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# NEW EVIDENCE ON INTEREST RATE AND FOREIGN EXCHANGE RATE MODELING

A Dissertation

Submitted to the Graduate Faculty of the University of New Orleans in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy in The Department of Financial Economics

by

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August 2003

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# DEDICATED

To my parents. Their encouragement, determination and assistance enabled me to bring this project to completion.

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## ABSTRACT

This dissertation empirically and theoretically investigates three interrelated issues of market anomalies in interest rates derivatives and foreign exchange rates. The first essay models the spot exchange rate as a decomposition of permanent and transitory components. Unlike extant analysis, the transitory component could be stationary or explosive. The second essay examines the market efficiency hypothesis in the foreign exchange markets and relates the rejection of forward rate unbiasedness hypothesis to the existence of risk premium not to the failure of rational expectation. The third essay examines the behavior of short-term riskless rate and models the risk free rate as a nonlinear trend stationary process.

While addressing these issues, these essays account for: (1) finite sample bias; (2) Unit root and other nonstationary behaviors; (3) the role of nonlinear trend; and (4) the interrelations between different behaviors.

Several new results have been gleaned from our analysis; we find that: (1) the spot exchange rates display a very slow mean aversion behavior, which implies the failure of the purchasing power parity; (2) there are positive autocorrelations across the long horizons overlapping returns increases overtime and then begin to decline at a very long horizon period; (3) the short-term riskless rate displays a nonlinear trend stationary process which is closer to driftless random walk behavior; (4) modifying the mean reverting short-term interest rates models to a nonlinear trend stationary shows an extreme improvement and outperforms all suggested models; (5) the traditional tests for rational expectations and market efficiency in the foreign exchange markets are subject to size distortions; (6) we relate the rejection of market efficiency in the foreign exchange markets documented across most currencies to the existence of risk premium not to the rejection of rational expectation hypothesis.

## INTRODUCTION

Over the last two decades, a large body of progress has been made in modeling the behavior of financial time series such as the spot foreign exchange rates and the short-term riskless rates. According to standard finance theory, both the spot exchange rates and the interest rates should be stationary processes. However, the main line of empirical research in the foreign exchange market continues to find evidence that the exchange rates display a unit root behavior, which is well known in the literature as the purchasing power parity puzzle. In the corresponding literature in interest rates derivatives, the stationary riskless rate models, whether they are arbitrage or general equilibrium, like Vasicek (1977) and Cox, Ingresoll and Ross (1985), still cannot fit the data generating process of the short-term interest rate. However, the random walk models like Cox and Ingresoll (1975) and Cox, Ingresoll and Ross (1980) outperform the mean reverting models especially if they have high volatility elasticity (a puzzle).

This dissertation contains three interrelated essays. The first develops a myopic fads model of prices that can explain the econometric behavior of the exchange rates spot exchange rates. Unlike extant analysis, the transitory component could be stationary or explosive. The existence of the transitory component of exchange rates implies that the holding periods returns are predictable. We find that the long horizon overlapping exchange rates returns display positive autocorrelations with a humped shaped pattern, which implies that the exchange rates display a very slow explosive process. Consistent with the fads model, the predictability cannot be captured across short period horizons. However, the results are inconsistent with the fads model in the since that the exchange rates display large swings about fundamentals. Instead, our results imply that the exchange rates display long swings i.e. the purchasing power parity does not hold.

The second essay examines the role of finite sample bias in tests for rational expectation and market efficiency in the foreign exchange market. In rationality and market efficiency hypotheses testing, the parameter estimates of regression equations are biased since the true mean of the predictors are unknown. A joint test of rationality and zero risk premia are conducted based on bootstrapping OLS and LAD estimators using data of six major currencies. After controlling for autocorrelation and heteroskedasticity, the results show that the bias is large enough to affect the statistical inference in empirical studies, and should be accounted for when testing for market efficiency. The rational expectation hypothesis cannot be rejected across all currencies. We relate the rejection of market efficiency in the foreign exchange market to the existence of foreign exchange risk premium.

The third paper examines a set of continuous short-term interest rate models and finds that previous work overstates the evidence of the level effect. The overstatement is mainly due to the stationarity assumption about the stochastic behavior of interest rates. Comprehensive diagnostic tests suggest that the short-term interest rate is nonlinear trend stationary. Based on this finding, I employ the Hodric-Prescot filter to locate the stationary component of the interest rate and compare the continuous time models based on this component. We document a substantial improvement across all mean reverting models and a substantial worsening across most of the heteroscedastic models. Finally, we modify the CIR SR model by replacing the drift term with the long-term nonlinear trend. We find that the modified model outperforms all competing models.

## ESSAY I

## Mean Aversion Down the Foreign Exchange Market

## I. Introduction

Over the last twenty years and more, a large body of progress has been made in the empirical studies of both the nominal and real exchange rates. However, the main line of empirical research continues to model the exchange rates as a unit root processes after the findings of Roll (1979), Frankel (1981), Alder and Lehman (1983), Hakkio (1984) and others. One implication of the unit root hypothesis is that a shock to real exchange rates induces a persistent movement and cannot revert as time passes. However, the traditional disequilibrium theory of exchange rate predicts that commodities prices adjust gradually and the nominal exchange rate adjusts eventually to a monetary shock; as a result, the real exchange rates are mean reverting (a Puzzle).

On the other hand, Hakkio (1986), who cannot reject the hypothesis of random walk, argues that the conventional unit root tests have a deviating local power against the alternative depending on the size of the autocorrelation. In a more elegant way, Sims (1988) argues that the use of the conventional unit root tests in the foreign exchange data are biased toward the acceptance of the random walk hypothesis if the size of the autocorrelation is large. As an alternative, he introduces a test that can discriminate

between the random walk and the slowly mean reverting behavior in the data generating process.

In the last decade there was a considerable amount of literature, which provide evidence that the foreign exchange spot rates do not have a unit root but a stationary behavior (see Huizinga (1987), Rogoff (1996), Wu and Chen (1998), and Papell and Theodoridis (1998), for example). Engle (2000) strongly contradicts this view; he argues that the power of the unit root tests used in such studies is very low and subject to large size biases. However, we should mention that the unit root tests would also be biased toward the acceptance of random walk if the exchange rates display a crashed or stochastic bubble behavior, since such behaviors are not included in the alternative hypothesis (see Evans (1991) and Charemza and Deadman (1995), for example).

In the corresponding literature in asset pricing, the long-horizon predictability conditional on past returns has been the focus of many studies of stock market efficiency. Following Summers (1986) it is well known that market efficiency tests have low power against the alternative where the fad behavior is difficult to be detected by examining the short horizons autocorrelations. Based on the variance ratio test of Cochrane (1988), Poterba and Summers (1988) and Lo and Mackinly (1988) find that the variance of returns increases at a rate which is less than proportional to the holding period. This implies that a substantial part of the variance in monthly returns is due to a predictable component. Fama and French (1988), using a univariate autoregression of multiyear stock returns, also present evidence of mean reverting behavior and conclude that about 35% of stock

variation is predictable from past returns. Further, Kim, Nelson, and Startz (1991), use stratified randomization to account for non-normality in the data, find that mean reversion is only a pre-war phenomenon. The autoregression parameter flips to be positive for the post-war period (mean aversion).

In this work, we first modify Fama and French (1988), hereafter F&F, model to allow for mean averting behavior in the exchange rates. Then, we implement the long horizon autoregression of F&F to examine the issue of predictability in the foreign exchange market in the sense of the fads model and test whether the long-horizon "buy low sell high" strategy can provide an abnormal return. Also, we perform a spectral-based test for each holding period return and test the hypothesis that the long horizon return is a sequence of serially independent process. However, we should mention that unlike the stock market, the existence of an abnormal return in the foreign exchange market does not necessarily imply that the market is irrational unless the exchange rate is an explosive process. For example, the traditional disequilibrium model of Dornbush (1976) implies that an unanticipated monetary shock causes the exchange rate to overshoot its long run equilibrium if the goods market prices are sticky, and as time elapses, the process returns back to its fundamentals. As a result, the participants in the foreign exchange market would still be rational. Conversely, another point of view is that the real exchange rate sometimes tends to deviate so far from its mean that the macroeconomic measurable fundamentals (like foreign demands on domestic goods) seem insufficient to derive the real exchange rates to its mean. Thus, we expect a speculative bubble or irrational

behavior in the foreign exchange market (see Frankel and Froot (1990), and Engel and Hamitlton (1990), for example).

To the best of our knowledge, this is the first paper that examines the long horizon predictability of returns in the foreign exchange market, which helps to fill in the gap in the empirical work. The fads model of Summers (1986) and Frankel and Froot (1990) suggests that the foreign exchange rates display large swings away from their long-run equilibrium (like the purchasing parity); as a result, even if the stationary component exists, it could be difficult to be detected by examining the short horizon periods as in previous studies. However, notice that the assumption of the fads model developed by Summers (1986) and Frankel and Froot (1990), sets a restriction that the transitory component of the spot exchange rate is stationary, therefore, it cannot explain the long swings in the exchange rate suggested by Engel and Hamilton (1990).

We use the Moving Blocks Bootstrap (MBB) suggested by Fitzenberg (1998). With minimal requirements on the data-generating process allowing for non-normality, which is evident of foreign exchange rates (see, Clark (1973), Tauchen and Pitts (1983), and Phillips (1996, 1997), for example). The new approach is robust to both the clustering heteroscedasticity and autocorrelations that result from the overlapping of returns (Hansen and Hodrick (1980), Kim, Nelson and Startz (1988), McQueen (1992), Frankel and Froot (1994), Daniel (2001), and Hansen (2002)). Second, it is robust to small sample bias that become more severe as the horizon of returns becomes longer since the number of the nonoverlapping returns observations are monotonically decreasing with the return

horizon (see, Fama and French (1988), Poterba and Summers (1988), Kriby (1997), Ang and Bekaert (2001), and Campbell (2001), for example). The use of the MBB technique is critical since the literature is thrust to such an estimator that is simultaneously robust to non-normality, finite sample bias, autocorrelation and heteroscedasticity.

Fitzenberger (1998) established the ability of the MBB to provide asymptotic refinements when the errors are both heteroskedastic and autocorrelated. It is shown that the MBB heteroscedastic and autocorrelated consistent covariance matrix, hereafter HAC, is equivalent to the Barlett kernel suggested by Newey and West (1987) and performs better when the samples exhibit temporal dependence<sup>1</sup>.

The remainder of the paper is constructed as follows: Section II develops a simple model of exchange rates returns that allow for a stationary or an explosive component in the spot exchange rates and sheds light on the complications of testing mean reversion. Section III develops the moving blocks bootstrap, Section IV outlines the data used. In section V, we test the hypothesis of mean reversion and explain the results. Section VI Concludes.

## II. A simple model of exchange rates returns

In this section we develop a simple model of exchange rate returns that allow the transitory component to be either stationary or explosive. We interpret the implications of each case to the autoregression suggested by Fama and French (1988). It is important to note that if the foreign exchange rates have an explosive component as well as a

<sup>&</sup>lt;sup>1</sup> Andrews (1991) shows that Barlett kernel HAC matrix of Newey and West behaves poorly when the data exhibits temporal dependence. This problem is more critical when the least square is used.

random walk component, then the holding period returns can still be predicted even though the exchange rates are persistent to shocks. We also review the econometric complications related to the long horizon regression and the commonly employed estimation procedures.

#### II.A. The model

Consider the F&F model of decomposing the time series and let the log of the exchange rate at time t,  $p_t$ , be the sum of a permanent component,  $q_t$ , and a transitory component,  $z_t$ , where  $q_t$ , follows a first order autoregression process (AR1) with drift  $\mu$ ,

$$p_t = q_t + z_t \tag{1}$$

$$q_t = q_{t-1} + \mu + \eta_t \tag{2}$$

Where  $\eta_t$  is a time series of serially uncorrelated shocks.

Let  $z_t$  be a slowly decaying (Summers 1986) or averting transitory component with a driftless first order autoregression,

$$z_t = \theta z_{t-1} + \varepsilon_t \tag{3}$$

where  $\varepsilon_t$  is a sequence of i.i.d random variables with zero mean and positive variance,  $\sigma_{\varepsilon}^2$ , and  $\theta$  is some parameter below but close to 1 if the transitory component displays a slowly mean reverting behavior. However, if we allow the transitory component to be an explosive process, then the absolute value of  $\theta$  is some parameter above but close to 1. Equation (3) implies that the transitory component is created by adding  $\varepsilon_t$  to  $\theta z_{t-1}$ . With repeated substitution, it is strait-forward to show that

$$z_t = \sum_{j=0}^{t-1} \theta^j \varepsilon_{t-i}$$
(4a)

and

$$z_{t+K} = \sum_{j=0}^{K} \theta^{j} \varepsilon_{t+k-j}$$
(4b)

The variance and the autocovariance of the transitory component,  $z_t$ , are

$$\sigma_z^2 = \frac{\left(\theta^{2t} - 1\right)}{\left(\theta^2 - 1\right)} \sigma_\varepsilon^2 \tag{5a}$$

$$\operatorname{cov}(z_t, z_{t+K}) = \theta^K \sigma_z^2$$
(5b)

Notice that if the transitory component is stationary, then expressions 4a (4b) demonstrate that the weight applied to the first observation declines to zero as t increases. Likewise, the variance of the transitory component increases as t gets large and converges to

$$\sigma_z^2 = \frac{1}{\left(1 - \theta^2\right)} \sigma_\varepsilon^2 \tag{5c}$$

and the autocovariances converge to zero as K increases.

However, if the transitory component is explosive, then we can infer from 4a (4b) that the weight applied to the first observation increases exponentially as t(K) increases. Hence, the first observation has a variance contribution of order  $\theta^{2t}(\theta^{2(t+K)})$ . This implies that the transitory component becomes more volatile with longer horizon K and the autocovariances grow exponentially as K increases.

The slope of the autoregression of the actual change in the transitory component, denoted by  $\rho(K)$ , is

$$\frac{Cov(z_{t,t+K} - z_t, z_t - z_{t-K})}{\sigma^2 [z_t - z_{t-K}]}$$
(6)

and the covariance of the actual change in the transitory component in the numerator of (6) is

$$Cov(z_{t+k} - z_t, z_t - z_{t-K}) = -\sigma_z^2 + Cov(z_{t+K}, z_t) + Cov(z_t, z_{t-K}) + Cov(z_{t+K}, z_{t-K})$$
(7a)

$$Cov(z_{t+k} - z_t, z_t - z_{t-K}) = (\theta^K - 1)\sigma_{z_t}^2 + (\theta^K + \theta^{2K})\sigma_{z_{t-K}}^2$$
(7b)

Using the variance expression in (5a) we can write,

$$Cov(z_{t+k} - z_t, z_t - z_{t-K}) = \left(\frac{\left(\theta^{2t+K} + \theta^{2t-K} - 2\theta^K - \theta^{2K} + 1\right)}{\left(\theta^2 - 1\right)}\right)\sigma_{\varepsilon}^2$$
(7c)

Observe that if the transitory component is stationary then  $\sigma_{z_t}^2$  is approximately equal  $\sigma_{z_{t-k}}^2$  and the model predicts positive covariances for short horizons' overlapping returns. However, for longer horizons' returns, the covariances that appear in the last terms of (7a) will converge to zero and the covariance in the numerator of (6) will converge to  $-\sigma_{z_t}^2$  (i.e. the overlapping returns display negative covariances).

On the other hand, if the transitory component is explosive, the slope of the autoregression in (6) is no longer the autocorrelation of the actual change since  $\sigma_{z_t}^2 > \sigma_{z_{t-K}}^2$ . It can also be implied from (7b) that the numerator of (6) is always positive and grows exponentially with *K*.

The variance of the actual change in the transitory component is

$$\sigma^{2}(z_{t} - z_{t-K}) = \sigma^{2}_{z_{t}} + \sigma^{2}_{z_{t-K}} - 2Cov(z_{t}, z_{t-K})$$
(8a)

Using the variance expression in (5a), we can express the variance of the actual change in the transitory component in terms of the residuals variance:

$$\sigma^{2}(z_{t}-z_{t-K}) = \left(\frac{\theta^{2t-2K}+\theta^{2t}-2\theta^{2t-K}+2\theta^{K}-2}{\theta^{2}-1}\right)\sigma_{\varepsilon}^{2}$$
(8b)

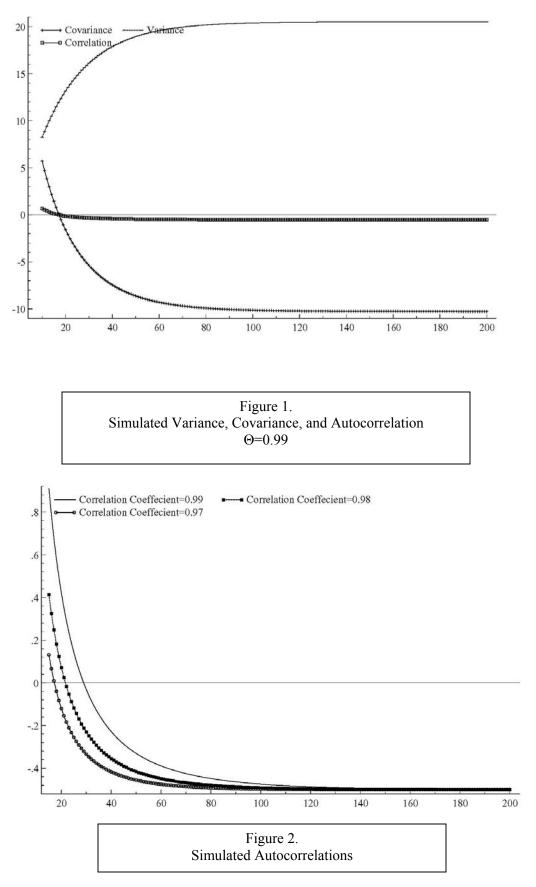
and the regression slope in (6) can be expressed as

$$\rho(K) = \frac{\left(\theta^{2t+K} + \theta^{2t-K} - 2\theta^{K} - \theta^{2K} + 1\right)}{\left(\theta^{2t-2K} + \theta^{2t} - 2\theta^{2t-K} + 2\theta^{K} - 2\right)}$$
(9)

Equation (7a) shows that if  $\theta$  is positive and close to but less than 1 (Summers 1986) then the variance of the actual change in the transitory component in (7a) will slowly approach  $2\sigma_z^2$  as the return horizon *K* increases and  $\rho(K)$  in (9) converges to -0.5. Conversely, if  $\theta$  is close to but larger than 1 then the variance expression in (8b) will be an increasing function in the return horizon *K* as is the covariance of the actual change. The behavior of the slope with respect to the return horizon depends on the growth rate of either the numerator or the denominator as *K* increases.

In order to gain insight into the behaviors of the autocorrelation, covariance and variance of the actual change in the transitory component, we simulate the model based on different values of  $\theta$ . Figure 1 plots the simulated values of the variance, covariance, and autocorrelation when  $\theta$ =0.99 against the return horizons based on the econometric specifications of (7c), (8b) and (9). As shown, the variance of the actual change in the stationary component is increasing at a decreasing rate until it reaches its maximum at the point where  $\theta^{\kappa}$  converges to zero. Moreover, the covariance is decreasing at decreasing rate and the sign of the covariance changes from positive to negative as the horizon *K* increases.

Figure 2 illustrates the simulated autocorrelation for several different values of  $\theta$  under the assumption that the bubble component is stationary. As the figure shows, the value of



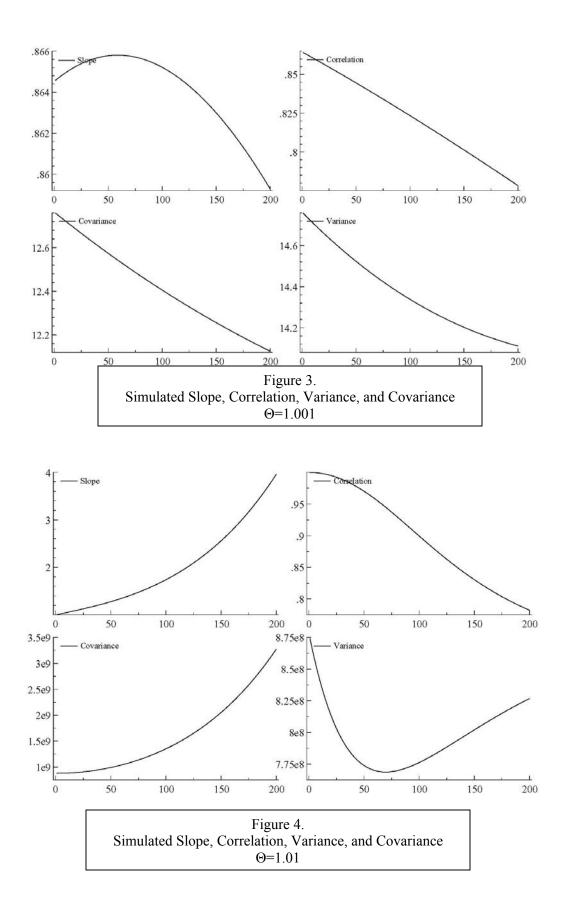
the autocorrelations converge to -0.5 as *K* increases. The value of  $\theta$  controls the degree of persistence; a smaller value of  $\theta$  implies faster convergence to -0.5.

Unlike the stationary case, if the bubble is explosive then the slope of the autoregression is no longer the autocorrelation of the actual change in the transitory component. In order to see this, remember that the autocorrelation of the actual change is

$$\frac{Cov(z_{t,t+K} - z_t, z_t - z_{t-K})}{\sqrt{\sigma^2 [z_{t+K} - z_t]} \sqrt{\sigma^2 [z_t - z_{t-K}]}}$$
(10)

Thus, it is straightforward to show that the denominator in (10) is greater than denominator of the autoregression in (6) since  $\sigma_{z_{+Kt}}^2 > \sigma_t^2 > \sigma_z^2$ .

Figure 3 illustrates the simulated slope, autocorrelation, covariance and variance for  $\theta$ =1.001 based on equations (9), (10), (7c) and (8b), respectively. As illustrated by the figure, we can see that the slope of the autoregression forms a humped-shaped pattern with respect to the horizon *K* in the first quadrant. Also, the autocorrelation forms a down-ward sloping curve and converges to zero as *K* increases. Moreover, we simulate the model with higher aversion parameter to see if the slope shape would be affected by the degree of aversion. Figure 4 plots the same econometric specifications where  $\theta$ =1.01. As shown, the slope of the actual change in the transitory component displays a *J*-shaped pattern where the slope becomes an increasing function of *K*. Unlike the autocorrelation, the value of the slope can now be any positive number without a bound; which is unrealistic Thus, we expect that the slope will lie between zero and one in the case of



mean averting behavior in the spot exchange rate, i.e., if the exchange rates are explosive processes, then the degree of aversion should be very slow.

We now use the model to characterize the overlapping returns in this subsection. Define the continually holding period return from t to t+K as the sum of the actual change of the random walk component and the transitory component from t+K to t, that is

$$R_{t,t+K} = [q_{t+K} - q_t] + [z_{t+K} - z_t]$$
(11)

As in F&F, we can infer the existence of the transitory component from the behavior of the holding period return by testing the slope coefficient of the autoregression

$$R_{t,t+K} = \alpha + \beta_K R_{t-K,t} + \varepsilon_{t+K}$$
(12)

where,

$$\beta(K) = \frac{Cov(R_{t+K_t}, R_{t-K})}{\sigma^2(R_{t-K})}$$
(13a)

since the transitory and the permanent components are linearly independent it is straight forward to show that

$$\beta(K) = \frac{Cov(z_{t+K} - z_t, z_t - z_{t-K})}{\sigma^2[z_t - z_{t-K}] + \sigma^2[q_t - q_{t-K}]}$$
(13b)

The proposed model has several interesting implications about the behavior of the spot exchange rate. First, if the spot rate is a random walk and does not have a transitory component, then the slope of the autoregression  $\beta(K)$  will be zero. Second, if the exchange rate does not have a permanent component and the transitory component is stationary, then  $\theta^{K}$  converges to zero as K increases (i.e.,  $\beta(K)$  converges to -0.5). Third, if the exchange rate has both permanent and stationary components, the autocorrelation of the holding period return will eventually converge to -0.5 and then gradually return to zero; the variance of the actual change in the random walk component will continuously grow like K, while the variance of the change in the stationary component converges to its maximum,  $2\sigma_z^2$ . This implies that the autocorrelations at long horizons will disappear gradually and  $\beta(K)$  will have a typical U-shaped pattern, which has also been documented by F&F. Fourth, if the transitory component is mean averting and the random walk component does not exist, then the slope of the autoregression in (9) is always positive; this is because the covariance in (7b) is positive. The model implies that the theoretical upper bound of  $\beta(K)$  will slowly converge to infinity as K increases, which is improbable unless the degree of aversion is small, as shown above. Finally, if the exchange rate is the sum of an explosive bubble and a random walk, then we expect that the variance of the white noise component,  $\sigma^2[q_{t+K}-q_t]$ , will grow at a lower rate than the variance of the actual change in the transitory component,  $\sigma^2[z_{t+K} - z_t]$ , at any value of K since the former grows exponentially and the latter grows like K. The variance of the white noise will eventually dominate the variance of the actual change in the transitory component since it continuously grows like K. This steers us to two possible implications about the behavior of the slope,  $\beta(K)$ . First, if the parameter  $\theta$  is large enough, then  $\beta(K)$  will increase with K; this is due to the covariance in the numerator growing faster than the variance of the holding period returns. Second, if  $\theta$  is close enough to 1, then initially the variance of the return will grow at a lower rate than the

covariance, but will eventually grow larger than the covariance because of the random walk effect at some large value of *K*. Thus, we expect a hump-shaped pattern for the slope  $\beta(K)$  in the first quadrant at higher degree of aversion.

#### **II.B.** Econometric issues

Four issues relating to the autoregression test are now in order. First, the small sample bias of the OLS estimate of  $\beta(K)$  since the true mean of the predictor is unknown in the finite sample. Kendall (1954) showed that the bias in the OLS estimate of  $\beta(K)$  is decreasing with the sample size and increasing with the value of the point estimate, in particular

$$E(\beta_{K} - \beta_{K}) = -(1 + 3\beta_{K})/n + O(n^{-2})$$
(14)

F&F correct for the bias in the least square estimate using Monte Carlo simulations; their results show that the bias is not large enough to affect the results. Using randomization simulation of Noreen (1989), Kim, Nelson, and Startz (1991) get the same results since the distribution does not affect the parameter estimate.<sup>2</sup> Campbell (2001) argues that the asymptotic critical values of the parameter estimates of the long horizon regression have significant size distortions and should be corrected.

Another issue concerns the methods used to correct for hetroscedasticity and autocorrelation in overlapping multiperiod returns. The problem of autocorrelation and heteroscedasticity leads to an inefficient least squares estimate since the standard error is

<sup>&</sup>lt;sup>2</sup> One of the differences between Monte Carlo simulation and randomization simulation is that the former assumes normality in random variables. The advantage of randomization over Monte Carlo is clear in the hypothesis testing if the return is not normally distributed.

biased and inconsistent<sup>3</sup>. F&F use the method of Hansen and Hodrick (1980), hereafter HH, which adjusts the autocorrelated standard error with a MA(K-1) error structure where K is the return horizon. In the case of multiperiod returns, this method may be inappropriate since positive definiteness of the covariance matrix may not exist. Also, they combine White's (1980) hetroscedastic consistence variance estimator with that of HH to solve for both problems. They find that the *t*-ratios are more dispersed than those of HH; thus, they report the t's based on HH alone. Richardson and Stock (1989) assume that the stock returns variance is stationary. Based on this assumption, they adjust the standard error for a form of stationary conditional heteroscedasticity. The resulting standard errors are so large as to provide a test statistic close to zero regardless of the point estimates<sup>4</sup>. However, Turner, Startz, and Nelson (1989) provide evidence that the variance of the long-horizon return is nonstationary. McQueen (1992) explicitly solves for the clustering heteroscedasticity results from overlapping returns using the GLS randomization test. According to his results, he cannot reject the random walk hypothesis. However, there are three problems with McQueen's method used to solve the problem of heteroscedasticity in the overlapping returns. First, the historical GLS results are inefficient since the first observation in the estimation is dropped and the problem becomes more severe as the return horizon increases. For example, in estimating the 10year horizon returns, the GLS drops the first 120 observations from the monthly returns. Second, the use of the Depression/World War II period as a weight in the GLS randomization test is adopted without a formal test of structural change in the returns

<sup>&</sup>lt;sup>3</sup> Standard error of the regression is downward biased if errors are correlated and upward biased if they are heteroscedastic.

<sup>&</sup>lt;sup>4</sup> Rechardson (1993) argues that properly adjusting for finite sample bias, hetroscedasticity, and autocorrelation in univariate autoregression may reverse many of the inferences of F&F (1988).

variance, which may underestimate the result of no mean reversion. Third, and most importantly, the GLS is inconsistent in the case of overlapping returns because of the lack of exogeneity; this is due to the fact that the multi-period nature induces high temporal dependence in the error term (See, Hansen and Hodrick (1980), Hansen (1982) and Hansen (2002), for further discussion).

#### III. Methodology

This section describes the moving blocks bootstrap (MBB) of Fitzenberger (1998) adopted in our paper for testing the long-horizon predictability in the holding periods returns. This methodology has several advantages; it is shown to adjust simultaneously for finite sample bias, autocorrelation, heteroscedasticity and non-normality in the foreign exchange returns.

Efron (1979) established the theory of Bootstrap to reduce the finite-sample bias and yield an approximation to the distribution of an estimator or test statistic that is at least as accurate as the approximation obtained from first-order asymptotic theory. Efron suggests two basic approaches to bootstrap the linear regression. One is to first fit the model and apply the bootstrap to the residuals. The resulting covariance estimate for this case is similar to that of the point estimate in the case of no finite sample bias, except for the degrees of freedom adjustment, (see Efron (1982), for further disussion). A second approach, with more general applicability is the Design-Matrix-Bootstrap, hereafter DMB. Under this approach, the entire vector of the dependent variable and the regressors are bootstrapped. So DMB provides a heteroscedastic consistent covariance matrix,

hereafter HC, since the errors are not resampled<sup>5</sup> (See, Horowitz (1995), for further discussion). Singh (1981) contradicts the asymptotic validity of the bootstrap covariance estimator in the case of autocorrelation. He notes that both approaches cannot provide an autocorrelated consistent estimate, hereafter AC, in the linear regression case. Wu (1986) claims that the Jackknife approach performs better than the bootstrap particularly in the case of heteroscedastic residuals. Lui (1988) introduce the wild bootstrap to solve the bias of the autoregression parameter. Mammen (1993) establishes the ability of the wild bootstrap to provide asymptotic refinements when the errors are heteroskedastic. Using Monte Carlo experimentation, Horowitz (1995) shows that wild bootstrap perform better than any version of HC estimator<sup>6</sup>.

The MBB approach adopted in our analysis is a general case of the block bootstrap introduced independently by the work of Kucsh (1989) and Liu and Singh (1992) to provide a HAC standard errors equivalent to the NW estimator. The MBB can be implemented by dividing the data into blocks and the bootstrap sample is obtained by sampling the overlapping blocks randomly with replacement. To describe the method of blocking the data, let the sample consist of observations  $\{y_{i}x_{i} : i=1, ..., T\}$ . With overlapping blocks of size b and length L = i+b-1, block B<sub>i</sub> is the entire vector of blocks  $(B_{i}^{y}, B_{i}^{x})$  with  $B_{j}^{y} = (y_{i}, ..., y_{l+b-l})$ ,  $b \times I$  vector, and  $B_{i}^{x}$ , a  $b \times k$  matrix of regressors. The bootstrap resample  $\{(y_{i}^{*}, x_{i}^{*}), ..., (y_{l}^{*}, x_{l}^{*})\}$  of size l = bm is generated by drawing m

<sup>&</sup>lt;sup>5</sup> Efron and Tabshirani (1986) claims that both approaches are asymptotically equivalent presumably when the covariance is assumed to be chosen from a probability distribution.

<sup>&</sup>lt;sup>6</sup> Horowitz (1995) compare the wild bootstrap with the jackknife approximation of the heteroskedasticityconsistent covariance matrix estimator of Mackinnon and White (1985) and the bootstrap method of resampling the dependent and the independent variables. The results show that the wild bootstrap provides a *t*-statistic with the lowest distortions.

*iid* ( $b \times [k+1]$ ) blocks, call them { $Z_1, ..., Z_m$ }, from q = T - b + 1 blocks  $B_i$ . The MBB resample is formed by laying { $Z_1$ , ...,  $Z_m$ } end-to-end in the order sample. The resulting vector, { $Z\tau v = (y_i^*, x_i^*)$ }, is the MBB sample, where  $\tau = [(i-1)/b] + 1$  and  $v = I - b \tau$ .

Fitzenberger (1998) compares the performance of MBB with Barlett kernel suggested by Newey and West (1987) and quadratic spectral kernel (QS) suggested by Andrews (1991) <sup>7</sup>. The Monte Carlo simulation shows that the MBB dominates both estimators in terms of variance bias and coverage properties especially when the data exhibit heteroskedasticity with increasing dependence i.e., the autocorrelation is rising<sup>8</sup>. The results show that the block size. However, the improvement of the root mean square error (RMSE) is not monotonic with increasing *b*. Consistent with Hall et al. (1995), the results show that the block size has to grow at the rate  $T^{1/3}$  to minimize the RMSE of the variance estimate. This is identical to Andrews' (1991) optimal bandwidth.

## IV. The Data

The raw data for nominal spot exchange rates consist of end-of-month observation (against U.S. Dollar) of the Pound Sterling, the Japanese Yen, the Swiss Franc, and the Canadian Dollar as given by the Data Resources Incorporated (DRI) database. The data are monthly and cover the period from January 1973 to May 2003. Following F&F, we transform the one-month returns into continuously compounded form. The resulting

<sup>&</sup>lt;sup>7</sup> Andrews (1991) discusses different Kernel HAC estimators used in the literature. He shows that quadratic spectral kernel covariance estimator (QS) dominates Barlett kernel suggested by Newey and West (1987) in terms of both RMSE and the true confidence interval performance.

<sup>&</sup>lt;sup>8</sup> Fitzenberger (1998) shows that the result is hold whether the MBB is centering around sample estimate or resample mean.

nominal returns are then adjusted for the inflation rate using the Consumer Price Index (CPI) to get the real returns. The overlapping monthly returns for each horizon are then calculated by summing up the monthly real returns.

Table 1 shows the means, median standard deviations, minimums, and maximums of the foreign exchange rates. The maximum levels of the exchange rates are about three times higher than their minimums. The measures of central tendency set approximately in the middle of the minimums and the maximums, this provides some evidence that these maximums and minimums are not outliers. The standard deviation of the Pound sterling is 0.295 that is the highest across all currencies. Although the standard deviation is a biased estimate to compare the fluctuations of the spot exchange rates since it is sensitive to the value of the currency, it offers some intuition about the behavior of the data.

## V. Empirical Results

In this section we first introduce the classical periodogram as a valid test of long horizon predictability in the foreign exchange returns. We then estimate and interpret the results based on the spectral analysis. Finally, we estimate the autoregression using the methodology described in section III and interpret the results.

## V.A. Spectral density analysis of the exchange rates returns

Since the spectral density of a time series is the Fourier transform of the covariance function, one could think of the periodogram to search for hidden periodicities in the overlapping returns. Our idea behind using the periodogram in examining the predictability of returns is based on the simple fact implied from the model that the

Summary statis	tics
----------------	------

Currency	Mean	Median	Standard deviation	Minimum	Maximum
Canadian Dollar	0.8031	0.8050	0.1064	0.6232	1.0410
Japanese yen	0.0064	0.0069	0.0022	0.0032	0.0118
Swiss Franc	0.5897	0.6081	0.1381	0.3050	0.8888
Pound Sterling	1.7307	1.6458	0.2950	1.0823	2.5855

demeaned expected returns equals the expected change in the transitory component. That is,

$$E_t(R_{t,t+K}) - K\mu = (\theta^{t+k} - 1)z_t$$
(15)

To set the ball rolling, if the exchange rate displays a random walk process, then the demeaned of the expected returns,  $E(R_{t,t+K} - K\mu)$ , must display a white noise process, which can simply be examined by looking at the sample spectrum of each return horizon.

The classical periodogram of the overlapping returns  $R_{t,t+K}$ , t=1, ..., T is given by

$$I(w_s) = \frac{2}{T} \left\langle \left( \sum_{j=1}^T R_{t,t+K} \cos w_s t \right)^2 + \left( \sum_{j=1}^T R_{t,t+K} \sin w_s t \right)^2 \right\rangle$$
(16a)

where,

$$w_s = \frac{2\pi s}{T} \tag{16b}$$

Since the periodogram can also be expressed as a multiple of the time series spectral density, if  $s \neq 0$ , we can write

$$I(w_{s}) = \frac{2}{T} \left\langle \left( \sum_{t=1}^{T} \left( R_{t,t+K} - ER_{t,t+K} \right) \cos w_{s} t \right)^{2} + \left( \sum_{j=1}^{T} \left( R_{t,t+K} - ER_{t,t+K} \right) \sin w_{s} t \right)^{2} \right\rangle$$
(17a)

As in Fuller (1996), the periodogram in (17a) can be expressed as

$$I(w_s) = \frac{2}{T} \left( \sum_{t=1}^{T} \sum_{j=1}^{T} \left( R_{t,t+K} - ER_{t,t+K} \right) \left( R_j - ER_{t,t+K} \right) \cos w_s t \right)$$
(17b)

and the sth periodogram ordinate is given by,

$$I(w_{ss}) = \begin{cases} 2T\overline{R}_{t,t+k}^2, & s = 0\\ 4\hat{rf}(w_s), & s = 1, 2, \dots m, \end{cases}$$
(18a)

where,

$$\hat{f}(w) = \frac{1}{2\pi} \left( \sum \frac{n - |n - p|}{n} \gamma(p) \cos wp \right).$$
(18b)

Due to the fact that periodogram ordinates are multiples of chi-square random variables, we can perform a robust test of the AR(1) fads model by examining each holding period return by standardizing the periodogram. Where under the null model:

$$R_{t,t+K} = K\mu + \varepsilon_{t,t+k} = E(R_{t,t+K}) + \varepsilon_{t+k}$$
(19a)

against the alternative fads model,

$$R_{t,t+K} = E(R_{t,t+K}) + B_1 \cos wt + B_2 \sin wt + \varepsilon_{t+k}, \qquad (19b)$$

where B1 and B2 are constants, and the standardized periodogram is given by,

$$I^{*}(w_{j}) = \frac{(m-1)I(w_{s})}{\sum_{j=1}^{m}I(w_{j}) - I(w_{s})}$$
(20)

where  $I^*(w_j)$  has  $F_{2,2m-2}$  for  $s \succ 0$ .

We search for the largest periodogram ordinate  $I(L_s)$  across each return horizon and test the hypothesis that this ordinate is reasonably the largest in a random sample of size *m* selected from a multiple of chi-square distribution function with two degrees of freedom using the following statistic suggested by Davis (1941)

$$\hbar = \frac{I(L_s)}{m \sum_{j=1}^m I(w_j)}.$$
(21)

Also, we employ the normalized cumulative periodogram suggested by Barlett (1966) to test the null model in (19a). This testing procedure takes the form

$$I^{*}(w_{j}) = \frac{\sum_{j=1}^{S} I(w_{j})}{\sum_{j=1}^{m} I(w_{j})}$$
(22)

which has a uniform (0,1) distribution function of an order sample size *m*-1. Hence, we apply the Kolmogorov-Smirnov test as suggested by Durbin (1967, 1969).

Intuitively, asymptotically, if the long horizon return is a white noise process, then the sth's periodogram ordinates have the same first moment. On the other hand, if it has a nonzero autocorrelation structure then the ordinates display different first moments.

Table 2 reports the suggested spectral based tests for the Canadian Dollar, the Japanese Yen, the Swiss Franc, and the Pound Sterling holding returns for 12-, 24-, 36-, 48-, 60-, 72-, 96- and 120- month holding periods. The main empirical results are as follows: i) the spectral estimates provide strong evidence against the white noise null for every holding period across all currencies, the one exception is the Kolomogrov-Smernov statistic at one year holding period return for the Swiss franc. ii) The significant values of the standardized periodogram are focused in the low frequency components of the holding

	Return Horizon (k)											
	12	24	36	48	60	72	96	120				
Canadian D	ollar											
F-Stat	57.781**	101.317**	130.446**	166.361**	203.187**	218.311**	252.392**	16.206**				
Ordinate	1	1	1	1	1	1	1	1				
Σ	9.443**	13.671**	15.587**	18.739**	21.356**	22.571**	24.971**	27.934**				
KS	0.291**	0.443**	0.513**	0.619**	0.708**	0.754**	0.828**	0.928**				
Japanese Y	en											
F-Stat	57.517**	77.626**	57.884**	91.547**	97.195**	212.212**	11.405**	191.694**				
Ordinate	4	4	3	3	1	1	1	1				
Σ	9.437**	11.892**	9.108**	13.794**	13.568**	23.367**	28.430**	23.631**				
KS	0.453**	0.682**	0.524**	0.212*	0.448**	0.774**	0.945**	0.784**				
Swiss Franc	<u>e</u>											
F-Stat	70.188**	80.826**	161.996**	211.244**	125.641**	54.683**	193.673**	160.950**				
Ordinate	7	3	3	3	3	3	1	1				
Σ	11.414**	11.896**	19.835**	22.323**	17.369**	9.220**	23.277**	22.649**				
KS	0.245	0.526**	0.831**	0.951**	0.890**	0.780**	0.766**	0.748**				
Pound Sterl	ling											
F-Stat	33.564**	81.869**	96.999**	69.936**	85.787**	123.768**	219.918**	222.890**				
Ordinate	4	4	4	4	1	1	1	1				
Σ	5.914	12.541**	14.184**	10.866**	13.095**	17.087**	21.624**	19.962**				
KS	0.294**	0.600**	0.737**	0.801**	0.429**	0.563**	0.592**	0.484**				

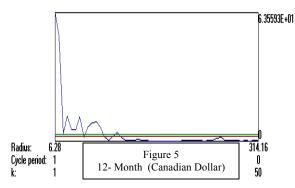
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Spectral Analysis Tests

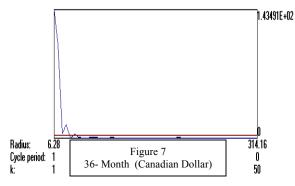
periods returns, particularly, for *S* between 1 and 7. iii) As the holding period increases the three spectral statistics becomes higher and more focused in the low frequencies, particularly at *S* equal one. This suggests more evidence of autocorrelations as the holding periods incease. iv) Comparing the last two columns of table 1 we find that value of the statistics begin to decline when the holding period is 120- month. The one exception is that of the Japanese yen when the statistics sharply decline at 96-month holding period. This behavior of periodograms statistics is consistent with the model implication that the absolute value of the holding returns autocorrelations begins to decline at a very large value of *K* as a result of the random walk variance effect.

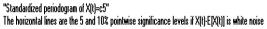
To get better insight about the behavior of periodograms with the horizon K. Figures 5 through 12 plot the normalized periodogram of each holding period return over the first fifty ordinates. For testing purposes, each of these figures also graphs the corresponding five and ten percent critical values of the F-statistic. As shown, the deviations of the periodogram above the five and ten percent critical values occur at a low frequency where the periodogram ordinates are between one and seven. Also, as the holding period of returns increases the periodogram becomes more flat at higher frequencies with higher value at low frequency. This provides evidence that holding returns are more predictable at longer horizons and these features stand out clearly with the fads model of Summers (1986).

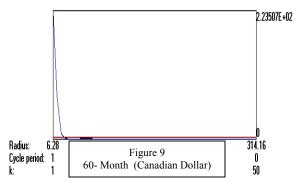
"Standardized periodogram of X(t)=c1" The horizontal lines are the 5 and 10% pointwise significance levels if X(t)-E[X(t)] is white noise



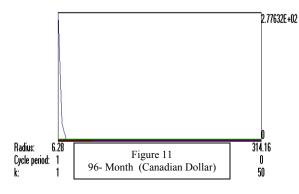
"Standardized periodogram of X(t)=c3" The horizontal lines are the 5 and 10% pointwise significance levels if X(t)-E[X(t)] is white noise



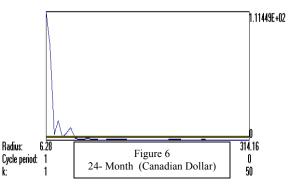




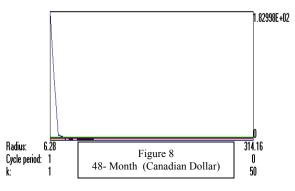
"Standardized periodogram of X(t)=c8" The horizontal lines are the 5 and 10% pointwise significance levels if X(t)-E[X(t)] is white noise



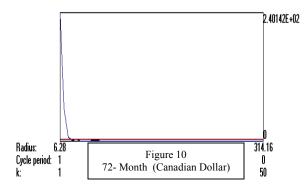
"Standardized periodogram of X(t)=c2" The horizontal lines are the 5 and 10% pointwise significance levels if X(t)-E[X(t)] is white noise

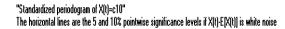


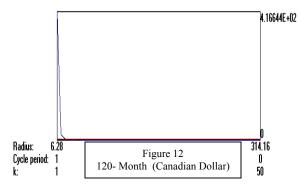
"Standardized periodogram of X(t)=c4" The horizontal lines are the 5 and 10% pointwise significance levels if X(t)-E(X(t)) is white noise



"Standardized periodogram of X(t)=c6" The horizontal lines are the 5 and 10% pointwise significance levels if X(t)-E[X(t)] is white noise







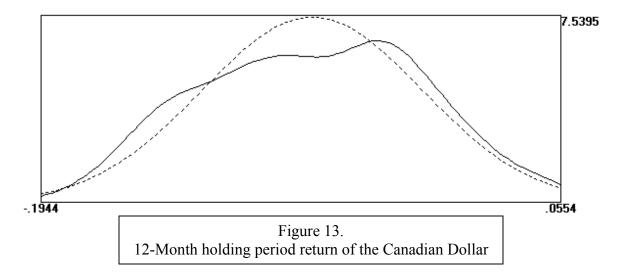
#### V.B. Regression analysis of the exchange rates returns

The holding period return and the error term implied by equation 12,  $\varepsilon_{t+K}$ , show excess kurtosis and fat-tail distribution. Figure 13 and 14 graph the standard normal kernel densities of 12-month holding period return and the error term for the Canadian Dollar compared to the density of the standard normal distribution  $N(M,S^2)$  where M is the sample mean and S is the sample standard error. Following Silverman (1986) we use the data based optimal bandwidth to estimate the normal kernel of the distribution.

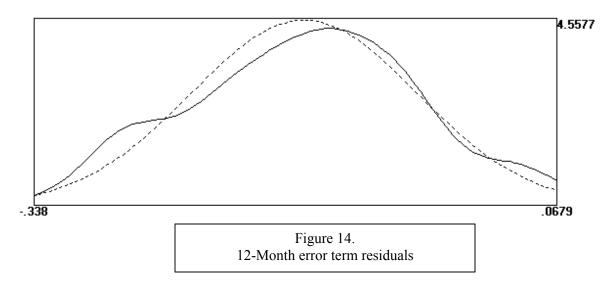
As shown, the excess kurtosis and fat-tailed facial appearance are obvious from the estimation for both the holding period return and the error term implied from OLS. This confirms the suitability of robust estimator in the present context. To save space, we only report the graphs for the Canadian Dollar, but parallel prototypes were observed for the other holding period returns and the other error terms across all currencies, which are available upon request.

We estimate the slopes of the autoregression in equation 12 for each return horizon from 3 to 120 months using both OLS and MBB for the Pound Sterling, the Japanese Yen, the Swiss Franc, and the Canadian Dollar. The OLS standard errors, labeled OLS-QS, are corrected for both autocorrelation and hetroscedasticity using the quadratic spectral kernel HAC estimator suggested by Andrews (1991). This test does not adjust for finite sample bias in the autoregression and the non-normality in the distribution of the holding periods returns. The robust MBB test described in section III.B. is estimated by setting

Kernel estimate of the density f(y) of Y = c1with bandwidth  $h = c.n^{-1/5}$ , where c = 1, compared with the corresponding normal density (dashec



Kernel estimate of the density f(y) of Y =  $c^2$ with bandwidth h =  $c.n^{-1/5}$ , where c = 1, compared with the corresponding normal density (dashed



an arbitrary numbers of blocks from 1 to  $15^9$ . The optimal number of blocks for each return horizon is selected by choosing the block size that minimizes the RMSE of the test variance based on 1000 resamples for each sample. Consistent with Hall and Horowitz (1995) the optimal number of blocks sets around 7, which is the inverse of the number of observations cube root.

Tables 3a and 3b report the results of random walk test for the period 1973 through 2003 using the tests described above. As shown, MBB reports slightly lower slopes and higher *p*-values than OLS-QS especially when the return horizons are small (3- and 6- month horizons).

The first three columns of each table examine the hypothesis of random walk for 3- 6and 12-month return horizons. If the foreign exchange rates display a unit root then, the model predicts that the value of  $\beta(K)$  is zero. For all currencies, OLS-QS reports statistically significant positive autocorrelations for those holding periods; an exception is the 3-month holding period returns of the Canadian dollar where the autocorrelation is insignificantly different from zero. This implies that foreign exchange rates display what is called in the literature as a bandwagon effect, i.e., the propensity of the exchange rates to continue to change in a path once taken. The most critical finding in studying these short horizons is that after adjusting for finite sample bias and fat tail distribution of returns the evidence against random walk behavior disappears. For example, the OLS-QS reports statistically significant positive autocorrelations for the Japanese Yen across 3-

<sup>&</sup>lt;sup>9</sup> When the number of blocks is set to one, i.e. the block size is 100% of the effective sample, the MBB becomes identical to DMB since DMB is a special case of MBB.

and 6- month holding period returns where the *p*-values are 0.009 and 0.005 respectively, which implies the rejection of the null hypothesis of random walk. However, after correcting for finite sample bias and the fat tail structure in the distribution of return, one cannot reject the hypothesis of random walk in favor to mean averting behavior where the MBB reports 0.054 and 0.105 *p*-values for the same return horizons, respectively. We conclude that momentum traders cannot outperform the market by considering 3-, 6-, and 12- month months holding period strategies. Thus, the foreign exchange market appears to be efficient in the short run and the bandwagon pattern of the exchange rates is just a methodological elusion.

Now we consider the slopes of the long horizon autoregression across 24-, 36-, 48-, 60-, 72-, 96-, and 120- month return horizons. As shown in the last seven columns of Tables' 2a and 2b, the slopes of the autoregressions are well above zero across all currencies and the evidence against the predictability of returns is disappeared. Three aspects of the results stand out. First, both OLS-QS and MBB report statistically significant positive autocorrelations. Second, both the slope and the  $R^2$  increase with the length of the holding period up to 96 months, then decrease. To illustrate, there is a peak at *K*=96 when *B*= 0.885 for the Canadian dollar, indicating that a 10 percent positive return over eight years is, on average, followed by a 8.85 percent positive return over the next eight years. The  $R^2$  in the regression is approximately 0.887. So the positive autocorrelation in the returns is consistent with that from the anomalies literature where a long-term "momentum" strategy earns persistent positive profits. Third, if investors perform the "buy low, sell high" strategy they would experience persistent negative profits since the spot exchange

# Table 3A.

OLS Quadratic Spectral Kernel (OLS-QS) and Moving Blocks Bootstrap (MBB) of the first order autoregression slope  $B_K$ ; Equally and Value weighted real returns The tests are based on the following specification

				Retur	n Horizon	(k)				
	3	6	12	24	36	48	60	72	96	120
Canadian I	Dollar									
OLS-QS:	<u>sonu</u>									
$\overline{B_K}$	0.076	0.217	0.349	0.727	0.851	0.891	0.894	0.891	0.885	0.856
P-value	0.173	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R-Square	0.000	.0441	0.119	0.539	0.725	0.806	0.849	0.862	0.902	0.886
MBB										
$B_K$	.073	.215	0.337	0.723	0.850	0.897	0.898	0.723	0.890	0.884
P-Value	.329	.055	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R-Square	0.010	0.065	0.253	0.674	0.814	0.847	0.859	0.674	0.877	0.920
Optimal	6	9	9	7	6	7	7	7	7	7
Number of										
Blocs										
Japanese Y	'en									
OLS-QS:										
$\overline{B_K}$	0.123	0.138	0.082	0.485	0.586	0.631	0.775	0.748	0.835	0.689
P-value	0.009	0.005	0.112	0.000	0.000	0.000	0.000	0.000	0.000	0.000
R-Square	0.012	0.016	0.004	0.240	0.352	0.405	0.598	0.566	0.697	0.573
MBB										
$\overline{B_K}$	0.123	0.135	0.078	0.481	0.588	0.637	0.776	0.755	0.840	0.69
P-value	0.054	0.105	0.514	0.000	0.000	0.000	0.000	0.000	0.000	0.00
R-Square	0.002	0.064	0.020	0.400	0.203	0.543	0.737	0.646	0.706	0.392
Optimal	6	7	7	8	6	5	7	5	5	7
Number of										
Blocs										

$$R_{t,t+K} = \alpha + \beta_K R_{t-K,t} + \varepsilon_t$$

# Table 3B.

OLS Quadratic Spectral Kernel (OLS-QS) and Moving Blocks Bootstrap (MBB) of the first order autoregression slope  $B_K$ ; Equally and Value weighted real returns The tests are based on the following specification

	Return Horizon (k) 3 6 12 24 36 48 60 72 96 120										
	3	6	12	24	36	48	60	72	96	120	
а · Б											
Swiss Fran OLS-QS:	nc										
$\frac{B_{LS}}{B_K}$	0.049	0.089	.0500	0.436	0.605	0.716	0.681	0.654	0.763	0.634	
P-value	0.363	0.101	0.359	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
R-	0.002	0.008	0.003	0.185	0.363	0.522	0.490	0.433	0.566	0.408	
Square											
MBB											
$\overline{B_K}$	0.056	0.081	0.034	0.432	0.608	0.712	.683	0.659	0.775	0.643	
P-Value	0.496	0.202	0.780	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
R-	0.000	0.006	0.016	0.1406	0.2919	0.3959	0.649	0.658	0.601	0.602	
Square											
Optimal	7	7	7	6	6	6	7	7	7	5	
Number											
of Blocs											
Pound Ste	rling										
OLS-QS:											
$\overline{B_K}$	0.073	0.063	0.113	0.568	0.641	0.680	0.764	0.792	0.849	0.782	
P-value	0.180	0.249	0.039	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
R-	0.002	0.003	0.000	0.332	0.464	0.505	0.617	0.6407	0.762	0.775	
Square											
MBB											
$B_K$	0.078	0.061	0.102	0.566	0.640	0.680	0.766	0.793	0.853	0.787	
P-value	0.309	0.474	0.398	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
R-	0.001	0.017	0.116	0.387	0.482	0.505	0.748	0.605	0.846	0.829	
Square											
Optimal	6	7	7	6	6	6	4	6	7	6	
Number											
of Blocs											

 $R_{t,t+K} = \alpha + \beta_K R_{t-K,t} + \varepsilon_t$ 

rates does not display mean reversion, that is, the purchasing power parity does not hold. Finally, the predictable component of the exchange rate cannot be captured by studying short-term horizons, i.e., the exchange rates display long swings about its long term mean.

Three possible explanations of the results are now in order. First, it could be related to the "peso problem" of Krasker (1980), where there are a sustained excess forward premia for long period of time resulting from investors' belief of low likelihood of large depreciation. Second, it could be related to investors' underreaction to monetary shocks. For example, Shiller (1985) suggest a market model where agents underreact to monetary policy. Third, it could be related to market participants' assumption that the central banks will reverse the error after large unanticipated monetary changes, not it has changed its money growth target. According to this view, there is a positive correlation between the growth of money supply and real interest rate because a large monetary expansion breeds the expectation of future contraction in credit and interest rate witch may take several years.

#### V.I. Conclusion

This paper proposes a more general and realistic parametric model that makes substantial progress in studying the behavior of the exchange rates. A model that allows the spot exchange rates to behave like stationary, explosive, or random walk is capable to explain the behavior of the long horizons returns in the foreign exchange markets. Using new functional techniques, we provide further evidence that long horizon returns display significant positive autocorrelations over restricted horizons between 24- and 120-month.

This suggests that the spot exchange rates display an explosive behavior. As of shorter period horizons, namely 3-,6-,and 12 month, we conclude that the holding period returns are conditionally unpredictable. The result support the idea of fads model that the predictability of exchange rates returns cannot be deducted by examining short horizons.

Some practical implications concerning trading strategies can be gleaned from our results. First, the "buy low sell high" strategy that supports the disequilibrium model of exchange rates determination provides negative abnormal returns. This suggests the failure of the purchasing power parity. Second, unlike the stock exchange, momentum traders cannot beat the market by considering short horizons forming periods of their currency portfolios. The results of MBB suggest insignificant autocorrelations in the holding period returns across all currencies under the study. We relate the findings of positive autocorrelations found in the previous studies to the finite sample bias and non Gaussian distribution of exchange rates returns. Finally, the long horizon returns display significant positive autocorrelations and a momentum strategy of forming periods 24-, 36-, 48-, 60-, 72-, 96-, and 120- month can provides positive abnormal returns. This particularly provides evidence that the spot exchange rates behave like an explosive process.

A few possible directions for future research are immediately suggested. It would be worthwhile to examine the forward premium unbiasedness hypothesis by modeling the spot exchange rate as an explosive process. Hasza (1977) set an error correction for the least squares estimator when the data generating process displays an explosive behavior. There are many issues unresolved in pricing foreign exchange options. It would be interesting to develop an options model where the underlying assets display an explosive stochastic bubble.

# Essay II

# Rational Expectations and Market Efficiency in Foreign Exchange Markets: the Role of Finite Sample Bias

### I. Introduction

The concept of market efficiency plays a prominent role in many theoretical models of exchange rate determination. A number of studies relate the rejection of the market efficiency hypothesis, hereafter MEH, in the foreign exchange market to rational expectation errors in which the expected foreign exchange rate is a biased estimator for the future spot rate, or to the existence of a risk premium. For example, Domingues (1986), Ito (1993), Cavaglia, Verschoor, and Wolf (1993), and Beng and Siong (1993) test the "unbiasedness" hypothesis in the sense that survey measures are unbiased forecast of actual future outcomes. Their results strongly reject the unbiasedness hypothesis for monthly data. On the other hand, Liu and Maddala (1992), using a direct cointegration method, cannot reject the rational expectations hypothesis, hereafter REH, for weakly and monthly data. They also test for market efficiency using survey data and relate the rejection of market efficiency to the existence of a risk premium in the foreign exchange market. Lewis (1994) and Evans (1995) relate the rejection of REH and MEH

to the peso problem<sup>10</sup>. Engel (1996) argues that the peso problem will be eliminated if the sample is large and the rational expectations hypothesis holds. Cornell (1989) argues that the rejection of the market efficiency hypothesis is related to a measurement error because researchers do not fully account for transaction costs when they test for these hypotheses<sup>11</sup>.

Consider the traditional test of market efficiency in which investors are risk neutral and the forward rate is an unbiased estimator of the expected future spot rate  $E(S_{t+1})$ . If we incorporate an additive risk premium,  $Rp_t$ , and invoke rational expectations then,

$$F_t = E(S_{t+1}) + Rp_t \tag{1}$$

$$S_{t+1} = E(S_{t+1}) + u_{t+1}.$$
 (2a)

Risk neutrality implies that  $Rp_t = 0$ . The risk premium is the speculators expected profit margin, which may also capture any transaction costs related to the forward contract. Rational expectations implies:

$$E\left(u_{t+1} / \Omega_t\right) = 0 \tag{2b}$$

where  $\Omega_t$  is the information set at period t. This representation includes the assumption that  $u_t$  are serially uncorrelated. Assuming that the risk premium is a positive constant plus a white noise element vector  $v_t$ :

$$Rp_t = \alpha + v_t. \tag{3}$$

Combining (1), (2) and (3) we obtain

<sup>&</sup>lt;sup>10</sup> The Peso problem poses additional difficulty when testing for bubbles in the foreign exchange market. Market participants are likely to form expectations about central bank intervention in mitigating large swings in (real) and nominal exchange rates and these expectations are unlikely to be measured correctly by an econometrician.

<sup>&</sup>lt;sup>11</sup> Cornell (1989) use the lagged forward discount in the right -hand side of equation (5) to mitigate the effect of measurement error; he cannot reject the hypothesis that  $\beta = 1$ .

$$S_{t+1} = \alpha + \beta F_t + \varepsilon_{t+1}. \tag{4}$$

This equation represents one of the traditional tests of the MEH shown in Longworth (1981). However, if  $F_t$  and  $S_t$  are unit root processes as documented in previous literature, there is no guarantee that  $\varepsilon_t$  is a white noise random element unless  $S_t$  and  $F_t$  are cointegrated.

Instead, Froot and Frankel (1989) suggest the following equation to avoid the nonstationarity problem:

$$\left(S_{t+1} - S_t\right) = \alpha + \beta \left(F_t - S_t\right) + \varepsilon_{t+1}.$$
(5)

Under the null of market efficiency,  $\alpha = 0$  and  $\beta = 1^{12}$ . As suggested by many scholars, this procedure may be inconsistent. Assume that both  $S_t$  and  $F_t$  are unit root processes of order 1. Consequently, the left-hand side of the equation is stationary but there is no guarantee that the right hand side is stationary. If this is the case, the asymptotic distribution of  $\beta$  will always be zero and the MEH will be rejected. On the other hand, if  $(S_{t+1} - S_t)$  and  $(F_t - S_t)$  are both stationary variables, then OLS estimation is inconsistent since the risk premium  $Rp_t$  is correlated with the forward rate<sup>13</sup>. Liu and Maddala (1992) suggest the following regression to solve the inconsistency problem in testing market efficiency:

$$F_t = \alpha' + \beta' S_{t+1} + \varepsilon_t \tag{6}$$

<sup>&</sup>lt;sup>12</sup> Based on the econometric specification of equation (5), Froot (1990) notes that about 75 published papers document that the estimate of  $\beta$  is less than zero.

<sup>&</sup>lt;sup>13</sup> Note that  $F_t = E(S_{t+1}) + RP_t$  where  $RP_t$  is the risk premium. Since  $S_{t+1} = E(S_{t+1}) + \varepsilon_{t+1}$ , plug in the forward rate equation we get,  $S_{t+1} = F_t + (\varepsilon_{t+1}-RP_t)$ .

$$F_t = \alpha' + \beta S_{t+1} + \varepsilon_t \tag{7}$$

$$S_{t+1} = \mu + \theta S_t + \eta_t \tag{8}$$

Stambaugh (1986, 1999) and Mankiw and Shapiro prove that this type of system is biased in finite samples because the exogenous variable is not fixed in repeated samples. Following Stambaugh, we suggest that the bias of the least squares estimate of  $\beta$  in (4) is proportional to the least squares estimate of  $\theta$  in (5). This is because the white noise random elements  $\varepsilon_t$  and  $\eta_t$  are contemporaneously correlated with covariance  $\sigma_{uv}$ . In particular,

$$E(\hat{\beta} - \beta) = \frac{\sigma_{uv}}{\sigma_v^2} E(\hat{\theta} - \theta)$$
(10)

The intuition is straightforward. If the predictor is  $S_t$  then  $\beta$  and  $\theta$  will have the same estimate and bias. Kendall (1954) suggests the following approximation for the bias of the least squares estimate of  $\theta$ :

$$E\left(\hat{\theta} - \theta\right) = -(1 + 3\theta)/n + O\left(n^{-2}\right)$$
(11)

Thus, the market efficiency hypothesis may be rejected as a result of the biased estimate of  $\beta$ . This bias will be stronger with more contemporaneous covariance between the errors, higher autocorrelation of  $\varepsilon_t$ , and smaller sample size.

Our method is fivefold. First, we run unit root and cointegration tests to determine the stochastic process of each series and to examine whether the spot, expected spot, and forward rates are cointegrated. Second, we investigate for the stability of individual parameters and the entire parameter vector using Andrews (1993) and Andrews and Ploberger (1994) structural break tests. Third we test for MEH, REH, and the existence of a risk premium using OLS and Least Absolute Deviations (LAD) models. The LAD estimator is optimal when the disturbances have the Laplace distribution. The LAD is preferred to least squares regression when (i) the data are leptokurtic (fat-tailed) and skewed, (ii) the errors are serially correlated (since there are exactly *K* zero residuals (for *K* right-hand-side variables), this is analogous to the least squares property that there are only *N-K* linearly independent residuals), (iii) the observations include extreme outliers, and (iv) when the endogeneity of the regressors exists<sup>14</sup>. The equation to be estimated is in the form

$$\log(S_{t+1}) = \alpha + \beta \ \log(S_t^e) + \varepsilon_{t+1}$$
(12)

when considering the REH test;

$$\log(S_{t+1}) = \alpha + \beta \quad \log(F_t) + u_{t+1} \tag{13}$$

when considering the MEH test; and

$$\log(F_t) - \log(S_t^e) = \lambda + v_{t+1} \tag{14}$$

<sup>&</sup>lt;sup>14</sup> The forward rate is endogenous even if it is predetermined since the true mean of the predictor is unknown.

when examining the existence of risk premium. For the fourth step, we find the bootstrap vector of the response variable for OLS and LAD estimators using the Freedman (1981, 1984) and Hall (1988) resampling method to solve for inconsistency and finite sample bias. Finally, we bootstrap the *F*-statistic conducted in four to jointly test for the MEH, REH, and the existence of a risk premium in the foreign exchange market.

Our contribution is in relating the rejection of the MEH in the foreign exchange market to the existence of risk premium not to the rejection of REH. Thus, we relate the rejection of REH commonly found in the literature to small sample bias that prior researchers have failed to take into account when conducting their tests of this hypothesis. The results show that the deviation of the coefficient estimates is too large, causing the value of the *F*- Statistic to be biased upward and thus explaining the rejection of REH.

The remainder of the paper is constructed as follows. Following the introduction in section I, Section II describes the data. Section III, develops the methodologies used. In section IV we test the REH and MEH and explain the results. Section V concludes.

#### II. Data

We used two separate data sets: the data for actual spot rates and the expected spot exchange rates that consist of end-of-month observations of the exchange rate (against U.S. Dollar) of the Pound Sterling, the Douche Mark, the Japanese Jen, the Swiss Franc, the French Franc, and the Canadian Dollar as given by Financial Times Currency Forecaster (FTCF). A team of 30 multinational companies and 15 forecasting service providers that comprise the currency–forecasting panel, provides the forecasts. The endof-month forward rates for the same currencies are as given by the Data Resources Incorporated (DRI) database. The data are monthly and span from February 1988 to May 2000.

#### III. Methodologies

This section describes Andrews (1993) and Andrews and Ploberger (1994) structural break tests, LAD inference, and bootstrap method adopted in our paper for testing for the REH and MEH. First, I briefly review structural break tests used in our study and then LAD is developed. Next, the bootstrap method is introduced.

#### **III.A.** Structural Change Test

Recent advances in the econometrics of structural break now allow for robust tests of parameter instability without assuming exogenous change points. Among the proposed structural break tests, the ones introduced by Andrews (1993) and Andrews and Ploberger (1994) are particularly attractive since they can be used to test for the stability of individual parameters or the entire parameter vector where the change points are endogenous<sup>15</sup>. However, Andrews (1993) introduces an elegant way to develop efficient asymptotic critical values for the test where the breakdate is unknown a priori. More recently, Hansen (1997) develops approximation methods to calculate P-values for the Andrews (1993) and Andrews and Ploberger (1994) tests. Moreover, the test has power against the alternative where the parameters may change gradually over time. For the analysis of this paper, I construct the Andrews (1993) *Sup* (Supremum) and Andrews and Ploberger (1994) *Exp* (Exponential) and *Ave* (Average) tests based on *F* statistic. In particular,

<sup>&</sup>lt;sup>15</sup> Andrews and Ploberger (1994) show that *Ave* (Average) and *Exp* (Exponential) tests have certain optimality relative to the *Sup* (Supremum) test proposed by Andrews (1993).

$$SupF = \sup_{\tilde{i} \le i \le \tilde{i}} F_i, \qquad (14a)$$

$$AveF = \frac{1}{\tilde{i} - \bar{i} + 1} \sum_{i=\tilde{i}}^{\bar{i}} F, \qquad (14b)$$

$$ExpF = \log\left(\frac{1}{\tilde{i} - \bar{i} + 1}\sum_{i=\tilde{i}}^{\bar{i}} \exp(0.5 \cdot F_i)\right)$$
(14c)

where i is some change point in an interval (i, i)<sup>16</sup>. Let  $\lambda$  is a prechange parameter vector and  $\lambda + \delta$  is a post-change parameter vector. The null hypothesis of interest is

$$H_0: \delta = 0 \tag{15a}$$

Against the alternative

$$H_a: \delta \neq 0 \tag{15b}$$

Where the hypothesis of no structural breaks in the entire parameter vector will be rejected if the p-values of those statistics computed by Hansen (1997) are below 5 percent.

#### **III.B.** LAD estimation

Least squares estimation gives disproportionate weight to large deviations in the calculation. This property becomes a disadvantage when the disturbances are not normally distributed, especially when the sample is small or moderate. Basset and Koenker (1978) argue that the LAD estimator can efficiently solve this problem that

<sup>&</sup>lt;sup>16</sup> Andrews (1993) suggests to consider all the breakdates in the interval v% to (1-v)% of the sample, where the trimming parameter v is typically between 5% and 15%. For the analysis of this paper, I use 15% trimming.

becomes severe as a result of outlying observations. Phillips (1991) argues that the LAD estimator makes modifications to the usual regression procedure to deal with the overlapping autocorrelations in the errors. This property is useful in testing the MEH hypothesis, where the duration of the futures maturity contract exceeds the time interval between observations (see, Hansen and Hodrick (1980), for further discussion). Also, Knight (1991) shows that the LAD estimator has desirable asymptotic properties when the data exhibits an autoregressive unit root, it is shown that the LAD estimator has a faster rate of convergence than the OLS. Moreover, Basset and Koenker (1978) demonstrate that the LAD has smaller asymptotic confidence ellipsoids than the OLS for any error distribution, that is more efficient. Further, the model is robust and consistent when the endogeneity of regressors exists, (see Basset and Koenker (1978), for further discussion).

To fix ideas, consider the VAR system in which  $Y_t$  and  $X_t$  are I(1) processes and can be cointegrated

$$Y_t = \beta' X_t + u_t \tag{16a}$$

$$X_t = X_{t-1} + v_t \tag{16b}$$

where  $u_t$  and  $v_t$  are stationary processes. The LAD estimator of *B* is the extremum estimator

$$\beta_{LAD} = \underset{\beta}{\operatorname{arg\,min}} \left[ \frac{\sum \left| Y_t - \beta' X_t \right|}{n} \right]$$
(17)

If the disturbances in the model are of the Laplace distribution,

$$f(\varepsilon_i) = \frac{1}{2\sigma} \exp\left(\frac{-|\varepsilon_i|}{2\sigma}\right)$$
(18)

then the LAD estimator is optimal since LAD is identical to the maximum likelihood estimator. Even if the distribution of the disturbances is not Laplace, the LAD estimator is still consistent since it is a special case of the quintile regression

$$\Pr{ob}[Y_i \le X_i^{\prime}\beta] = q \tag{19}$$

at q = 0.5. Namely, the LAD estimator estimates the median regression. Rogers (1993) suggests the following asymptotic covariance matrix of the quintile regression estimator,

$$VAR[b_q] = (X'X)^{-1} X'WX(X'X)^{-1}, \qquad (20)$$

where W is the diagonal weights matrix

$$d_{i} = \begin{cases} \left[\frac{q}{f(0)}\right]^{2} \text{ if } Y_{i} - X_{i}^{\dagger}\beta & \text{ is positive} \\ \left[\frac{1-q}{f(0)}\right]^{2}, \text{ otherwise} \end{cases}$$
(21)

and f(0) is the true density of the disturbances evaluated at 0 and is unknown. To get robust estimate of the disturbance density, we employ the kernel density estimator with standard normal kernel and optimal bandwidth

$$c = \left[ k^{-1} \left( 2\sqrt{\pi} \right)^k \sqrt{\det \sum} \int \left( trace \left[ \left( \frac{df(x)}{dxdx'} \right) \sum \right] \right)^2 dx \right]^{-1/(k+4)} n^{-1/(k+4)}$$
(22)

This modified LAD estimator is asymptotically consistent and free of all shortcomings of the least squares estimation of the foreign exchange data especially the autocorrelation that leads to inconsistent estimate in the case of lagged regressors. However, even though this methodology provides consistent estimates, since the endogeneity problem no longer exists, it is still bias in the finite sample and should be simulated.

#### **III.C. Bootstrap Biased adjustment**

A number of strategies exist for adjusting for bias in the point estimation. Monte Carlo techniques usually assume that the distribution function of the random errors is normal. In the case of foreign exchange it is known that the sampling distribution is leptokurtic, skewed and deviating from normality (see, Clark (1973) and Tauchen and Pitts (1983), for example). For our data, the series also exhibits serial correlation and heteroskedasticity. We draw residuals with replacement, which is called, bootstrapping (see, Efron (1979, 1982, 1987), for further discussion). Bootstrapping differs from randomization of Noreen (1989) only in that sampling is with replacement. Under random sampling with replacement, the resamples will vary randomly from the original sample, and the resulting estimates calculated from these resamples will likewise vary randomly from the point estimate. Babu and Singh (1983) show that the sampling

distribution of the point estimate will be identical to the population distribution function, in this since we expect that the peso problem will be mitigated since the consistency of  $\beta$  implies that

$$Cov(f_t - S_t, S_{t+1}^e - S_{t+1}) = 0$$
(23)

Freedman (1981, 1984) and Hall (1988) suggest the resampling of residuals to solve for the bias in the point estimates when the regressors and the errors are correlated. Singh (1981) contradicts the asymptotic validity of the bootstrap covariance estimator in the case of autocorrelation. He notes that both approaches suggested by Efron (1979, 1982) namely, the classical bootstrap and the Design Matrix Bootstrap (DMB) cannot provide autocorrelated consistent estimator, hereafter AC, in the OLS case The LAD estimator is appropriate in this case where the errors are uncorrelated and temporarily homogeneous. The bootstrap estimate of the LAD parameters and the asymptotic covariance matrix are

$$\beta = \frac{\sum_{l=1}^{R} \beta_{LAD}(r)}{R}$$
(24a)

$$Var[\beta_{LAD}] = \frac{1}{R} \sum_{r=1}^{R} (\beta_{LAD}(r) - \beta_{LAD}) (\beta_{LAD}(r) - \beta_{LAD})$$
(24b)

where  $\beta_{LAD}$  is the LAD estimator and  $\beta_{LAD}(r)$  is the rth LAD estimate of  $\beta_{LAD}$  drawn with replacement, from the original data set.

#### **IV.** Empirical Results

In this section, we first report the results of unit root tests for each series. Second, we report the stability tests on the basis of individual parameters and full parameter vector for tests specification in (12) and (13). Third, we compare the OLS and modified LAD performance in testing the MEH and REH. Finally, the bootstrap parameters estimates for

the overall sample period and different subperiods selected on the basis of the stability tests are reported.

#### **IV.A.** Unit Root and Cointegration

Before we apply our methodology, it is necessary to examine whether the spot rate and its expectations are stationary or unit root processes. If they are an I(1) process, as much of the literature suggests, they must be cointegrated. This will ensure that suggested estimators are not subject to the spurious regression problem suggested by Granger and Newbold (1974). For the sake of robustness, we imply two types of unit root tests. The first type is the conventional parametric tests, which include the Augmented Dickey-Fuller (1979) (ADF) test, the Phillips and Peron (1988) (PP) test, and the Augmented Weighted Symmetric (WS) tests. The WS test is a weighted, double-length regression. Pantula et al. (1994) argue that WS dominates the ADF and PP tests in terms of power. Following Campbell and Perron (1991) we include the trend for all the unit root tests. The Akaike Information criterion (AIC) is used to determine the optimal number of lags for all the tests<sup>17</sup>. The second type is the nonparametric test suggested by Breitung (2002). Two advantages of Breitung test over ADF, PP, and WS tests. The Monte Carlo simulations show that it is robust to structural breaks, and it is robust to model misspecifications since the asymptotic property is independent from the stationary component of the series.

Panel A of Table 1, 2, and 3 report the results of ADF, PP, and WS tests for the spot, expected spot, and forward rates, respectively. As shown across all tests we do not reject

<sup>&</sup>lt;sup>17</sup> The ADF,PP, and WS accurate asymptotic *p*-values are computed using MacKinnon (1994) approximation (robust to finite sample distortion).

the null of a unit root process for all of the series. We repeat the test for the first-order difference (the results are not reported here) for each series. The entire set of tests reject the null of a unit root process. Panel B of Table 1, 2, and 3 report the Breitung test for the two hypotheses. The first hypothesis, the test is denoted by B(n), where under the null the exchange rate is driftless unit root against the alternative of stationarity. The second hypothesis, the test is denoted TB(n), where under the null the exchange rate is a unit root with drift against the alternative of trend stationary. All tests demonstrate that the unit root hypothesis cannot be rejected. We conclude that the spot, expected spot and forward rates are I(1) processes.

For the purpose of testing the MEH and REH, the Engle-Granger (EG) cointegration test has been used to determine if the spot rate and its expectations are cointegrated. Table 4, Panel A reports the cointegration vectors and EG tests of the spot and expected spot rates. We reject the null hypothesis that the two series cannot be cointegrated. In Panel B, we run the test for the spot rate and the forward rate again; the results suggest that these two series are cointegrated<sup>18</sup>. Panel C suggests that the same results are reported for the forward rate and expected spot rate, one exception is Dutch mark. Since each currency spot rate is cointegrated with its forward and expected spot rates with cointegration vector (1,-1), we conclude that invoking the OLS and LAD regression in (12) and (13) is neither subject to the well known spurious regression problem suggested by Granger and Newbold (1974) nor to bootstrap inconsistency in the case of unit root variables

 $<sup>^{18}</sup>$  Hakkio and Rush (1989) using Engle and Granger's (1987) test, provide evidence that  $S_{t\!+\!1}$  and  $F_t$  are cointegrated.

Horvath and Watson (1995) develop a test for cointegration when the cointegrating vector is known. They strongly reject the null that  $F_t$  and  $S_t$  are not cointegrated.

entional Unit F	Root Tests				
		Currency			
UK	DM	YEN	SW	FF	CAN
-2.778	-2 404	-1 590	-2.384	-2.218	-2.563
					0.331
	3	6	2	6	3
-2.789	-1.894	-1.245	-2.251	-2.109	-2.236
0.200	0.657	0.901	0.461	0.540	0.432
2	2	6	2	5	2
-15.978	-9.081	-6.098	-12.970	-11.115	-12.073
0.154	0.500	0.737	0.267	0.365	0.259
2	2	6	2	5	2
parametric tests	3	~			
		•			
UK	DM	YEN	SW	FF	CAN
0.024	0.016	0.0373	0.0233	0.0145	0.0852
	0.010	0.0137			0.0092
	UK -2.778 0.152 2 -2.789 0.200 2 -15.978 0.154 2	-2.778       -2.404         0.152       0.354         2       3         -2.789       -1.894         0.200       0.657         2       2         -15.978       -9.081         0.154       0.500         2       2         parametric tests       UK         UK       DM         0.024       0.016	UK         DM         Currency YEN           -2.778         -2.404         -1.590           0.152         0.354         0.862           2         3         6           -2.789         -1.894         -1.245           0.200         0.657         0.901           2         2         6           -15.978         -9.081         -6.098           0.154         0.500         0.737           2         2         6           Currency           UK         DM         YEN           0.024         0.016         0.0373	UK         DM         YEN         SW $-2.778$ $-2.404$ $-1.590$ $-2.384$ $0.152$ $0.354$ $0.862$ $0.368$ $2$ $3$ $6$ $2$ $-2.789$ $-1.894$ $-1.245$ $-2.251$ $0.200$ $0.657$ $0.901$ $0.461$ $2$ $2$ $6$ $2$ $-15.978$ $-9.081$ $-6.098$ $-12.970$ $0.154$ $0.500$ $0.737$ $0.267$ $2$ $2$ $6$ $2$ Currency           UK         DM           Currency           UK         DM           SW $0.024$ $0.016$ $0.0373$ $0.0233$	UK         DM         YEN         SW         FF           -2.778         -2.404         -1.590         -2.384         -2.218           0.152         0.354         0.862         0.368         0.489           2         3         6         2         6           -2.789         -1.894         -1.245         -2.251         -2.109           0.200         0.657         0.901         0.461         0.540           2         2         6         2         5           -15.978         -9.081         -6.098         -12.970         -11.115           0.154         0.500         0.737         0.267         0.365           2         2         6         2         5

Table 1.
Unit root tests on the logarithm of spot exchange rates $(S_t)$ .

Note: unlike the conventional unit root tests the hypothesis of unit under Bretung (2002) approach is rejected if the test statistic is below the corresponding critical value.

Notice that Breitung (2002) only reports the critical values for n = 100, n = 250 and n = 500. However, the critical values used in this study are 0.01004, 0.00342 for 5% significance level for the alternative of stationary and trend stationary, respectively are based on linear interpolation.

Table 2.
Unit root tests on the logarithm of the expected spot exchange rates $(S_t^e)$

			Currency			
Statistics	UK	DM	YEN	SW	FF	CAN
WO	2 0 2 2	2 406	1 670	2 00 4	2 (20	0.550
WS	-2.922	-2.406	-1.572	-2.884	-2.620	-2.553
P-Value	0.104	0.353	0.868	0.115	0.224	0.301
Optimal	2	2	6	3	2	2
Number of						
Lags						
ADF	-2.841	-2.178	171	-2.643	-2.423	2.231
P-Value	0.182	0.502	0.916	0.260	0.367	0.453
Optimal	2	2	6	2	2	2
Number of						
Lags						
PP	-16.357	-10.438	-6.13081	-15.027	-13.362	-11.989
P-Value	0.143	0.407	0.735	0.184	0.250	0.354
Optimal	2	2	6	2	2	2
Number of						
Lags						
Panel B. Nonr	parametric Tests	3				
Tuner D Trong		,	Currency			
Statistics	UK	DM	YEN	SW	FF	CAN
B(n)	0.02672	0.01446	0.03523	0.02051	0.01452	0.08522
TB(n)	0.00772	0.01029	0.01373	0.00613	0.00899	0.00925

Panel A ·	Conventional	Unit Root Tests	
	Conventional		

Note: unlike the conventional unit root tests the hypothesis of unit under Bretung (2002) approach is rejected if the test statistic is below the corresponding critical value.

Notice that Breitung (2002) only reports the critical values for n = 100, n = 250 and n = 500. However, the critical values used in this study are 0.01004, 0.00342 for 5% significance level for the alternative of stationary and trend stationary, respectively are based on linear interpolation.

			Currency			
Statistics	UK	DM	YEN	SW	FF	CAN
WO	1 075	1 214	1.011	1 4 4 0	1 254	1 200
WS	-1.875	-1.214	-1.011	-1.448	-1.354	-1.306
P-Value	0.728	0.961	0.972	0.905	0.927	0.935
Optimal	3	2	2	2	3	2
Number of						
Lags	1 (0)	1 5 4 0	1 00 4	2 002	1 000	1 70 4
ADF	-1.696	-1.548	-1.234	-2.093	-1.980	-1.724
P-Value	0.752	0.793	0.903	0.550	0.611	0.753
Optimal	3	2	2	2	2	2
Number of						
Lags						
PP	-6.68	-6.015	-5.082	-9.535	-9.281	-7.604
P-Value	0.691	0.780	0.815	0.468	0.486	0.563
Optimal	3	2	2	2	2	2
Number of						
Lags						
Panel B: Nonp	parametric tests					
			Currency			
Statistics	UK	DM	YEN	SW	FF	CAN
$\mathbf{D}(\cdot)$	0.01251	0.01717	0.012(4	0.02405	0.00740	0.00405
B(n)	0.21351	0.01717	0.01364	0.02405	0.02740	0.08495
TB(n)	0.00693	0.01717	0.02336	0.00650	0.00548	0.00879

Table 3. Unit root tests on the logarithm of the forward exchange rates  $(F_{t+1})$ 

Note: unlike the conventional unit root tests the hypothesis of unit root under Bretung (2002) approach is rejected if the test statistic is below the corresponding critical value.

Notice that Breitung (2002) only reports the critical values for n = 100, n = 250 and n = 500. However, the critical values used in this study are 0.01004, 0.00342 for 5% significance level for the alternative of stationary and trend stationary, respectively are based on linear interpolation.

Currencies								
Statistics	UK	DM	Yen	SW	FF			
$S_{t+1} \& ES_t$	-6.502	-6.084	-0.388	-5.889	-4.756			
P-Value	0000	0.000	0.386	0.002	0.021			
Optimal Number of	2	2	7	2	5			
Lags Cointegration vector	1.0 ,874	1.0 ,944	1.0 ,977	1.0 ,902	1.0,926			
$F_t \& ES_t$	-7.736	-3.379	-4.238	-5.596	-5.009			
P-Value	0.000	0.129	0.013	0.000	0.000			
Optimal Number of Lags	2	2	4	2	3			
Cointegration vector	1.0, 0.955	1.0, -0.992	1.0, -0.975	1.0, -0.846	1.0, -0.944			
$S_{t+1}$ & $F_t$								
P-Value	0.014	0.002	0.004	0.013	0.011			
Optimal Number of Lags	3	3	4	3	3			
Cointegration vector	1.0,-0.809	1.0-0.921	1.0,-0.904	1.0,-0.884	1.0,-0.875			

Table 4. Engle-Granger contegration (EG) tests and Contigration vectors for the logarithm of the spot exchange rate, its expectation, and the forward rate  $(S_{t+l}, S_t^e \text{ and } F_t)$ .

suggested by Philips (2003) since the errors are stationary. However, bootstrapping using OLS residuals cannot provide HAC variance if the errors are autocorrelated.

#### IV.B. Model Stability Test

Table 4 reports the results of the structural break tests for the econometric specification in (12) and (13). To allow for sufficient degrees of freedom, the trimming parameters are set to 15% and 30% of the effective sample when Andrews (1993) and Andrews and Ploberger (1994) are considered, respectively.

The findings in Table 4 provide strong evidence that both MEH and REH tests on the Japanese yen displays a structural break at the beginning of 1996. The results are robust across the *Sup, Ave,* and *Exp* tests and all *p*-values indicate a rejection of the null hypothesis of parameter stability at 5% and 1% significance level. For the Pound Sterling, the Deutsche Mark, the Swiss Franc, the French Franc, and the Canadian Dollar, we find no evidence of the parameters' instability; none of the *p*-values are below the 5% significance level.

To gain some insight into the nature of the instability, Figures 1-4 plot the values of the *Sup F* test statistic supported by Andrews' (1993) 5% critical value for the null hypothesis of no structural change in the full parameter vector for the REH tests for the Pound Sterling, the Japanese Yen, the French Franc and the Swiss Franc<sup>19</sup>. For the Japanese Yen, the single peak of the *Sup F* test statistic occurred in March 1996.at the height of the depression on March 1933.

<sup>&</sup>lt;sup>19</sup> To save space we do not report the graphs of the Canadian Dollar and the Deutsche mark, we also do not report the graphs related to the MEH test. These are available upon request.

The following econometric specifications are used to test for MEH and REH across the subsamples, respectively:

$$\log(S_{t+1}) = E\alpha_e + L \alpha_L + E\beta_e \log(F_t) + L\beta_l \log(F_t) + \varepsilon_{t+1}$$
(59a)

$$\log(S_{t+1}) = E\alpha_e + L \ \alpha_L + E\beta_e \log(S_t^e) + L\beta_l \log(S_t^e) + \varepsilon_{t+1}$$
(59b)

where, E=1 if t is before the break and 0 otherwise, L=1 if t is after the break and zero otherwise. The slope coefficients' subscripts e or 1 indicates before and after subperiod coefficients (i.e.,  $\beta_e$  is the slope of the test before the break).

#### IV. Robust Test of MEH and REH

The foreign exchange data show clear auotocorrelation and heteroskedasticity, which should be taken into account in standard errors when OLS is used. Newey and West (1987), hereafter NW, suggest a consistent variance estimator which is positive semidefinite to deal with auotocorerelation. The spectral density kernel is used to ensure positive definiteness of the variance matrix when the number of moving averages is greater than zero. The process is conducted by weighting the sample autocorrelations with weight w(j,m) = 1-[j/m + 1], where j is the lag and m is the maximum lag. Following Cochrane (1991), we use 2(K-1) numbers of moving averages, where k is the number of each forecast horizon. For heteroscedasticity, we use White's consistence variance estimator with a Jackknife approximation suggested by Mackinnon and White (1985)<sup>20</sup>. In the case of the modified LAD, the normal kernel density is used to provide an efficient

<sup>&</sup>lt;sup>20</sup> Mackinnon and White (1985) provide evidence that the Jackknife approximation provides *t*-statistics with smallest distortions than any version of a heteroscedastic consistent variance covariance matrix.

estimator. The autocorrelation also does not exist since there are exactly *K* zero residuals for *K* right-hand-side variables.

Figure 6 graphs the standard normal kernel densities of the spot exchange rates and the error term (u) implied by equation 15 for the Pound Sterling. As shown, the excess kurtosis and fat-tailed facial appearance are obvious from the estimation for both the exchange rate and the error term implied from the quintile regression for the REH. To save space, we only report the graphs for the Pound Sterling, but parallel prototypes were observed for the other exchange rates and the other error terms for the MEH, which are available upon request.

The rational expectations hypothesis was estimated for each of the six currencies by OLS and modified LAD. The results are shown in Table 5. In the case of the LAD estimator, the value of  $\beta$  is much closer to 1. For example, in the case of the one-month Pound Sterling  $\beta_{\text{LAD}} = 0.88$  and  $\beta_{\text{OLS}} = 0.87$ . (Note, however that both estimators are biased since the predictor is not fixed in the repeated sample, and OLS is inconsistent as a result of the endogeneity problem and autocorrelation). The *F*-ratio for testing the joint hypothesis, H<sub>0</sub>:  $\alpha = 0$ ,  $\beta = 1$ , are given in Table 5. In the case of the Pound Sterling, Deutsche Mark, and Swiss Franc, we reject the null hypothesis of rational expectations unbiasedness using both OLS and LAD for the one-month forecast horizon. The t-ratio for testing the hypothesis, H<sub>0</sub>: b=1, is also rejected across the three currencies using both estimators.

			G			
			Currency			
Statistics	UK	DM	YEN	SW	FF	CAN
α	0.061	0.024	-0.087	0.035	0.187	-0.009*
t-Stat	-2.578*	1.343	0.740	3.052*	2.244*	-0.242
β	0.878	0.950	0.981	0.900	0.891	1.008*
t-Stat	-2.377*	-1.405	-0.745	-3.046*	-2.233*	0.633
F-Stat	4.810*	1.025	0.292	3.782*	2.550	2.673
Donal D. I A	D					
Panel B: LA	١D		Currency			
	JD UK	DM	Currency YEN	SW	FF	CAN
Panel B: LA Statistics α		DM 0.019	5	SW 0.048	FF 0.161	CAN -0.065
Statistics	UK		YEN			
Statistics α <i>t</i> -Stat	UK -0.060	0.019	-0.034	0.048	0.161	-0.065
Statistics α	UK -0.060 -4.418*	0.019 1.499	YEN -0.034 -0.588	0.048 5.039*	0.161 3.291*	-0.065 -2.366*

## Table 5. OLS and LAD estimation of the rational expectation hypothesis $(S_{t+1}) = \alpha + \beta(S_t^e) + \varepsilon_t$

Note: the number on the parenthesis is t-test for  $H_0$ :  $\alpha=0$ ,  $H_0$ :  $\beta=1$ . F-test for  $H_0$ :  $\alpha=0$ ,  $H_0$ :  $\beta=1$  \* null hypothesis is rejected at 5% significance level.

The significance level of F(2,137) is 3.0642.

Newey West consistent estimate is used to adjust for autocorrelation.

White hetroscedastic consistent estimate with jackknife approximation is used to adjust for heteroscedaticity

Table 6 Panel A and B report the bias adjusted OLS and LAD estimates using the bootstrap method. For all currencies, the value of  $\alpha$  is close to zero and the value of  $\beta$ converges to 1. For example, in the case of the Pound Sterling  $\alpha_{OLS} = 0.00018$ ,  $\alpha_{LAD} = -$ 0.0051,  $b_{OLS} = 1.0007$ , and  $b_{LAD} = 1.00248$ . Table 9 reports the bootstrapped adjusted confidence interval for the hypotheses,  $H_{0:} \alpha = 0$  and  $H_{0:} \beta = 1$ . Across all currencies, we cannot reject the null that investors in the foreign exchange market are rational. The gains in efficiency from the bootstrap confidence interval are evident since we reject the same hypothesis for the Pound Sterling, the Deutsche Mark, the Swiss Franc, and the French Franc before the finite bias adjustment. The joint hypothesis,  $H_0$ ,  $\alpha = 0$ ,  $\beta = 1$ , also cannot be rejected for all the currencies after the bias adjustment. The empirical levels of the Ftests based on the asymptotic critical values are smaller than the bootstrap estimates of the F-ratio. The critical values of the bootstrap F distribution shift widely to the right relative to the asymptotic F distribution. Instead, all of the nominal values of the calculated F-ratios using a one-month horizon lie inside the acceptance region of the empirical bootstrap confidence interval. We conclude that the REH is strongly accepted for a one-month horizon.

Table 7 reports the OLS and LAD estimates of the efficient market hypothesis. In all cases the value of  $\beta$  resulting from LAD estimation is closer to 1. As mentioned above, the value of  $\beta$  has an important interpretation on the statistical property of the risk

$$Rp_{t,k} = F_{t,k} - E_t(S_{t+k}) = (1-b)F_{t,k} - \alpha - E_t(u_{t+k})$$

## Table 6. Bootstrap estimates for OLS and LAD methods of the rational expectation hypothesis $(S_{t+1}) = \alpha + \beta(S_t^e) + \varepsilon_t$

			Currency			
Statistics	UK	DM	Yen	SW	FF	CAN
Boot α	0.000	-0.000	0.000	-0.000	-0.001	-0.001
Bias	-0.059	-0.024	0.086	-0.034	-0.185	-0.008
estimate						
Boot β	1.000	1.001	1.000	0.999	1.002	1.011
Bias	-0.122	-0.059	-0.019	-0.099	-0.1113	-0.000
estimate		+	+	+		+
Boot F	5.015 <sup>†</sup>	3.386 <sup>†</sup>	3.608 <sup>†</sup>	3.951 <sup>†</sup>	4.306 <sup>†</sup>	$2.872^{\dagger}$
Panel B: Bi	as adjusted LA	D	Curronau			
			Currency YEN		FF	CAN
Panel B: Bi Statistics	as adjusted LA	D DM	Currency YEN	SW	FF	CAN
			•		FF 0.141	CAN -0.007
Statistics	UK	DM	YEN	SW		
Statistics Boot α	-0.058	DM 0.023	YEN -0.0309	SW 0.041	0.141	-0.007
Statistics Boot α Bias	-0.058	DM 0.023	YEN -0.0309	SW 0.041	0.141	-0.007
Statistics Boot α Bias estimate	-0.058 0.001	DM 0.023 -0.005	YEN -0.0309 -0.0266	SW 0.041 0.007	0.141 0.020	-0.007 -0.016

Note: boot F is the bootstrap confidence interval centered at 0 and 1. <sup>†</sup> Indicates that the point estimate lies inside the bootstrap confidence interval.

Panel A: OLS	)					
			Currency			
Statistics	UK	DM	Yen	SW	FF	CAN
α	0.031	0.041	-0.194	-0.022	-0.084	0.003
<i>t</i> -Stat	3.897*	2.148*	-3.592*	-4.297*	-1.555	3.660*
β	0.934	0.921	0.960	0.953	0.984	1.012
<i>t</i> -Stat	-3.642*	-2.088*	-3.541*	-4.103*	-1.545	4.258*
F-Stat	10.081*	6.732*	8.557*	9.259*	4.627*	9.706*
			0.007	,		2.100
Panel B: LAI	)		Currency			
Panel B: LAI Statistics	D UK	DM		SW	FF	CAN
Statistics	UK	DM	Currency Yen	SW	FF	CAN
			Currency			
Statistics α	UK 0.025	DM 0.043	Currency Yen -0.209	SW -0.013	FF 0167	CAN 0.007
Statistics α <i>t</i> -Stat	UK 0.025 8.673*	DM 0.043 3.080*	Currency Yen -0.209 -9.773*	SW -0.013 -1.587	FF 0167 -2.077*	CAN 0.007 4.304*

## Table 7. OLS and LAD estimation of the efficient market hypothesis. $(S_{t+1}) = \alpha + \beta(F_t) + \varepsilon_t$

Note: the number on the parenthesis is t-test for  $H_0$ :  $\alpha=0$ ,  $H_0$ :  $\beta=1$ . F-test for  $H_0$ :  $\alpha=0$ ,  $H_0$ :  $\beta=1$ 

\* Null hypothesis is rejected at 5% significance level.

The significance level of F (2,2102) is 3.0642.

Newey West consistent estimate is used to adjust for autocorrelation.

White hetroscedastic consistent estimate with jackknife approximation is used to adjust for heteroscedaticity

The risk premium,  $Rp_{t,k}$ , is non-stationary if the null hypothesis,  $H_0$ :  $\beta = 1$  is rejected. For all currencies the hypothesis is rejected at the 5% level using OLS. We find little evidence of a stationary risk premium when the modified LAD is used; the hypothesis is accepted for the French Franc only. For all currencies, both OLS and modified LAD fail to reject the hypothesis  $H_0$ :  $\alpha = 0$ . Consistent with Naka and Whitney (1995), Norrbin and Refett (1996), and Hai et al. (1998), we also reject the joint hypothesis of market efficiency (i.e.  $H_0$ :  $\alpha = 0$ ,  $\beta = 1$ ). The *F*-ratio constructed from both the OLS and LAD estimator is too large to be accepted.

Again, we adjust for the small sample bias and reconstruct the test using the bootstrap technique. The results show that the value of  $\beta$  converges substantially to 1, and are robust across all currencies, except for the Canadian Dollar. For example, in the case of the Pound Sterling, the bootstrap estimates are 0.991 and 0.997 based on OLS and LAD, respectively. The last column of Table 8 reports the bias adjusted parameters values for the Canadian Dollar. In this case, the bootstrap shows substantial improvement on the basis of OLS. The value of the parameter  $\beta$  increased from 0.047 to 0.331, although it is still small. On the other hand, the opposite finite bias direction is documented in the case of LAD, where the value of the parameter  $\beta$  displays downward divergence from the value of 1 toward 0.

## Table 8. Bootstrap estimates for OLS and LAD methods of the market efficiency hypothesis $(S_{t+1}) = \alpha + \beta(F_t) + \varepsilon_t$

			Currency			
Statistics	Uk pond	DM	Yen	SW	FF	CAN
Boot a	0.027	0.032	-0.219	-0.019	0.028	0.000
Bias estimate	0.023	-0.015	0.026	0.004	-0.163	-0.003
Boot β	0.998	0.923	0.954	0.965	0.984	1.017
Bias estimate	-0.063	0.001	0.003	-0.024	0.007	-0.005
Boot F	4.446	2.877	3.420	4.561	$3.058^{\dagger}$	2.959

	Panel A:	Bias	adjusted	OLS	method
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Panel B: Bias adjusted LAD (on median) method

			Currency			
Statistics	Uk pond	DM	Yen	SW	FF	CAN
Boot α	0.001	0.049	-0.218	0.024	0.002	0.000
Bias estimate	0.023	-0.005	0.095	-0.025	-0.025	-0.007
Boot β	0.961	0.904	0.954	0.985	1.001	1.034
Bias estimate	0017	0.053	0.002	0.000	0.011	1.012
Boot F	5.051	8.241	4.650	$15.598^{\dagger}$	$12.460^{\dagger}$	8.221

Note: boot F is the bootstrap confidence interval centered at 0 and 1.

<sup>†</sup> Indicates that the point estimate lies inside the bootstrap critical value.

Table 9 reports the bootstrap confidence interval for the stationary risk premium hypothesis; the hypothesis cannot be rejected using the percentile bootstrap confidence interval across all currencies, the Canadian Dollar is an exception. Consistent with Naka and Whitney (1995) and Wu and Chen (1998) we conclude that the risk premium is stationary. For the hypothesis H<sub>0</sub>:  $\alpha = 0$ , the point estimate of  $\alpha$  lies inside the percentile bootstrap confidence interval for both OLS and LAD estimators. The results fail to reject the forward rate unbiasedness hypothesis across all currencies. We conclude that the risk premium is previous literature are mainly subject to the finite sample bias.

As a robust check we turn to employ the parametric and the nonparametric unit root tests to examine the stationarity of forward premium defined as the error term of equation 13 that is  $u_t = S_{t+1} - F_t$ . Table 10 reports only the Breitung (2002) nonparametric test<sup>21</sup>. Consistent with Wu and Chen (1998), who used the mean group approach of Im et. el. (1995), we cannot reject the hypothesis that  $u_t$  is a stationary process.

For the joint test of market efficiency, i.e.  $H_0$ :  $\alpha = 0$ ,  $\beta = 1$ , the bootstrap bias adjustment fails to accept the null hypothesis for the Pound Sterling, the Deutsche Mark, the Japanese Yen and the Canadian Dollar. As shown in Table 9, we observe that the critical values of the bootstrap *F* distribution are shifted to the right relative to the asymptotic *F* distribution, but the *F*-ratios constructed from the point estimates for those four currencies are too high to lie inside the acceptance region. However, for both the Swiss

<sup>&</sup>lt;sup>21</sup> We also employ the ADF, PP, and WS tests. Consistent with Crowder (1994), the naïve tests fail to reject the non-stationary forward premia for most countries.

Т	able	9.

Bootstrap adjusted confidence interval for OLS and LAD estimates of the REH and MEH

Panel A: REH						
			Currencies			
Statistics	UK	DM	YEN	SW	FF	CAN
OLS confidence	ce interval					
Upper tail α	-0.038	-0.033	-0.240	-0.026	-0.130	-0.006
Lower tail $\alpha$	0.036	0.032	0.218	0.024	0.130	1.027
Upper tail β	0.923	0.934	0.953	0.930	0.923	0.975
Lower tail $\beta$	1.072	1.061	1.049	1.071	1.074	1.008
LAD confiden	ce interval					
Upper tail α	-0.035	-0.027	-0.242	-0.040	0.156	-0.008
Lower tail $\alpha$	0.038	1.014	0.318	0.013	0.116	0.009
Upper tail β	1.031	0.944	1.007	1.024	0.906	0.970
Lower tail $\beta$	1.883	0.947	1.050	0.996	1.090	1.029
Panel B: MEH			Currencies			
Statistics	UK	DM	YEN	SW	FF	CAN
OLS confidence	<u>ce interval</u>					
Upper tail $\alpha$	-0.008	-0.037	-0.057	-0.019	-0.019	-0.001
Lower tail $\alpha$	0.010	0.034	0.060	0.023	0.020	0.001
Upper tail β	0.922	0.976	0.942	0.953	0.973	0.992
Lower tail $\beta$	0.963	0.995	0.967	0.977	0.996	1.007
LAD confiden	ce interval					
Upper tail α	-0.008	-0.037	-0.049	-0.007	-0.024	-0.002
Lower tail $\alpha$	0.011	0.047	0.047	0.005	0.037	0.002
Upper tail β	0.938	0.906	0.944	0.949	0.978	0.991

Note: upper tail  $\alpha$  is the bootstrap confidence interval at 2.5 percentile where the lower tail is at 97.5 percentile centered at 0.

Upper tail  $\beta$  is the bootstrap confidence interval at 2.5 percentile where the lower tail is at 97.5 percentile centered at 1.

## Table 10. Bretung nonparametric unit root test for the hypothesis of forward-premia stationary hypothesis

Currencies									
Statistic	UK	DM	YEN	SW	FF	CAN			
B(n)	0.00096	0.00544	0.00042	0.00040	0.00179	0.00168			
TB(n)	0.00085	$0.00358^{\dagger}$	0.00042	0.00037	0.00101	0.00038			
Notes:									

Notes:

\*Implies that the unit root hypothesis cannot be rejected at 5% significance level.

<sup>†</sup>Implies that the unit root hypothesis is rejected at 5% but accepted at 10% significance level.

Unlike the conventional unit root tests the hypothesis of the unit root under Bretung (2002) approach is rejected if the test statistic is below the corresponding critical value.

Notice that Breitung (2002) only reports the critical values for n = 100, n = 250 and n = 500. However, the critical values used in this study are 0.01004, 0.00342 for 5% significance level for the alternative of stationary and trend stationary, respectively are based on linear interpolation.

Franc and French Franc, the *F*-ratio constructed from the LAD estimator lies inside the bootstrap critical region. We conclude that the market for both the French Franc and Swiss Franc foreign exchanges can be considered as efficient.

Table 11 reports the results of REH and MEH for the Japanese Yen based only on the LAD estimator after adjusting for the structural break found in section IV.B. As shown, The REH hypothesis is fairly accepted before and after finite bias adjustment. Comparing the results of table 6 with the one gleaned after adjusting for the structural break, we can conclude that the decision about REH hypothesis could be subject to structural break (breaks) since the hypothesis is rejected for whole sample period in the case of the Japanese Yen before the finite bias adjustment. Nevertheless, the conclusion is justified that the investors in the market of the Japanese Yen are rational. For the MEH test, we still cannot reject the hypotheses of forward rate unbiasedness across the subsamples but the joint hypothesis of zero risk premia and forward rates unbiasedness is still rejected across the subsamples.

Table 11.
LAD and bootstrap LAD Subsample REH and MEH estimation for the Japanese Yen

	1989 through 1996		1996 thro	ough 2000
Parameters	α	В	α	β
LAD	0953†	$1.019^{\dagger}$	$-0.078^{\dagger}$	1.016 <sup>†</sup>
t-Stat	-1.090	0.280	1.016	0.898
F- Stat	$0.610^{\dagger}$		0.404*	
Boot.	-0.098	1.020	-0.089	1.018
Upper tail	-0.227	0.951	-0.229	0.941
Lower tail	0.225	1.047	1.047	1.047
Boot F	9.571 <sup>†</sup>		$10.176^{\dagger}$	
Panel B. MEH				
Panel B: MEH	1989 through 1996		1996 thro	ough 2000
	1989 through 1996 α	В	1996 thro	ough 2000 β
Parameters	-	B 0.936		-
Parameters LAD	α	_	α	β
Parameters LAD <i>t</i> -Stat	α 0.023	0.936	α -0.432	β 0.910
Parameters LAD <i>t</i> -Stat <i>F</i> - Stat	α 0.023 5.049*	0.936	α -0.432 -7.006*	β 0.910
Panel B: MEH Parameters LAD <i>t</i> -Stat <i>F</i> - Stat Boot. Upper tail	α 0.023 5.049* 36.632*	0.936 -5.983*	α -0.432 -7.006* 35.447*	β 0.910 -6.918
Parameters LAD <i>t</i> -Stat <i>F</i> - Stat Boot.	α 0.023 5.049* 36.632* -0.014	0.936 -5.983* 0.941	α -0.432 -7.006* 35.447* -0.301	β 0.910 -6.918 0.919

<sup>†</sup>Implies that the point estimate lies inside the bootstrap confidence interval. \*Null hypothesis is rejected at 5% significance level The explicit test of the existence of risk premium is conducted based on the econometric specification of equation 14. Table 12 reports the result of the test based on both OLS and LAD estimators. As shown, both the OLS and LAD estimates of the mean and the median, respectively, suggest the existence of exchange risk for the Pound Sterling, the Deutsche Mark, the Japanese Yen, and the Canadian Dollar. This suggests that foreign exchange market participants in these markets are risk averse. On the other hand, we did not document a significant risk premium for both the French franc and Swiss franc, where the hypothesis is fairly rejected across using the LAD estimator. The results of table 12 are of considerable interest since the MEH is fairly accepted only for the those two currencies after biased adjustment.

We also employ Breitung's (2002) nonparametric test to test for cointegration between  $S_t^e$  and  $F_t$  by examining the stationarity of the first log difference of the forward rate and the expected spot rate that is,  $\varepsilon_t = S_t^e - F_t$ . Table 10 Panel B shows that  $\varepsilon_t$  is a random walk stochastic process for the Pound Sterling, the Deutsche Mark, the Japanese Yen and the Canadian Dollar. We conclude that the joint test of market efficiency is rejected for those four currencies because of the existence of non-stationary risk premia. However, the stationarity of risk premia is documented in the case of the French Franc and the Swiss Franc, where the unit root hypothesis is fairly rejected. We conclude that the foreign exchange market for both the Swiss Franc and the French Franc are efficient because the exchange risk display stationary behavior in those markets.

### Table 12. Bretung nonparametric unit root test for the hypothesis of forward-premia stationary and the existence of risk premia

Statistics	UK	DM	YEN	SW	FF	CAN
OLS-Intercept	0.002	0.003	0.001	0.002	0.001	-0.048
P-Value	0.572	0.717	0.372	0.639	0.495	0.264
LAD-Intercept	0.001	0.001	0.007	0.001	0.001	-0.001
P-Value	0.001	0.000	0.000	0.653	0.118	0.740
Panel B: Sation Statistics	arity of risk p UK	oremia test DM	YEN	SW	FF	CAN
Statistics	ÖK	DM	I LIV	5 11	11	CITIV
B(n)	0.01302	0.01122	0.02611	0.01112*	0.00176*	0.01173

Notes:

\*Implies that the unit root hypothesis cannot be rejected at 5% significance level.

<sup>†</sup>Implies that the unit root hypothesis is rejected at 5% but accepted at 10% significance level.

Unlike the conventional unit root tests the hypothesis of the unit root under Bretung (2002) approach is rejected if the test statistic is below the corresponding critical value.

Notice that Breitung (2002) only reports the critical values for n = 100, n = 250 and n = 500. However, the critical values used in this study are 0.01004, 0.00342 for 5% significance level for the alternative of stationary and trend stationary, respectively are based on linear interpolation.

#### V. Conclusion

This paper employs monthly data over an 11-year period for six major currencies to examine the effect of small sample bias in testing for rational expectations and market efficiency in the foreign exchange markets. It appears that the *F*-ratios from traditional regressions understate the estimates of REH and MEH due to the biasedness and inconsistency in the parameter estimates. We invoke the bootstrap methodology suggested by Freedman (1981, 1984) and Hall (1988) to estimate and solve for the finite sample bias. The main finding of the paper is that the estimated bias is large enough to affect the statistical inference toward the rejection of REH and MEH.

Some practical implications concerning MEH and REH can be gleaned from our results. In particular, the REH is accepted for the six major world currencies (the Pound Sterling, the Deutsche Mark, the Japanese Yen, the Swiss Franc, the French Franc, and the Canadian Dollar) using monthly data. The critical values of the bootstrapped F distribution are shifted to the right of the nominal F ratio constructed from the regressions. According to the parameter estimates, the value of  $\beta$  converges to 1 and the value of a is close to zero in the case of the REH. For the MEH, the value of  $\beta$  converges to 1 and the hypothesis of a stationary risk premium is accepted across most of the currencies. The results provide some support to the market efficiency hypothesis even though the joint hypothesis was rejected for four out of six currencies. Furthermore, our results strongly relate the rejection of the market efficiency hypothesis in some currencies to the existence of a risk premium, not to the failure of the rational expectations hypothesis. Moreover, our results are robust to the paso problem; the effect of the

problem will be mitigated if it still exists since the bootstrap sampling distribution of the point estimate is identical to the population distribution function.

## ESSAY III

# An Empirical Examination of the Short-Term Riskless Rate Models: the Role of the Nonlinear Trend Component

#### I. Introduction

The dynamic behavior of the interest rate has been the focus of many theoretical models of term structure. In most of the interest rate models, the behavior of the interest rate  $\{r_t, t \ge 0\}$  is a diffusion process developed in a continuous time setting, where it is determined by the behavior of a set of exogenous variables obeys the following Markov stochastic differential equation:

$$dr_t = \mu(r_t, \theta)dt + \sigma(r_t, \theta)dZ_t$$
(1a)

where  $\{Z_t, t \ge 0\}$  is a standard Brownian motion,  $\mu(r_t, \theta)$  is the drift (instantaneous mean) function, and  $\sigma(r_t, \theta)$  is the diffusion (instantaneous standard deviation) function.

Since Chan, Karolyi, Longstaf and Sanders (1992), hereafter (CKLS), finding that models allow the volatility of interest rate changes to be heteroscedastic and highly sensitive to interest rate changes tend to perform superior relative to other models. However, several departures from this conclusion have been documented. One category of departures is regarding the mean-reverting behavior of the level of the interest rate. Based on his nonparametric point estimate that accounts for nonstationary deviations,

Bandi (2002) provides evidence that the short-term interest rate displays a martingale behavior over a range between 3% and 15%. Thus, the level of the interest rate seems less predictable and none of the restricted interest rate models have been accepted. On the other hand, based on the Efficient Method of Moments (EMM) and econometric specification that are composed of both a level effect and a stochastic volatility factor, And ersen and Lund (1997) provide point estimates of  $\gamma$  that are close to 0.5. The diagnostic tests result in an interest rate that is mean reverting and therefore supports the (modified) Cox, Ingersoll, and Ross (1985), hereafter CIR SR, model (i.e., the CIR SR model with stochastic volatility). Colony et al. (1997) argues that high-volatility elasticity tends to induce stationarity in the interest rate stochastic process, which may explain the out performance of the high volatility elasticity models over the homoscedastic models. Also, a substantial reduction in the importance of the level effect is documented by Koedjik et al. (1994), and Brenner et al. (1994) who use the (generalized) autoregressive conditional heteroscedasticity models introduced by Engle (1982) and Bollerslev (1986) to estimate the models considered by CKLS. On the other hand, Ait-Sahalia (1996a) and Jiang (1998) mainly relate the rejection of homoscedastic mean reverting models to the linearity of the drift that these models assume.

In this paper, we re-examine a variety of term structure models by analyzing separately the level and the stationary component of the short-term interest rate. Applying a variety of unit root tests, we strongly conclude that the short-term interest rate is a nonlinear trend-stationary stochastic process. Based on this finding, we employ the Hodrick-Prescot filter to capture the stationary and the nonlinear trend components. Our analysis based on the stationary component shows that previous work overstates the evidence of the level effect. The result shows a substantial improvement for the CIR SR model in capturing the stationary component of the interest rate. Given that the interest rate level is nonlinear trend stationary, we develop the drift term in the CIR SR model to capture this stochastic process. Extensive diagnostic tests show extreme improvement in the CIR SR model to a nonlinear trend-stationary process. Moreover, we argue that our model outperforms all continuous-time models of short-term interest dynamics.

The remainder of the paper is constructed as follows. Section II reviews the models to be considered. Section III outlines the data used. Section IV develops the structural break tests of Andrews (1993) and Andrews and Ploberger (1994), and the Generalized Method of Moments introduced by Hansen (1982). In Section V, we compare models on the basis of the level and the stationary component of interest rate, and examine the stability of the generalized CIR SR model. Next, models comparison is performed across subperiods. In Section VI, we introduce and estimate our model. Section VII concludes.

#### II. The Models

The models to be estimated in this paper are based on the following parameterization of (1a)

$$dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dZ.$$
(1b)

The specification of each model can easily be implemented by setting a number of restrictions on the parameters ( $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\sigma$ ). The first model under consideration has the form:

$$dr = (\alpha)dt + \sigma dZ \tag{2}$$

when Merton (1973) is considered. Thus, the riskless rate is a Gaussian process. Also, The movements of the short-term risk free rates at different maturities are perfectly correlated as the difference between any two of them is deterministic. The second model is introduced by Vasicek (1977) and the stochastic differential equation (SDE) takes the form:

$$dr = (\alpha + \beta r)dt + \sigma dZ \tag{3}$$

where  $\alpha$ ,  $\beta$  and  $\sigma$  are constants. The SDE is an Ornstien-Uhlenbeck process composed of Brownian motion and a restoring drift that pushes it downwards when the process is above  $\alpha/\beta$  and upwards when it is below. Thus, The distribution of the process is mean reverting and converges in equilibrium to a normally distributed mean  $\alpha/\beta$  and variance  $\sigma^2/2\beta^{22}$ . As in Merton (1973), the only source of randomness is the Brownian motion, which is a process over time not over maturity. Thus, the Merton model can be considered as a special case of the Vasicek model when the parameter  $\beta$  is restricted to zero.

The third model is the mean reverting process introduced by Cox, Ingresoll and Ross (CIR SR) (1985). The instantaneous rate's stochastic differential equation is

$$dr = (\alpha + \beta r)dt + \sigma r^{1/2}dZ$$
(4)

where  $\alpha$ ,  $\beta$ , and  $\sigma$  are deterministic functions of time. The process is composed of Brownian motion and a restoring force, the drift term, that moves toward the expected value of  $\alpha/\beta$ . The volatility term is decreasing with *r*, so allowing  $\alpha$  to prevent *r* from going below zero. This condition holds as long as  $\alpha \ge 1/2\sigma^2$ . The CIR model has been

<sup>&</sup>lt;sup>22</sup> The convergence of the Ornstien-Uhlenbeck distribution to its mean does not imply that the short-term riskless rate is a mean reverting process, only that the distribution is.

widely used in pricing interest contingent claims, as in Ramaswamy and Sundaresan's

(1986) futures pricing model and Longstaff's (1990) yield option valuation model.

The fourth model appears in Brennan and Schwartz (1977) in modeling savings and callable bonds and has also been used for discount bond valuation in Dothan (1978). The SDE is a driftless Brownian motion that allows the volatility term to be proportional to riskless rate. The SDE is

$$dr = \sigma r dZ \tag{5}$$

The fifth model is used by March and Rosenfeld (1983) in deriving an equilibrium model for bond prices. The stochastic process of the riskless rate is simply the geometric Brownian motion (GBM) process introduced by Black and Scholes (1973). The SDE of the riskless rate takes the form

$$dr = \beta r dt + \sigma r \ dZ \tag{6}$$

where the riskless rate follows an arithmetic random walk with *i.i.d.* increments. In contrast, we expect that the GBM model will perform better when it is estimated using the level of interest rates as compared to the stationary component. The idea behind this is related Bierens (2000) who argues that a nonlinear stochastic process is likely to act as a unit root process.

The sixth model is considered by Brennan and Schwartz (1980) in deriving a model for pricing discount bond options. The stochastic differential equation for the model is

$$dr = (\alpha + \beta r)dt + \sigma r dZ \tag{7}$$

As in both the Black and Scholes (1973) and Dothan (1978) models, the volatility term is deterministic and proportional to *r*. Thus, the Brennan and Schwartz, GBM, and Dothan models, can be nested within the CIR model by the following parameter restrictions  $\gamma = 1$ ,  $\alpha = 0$  and  $\gamma = 1$ , and  $\alpha = 0$ ,  $\beta = 0$  and  $\gamma = 1$ , respectively.

The seventh model is used by CIR (1980) in analyzing variable rate loan contracts. The SDE of the model is a driftless Brownian motion that takes the form

$$dr = \sigma r^{3/2} dZ \tag{8}$$

The volatility term is set to smaller at a decreasing rate as r approaches zero, thus preventing r from going below zero. The model also appears in Costantinides and Ingersoll (1984) in pricing taxable bonds. The eighth model is introduced by Cox (1975) and Cox and Ross (1976). The SDE of the model is constant elasticity of variance diffusion process that takes the form

$$dr = \beta r dt + \sigma r^{\gamma} dZ \tag{9}$$

This model can be nested within CIR (1980) by parameters restrictions  $\beta = 0$  and  $\gamma = 3/2$ .

The ninth model is introduced by Black and Karasinski (1991) in pricing discount bond options when the riskless rate is log normally distributed. The stochastic process of the natural logarithm of the riskless rate, denoted by X, is

$$dX = (\alpha + \beta X)dt + \sigma dZ \tag{10a}$$

Unlike the Vasicek model, the parameters  $\alpha$ ,  $\beta$  and  $\sigma$  are deterministic functions of time and the instantaneous riskless rate is

$$r = e^X \tag{10b}$$

The SDE of the model is an Ornstien-Uhlenbeck process and the logarithm of the riskless rate is normally distributed and drifts towards the current mean of  $\alpha/\beta$ . Additionally, the riskless rate itself is mean reverting and always positive.

#### III. Methodologies

This section describes the structural break tests of Andrews (1993) and Andrews and Ploberger (1994) and the generalized method of moments (GMM) of Hansen (1982) adopted in our paper for testing and comparing continuous-time models of the short term interest rate. First, I briefly review the structural break tests introduced by Andrews (1993) and Andrews and Ploberger (1994). Next, the GMM is developed.

#### **III.A.** Structural Change Tests

Recent advances in the econometrics of structural break now allow for robust tests of parameter instability without assuming exogenous change points. Among the proposed structural break tests, those introduced by Andrews (1993) and Andrews and Ploberger (1994) are particularly attractive for many reasons. First, the tests can be constructed for nonlinear methods of estimation, such as GMM. Second, they can be used to test for the stability of individual parameters or the entire parameter vector where the change points are endogenous<sup>23</sup>. Moreover, Andrews (1993) introduces an elegant way to develop efficient asymptotic critical values for the test where the breakdate is unknown a priori. More recently, Hansen (1997) develops approximation methods to calculate *P*-values for the Andrews (1993) and Andrews and Ploberger (1994) tests. Furthermore, the test has power against the alternative where the parameters may change gradually overtime. For

 $<sup>^{23}</sup>$  Andrews and Ploberger (1994) show that *Ave* (Average) and *Exp* (Exponential) tests have certain optimality relative to the *Sup* (Supremum) test proposed by Andrews (1993).

the analysis of this paper, we construct the Andrews (1993) *Sup* (Supremum) and Andrews and Ploberger (1994) *Exp* (Exponential) and *Ave* (Average) tests based on the Wald statistic. In particular,

$$SupW = \sup_{\tilde{i} \le i \le \tilde{i}} W_i, \tag{11}$$

$$AveW = \frac{1}{\tilde{i} - \bar{i} + 1} \sum_{i=\tilde{i}}^{\bar{i}} W, \qquad (12)$$

$$ExpF = \log\left(\frac{1}{\widetilde{i} - \overline{i} + 1}\sum_{i=\widetilde{i}}^{\overline{i}} \exp(0.5 \ . \ W_i)\right)$$
(13)

where i is some change point in an interval (i, i)<sup>24</sup>. Let  $\lambda$  be a prechange parameter vector and  $\lambda + \delta$  be a post-change parameter vector. The null hypothesis of interest is

$$H_0:\delta=0$$

Against the alternative

$$H_a: \delta \neq 0$$

Where the hypothesis of no structural breaks in the entire parameter vector will be rejected if the p-values of those statistics computed by Hansen (1997) are below 5 percent.

#### **III.B.** Generalized Method of Moments

Following Brennan and Schwartz (1982) and Dietrich-Campbell and Schwartz (1986), the parameters of the continuous-time stochastic process can be estimated using the following discrete-time econometric specification

<sup>&</sup>lt;sup>24</sup> Andrews (1993) suggests to consider all the breakdates in the interval v% to (1-v)% of the sample, where the trimming parameter v is typically between 5% and 15%. For the analysis of this paper, we use 15% trimming.

$$r_{t+1} - r_t = \alpha + \beta r_t + \varepsilon_{t+1} \tag{14}$$

$$E[\varepsilon_{t+1}] = 0, \qquad E[\varepsilon_{t+1}^{2}] = \sigma^{2} r_{t}^{2\gamma}.$$
(15)

This requires a nonlinear estimation procedure that can estimate the parameters simultaneously. Hansen (1982) has introduced the Generalized Method of Moments (GMM) that can handle (14) and (15) as a set of moment equations. The technique has relevant characteristics for the case at hand. First, the GMM procedure does not require any distribution assumption about the model. This seems attractive in testing term structure models since each model assumes a different distribution for the riskless rate. Second, the covariance matrix of GMM is heteroscedastic consistent since it includes the interaction terms of the residuals and the derivatives with respect to the estimators. This is important because the riskless rate seems to be dependent on its conditional volatility<sup>25</sup>. Third, the GMM standard errors are autocorrelated consistent when spectral density is used to ensure positive definiteness of the covariance matrix in the case of serially correlated residuals. Fourth, the GMM procedure is widely used to test term-structure models (see for example, Harvey (1988), Longstaff (1989), Chan, Karolyi, Longstaff, and Sanders (1992), and Gibbons and Ramaswamy (1993)).

The econometric specification of (14) and (15) can be fit into a GMM framework as follows. Let the noise term  $\varepsilon_{t+1}$  in (14) be:

$$\varepsilon_{t+1} = [r_{t+1} - r_t - \alpha - \beta r_t] \tag{16}$$

Let  $\lambda$  defined to be the entire parameter vector of  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\sigma$ . Let the vector  $h(x_{t+1}, \lambda)$  be:

<sup>&</sup>lt;sup>25</sup> Engle, Lilien and Robins (1987) argue that the GMM outperforms ARCH-M when the data depends on its conditional variance since ARCH models require the variance to be specified a priori.

$$h(x_{t+1},\lambda) = \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon_{t+1}r_t \\ \varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma} \\ (\varepsilon_{t+1}^2 - \sigma^2 r_t^{2\gamma})r_t \end{bmatrix}$$
(17)

where  $h: R^2 \times R^4 \to R^2$ ,  $x_{t+1} = [r_{t+1} - r_t, r_t]'$  is 2-by-1 vector of variables, and  $\lambda = [\alpha, \beta, \sigma^2, \gamma]'$  is a 4-by-1 vector of parameters of the unrestricted model.

Define  $Y_t$  to be a *K*-dimensional vector of instrumental variables with finite variance that are included in the information set, and let the function *f* be

$$f(x_{t+1}, Y_t, \lambda) = h(x_{t+1}, \lambda) \otimes Y_t, \qquad (18)$$

where  $\otimes$  is the Kronecker product. The minimum distance estimator of  $\lambda$  that is defined by the unconditional expectation of (18) can be estimated by minimizing the GMM criterion of the form,

$$\frac{1}{T} \left[ f(\boldsymbol{x}_{t+1}, \boldsymbol{Y}_{t}, \boldsymbol{\lambda}) \right]' \boldsymbol{W}_{T} \frac{1}{T} \left[ f(\boldsymbol{x}_{t+1}, \boldsymbol{Y}_{t}, \boldsymbol{\lambda}) \right]$$
(19)

where  $W_T$  is a consistent estimate of  $\left( \operatorname{var}[(1/T)(f(x_{t+1}, Y_t, \lambda)_t)] \right)^{-1}$ .

To test for restrictions validity among the models, the GMM test of overidentifying restrictions is used. Let L be the number of moments restrictions that is greater than the elements in the parameter vector. The minimized GMM criterion in (19) can be used to test if the remaining, L-K, linearly independent moments conditions are zero. Under the null hypothesis that these restrictions are valid,

$$\frac{1}{T} \left[ f\left(x_{t+1}, Y_t, \lambda\right) \right]' W_T \frac{1}{T} \left[ f\left(x_{t+1}, Y_t, \lambda\right) \right]^a \chi^2_{L-k}$$
(20)

We use the chi-square statistic in (20) to test the models' restrictions validity.

#### IV. The Data

The overlapping raw data for the three month Treasury bill rate consists of end-of-month observations from January 1934 until July 2002 and are collected from the Federal Reserve Economic Database (FRED) of the Federal Reserve Bank of St. Louis. The data is monthly and the total number of observations is 826. Three reasons for using the three month Treasury bill rate are now in order. First, it is available and costless. Second, it has been used before in previous literature (see for example Anderson and Lund (1997) and Santon (1997)). Third, and most importantly, Chapman et. al. (1999) show that the proxy problem is trivial in the case of single-factor models for parameter estimates.

Table 1 shows the means, median standard deviations, and first 5 autocorrelation of the three-month yield and the three-month changes in the yield. The mean of the short-term riskless rate is 4.01% with a standard deviation of 3.21%. Although the slow mean reverting behavior of the short-term riskless rates is obvious. This offers some evidence that process has a stationary behavior.

Since the data are overlapping, the autocorrelation problem may exist<sup>26</sup>. We invoke the Parzen spectral density autocorrelated consistent estimator suggested by Gallant (1987) to insure positive definiteness of the covariance matrix in case of autocorrelation<sup>27</sup>. For

<sup>&</sup>lt;sup>26</sup> See Hansen and Hordrick (1980) for further discussion on the autocorrelation problem resulting from overlapping data.

<sup>&</sup>lt;sup>27</sup> Andrews (1991) discusses different types of autocorrelated consistent (AC) estimators. He argues that the Parzen kernel outperforms the Barlett kernel as suggested by Newey and West (1987), especially when the data exhibits temporal dependence.

Variables N	Mean	Standard Deviation	ρ <sub>1</sub>	ρ <sub>2</sub>	ρ <sub>3</sub>	ρ <sub>4</sub>	ρ <sub>5</sub>
<i>r</i> <sub>t</sub>	0.004	0.032	0.992	-0.633	0.249	-0.038	0.013
$r_t - r_{t+1}$	0.000	0.003	0.407	-0.227	0.032	-0.029	0.036

Table 1 Summary Statistics

heteroscedasticity, we use a heteroscedastic consistent covariance matrix (HC) suggested by MacKinnon and White (1985) with Jacknife approximation<sup>28</sup>.

In order to determine whether the interest rate is stationary, trend stationary, nonlinear trend stationary, break stationary or a unit root process, we employ several different types of unit root tests. The first type, in which the unit root under the null against the alternative of stationary and linear trend stationary, includes the traditional Augmented Dickey-Fuller (ADF) test, the Phillips an Perron (1988) (PP) test, and the Augmented Weighted Symmetric (WS) tests. The WS test is a weighted, double-length regression. Pantula et al. (1994) argue that the WS test dominates the ADF and PP tests in terms of power. The Akaike Information criterion (AIC) is used to determine the optimal number of lags for all the tests<sup>29</sup>. We also employ the non-parametric test of Breitung (2002) for two hypotheses where under the first that the interest rate is a driftless unit root and under the second that it is a unit root with drift against the alternatives that the level of the interest rate is stationary and linear trend stationary, respectively. There are two advantages of the Breitung test over the ADF, PP, and WS tests. The Monte Carlo simulations show that it is robust to structural break, and to model misspecifications since the asymptotic property is independent from the stationary component of the series. Finally, we employ Bierens (1997) test where the unit root with drift hypothesis will be tested against the alternative of nonlinear trend stationary.

<sup>&</sup>lt;sup>28</sup> MacKinnon and White (1985) argue that HC with jackknife approximation outperforms any type of White HC estimators and it is wise to <del>be</del> use<del>d</del> it even if-heteroscedasticity does not exist.

<sup>&</sup>lt;sup>29</sup> The ADF, PP, and WS accurate asymptotic *p*-values are computed using MacKinnon (1994) approximation (robust to size distortion).

Table 2, panel A reports the results of the ADF, PP, and WS tests for the riskless rate. Across all tests we cannot reject the null of a unit root  $process^{30}$ . The *P*-values of all of the tests are so high, which provides evidence that the short-term interest rate is not stationary, break stationary or linear trend stationary<sup>31</sup>. However, the unit root tests have low local power against the alternative since the data could also be break stationary or nonlinear trend stationary<sup>32</sup>. To test if the short-term interest rate is nonlinear trend stationary, panel B reports the Bierens tests. Regarding the Breituring test, the unit root hypothesis cannot be rejected in favor of stationarity or trend stationarity (the result is robust to structural breaks). The last two lines of panel B reports both the T(m) and A(m)nonparametric tests suggested by Bierens (1997); the value of both statistics (-3.985 and -40.220 respectively) are higher than their corresponding right-hand sided critical values (-3.97 and -27.20, respectively). To take into account any possibility of size destortion possible size distortion, we simulate the *p*-values of the test using the Wild bootstrap, as shown in the last two lines of panel B. The *p*-value of the T(m) statistic is (0.049), and for A(m) test is (0.040). Consistent with Bierens (1997, 2000), we conclude that short-term interest rate is nonlinear trend-stationary<sup>33,34</sup>.

<sup>&</sup>lt;sup>30</sup> Peron (1989) concludes that the nominal interest rate is a unit root process; the same conclusion is also documented by Ai-Sahalia (1996b). On the other hand, according to Bierens (1997), this cannot be realistic for two reasons. First, if the interest rate is driftless random walk then it must allow for negative values. Second, if it is a random walk with positive drift then it would converge to infinity, which is improbable. <sup>31</sup> See Perron (1989) for further discussion.

<sup>&</sup>lt;sup>32</sup> Phillips (1998) argues that the lack of consensus in the empirical work regarding the Fisher effect is mainly related to the apparent nonlinear trend-stationary behavior in the nominal interest rate.

<sup>&</sup>lt;sup>33</sup> Bierens (2000) provides evidence that the nominal interest rate and inflation rate are nonlinearly cotrended.

<sup>&</sup>lt;sup>34</sup> Bierens (1997) tests the hypothesis that the interest rate is a unit root process against the alternative of nonlinear trend-stationary. The intensive diagnostic tests suggest that the short-term interest rate is nonlinear trend-stationary.

Panel A: Parametric tests			
	WS	ADF	РР
	-2.4449 (0.327)	-2.2419 (0.466)	-12.39766 (0.295)
Panel B: Nonparametric tests	Breitung	Beirens $A(m)$	Beirens <i>T</i> ( <i>m</i> )
	$\begin{array}{c} 0.0104 \\ 0.0035* \\ (0.421)^{\dagger} \end{array}$	-40.220 -27.200* (0.049) <sup>†</sup>	-3.985 -3.971* (0.040) <sup>†</sup>

Table 2Unit root tests on the short-term interest rate

Note: For Beirens A(m) and T(m) tests the actual value of the statistics must lie in the right hand side of the distribution to conclude that the riskless rate is nonlinear trend stationary.

\* 5% right-hand sided critical value

† Simulated *P*-value based on 1000 replications drown from the normal distribution with zero mean and OLS squared residuals variances (the Wild bootstrap).

In order to gain insight into the nature of the riskless rate, Figure 1 plots the short-term interest rate and the HP-Trend. This provides evidence that the interest rate is stationary about the dynamic trend (i.e., the interest rate itself is not stationary).

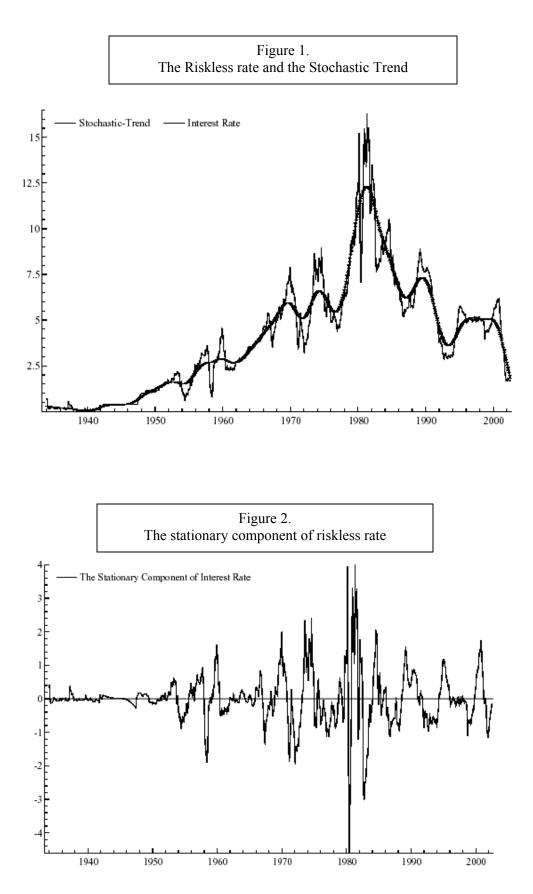
The fluctuation process of the detrended short-term interest rate can be illustrated using Figure 2. As shown, the stationary component is reverting systematically about its zero mean except for the period spanning from September 1979 to July 1982, where substantial changes occur in the variability of the interest rate around the trend. This may suggest a structural break during that period when the Federal Reserve targeted the inflation rate after September 1979.

#### V. The Empirical Results

In this section, we first estimate and compare the unrestricted and nine restricted models of the short-term interest rate process by employing the GMM technique. We compare the performance of the restricted models with each other and with the unrestricted model. Next, we repeat the estimation and the comparison by employing the detrended short-term interest rate and comparing the results with those obtained from the non-detrended estimation. We also employ the *Sup* statistic of Andrews (1993) and *Ave* and *Exp* statistics of Andrews and Ploberger (1994) to test if the unrestricted model displays a structural break during the study period. Finally, we reestimate the models after adjusting for structural break(s) and compare the results.

#### V.A. 1934 through 2002 Sample

Table 3 reports the parameter estimates, the asymptotic *P*-values of the individual parameters, and the GMM overidentifying  $(X^2)$  test, for the unrestricted model and nine



## Table 3. GMM Estimate of Unrestricted and Nine Restricted Models of Short-Term Interest Rate Dynamics Coefficients, Their Significance Levels, and Overidentifing Test for the Period (1934 Through 2002)

Model	α ( <i>p</i> -value)	β ( <i>p</i> -value)	$\sigma^2$ ( <i>p</i> -value)	γ ( <i>p</i> -value)	OverIdentifying Test (p-value)	Sensitively of conditional Variance to
					(p-value)	10% increase in interest
						rate
Unrestricted	0.0100	0.0010	0.0018	1.1849	13.2035	0.338
	(0.3185)	(0.8065)	(0.2050)	(0.0000)	(0.0103)	
Merton	-0.0077	-	0.02470	0.0	115.5494	-
	0.2277		(0.0000)	0.0	(0.0000)	
Vsicek	0.0798	-0.0245	0.0111	0.0	127.8952	-
VSICCK	(0.0444)	(0.0292)	(0.0189)	0.0	(0.0000)	
	(0.011)	(0.02)2)	(0.010))		16.1650	0.100
CIR SR	0.0137	-0.0019	0.0155	0.5		
	(0.1579)	(0.6373)	(0.0000)		(0.0063)	
Dether	0.0	0.0	0.0022	1.0	20.2008	0.210
Dothan	0.0	0.0	0.0033 (0.0000)	1.0	20.2098 (0.0051)	
			(0.0000)		(0.0031)	
GBM	0.0	0.0028	0.0033	1.0	18.9062	0.210
		(0.28152)	(0.0000)		(0.0043)	
Brennan-	0.01067	-0.0007	0.0033	1.0	17.1727	0.210
Schwrtz	(0.2733)	(0.8575)	(0.0000)		(0.0042)	
CIR VR	0.0	0.0	0.0075	1.5	19.2035	0.331
			(0.0000)		(0.0076)	
CEV	0.0	0.0034	0.0018	1.2425	14.9068	0.398
		(0.1918)	(0.1686)	(0.0000)	(0.0107)	
Black-	0.0024	-0.0026	0.0054	0.0	17.2788	-
Karasinski	(0.7466)	(0.5719)	(0.0000)		(0.0040)	

restricted models, along with the sensitivity of the conditional variance to the level of interest rate implied by each model. As shown, for almost every model, the *P*-Values of the GMM criterion test is below 0.05, which suggests that all models are misspecified. Our results are consistent with Ait-Sahalia (1996a) and Bandi (2002), which argue that none of the models are correctly specified. The unrestricted model and the CEV model have the lowest goodness of fit statistic, but are still rejected. Consistent with CKLS, for most models except Vasicek, none of the parameters in the drift term is significant. However, no robust conclusion can be gleaned about the linear mean reverting behavior in this case; this is because the GMM overidentifying test fails to accept the null hypothesis, which provides an inconsistent estimate of the parameters (Ait-Sahalia 1996a). Ignoring the inconsistency in the entire parameter vector, one could reject the linearity specification of the drift term<sup>35</sup>.

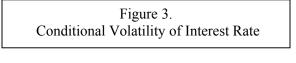
We also find that the level of interest rate is an important determinate of the conditional volatility; the estimate of the unrestricted parameter  $\gamma$  is 1.1849. Moreover, the CIR VR model with  $\gamma$ =1.5 reports a higher unconditional variance ( $\sigma^2$ ) than those models which assume that  $\gamma$ =1 but with a lower unconditional variance ( $\sigma^2$ ) than that of CIR SR which assumes  $\gamma$ =1/2<sup>36</sup>. This could imply that models with higher volatility elasticity report a lower unconditional variance. To ensure that the CIR VR model implies higher conditional volatility sensitivity to the level of interest rate than other models, the last

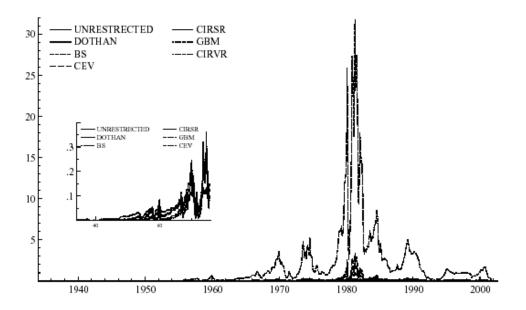
<sup>&</sup>lt;sup>35</sup> Inconsistent with CKLS, Bandi (2002) finds that the parameters in the drift term are significant and concludes that the drift term has a linear structure.

<sup>&</sup>lt;sup>36</sup> Note that models that assume a lower parameter  $\gamma$  do not necessarily mean that the conditional variance is less sensitive to the level of the interest rate since the value of the unconditional variance could mitigate the effect of a higher  $\gamma$  unless those models have a lower unconditional variance. The idea is clear since CIR SR model reports a higher  $\sigma$  than the CIR VR model, and the CEV model reports the lowest unconditional variance in comparison to the other heteroscedastic models.

column in Table 3 reports the effect of 10% increases in short-term interest rate on conditional volatility. The results suggest that models with a higher parameter  $\gamma$  imply higher conditional variance sensitivity to interest rate changes. However, the estimated unconditional volatility for the CEV model is very small, which implies that models with a higher parameter  $\gamma$  may display lower conditional volatility. By contrast, volatility as a measure of the expected conditional variance displays a very interesting dynamic structure. The CEV model, with highest parameter  $\gamma = 1.242$ , displays a comparable implied volatility level to models that report lower sensitivity to interest rate changes. The only extreme volatility measure is that of CIR VR, which has a much higher volatility than the others models. To gain more insight into the volatility behavior, figure 3 plots the short-term interest rate with the implied volatility generated from each model in order.

Given the nonlinear trend component in interest rate, we now compare the performance of the nine models by considering only the stationary component. Following Granger and Newbold (1974) the bias in the *t*-statistic that appears to be significant can now be removed. Notice that we do not try to evaluate the performance of the models in capturing the stochastic behavior of the interest rate. We only try to examine the performance of the mean reverting models in capturing the behavior of the stationary component of the short-term interest rate and how the volatility elasticity will be affected. Following Conley et al. (1997), we expect that the volatility elasticity will increase if the high volatility elasticity tends to induce stationarity.





We invoke the GMM framework on the moments in (17). The minimized values of the quadratic form in (20) for the various models are reported in table 4. As expected, all mean reverting processes show considerable improvement in capturing the stationary component of the short-term interest rate. Consequently, the unrestricted model shows considerable improvement but it is still misspecified; the CIR SR model is marginally accepted. Comparing the mean reverting processes with each other, we find that the Vasicek, Brennan-Schwartz and Black-Karasinski models outperform the unrestricted and CIR SR models. Also, the results imply that those models have the same power in capturing the stationary component of the interest rate (the GMM goodness of fit statistics are about 2.8 across the three models). Conversely, there is a significant increase in the volatility elasticity implied by both the unrestricted and the CEV models; the value of the parameter  $\gamma$  for the CEV model increased from 1.242 to 5.84797. This is consistent with Conley et. al. (1997) argument that volatility induces stationarity. There is considerable worsening in all models, except the GBM, that assume no stationarity in the interest rate. For example, the *p*-value of the goodness-of fit statistic of the CIR VR model decreased from 0.0076 to 0.0006, which decreases the probability of accepting the model.

As shown in the second column of table 4, the hypothesis that  $\alpha = 0$  is fairly accepted across all correctly specified models. Comparing the result with the one gleaned from the level of short-term riskless rate, we cannot reject the hypothesis that  $\alpha = 0$  for all mean reverting models, which shows considerable improvement in the detrended data. Also, there is significant variation in the goodness-of-fit statistic between the unrestricted

## Table 4. GMM Estimate of Unrestricted and Nine Restricted Models of De-Trended Short-Term Interest Rate Dynamics Coefficients, Their Significance Levels, and Overidentifing Test for the Period (1934 Through 2002)

Model	α	β	$\sigma^2$	γ	OverIdentifying
	( <i>p</i> -value)	( <i>p</i> -value)	( <i>p</i> -value)	(p-	Test
	- /	<b>-</b> '	- /	value)	( <i>p</i> -value)
Unrestricted	0.7982	-0.1578	0.0006	3.5173	10.6888
	(0.0000)	(0.0000)	(0.6082)	(0.0000)	(0.0302)
Merton	0.19088	-	0.0553	-	17.9355
	(0.0515)		(0.6081)		(0.0001)
Vsicek	0.0517	-0.15723	0.1132	-	2.8715
	(0.4445)	(0.0000)	(0.2673)		(0.0901)
CIR SR	0.0799	-0.15956	0.0059	0.5	3.9513
	(0.1830)	(0.00003)	(0.9299)		(0.0468)
Dothan	-	_	0.0660	1.0	19.0797
			(0.1061)		(0.0002)
GBM	-	-0.1655	0.03720	1.0	3.2532
		(0.0000)	(0.0965)		(0.1966)
Brennan-	0.04812	-0.15866	0.0284	1.0	2.8241
Schwrtz	(0.4829)	(0.0000)	(0.2548)		(0.0928)
CIR VR	-	-	0.0216	1.5	17.08223
					(0.0006)
			(0.0936)		
CEV	-	-0.14724	0.0001	5.8479	2.0595
		(0.7591)	(0.9998)	(0.9966)	(0.0000)
Black-	0.7531	-0.15120	0.0184	0.0	9.4972
Karasinski	(0.0000)	(0.0000)	(0.0000)		(0.0908)

model and the Vasicek, CIR SR, Brennan-Schwartz, and Black-Karasinski models. The unrestricted model reports a much lower *P*-Value than all of the mean reverting processes; the same result holds when comparing the performance of the GBM model and the Brennan-Schwartz model. This is considerably important since the CEV model and the GBM model are nested within the unrestricted model and the Brennan-Schwartz model, respectively, by setting the parameter restriction  $\alpha = 0$ . Also, the GBM model (which is an arithmetic random walk) tends to outperform all mean reverting processes in capturing the stationary component; this is consistent because the stationary component has a zero mean and the GBM allows the process to revert about a zero mean. Thus, the data appears to support the choice of  $\alpha = 0$ . We conclude that the existence of the parameter  $\alpha$  in any model substantially reduces the model's power in capturing the stationary component.

Also, The Vasicek model substantially outperforms the Merton model; the GBM model is correctly specified while the Dothan model is not. The Merton and Dothan models are nested within the Vasicek model and the GBM model, respectively, by setting the parameter restriction  $\beta = 0$ . The outperformance of the Vasicek and GBM models over their nestable models is evident of the valid specification of the parameter  $\beta$  in capturing the cyclical component of interest rate. Also, across all correctly specified models and most of incorrectly specified models the parameter  $\beta$  is significant.

The results based on the stationary component of the interest rate provide an interesting feature about the volatility implied by the level of interest rates. As mentioned above, we

document higher values of  $\gamma$  for both the unrestricted and CEV model, which implies higher conditional variance sensitivity to the level of interest rates<sup>37</sup>. However, the estimated unconditional variance declines substantially across all models that assume conditional heteroscedasticity, except for the CIR SR and CEV model, more specifically, across all models that assume or imply  $\gamma \geq 1$ . The results are robust across all constant and deterministic variance models. Taken together, the results suggest that the stationary component of the interest rate implies a lower level of instantaneous volatility than that implied from the corresponding interest rate level<sup>38</sup>.

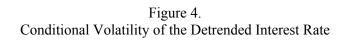
Figure 4 plots the implied volatility generated from each model based on the estimated parameters from the stationary component series. As shown, the implied volatility deviations among the models are much smaller, especially for the CEV model. For example, the CEV implied volatility displays slightly lower implied volatility than the CIR SR and Dothan models and very close to the rest when the level of interest rate is stable; it displays a slightly higher volatility than all models when the level of interest rate fluctuates. Comparing Figure 3 with 4, the implied volatility measure on the basis of stationary component seems much more convincing.

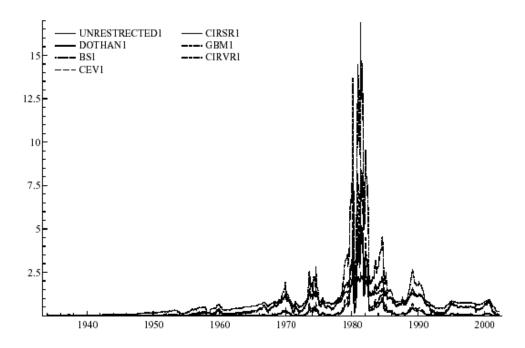
### V.B. Model Stability Test

Table 5 reports the results of the structural break tests for the unrestricted model of interest rate dynamics. Following Andrews (1993) and Andrews and Ploberger (1994), to allow for sufficient degrees of freedom, the trimming parameter is set to 15% and 30% of the effective sample, respectively.

<sup>&</sup>lt;sup>37</sup> This is consistent since the stationary component is the main source of volatility.

<sup>&</sup>lt;sup>38</sup> This must hold since the level of interest rate is in the interval [0,1].





Period	Andrews Sup W Test (Middle 85%)	Andrews / Ploberger Average W Test (Middle 80%)	Andrews / Ploberger Exponential W Test (Middle 80%)	Max date
1/1934 – 7/2002	28.3689 (0.0000)	5.7466 (0.0215)	8.052 (0.0000)	April, 1981
1/1934 - 4/1981	12.2038 (0.0262)	5.5669 (0.0332)	3.9403 (0.0220)	June, 1973
1/1934 - 6/1973	10.024 (0.0563)	4.3942 (0.0792)	3.0539 (0.0544)	October, 1969

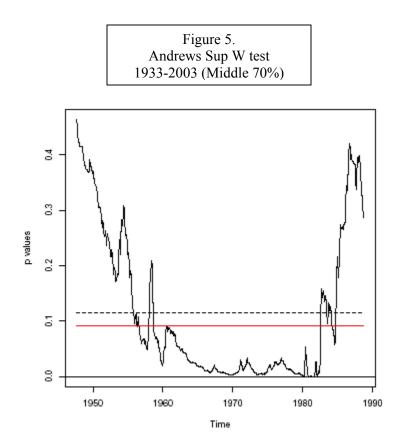
Table 5Hypothesis ofNo Structural Break in the Entire Parameter Vector of The Unrestricted Model

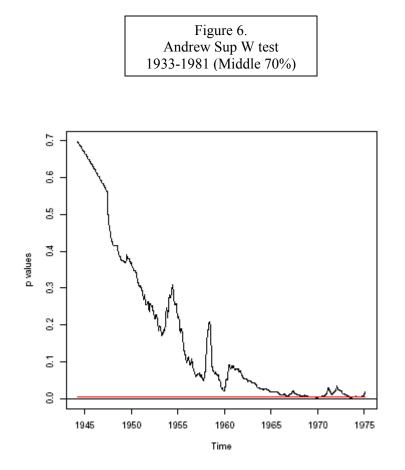
The findings in Table 5 provide strong evidence that the unrestricted model displays a structural break at the beginning of 1981 when the interest rate begins to decline substantially. The results are robust across the *Sup*, *Ave*, and *Exp* tests and all *p*-values indicate a rejection of the null hypothesis of parameter stability at the 5% and 1% significance level.

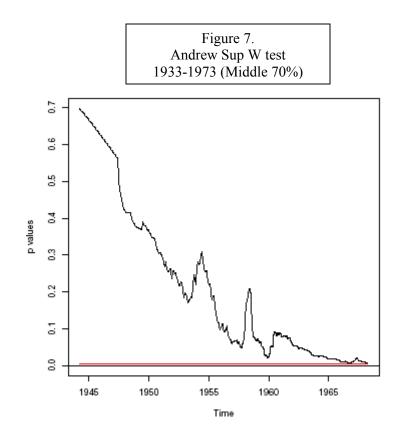
To gain insight into the nature of the instability, Figure 5 graphs Hansen (1997) *p*-values of the *Sup W* statistic supported by a 5% critical value for the null hypothesis of no structural change in the full parameter vector for the unrestricted model. As shown, the value of the Sup *W* statistic suggests three periods that could display structural change. The first is at the beginning of 1970, the second is at the mid of 1974 and the third occurs when the *Sup* statistic becomes highly fluctuating during the period between October 1979 and October 1981, which is commonly known as the 'Monetary Experiment' period of the Federal Reserve System. However, the *p*-value of the *Sup W* statistic in the third period, reaches its single trough on April 1981. Following Andrews (1993), we only consider the point where the maximum value of the statistic occurs as a single structural break for the model (i.e., April 1981)<sup>39</sup>.

To test if the model displays another structural break(s), we exclude the period after April 1981 from the effective sample and run the test again. The result is reported in the third column of Table 4. As shown, the test suggest that another structural break occurred in

<sup>&</sup>lt;sup>39</sup> Andrews (1993) and Andrews and Ploberger (1994) tests are designed to test the hypothesis of no structural change against the alternative of one structural break. Consequently, it will be more powerful to do the test several times by excluding the periods after the breaks. This is appropriate since the critical value of the test depends on the sample size and the estimated parameters.







June 1973 when the Fed begins to implement the federal-fund operating procedure for the period spanning from September 1972 to October 1979. We also follow the same procedure to test for the existence of a third break. As shown in graph 7, the *Sup* statistic suggests that the third break is in October 1969. However, both the *Ave* and *Exp* statistics cannot reject the hypothesis of parameter stability during this subperiod. Following Andrews and Ploberger (1994), only the rejected hypotheses by the *Ave* and *Exp* statistics are considered. Thus, we consider the period spanning from January 1934 to June 1973 as one subperiod. Taken together, graphs 5 through 7 and Table 4 provide the date and the nature of the structural break across the whole sample.

## V.C. Subperiod analysis

Table 5 reports the results of the GMM estimation for the considered models for the first subperiod from January 1934 to June 1973, or the end of the Britton Woods exchangerate system (we choose some period after the end of the system, namely 1973, as suggested by the structural break analysis). As shown, none of the term structural models have the ability to describe the stochastic behavior of the short-term interest rate during this period. Notice, also, that the heteroscedastic models, such as the unrestricted and CEV, report low volatility elasticity, 0.600 and 0.616, respectively, between a 0 and 9% level of interest rates. As shown in figure 1, we can infer that the level of interest rates has a positive growth component and it mean reverts strongly when it is far from its growth component. As shown in table 7, the same conclusion can be drawn from analyzing the third period from 1981 to 2002 where the interest rate had a negative growth component. This can be considered as a complement to Bandi (2002) and AitSahalia's (1996b) conclusion that the process tends to revert to its mean only when it approaches its upper bound range.

However, as shown in table 6, the performance of the models differs considerably for the period of highly fluctuating interest rates, i.e. when the fed begins to implement the federal-fund operating procedure from 1973 through 1981. The p-value of the  $(X^2)$ goodness-of fit statistic of each model is above 0.05, which implies that models under consideration are correctly specified. Consistent with CKLS, the CEV model outperforms the other competing models. On the other hand, the mean reverting models, such as Vasicek, CIR SR, and Black-Karasinski, perform poor in comparison to the other models even though they are correctly specified. One interpretation of this finding is that models of high volatility elasticity tend to outperform mean reverting models during this subperiod because the period represents high volatility elasticity. Yet this alone cannot explain why an arithmetic random walk process like Merton (1973) outperforms all of the mean reverting models including the unrestricted model during this subperiod. This also cannot explain why all random walk models with a volatility elasticity parameter restricted to 1 (Dothan, GBM, Brennan-Schwartz) tend to outperform the CIR VR model, which is a random walk process of higher restricted parameter volatility ( $\gamma$ =1.5). Our firm conclusion is that models with higher parameter volatility tend to outperform the mean reverting models across this subperiod because those models are random walk processes and the short-term interest rate is not stationary during this period. Also, the unrestricted mean reverting model tends to outperform the Vasicek, CIR SR, and Black-Karasinski models because it induces stationarity to the process as a result of high volatility elasticity

Model	α ( <i>p</i> -value)	β ( <i>p</i> -value)	$\sigma^2$ ( <i>p</i> -value)	γ ( <i>p</i> -value)	OverIdentifying Test (p-value)
Unrestricted	0.0023	0.0106	0.0087	0.6002	<u>19.07251</u>
omostrotou	(0.7421)	(0.0610)	(0.0536)	(0.0014)	(0.0007)
Merton	0.0125	-	0.0054	-	53.6871
	(0.0023)		(0.00002)		(0.0001)
Vsicek	0.0095	0.0062	0.0123	-	40.2910
	(0.1453)	(0.2380)	(0.0000)		(0.0000)
CIR SR	0.0045	0.0093	0.0116	0.5	22.1136
	(0.4735)	(0.0788)	(0.0000)		(0.0004)
Dothan	-	-	0.0026	1.0	30.2123
			(0.0000)		(0.0000)
GBM	-	0.0108	0.0027	1.0	23.3978
		(0.0086)	(0.0000)		(0.0006)
Brennan-	-0.0014	0.01216	0.0026	1.0	20.2354
Schwrtz	(0.8222)	(0.0288)	(0.0000)		(0.0011)
CIR VR	-	-	0.0057	1.5	29.3079
			(0.0000)		(0.0001)
CEV	-	0.0114	0.0086	0.6164	0.033705
		(0.0062)	(0.0322)	(0.0002)	(0.0006)
Black-	0.0045	-0.0002	0.0065	0.0	18.0489
Karasinski	(0.5773)	(0.9686)	(0.9287)		(0.0028)

Table 6. GMM Estimate of the Unrestricted and Nine Restricted Models of Short-Term Interest Rate Dynamics Coefficients, Their Significance Levels, and Overidentifing Test for the Period (1934 Through 1973)

Model	α ( <i>p</i> -value)	β ( <i>p</i> -value)	$\sigma^2$ ( <i>p</i> -value)	γ (p-value)	OverIdentifying Test (p-value)
Unrestricted	0.0364	0.0011	0.0007	1.4574	7.3417
	(0.8603)	(0.9723)	(0.4291)	(0.0000)	(0.1188)
Merton	0.0474	-	0.1445	-	9.7688
	(0.3036)		(0.0000)		(0.1347)
Vsicek	0.1919	-0.0224	0.17900	-	10.8916
	(0.3183)	(0.4664)	(0.0000)		(0.0535)
CIR SR	0.1921	-0.0225	0.0287	0.5	10.1825
	(0.3140)	(0.4604)	(0.0000)		(0.0702)
Dothan	-	-	0.0043	1.0	11.2764
			(0.0000)		(0.1270)
GBM	-	0.0083	0.0044	1.0	9.7399
		(0.2400)	(0.0000)		(0.1360)
Brennan-	0.1513	-0.0166	0.0042	1.0	8.1456
Schwrtz	(0.4059)	(0.5652)	(0.0000)		(0.1483)
CIR VR	-	-	0.01148	1.5	12.0959
			(0.0000)		(0.097)
CEV	-	0.0094	0.0005	1.5135	7.9686
		(0.1783)	(0.3202)	(0.0002)	(0.1579)
Black-	0.013216	-0.0030	10.5524	0.0	10.5524
Karasinski	(0.7576)	(0.8925)	(0.9287)		(0.0610)

Table 7 GMM Estimate of the Unrestricted and Nine Restricted Models of Short-Term Interest Rate Dynamics Coefficients, Their Significance Levels, and Overidentifing Test for the Period (1973 Through 1981)

# Table 8 GMM Estimate of the Unrestricted and Nine Restricted Models of Short-Term Interest Rate Dynamics Coefficients, Their Significance Levels, and Overidentifing Test for the Period (1981 Through 2002)

Model	α	β	$\sigma^2$	γ	OverIdentifying
	( <i>p</i> -value)	( <i>p</i> -value)	(p-value)	( <i>p</i> -value)	Test
					(p-value)
Unrestricted	0.0023	0.0106	0.0087	0.6002	19.0725
	(0.7421)	(0.0610)	(0.0536)	(0.0014)	(0.0007)
Merton	0.0131	-	0.0091	-	33.7344
	(0.0167)		(0.0002)		(0.0000)
Vsicek	0.0095	0.0062	0.01235	-	40.2910
	(0.1453)	(0.2380)	(0.0000)		(0.0000)
CIR SR	0.0044	0.0093	0.0116	0.5	22.1136
	(0.4735)	(0.0788)	(0.0000)		(0.0004)
Dothan	-	_	0.0026	1.0	30.2123
			(0.0000)		(0.0000)
GBM	-	0.0108	0.0027	1.0	23.3978
		(0.0086)	(0.0000)		(0.0006)
Brennan-	-0.0014	0.0121	0.0026	1.0	20.2354
Schwrtz	(0.8222)	(0.0288)	(0.0000)		(0.0011)
CIR VR	-	_	0.0057	1.5	29.3079
			(0.0000)		(0.0001)
CEV	-	0.01147	0.0086	0.61643	21.6277
		(0.0062)	(0.0322)	(0.0002)	(0.0006)
Black-	0.0045	-0.00026	0.0065	0.0	18.0489
Karasinski	(0.5773)	(0.9686)	(0.9287)		(0.0028)

during this subperiod. Comparing the results gleaned from Table 7 with the results from Tables 6 and 8, we conclude that the unrestricted model cannot outperform the other mean reverting models during the first and last subperiods because the first and last subperiods display low volatility (i.e., no volatility inducement). Also The random walk process cannot capture the short-term interest rate during the first and the last subperiods because of two reasons. First, the process is not a random walk but nonlinear trend stationary. Second, there is no stationarity inducement during these two subperiods.

Finally, the short-term riskless rate displays a regime shifting behavior and the finding that high elastic volatility models outperform others is extremely sensitive to the sample period and largely due to the second subperiod.

#### VI. A Simple Model of Short-Term Interest Rate with Dynamic Trend

In this model we assume that the level of short-term interest rate is nonlinear trend stationary. As a result, it is mean reverting to its long-term conditional mean (the dynamic trend) denoted by  $(m_t)$ . Following Hodrick and Prescot (1981, 1997), let the short-term interest rate composed of the stationary component (c) and the dynamic trend  $(m_t)$ 

$$r_t = c_t + m_t \qquad t = 1, \dots, T \tag{21}$$

The trend component  $m_i$  is determined by solving the loss function:

$$\sum_{t=1}^{T} [c(t)]^2 + \lambda \sum_{t=2}^{T} [\nabla^2 m_t]^2$$
(22)

given (21), where,  $\nabla^2 m_t$  is the second order lag operator

$$\nabla^2 m_t = (1 - L)^2 m_t = [m(t+1) - m(t)] - [m(t) - m(t-1)]$$
(23)

where L is the lag operator, such that  $L^{i}m_{t} = m_{t-i}$ , and  $\nabla$  is the regular difference.

The first term in (22) penalizes the poorness of fit, while the second term penalizes the variability in the trend. The, HP-parameter  $\lambda$  is a positive parameter that balances the trade off between the two decisive factors. In this framework, the value of the parameter  $\lambda$  is determined by assuming the probability distribution where

$$c(t) - c(t-1) \sim IN(0, \sigma_c^2)$$
(24a)

$$m(t+1) - c(t) \sim IN(0, \sigma_m^2)$$
(24b)

and solving the loss function in (22) when  $\lambda = (\sigma_c^2 / \sigma_m^2)$ .

A useful insight of the filter can be derived from its representation of time domain by considering the case of a finite sample in which the trend component can be presented as,

$$m_{t} = \sum_{i=1}^{T} w_{it} r_{i} = M(L) r_{t}$$
(25)

that is  $m_t$  is a two side weighted moving average of the short-term interest rate. The stationary component is,

$$c_t = \left[1 - \sum_{i=1}^T w_i\right] r_t = C(L)r_t$$
(26a)

To set the ball rolling, let the stationary component follows the following AR(1) specification

$$c_{t+1} = \gamma (r_t - m_t) + v_{t+1}$$
 (26b)

Subtracting  $r_t$  from both sides of 21 after taking one period ahead and plugging (26b) specification we get

$$r_{t+1} - r_t = \gamma (r_t - m_t) + m_{t+1} - r_t + v_{t+1}$$
(27)

where  $v_{t+1}$  is now an inverted ARMA process.

We follow the CIR SR model to ensure a positive interest rate. Thus, our model is similar to the CIR SR model except in that the level of interest rates is mean reverting about its growth component. For simplicity, consider the CIR SR with the following drift function

$$\mu(\theta, r_t, m_t) = \left(\beta r_t + \gamma \sum_{i=1}^T w_{it} r_i + \sum_{i=1}^T w_{it+1} r_i\right) dt$$
(28)

To estimate the model, we replace the moments condition in (17) by the following vector

$$h(x_{t+1}, \lambda) = \begin{bmatrix} v_{t+1} \\ v_{t+1}r_t \\ v_{t+1}m_t \\ v_{t+1}^2 - \sigma^2 r_t^{2\gamma} \\ (v_{t+1}^2 - \sigma^2 r_t^{2\gamma})r_t \\ (v_{t+1}^2 - \sigma^2 r_t^{2\gamma})m_t \end{bmatrix}$$
(29)

where  $v_t$  is a noise term that takes the following econometric specification

$$v_{t+1} = [r_{t+1} - r_t - \beta r_t + \gamma m_t - m_{t+1}]$$
(30)

We examine three different specifications for the model by setting a number of restrictions on the parameter  $\gamma$ . Particularly, we examine, the unrestricted CIR SR, CIR SR, and Vasicek models. We invoke the GMM in (29) and compare the performance of our model to others.

Table 9 reports the GMM estimation of the nonlinear trend stationary model. As shown, the value of the parameter  $\gamma$  sharply declines from 1.184 to 0.889 when the unrestricted model is modified to nonlinear trend stationary. Also the goodness of fit statistic shows

Table 9. GMM Estimate of Modefied CIR SR Model of Short-Term Interest Rate Dynamics Coefficients, Their Significance Levels, and Overidentifing Test for the Period (1934 Through 2002)

Model	$\beta$ ( <i>p</i> -value)	$\Theta$ ( <i>p</i> -value)	$\sigma^2$ ( <i>p</i> -value)	$\gamma$ ( <i>p</i> -value)	Overidentifying Test (p-value)
Unrestricted	-0.0790	0.9212	0.0027	0.8892	5.8527
	(0.0001)	(0.0000)	(0.6360)	(0.0268)	(0.1190)
Vsicek	-0.4883	0.5092	0.0189	0.0386	27.1339
	(0.0002)	(0.00008)	(0.0012)	(0.0000)	(0.0000)
CIR SR	-0.083382	0.91583	0.000139	0.5	11.3021
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1259)

extreme improvement where  $(X^2)$  statistic declines from 13.203 to 5.852. Also, the nonlinear trend stationary CIR SR shows extreme improvement and the *p*-value of the  $(X^2)$  goodness-of fit statistic sharply increases from 0.0063 to 0.127, implying that the model cannot be rejected at even 88% confidence interval. Across the three suggested models, the value of the parameter  $\beta$  is approximately equal ( $\theta$ -1) which is negative and significantly different from zero implying mean reversion about the trend.

#### VII. Conclusion

In this paper we employ a wide range of unit root tests to evaluate the stationary hypothesis of the short-term riskless rate. Consistent with Bandi (2002) and Perron (1989), the random walk hypothesis is strongly accepted when it is tested against the stationary and linear trend stationary hypotheses. However, when the random walk hypothesis is tested against the alternative of non-linear trend stationary, the unit root hypothesis is no longer accepted (the result is robust to structural break and size distortions). Consistent with Beirens (1997) and Philips (1998), we conclude that the riskless rate is nonlinear trend stationary.

Based on the finding of nonlinear trend stationarity in the level of the short-term interest rate, we locate the stationary component of the process using the HP filter and examine the performance of 10 models of short-term riskless rate. The analysis shows that all mean reverting models (even those models which assume homoscedastic interest rate volatility) show extreme improvement in capturing the cyclical component of the interest rate. In contrast, we document a significant performance decline in all models which assume the level effect, except for the CIR SR model which shows significant improvement. Moreover, the long-term constant or deterministic mean converges to zero in all models that assume mean reversion; this implies invalidity of the linearity assumptions about this parameter. Based on this finding, we modify the CIR SR model by replacing the constant drift with the stochastic trend (the long-term dynamic mean of the interest rate). The results imply a substantial improvement in the CIR SR model toward a nonlinear trend stationary process.

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Vita

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# DOCTORAL EXAMINATION REPORT

CANDIDATE: Haitham Al-Zoubi

MAJOR FIELD: Financial Economics

TITLE OF DISSERTATION:

"New Evidence on Interest Rate and Foreign Exchange Rate Modeling"

APPROVED:

14d. Habin of Hessen

Major Professor & Co-Chair M. Kabir Hassan

n

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DATE OF EXAMINATION: July 3, 2003