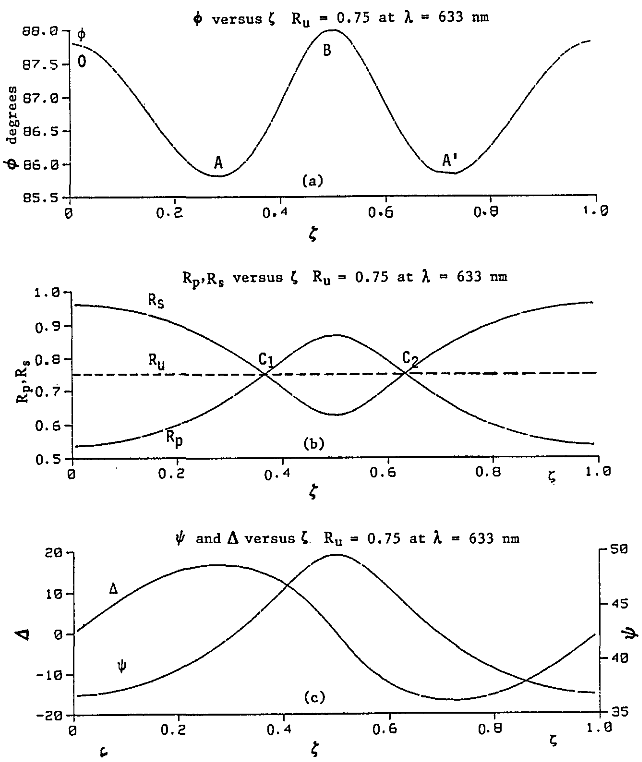

Fig. 5. Same as Fig. 3 except that $\mathscr{R}_u = 0.75$.

unpolarized light reflectances of 0.75, 0.6667, and 0.50, respectively, in order to accomplish equipartition of energy among the four detectors when the input light is unpolarized.[5] The latter equipartition condition is intuitively desirable but not essential for the operation of the FDP. For this reason, results are presented for the SiO_2–Si film–substrate system (corresponding to oxide-layer-coated Si detectors) at two laser wavelengths (633 and 337 nm) and for $R_g = 0.5$, 0.6667, and 0.75.

Other possible applications of this work include beam splitters that use transparent thin films on dielectric surfaces with specified reflectance and transmittance and reflectance standards that use dielectric-coated (protected) metal surfaces.

2. SiO_2 Coatings on Si Detectors at $\lambda = 633$ nm

The complex refractive indices of SiO_2 and Si at the (He–Ne laser) wavelength $\lambda = 633$ nm are taken to be $N_1 = 1.46–j0$ and $N_2 = 3.85–j0.02$, respectively.[6,7] For a given $\mathscr{R}_g$, Eq. (4) is more easily solved by assuming ϕ and iterating on ζ until the equation is satisfied. All the multiple solutions in the reduced thickness interval $0 \leq \zeta < 1$ are determined at a given ϕ. All the reflection properties are periodic with film thickness d with period D_ϕ, so that, if (ζ, ϕ) is a solution of Eq. (4), $(\zeta + m, \phi)$ is also a solution, where m is an integer. These higher thickness solutions are not desirable practically.

Figure 3(a) shows the locus of all possible solutions of Eq. (4) in the ζ, ϕ plane for $\mathscr{R}_u = 0.5$ (third detector of the FDP). Solutions exist over an angular interval of $\approx 6°$, $78.95° \leq \phi \leq 84.87°$. Special points of interest are marked by O, A, A′, and B. Obviously O corresponds to the bare Si substrate. The design represented by point A is important in that it corresponds to the lowest angle ($\phi_A = 78.95°$) at which $\mathscr{R}_u = 0.5$ can be attained by the SiO_2–Si system at 633 nm. Interestingly, the associated normalized film thickness $\zeta_A = 0.25$ corresponds to a layer of ⅛th wave optical thickness. Point A′ represents an almost equivalent design at which $\zeta_{A'} \cong 1 - \zeta_A = 0.75$, and $\phi_{A'} \cong \phi_A$ ($\phi_{A'}$ is slightly $> \phi_A$). At B, $\zeta = 0.5$ (or quarter-wave layer) and $\mathscr{R}_u = 0.5$ is achieved at the highest angle ($\phi_B = 84.87°$).

In Fig. 3(b) the p and s reflectances $\mathscr{R}_p$ and $\mathscr{R}_s$ are plotted versus ζ for every point along the solution curve of Fig. 3(a). $\mathscr{R}_u = (\mathscr{R}_p + \mathscr{R}_s)/2$ is a constant = 0.5 independent of ζ as required by design. Points C_1 and C_2, where $\mathscr{R}_p = \mathscr{R}_s = \mathscr{R}_u = 0.5$, represent unacceptable designs because, in such conditions, the instrument matrix of the FDP becomes singular.[5]

Figure 3(c) shows the associated ψ and Δ versus ζ. For polarimetric analysis using the FDP, Δ of the third detector may assume any value. However, ψ must differ sufficiently from 45°. This is the case for the design of point A, where $\psi_A = 33.92°$ and ϕ is minimum.

Figure 4 shows the corresponding results when $\mathscr{R}_u = 0.667$, which is the reflectance required for the second detector of the FDP. The range of ϕ over which Eq. (4) has a solution is $83.99° \leq \phi \leq 87.13°$. The preferred design corresponds to point A where ϕ is at a minimum, $\phi_A = 83.99°$, and $\zeta_A = 0.27$, i.e., the required SiO_2 film has a thickness slightly > ⅛th wave. The differential reflection phase shift Δ at the second detector is important.[5] Fortunately, $\Delta_A = 24.00°$ is as far as possible from the singular value of 0. Also, $\psi_A = 39.82°$ is sufficiently removed from the singular value of 45°.

Figure 5 shows the results for $\mathscr{R}_u = 0.75$ that correspond to the first detector of the FDP. The

Table I. Characteristics of SiO_2-Coated Si Detectors that Achieve Equipartition of Energy for Incident Unpolarized Light of Wavelength $\lambda = 633$ nm

Detector	$\mathscr{R}_u$	ζ	d (nm)	ϕ°	ψ°	Δ°	L (mm)
1st (D_0)	0.750	0.27	80.14	85.82	41.05	16.62	13.71
2nd (D_1)	0.667	0.27	79.94	83.99	39.82	24.00	9.56
3rd (D_2)	0.500	0.25	73.20	78.95	33.92	45.24	5.21

$\mathscr{R}_u$ is the unpolarized light reflectance, ζ and d are the normalized and metric thicknesses of the SiO_2 film, ϕ is the angle of incidence, ψ and Δ are the surface ellipsometric parameters, and L gives the length of the major axis of the elliptical spot cast on the detector surface by a 1-mm-diameter beam.

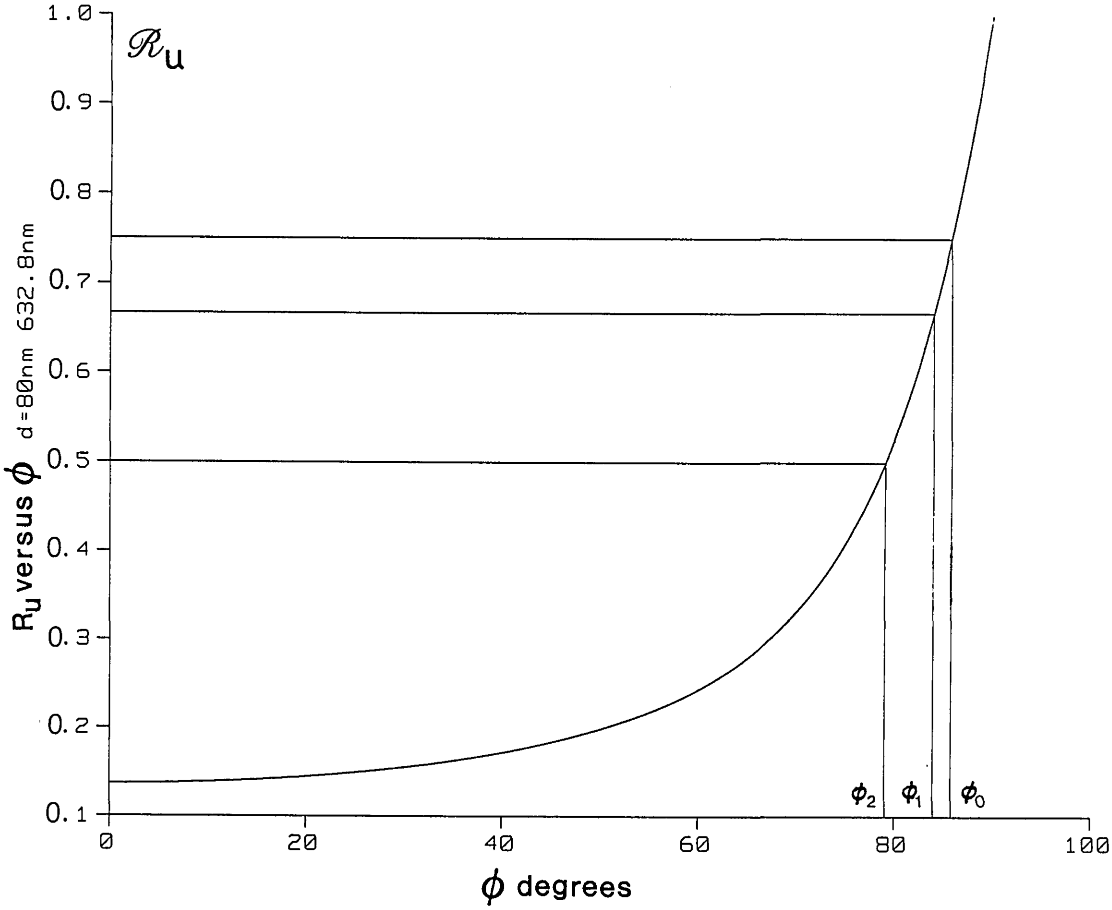

Fig. 6. Unpolarized light reflectance $\mathscr{R}_u$ versus angle of incidence ϕ for an 80-nm SiO_2 film on Si at $\lambda = 633$ nm. ϕ_0, ϕ_1, and ϕ_2 are the angles of incidence at which $\mathscr{R}_u = 0.75$, 0.667, and 0.50, respectively.

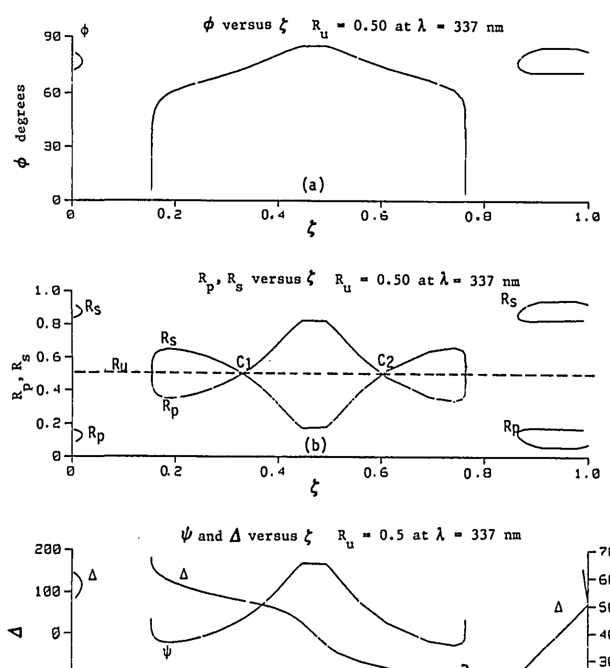

Fig. 7. Same as Fig. 3 except that here $\lambda = 337$ nm.

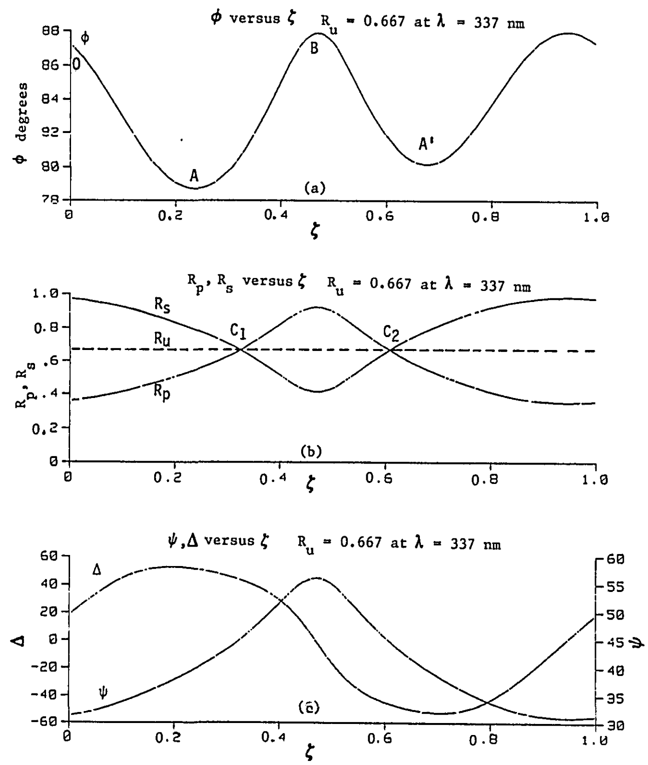

Fig. 8. Same as Fig. 3 except that here $\lambda = 337$ nm and $\mathscr{R}_u = 0.667$.

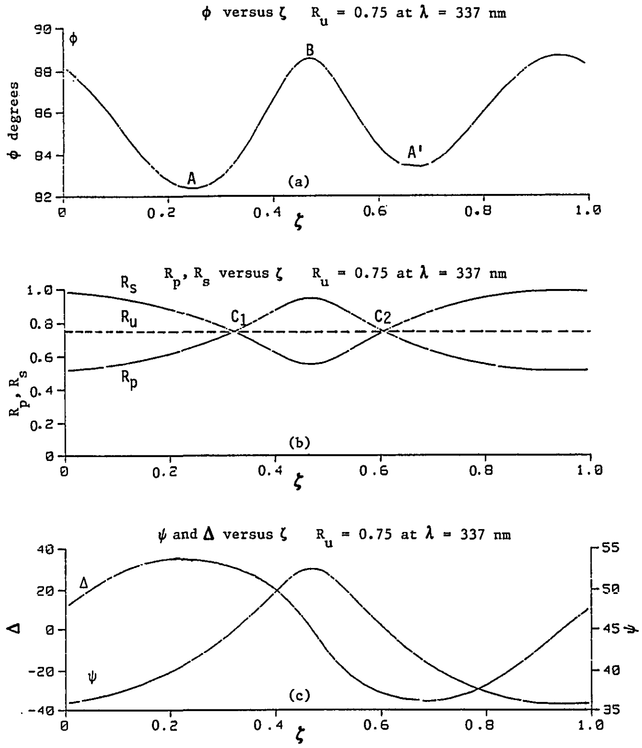

Fig. 9. Same as Fig. 3 except that here $\lambda = 337$ nm and $\mathscr{R}_u = 0.75$.

increased reflectance causes the angular interval over which Eq. (4) has a solution to narrow further and move toward higher values: $85.82° \leq \phi \leq 87.99°$. As before, A is the desirable operating point where $\phi_A = 85.82°$ (minimum), $\zeta_A = 0.27$, and $\psi_A = 41.05°$, which is sufficiently far from the singular value of 45°.

Table 1 summarizes the data for the coatings that produce the desired reflectances at the least possible angles of incidence for a FDP that uses Si detectors and operates at 633 nm. The last column of the table lists the length L (in millimeters) of the major axis of the elliptical spot cast on the detector surface when it intercepts a 1-mm-diameter beam at the indicated angles of incidence. If a rectangular detector is used, its long side must be $>L$ and it must be oriented parallel to the plane of incidence.

3. Simple Design with Identically Coated Detectors

From Table 1 it is apparent that the thicknesses of the SiO_2 films on the three Si detectors are nearly equal. This suggests a simple solution in which all the detectors are identically coated by a SiO_2 film of the same thickness, $d = 80$ nm. Figure 6 is a graph of $\mathscr{R}_u$ versus ϕ for an 80-nm SiO_2 film on Si at 633-nm wavelength. The three lines that represent $\mathscr{R}_u = 0.5$, 0.667, and 0.75 parallel to the ϕ axis are superimposed in Fig. 6 and intersect the $\mathscr{R}_u$-versus-ϕ curve at the operating angles of incidence of 85.7°, 83.9°, and 78.8°, respectively. These angles differ little from those listed in Table 1.

The instrument matrix $\mathscr{A}$, which relates the output current vector $\mathscr{I}$ of the FDP to the input Stokes vector $\mathscr{S}$ by $\mathscr{I} = \mathscr{A}\mathscr{S}$, is calculated as described in Ref. 5 for the identically coated first three detectors (80-nm SiO_2 film) and with each detector set at the incidence angle specified in Fig. 6. We assume optimum rotations of 45° between the second and the first and between the third and the second planes of incidence, and we assume unit responsivity for each detector. The last detector is considered to be coated with an antireflection layer of Si_3N_4 of refractive index 1.91 and of $\approx \lambda/4$ optical thickness ($d = 80$ nm also). This resulting matrix is given below:

$$\mathscr{A} = \begin{bmatrix} 0.254 & 0.105 & 0.000 & 0.000 \\ 0.251 & -0.035 & 0.092 & 0.029 \\ 0.260 & -0.107 & -0.056 & 0.017 \\ 0.235 & 0.037 & -0.036 & -0.045 \end{bmatrix}. \quad (6)$$

The fact that all the elements of the first column of $\mathscr{A}$ are nearly equal (and ≈ 0.25) indicates that energy equipartition among the detectors has been essentially attained.[8] The polarization sensitivities of the individual detectors are determined by the lengths of the respective normalized projection vectors[9] which, from Eq. (6), are 0.415, 0.411, 0.469, and 0.290, for the 1st, 2nd, 3rd, and 4th detectors, respectively.

4. SiO_2 Coatings on Si Detectors at $\lambda = 337$ nm

The complex indices of refraction of SiO_2 and Si at this (Ar-ion laser) wavelength are $1.48–j0$ and $5.179–j3.039$, respectively, and the bare-substrate Si reflectance at normal incidence is $\overline{\mathscr{R}}_0 = 0.563$. For $\mathscr{R}_u = 0.5$ (the 3rd detector of the FDP), the constraint on ζ and ϕ such that Eq. (4) is satisfied is shown in Fig. 7(a). The nature of the locus of the (ζ, ϕ) solution pair at $\lambda = 337$ nm is significantly different from that shown in Fig. 3(a) for the same $\mathscr{R}_u = 0.5$ at $\lambda = 633$ nm. The reason is that $\mathscr{R}_u < \overline{\mathscr{R}}_0$ and Eq. (4) can be satisfied at all angles of incidence from 0 to 85.0°. For $0 < \phi < 70°$, Eq. (4) has two solutions for ζ; for $71.0 < \phi < 83.0°$, Eq. (4) has four solutions; and for $84.0 < \phi <$

Table II. Characteristics of SiO_2-Coated Si Detectors that Achieve Equipartition of Energy for Incident Unpolarized Light of Wavelength $\lambda = 337$ nm

Detector	$\mathscr{R}_u$	ζ	d (nm)	$\phi°$	$\psi°$	$\Delta°$	L (mm)
1st (D_0)	0.750	0.24	36.8	82.39	41.24	34.84	7.55
2nd (D_1)	0.667	0.23	26.4	78.73	39.45	51.43	5.11
3rd (D_2)	0.500	0.20	24.2	60.00	36.32	−55.30	2.00

$\mathscr{R}_u$ is the unpolarized light reflectance, ζ and d are the normalized and metric thicknesses of the SiO_2 film, ϕ is the angle of incidence, ψ and Δ are the surface ellipsometric parameters, and L gives the length of the major axis of the elliptical spot cast on the detector surface by a 1-mm-diameter beam.

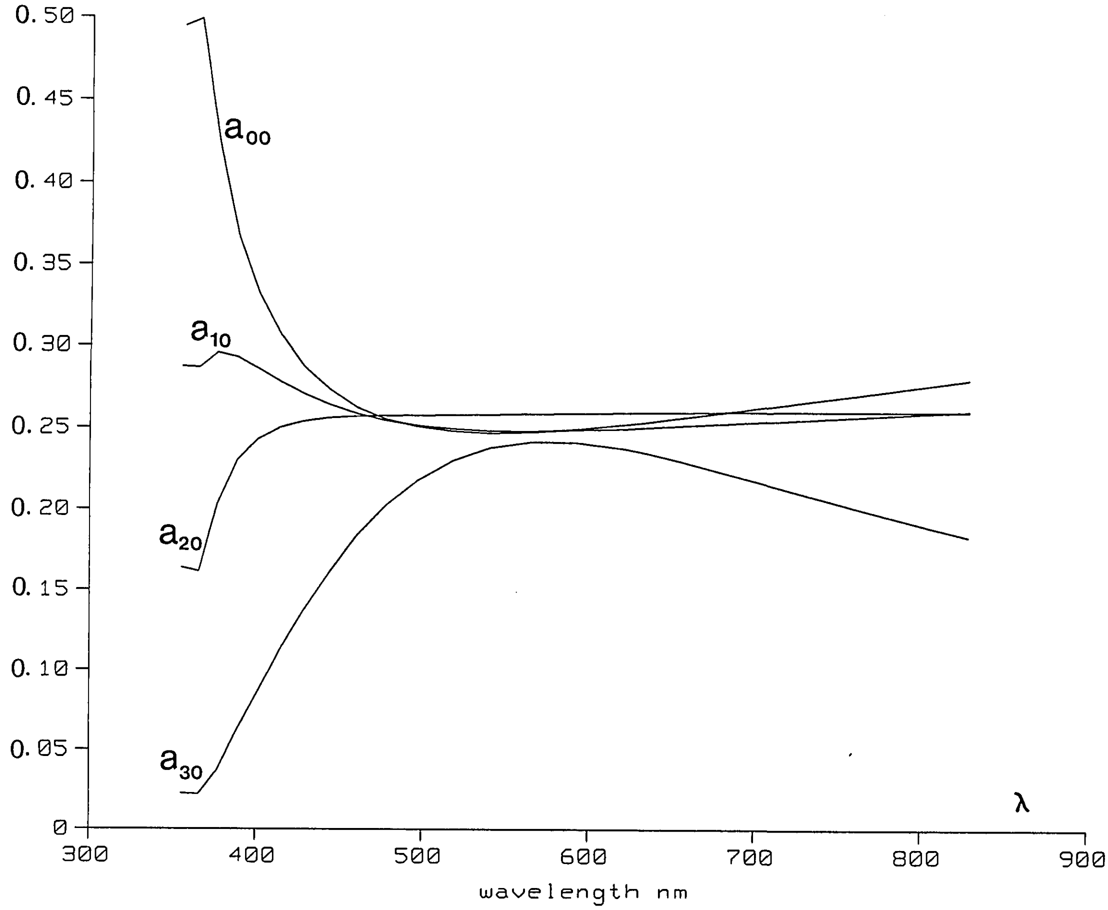

Fig. 10. Spectral dependence of the elements a_{00}, a_{10}, a_{20}, and a_{30} of the first column of the normalized instrument matrix $\mathscr{A}$ of a FDP that uses Si detectors with an 80-nm SiO_2 film on the first three detectors and an 80-nm Si_3N_4 film on the last. Near equality of the four elements indicates near equipartition of energy among the four detectors for incident unpolarized light.

85.0°, Eq. (4) again has two solutions only in the $0 < \zeta < 1$ range.

Figures 7(b) and 7(c) show the associated $(\mathscr{R}_p, \mathscr{R}_s)$ and (ψ, Δ) plotted versus ζ, respectively. A good operating point is that marked by A at which $\zeta_A = 0.2$, $\phi_A = 60°$, and $\psi_A = 36°$. Figure 8 shows the results for $\mathscr{R}_u = 0.667$ (2nd detector of the FDP) and $\lambda = 337$ nm. Because $\mathscr{R}_u > \mathscr{R}_0$, the trajectory of the point (ζ, ϕ) that represents a solution of Eq. (4) is confined within a limited range of ϕ, $78.7° \leq \phi \leq 87.9°$, Fig. 8(a). Note that $\phi_{A'} \neq \phi_A$ and $\zeta_{A'} \neq 1 - \zeta_A$ by a significant margin because of the large extinction coefficient of Si ($k_2 = 3.039$) at this near-UV wavelength [compare these results with those shown in Fig. 3(a)]. The best operating point is A where ϕ is minimum, $\phi_A = 78.7°$, $\zeta_A = 0.23$, $\psi_A = 39.45°$, and $\Delta_A = 51.43°$; $\sin\Delta_A = 0.79$ is near the optimum value of 1 and ψ_A is adequately removed from the singular value of 45°.

Finally, Fig. 9 shows the results for $\mathscr{R}_u = 0.75$ (1st detector of the FDP) and $\lambda = 337$ nm. The suggested operating point is again marked by A, where $\phi_A = 82.4°$, $\zeta_A = 0.24$, and $\psi_A = 41.24°$. The latter value of ψ_A indicates weak polarization sensitivity for the 1st detector. Δ at the 1st (and 3rd) detector has no bearing on the singularity condition of the instrument matrix.[5] Table 2 summarizes the design data for SiO_2-coated Si detectors that have the required reflectances for energy equipartition in the FDP with unpolarized incident light at $\lambda = 337$ nm. The 4th detector should be antireflection coated to accomplish equipartition. A Si_3N_4 layer, $N_1 = 2.00$ and $d = 37$ nm, reduces the normal-incidence reflectance of Si from 56.3% to 8.05% at 337 nm.

Table 2 also suggests a simple design in which the first three detectors are coated by a SiO_2 film of the same thickness, say $d = 40$ nm. By use of a construction similar to that in Fig. 6, we determine the angles of incidence at the 1st, 2nd, and 3rd detectors to be 82.42°, 78.90°, and 67.65° to achieve reflectances of 0.75, 0.667, and 0.50, respectively.

5. Operation over an Extended Spectral Range

Suppose that the simple monochromatic FDP design with identical coatings (80-nm SiO_2 film) on the first three detectors and with an antireflection layer (80-nm Si_3N_4 film) on the last detector, described in Section 3, is operated over an extended spectral range, $335 \leq \lambda \leq 829$ nm, without changing the detectors or the geometric configuration (i.e., light path). It is then of

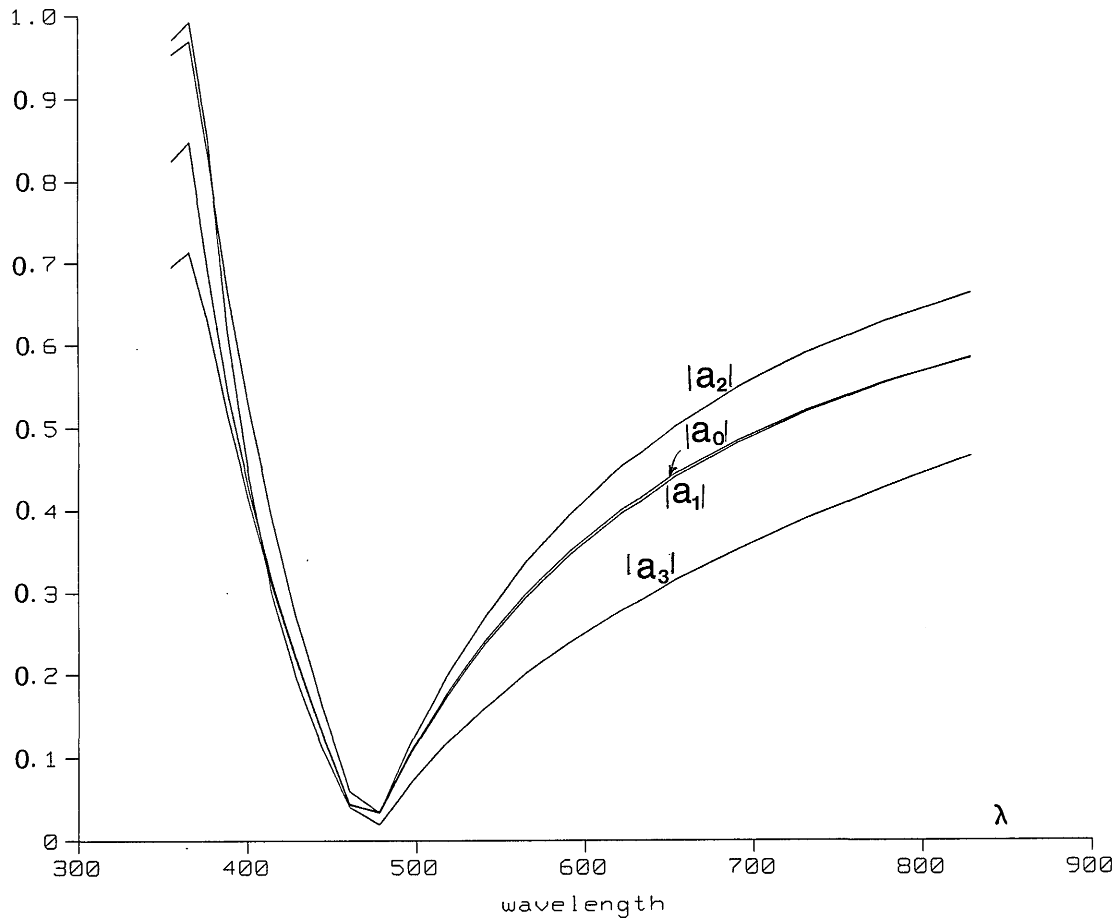

Fig. 11. Lengths of the normalized projection vectors $|\mathbf{a}_0|$, $|\mathbf{a}_1|$, $|\mathbf{a}_2|$, and $|\mathbf{a}_3|$ versus wavelength λ for a FDP of Si detectors with coatings and incident angles as specified in Fig. 10 (also see text).

interest to examine the effect of changing λ on the energy equipartition condition, the lengths of the normalized projection vectors, and the determinant of the instrument matrix. We have performed such calculations by using a previously published analysis[5] and by taking proper account of the dispersion of the optical properties of the coating materials (SiO_2 and Si_3N_4) and of the Si substrate.[6,7]

Figure 10shows the spectral dependence of the elements a_{00}, a_{10}, a_{20}, and a_{30} of the first column of the $\mathscr{A}$ matrix that determine the relative amounts of light absorbed by the 1st, 2nd, 3rd, and 4th detectors, respectively. Note that the four elements are nearly equal (and ≈ 0.25) for $500 \leq \lambda \leq 700$ nm, so that equipartition is satisfied reasonably well over this range, but that they diverge at shorter and longer wavelengths.

Figure 11 shows the lengths of the four normalized projection vectors $|\mathbf{a}_0|$, $|\mathbf{a}_1|$, $|\mathbf{a}_2|$, and $|\mathbf{a}_3|$ as functions of λ. The polarization sensitivity of all the detectors is minimum in the $450 \leq \lambda \leq 500$-nm range and increases substantially at shorter and longer wavelengths.

Finally, Fig. 12 shows the spectral response of the normalized determinant[5] $\det \mathscr{A}/(\Pi_{i=0}{}^{3}k_i)$, in which the denominator is the product of the responsivities of all four detectors. The determinant is zero, hence $\mathscr{A}$ is strictly singular at two wavelengths, $\lambda_1 = 360$ nm and $\lambda_2 = 470$ nm. Therefore, the FDP can be operated over the entire spectrum with the exception of the immediate neighborhoods of λ_1 and λ_2.

6. Summary

The constraint on the angle of incidence and the thickness of a transparent thin film on an absorbing substrate is determined such that a specified reflectance level for incident unpolarized light is achieved. Specific application is made to coatings on the 1st, 2nd, and 3rd detectors of the FDP that yield reflectances of 0.75, 0.667, and 0.5, respectively, at the least possible angles of incidence, without rendering the instrument matrix singular. These reflectance levels lead to the desirable condition of energy equipartition among the four detectors (assuming that the last detector is antireflection coated) for incident unpolarized light. Designs have been presented at two laser wavelengths (633 and 337 nm) and the operation of a monochromatic design over an extended spectral range has also been investigated.

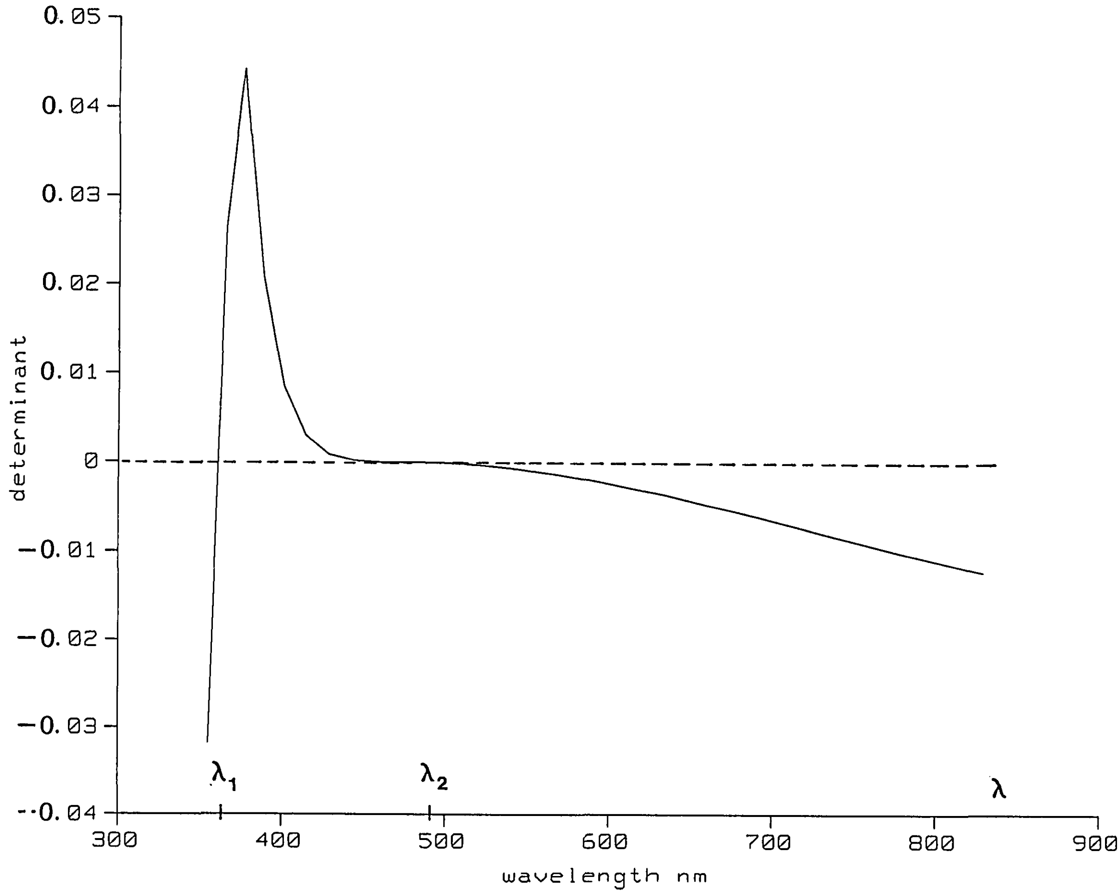

Fig. 12. Normalized determinant of the FDP instrument matrix versus wavelength λ. The Si detectors are coated and set at incident angles that are specified in the text. Singularity occurs at λ_1 and λ_2.

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References and Notes

1. The discussion applies more generally to incident partially circularly polarized light with any degree of polarization.
2. All media are assumed to be homogeneous, linear, optically isotropic, and nonmagnetic.
3. See, for example, W. R. Hunter, "Errors in using the reflectance vs angle of incidence method for measuring optical constants," J. Opt. Soc. Am. **55,** 1197–1204 (1965).
4. See, for example, R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, Amsterdam, 1987).
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8. An additional condition for strict equipartition as defined in Ref. 5 is that Δ_1 at D_1 be optimum (= ±90°), which is not satisfied by the simple design under consideration.
9. R. M. A. Azzam, "Instrument matrix of the four-detector photopolarimeter: physical meaning of its rows and columns and constraints on its elements," J. Opt. Soc. Am. A **7,** 87–91 (1990).