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Quadratic Term Structure Models with Jumps in Incomplete Currency Markets

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Abstract

We propose a multi-currency quadratic term structure model that allows for several sources of market incompleteness. A new feature of the model is the jump-quadratic dynamics of the exchange rates that simultaneously generate greater flexibility in the time-varying risk premium and excessive currency volatility. Our model empirically outperforms the complete market quadratic and affine multi-currency diffusion models. It accounts for the forward premium anomaly with reasonable market price of risks. The market incompleteness consists of idiosyncratic diffusion-like innovations and jump discontinuities. We find that the jumps dominate the variations in the currency returns and produce most of the excessive currency volatility.

JEL Classification: F31, E43, D52, and C14

Keywords: Quadratic Term Structure, Incomplete Markets, Jumps, and Excess Volatility

The majority of assets traded on the world financial markets are exposed to nominal interest rates and exchange rate fluctuations. However, the joint dynamics of the interest rates and exchange rates continue to be an unresolved issue in international finance. A major challenge is how to simultaneously explain the forward premium anomaly, the nonlinearities in the interest-rate process, the excessive volatility of currency returns, and the jump discontinuities in these returns. Several studies address one or two of these features, while the others remain unresolved.

Inci and Lu (2004) show that quadratic models are equipped to capture deviations from the uncovered interest-rate parity. As is well-known, uncovered interest-rate parity requires that expected changes in the nominal value of a foreign currency be positively related to nominal domestic less foreign currency interest rates. Yet, empirical evidence shows that low (high) interest rate currencies tend to depreciate (appreciate).[1] This departure from uncovered interest -rate parity, which is also known as the forward premium anomaly, implies that the slope coefficient in the linear projection of the expected foreign currency returns onto the forward premium is negative and significantly different from unity. Fama (1984) attributes this forward premium anomaly to a time-varying foreign exchange risk premium that is negatively correlated with expected depreciation rates and has a larger variance.

As noted in Backus et al. (2001) and Bansal (1997), the literature relying on asset pricing theories, covariance between money supply and output, market imperfections and/or non-additive preferences, has been unsuccessful in producing a risk premium that satisfies these Fama conditions.[2] Recently, several studies have resorted to international affine term structure models of interest rates to explain the forward premium anomaly [e.g., Ahn (2004), Bansal (1997), Frachot (1996), Nielsen and Saá-Requejo (1993), and Saá-Requejo (1994)]. To ensure positivity of the nominal interest rates, most adapt the Cox, Ingersoll, and Ross

[1] See, for example, e.g., Backus et al. (2001), Bansal (1997), Bilson (1981), Fama (1984), Hodrick and Srivastava (1986), and Hsieh (1984).

[2] See, for example, Backus, Gregory, and Telmer (1993), Bansal et al. (1995), Bekaert (1996), Bekaert, Hodrick, and Marshall (1997), Cumby (1988), Domowitz and Hakkio (1985), Frankel and Engel (1984), Hollifield and Uppal (1997), and Mark (1988).

(1985, hereafter CIR) affine term structure model to a multi-currency setting. Backus et al. (2001) consider a more general class of multi-currency affine models by adapting Duffie and Kan's (1996) affine yield models, and find that these models have difficulty accounting for the anomaly. They show that these models either admit negative nominal interest rates with a positive probability or provide counterfactual large values for the market price of risks.

In contrast, Inci and Lu (2004) find that quadratic term structure models of interest rates can successfully explain the forward premium anomaly with reasonable market price of risks. Their multi-currency quadratic model is an extension of Constantinides (1992) to an international setting. It allows for a more flexible correlations structure among the state variables and guarantees positive interest rates. Inci and Lu also find that the quadratic term structure model can provide a good description of the interest rate and exchange rate movements. However, their quadratic model fails to outperform the random-walk model of exchange rates. The possible explanations for such a failure are as follows. First, the exchange rate dynamics in their models is fully determined by domestic and foreign pricing kernels. Brandt and Santa-Clara (2002) find that the pricing kernels alone cannot reproduce the excessive volatility in the currency returns. They alternatively use market incompleteness to generate the excess volatility. Second, the quadratic model used in Inci and Lu does not allow for jump discontinuities in the currency returns. Yet, the empirical evidence strongly supports the presence of jumps in the exchange rate movements.[3]

In this paper, we propose a multi-currency quadratic model that accommodates for both jump discontinuities and excess currency volatility in incomplete markets. We generalize the multi-currency quadratic term structure model in Inci and Lu (2004) by using the comprehensive quadratic term structure model developed in Ahn et al. (2002), accommodating country-specific and global risk factors, allowing for incompleteness in the currency markets, and introducing jump-quadratic exchange rate dynamics.[4] As in Brandt and Santa-Clara

[3] See, for example, Akgiray and Booth (1988), Jorion (1988), Bates (1996), Campa et al.(1998), Chang and Kim (2001), and Daal and Madan (2003).

[4] Ahn et al. (2002) show how their all-encompassing quadratic term structure model can nest other

(2002), the currency premium consists of an interest-rate risk premium and a pure currency risk premium. In contrast to Brandt and Santa-Clara, the interest-rate risk premium is non-linear in the state variables, which allows for greater flexibility and variability in the currency premium. We further complement Brandt and Santa-Clara by allowing jump discontinuities to be an additional source of market incompleteness.

We examine whether our quadratic model can simultaneously explain the forward premium anomaly, describe the jump behavior, and capture the excessive variation in the exchange rates. We use the efficient method of moments (EMM) of Gallant and Tauchen (1996) to estimate and test the quadratic models and their affine counterparts. The EMM is advantageous because it accommodates for the unobservability of the state variables, avoids issues of discretization bias, and provides consistent and efficient estimates. To examine the role of the factor structure and the contribution of market incompleteness, four parametrizations of the general quadratic model are investigated. The data consists of weekly currencies and interest rates for the United States, United Kingdom, Germany, Canada, and Japan.

The main results can be summarized as follows. First, the goodness-of-fit results indicate that our model outperforms the complete market quadratic and affine multi-currency models. For all currencies, except the Japanese yen, we find that the overidentifying restrictions of the model cannot be rejected by the joint interest-rate and exchange rate data. Second, consistent with Inci and Lu (2004), we find that the multi-currency quadratic model can provide a great improvement in accounting for the forward premium anomaly relative to the affine model. In addition, accommodating for positive and negative correlations among the state variables in the quadratic model significantly reduces the burden on the market price of risks. A new insight is the fact that the quadratic model can explain the anomaly with only pure country-specific risk factors, while still maintaining strictly positive nominal interest rates.

Third, we find in most cases that the pure currency risk premium dominates the interest-

quadratic models, e.g., Beaglehole and Tenney (1991), Constantinides (1992), and Longstaff (1989).

rate risk premium. In contrast to Brandt and Santa-Clara (2002), the contribution of the interest-rate risk premium is economically significant for all the currencies. The substantial contribution of interest-rate premium is due to the fact that the components of the market price of interest-rate risks in the quadratic model are completely disentangled from the factor volatilities. This specification allows for more flexibility and variability in the interest-rate risk premium. Fourth, except for Canada, we find that the domestic and foreign pricing kernels can account for only a small portion of the high variance of the currency returns. This limitation of the domestic and foreign pricing kernels implies a high degree of incompleteness in the currency markets. Fifth, for all the currencies, the high degree of market incompleteness is induced primarily by the jump discontinuities. We find that jumps dominate the variations in the currency returns and produce most of the excessive volatility. We note, however, that jumps are not the only source of market incompleteness. Variance decomposition indicates that a substantial portion of the market incompleteness and excess volatility are generated by diffusion-like innovations that are not priced in these currency markets.

The paper is organized as follows. Section 1 presents a complete characterization of the general quadratic model with incomplete markets and jump effects. We describe the pricing kernels, the sources of market incompleteness, and the dynamics of the interest rates and the currency returns. Section 2 presents the parametric special cases, the data, and the estimation methods. In Section 3, we discuss the results and Section 4 concludes.

1 The General Model

This section presents a complete characterization of a broad class of international quadratic term structure models with jump-diffusion exchange rates (IQ-JD) in an arbitrage-free two-country economy that has incomplete financial markets. The general IQ-JD model encompasses both country-specific and global factors, and allows for maximal flexibility in the correlation structure between domestic and foreign nominal interest rates. In addition, it accommodates stochastic volatility and nonsystematic jump discontinuities in the exchange rate movements. The time-varying foreign exchange risk premium compensates for both interest-rate related risk and pure currency risk.

1.1 Quadratic domestic and foreign nominal interest rates

Without loss of generality, we consider a continuous-time two-country economy that is free of arbitrage opportunities.[5] Uncertainty in this economy is described by a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$, where Ω is the sample space, $\mathcal{F}$ is the $\sigma-algebra$ of measurable events, $\mathbb{P}$ is the physical probability measure, and $\mathbb{F} = \{\mathcal{F}(t) : t \geq 0\}$ is the augmented filtration.

As in Ahn et al. (2002), we assume that the nominal instantaneous interest rate, $r_i(t)$, is a quadratic function of the state variables,

$$r_i(t) = \alpha_i + \beta_i^{'} Y(t) + Y(t)^{'} \Psi_i Y(t), \quad \text{for } i = \{D, F\}, \tag{1}$$

where $i = \{D, F\}$ is the index for the domestic and the foreign country, α_i is a strictly positive constant, β_i is an N-dimensional vector of constants, Ψ_i is an $N \times N$ diagonal matrix of constants, and $Y(t)$ is an N-dimensional vector of unobservable state variables. The parameters in equation (1) are restricted such that $\alpha_i - \frac{1}{4}\beta_i^{'}\Psi_i^{-1}\beta_i \geq 0_N$, and Ψ_i is a positive semi-definite matrix. These restrictions ensure that $r_i(t) \geq 0$, which is an appealing feature that generally is unattainable by affine models..

The N-dimensional vector of unobservable state variables, $Y(t)$, is partitioned as $Y \equiv \left(Y^D, Y^F, Y^C\right)^{'}$, where Y^D and Y^F are $k \times 1$ vectors of country-specific, idiosyncratic factors, and Y^C is an $(N - 2k) \times 1$ vector of worldwide, common factors. To isolate the impact of each factor structure, we assume without loss of generality that the country-specific factors, Y^D and Y^F, are uncorrelated with each other and with the common factors, Y^C. We assume

[5] We can easily allow for K number of foreign countries in our framework. However, such a treatment will complicate the presentation without providing any additional insight.

that $Y(t)$ follows a multivariate mean-reverting Gaussian process,

$$dY(t) = [\mu - \xi Y(t)]\, dt + \Sigma dW(t), \tag{2}$$

where μ is an N-dimensional vector of constants, $W(t)$ is an N-dimensional vector of standard Brownian motions, ξ and Σ are block diagonal $N \times N$ matrices with

$$\xi = \begin{bmatrix} \xi^D_{k\times k} & 0_{k\times k} & 0_{k\times(N-2k)} \\ 0_{k\times k} & \xi^F_{k\times k} & 0_{k\times(N-2k)} \\ 0_{(N-2k)\times k} & 0_{(N-2k)\times k} & \xi^C_{(N-2k)\times(N-2k)} \end{bmatrix}, \tag{3}$$

$$\Sigma = \begin{bmatrix} \Sigma^D_{k\times k} & 0_{k\times k} & 0_{k\times(N-2k)} \\ 0_{k\times k} & \Sigma^F_{k\times k} & 0_{k\times(N-2k)} \\ 0_{(N-2k)\times k} & 0_{(N-2k)\times k} & \Sigma^C_{(N-2k)\times(N-2k)} \end{bmatrix}, \tag{4}$$

where ξ is the speed of adjustment matrix, the square submatrices ξ^D, ξ^F, and ξ^C are diagonalizable and have positive eigenvalues, and $\Sigma\Sigma'$ is the variance-covariance of the state variables, $Y(t)$. In Appendix A, we present the Gaussian transition densities for the state variables.

The orthogonality of Y^D, Y^F, and Y^C does not preclude interdependencies and interactions among the state variables. Our setup is flexible enough to allow for interdependent state variables through the off-diagonal elements of the submatrices ξ^i, ξ^C, Σ^i, and Σ^C in equations (3) and (4). The country-specific factors can be interdependent within each respective country, but not across countries, and the common factors can fully interact with each other. We observe that the correlations within each type of state variable can be either positive or negative without hampering the positivity of the nominal interest rates.

The partitioning of the state variables into Y^D, Y^F, and Y^C allows us to classify the IQ-JDs into three broad subfamilies according to the index k, which represents the number of idiosyncratic factors in each country. First, when k is zero, $r_i(t)$ is determined only by the global factors, Y^C. Second, when k is between zero and $N/2$, $r_i(t)$ depends on both the country-specific factors, Y^i, and the common factors, Y^C. Third, for k equal to $N/2$, only the country-specific factors, Y^i, generate $r_i(t)$.

This classification scheme imposes the following restrictions on the elements of the parameter vector, β_i, and the matrix, Ψ_i, in equation (1):

$$\begin{aligned} \beta_{D,j} &= 0, \quad \Psi_{D,jj} = 0, \quad k+1 \leq j \leq 2k, \\ \beta_{F,j} &= 0, \quad \Psi_{F,jj} = 0, \quad 1 \leq j \leq k, \quad \text{for } k \geq 1. \end{aligned} \tag{5}$$

These parameter restrictions imply that the foreign (domestic) idiosyncratic factors do not affect the domestic (foreign) nominal interest rates. Therefore, the correlation among the cross-country interest rates is only determined by the common state variables, Y^C, and the corresponding coefficients in $\beta_{i,j}$ and $\Psi_{i,jj}$. Since both the coefficient $\beta_{i,j}$ and the correlation among the common state variables can take on either sign, we observe that the general IQ-JD can accommodate both positive and negative correlations between the nominal interest rates across countries. In contrast, affine models only allow for positive correlation between the cross-country interest rates [e.g., Bakshi and Chen (1997), and Nielsen and Saá-Requejo (1993)].

1.2 Pricing kernels, market price of risks, and cross-country yields

The absence of arbitrage opportunities guarantees the existence of a nominal pricing kernel $M_i(t)$.[6] To obtain a general $N+1$-factor IQ-JD, we directly specify the process for $M_i(t)$,

$$\frac{dM_i(t)}{M_i(t)} = -r_i(t)\,dt - \mathbf{1}_N'\left[\Lambda_i(t) \circ dW(t)\right] - \Lambda_{c,i}dW_c(t), \quad \text{for } i = \{D, F\}, \tag{6}$$

[6] The pricing kernel approach has been used extensively in the finance literature, e.g., Ahn et al. (2002), Constantinides (1992), Dai and Singleton (2000), and Dai and Singleton (2002).

where $\circ$ is the Hadamard product, $\Lambda_i(t)$ is the $N \times 1$ vector of the market price of factor risks,

$$\Lambda_i(t) = \eta_{0i} + \eta_{1i} \circ Y(t), \tag{7}$$

η_{0i} and η_{1i} are N-dimensional vectors of constants, $\Lambda_{c,i}$ is the market price of pure currency risk,

$$\Lambda_{c,i}(t) = \sigma_{c,i}\sqrt{X(t)}, \tag{8}$$

where $X(t)$ is a state variable that follows a square-root process,

$$dX(t) = [\theta - \kappa X(t)]\,dt + \sigma\sqrt{X(t)}dW_c(t),$$

and $W_c(t)$ is a one-dimensional Brownian motion that is orthogonal to $W(t)$. The specification of the pricing kernels in equation (6) accommodates two independent sources of systematic risks. The process $W(t)$ generates the factor risks associated with the interest rates and $W_e(t)$ governs the pure currency risks. As in Brandt and Santa-Clara (2002), we introduce a pure currency component in the pricing kernels to allow for other risk factors besides the interest-rate risk to determine the currency premium. In this respect, Inci and Lu (2004) find that the exchange rates are affected by other factors that are not in the interest rate dynamics.

Consistent with our classification scheme, we set k elements of η_{0i} and η_{1i} equal to zero,

$$\begin{aligned} \eta_{0D,j} &= 0, \quad \eta_{1D,j} = 0, \quad k+1 \le j \le 2k, \\ \eta_{0F,j} &= 0, \quad \eta_{1F,j} = 0, \quad 1 \le j \le k, \quad \text{for } k \ge 1. \end{aligned} \tag{9}$$

These restrictions imply that the risks associated with the country-specific factors, Y^i, are only priced in the i-th country, whereas the global factor risks are priced in both countries.

In equation (7), the state variables, $Y(t)$, affect the market price of risks directly, and not through the factor volatilities. As a result, there is greater flexibility in the variation and sign of the market price of risks, $\Lambda_i(t)$. We note that all the individual elements of $\Lambda_i(t)$ can easily switch sign over time, without violating the admissibility conditions of the IQ-JD model and with fully heteroscedastic interest-rate volatilities. Duffee (2002) notes that such a high degree of flexibility cannot be attained by completely affine models because of the tight link between the market price of risks and factor volatilities.

The cross-country yields are a straightforward extension of Ahn et al. (2002) to an international setting. First, we follow Ahn et al. and use the Girsanov theorem to specify the risk-neutral process for the state variables as perceived in the i-th country,

$$\begin{aligned} dY(t) &= [\mu - \xi Y(t) - \Sigma\Lambda_i(t)]\,dt + \Sigma d\widetilde{W}_i(t) \\ &\triangleq [(\mu - \delta_{0i}) - (\xi + \delta_{1i})\,Y(t)]\,dt + \Sigma d\widetilde{W}_i(t) \end{aligned} \tag{10}$$

where $\delta_{0i} = \Sigma\eta_{0i}$, $\delta_{1i} = \Sigma\mathrm{diag}\left[\eta_{1i,j}\right]_N$, and $\widetilde{W}_i(t) = W(t) + \int_0^t \Lambda_i(s)\,ds$ is a standard Brownian motion in $\mathbb{R}^N$ under a risk-neutral probability measure, $\mathbb{Q}_i$.[7] Applying standard Itô's lemma, the fundamental valuation equation for a nominal interest-rate contingent claim, $V_i(r_i(t), t, \tau)$, with τ periods until maturity is given by

$$\begin{aligned} 0 &= \frac{1}{2}\mathrm{tr}\left(\Sigma\Sigma' V_{i,yy}\right) + [(\mu - \delta_{0i}) - (\xi + \delta_{1i})\,Y(t)]\,V_{i,y} \\ &\quad + V_{i,t} - r_i(t)V_i, \end{aligned} \tag{11}$$

where the additional subscripts on V_i denote partial derivatives with respect to the corresponding variables. The boundary conditions of this PDE are provided by the contractual provisions of the contingent claim and are particular for each type.

For the term structure of nominal interest rates, we let $P_i(t, \tau)$ be the price of a zero-coupon bond that pays \$1 at maturity $T = t + \tau$ in the i-th country. Under the risk-neutral

[7] Harrison and Kreps (1979), and Harrison and Pliska (1981) show that there exists at least one equivalent martingale measure, $\mathbb{Q}$, when the markets are free of arbitrage opportunities.

$\mathbb{Q}_i$-measure, the bond price $P_i(t,\tau)$ must satisfy the PDE in equation (11), subject to the boundary condition,

$$P_i\left(t,0\right)=1. \tag{12}$$

As shown in Ahn et al., solving the PDE subject to this boundary condition results in a bond price that is exponential quadratic in the state variables,

$$P_i(t,\tau)=\exp\left[A_i\left(\tau\right)+B_i\left(\tau\right)^{'}Y\left(t\right)+Y\left(t\right)^{'}C_i\left(\tau\right)Y\left(t\right)\right], \tag{13}$$

where $A_i\left(\tau\right)$, $B_i\left(\tau\right)$, and $C_i\left(\tau\right)$ are the solutions of the following system of ordinary differential equations:

$$\begin{aligned}
\frac{dC_i}{d\tau} &= 2C_i\Sigma\Sigma^{'}C_i-C_i\left(\xi+\delta_{1i}\right)-\left(\xi+\delta_{1i}\right)^{'}C_i-\Psi_i \\
\frac{dB_i}{d\tau} &= 2C_i\Sigma\Sigma^{'}B_i-\left(\xi+\delta_{1i}\right)^{'}B_i+2C_i\left(\mu-\delta_{0i}\right)-\beta_i \\
\frac{dA_i}{d\tau} &= \mathrm{tr}\left[\Sigma\Sigma^{'}C_i\right]+\frac{1}{2}B_i^{'}\Sigma\Sigma^{'}B_i+B_i^{'}\left(\mu-\delta_{0i}\right)-\alpha_i,
\end{aligned} \tag{14}$$

with initial conditions $A_i\left(0\right)=0$, $B_i\left(0\right)=0_N$, and $C_i\left(0\right)=0_{N\times N}$. The exponential quadratic bond price in equation (13) implies that the yield-to-maturity, $yt_i\left(t,\tau\right)$, is a quadratic function of the state variables,

$$yt_i\left(t,\tau\right)=\frac{1}{\tau}\left[-A_i\left(\tau\right)-B_i\left(\tau\right)^{'}Y\left(t\right)-Y\left(t\right)^{'}C_i\left(\tau\right)Y\left(t\right)\right]. \tag{15}$$

To obtain domestic factor loadings that are consistent with our classification scheme of the international factor structures, we let the j-th element of $B_D\left(\tau\right)$ and the j-th diagonal element of $C_D\left(\tau\right)$ be equal to zero for $k+1\leq j\leq 2k$. For the foreign bond price, we set the j-th element of $B_F\left(\tau\right)$ and the j-th diagonal element of $C_F\left(\tau\right)$ equal to zero for $1\leq j\leq k$.

1.3 Exchange Rates Dynamics and Currency Premium

A full characterization of the general IQ-JD requires the specification of the exchange rate dynamics. In the absence of arbitrage opportunities, Ahn (1997), Backus et al. (2001), and Inci and Lu (2004) show that exchange rates 'equalize' domestic and foreign pricing kernels in complete markets. As noted in Brandt and Santa-Clara (2002), exchange rates are not fully determined by the two pricing kernels when the financial markets are incomplete. To allow for nonsystematic jump and excess volatility risk, we assume therefore that the exchange rates are related as follows to the pricing kernels:

$$\frac{S(t)}{S(0)} = \frac{M_F(0,t)}{M_D(0,t)} Z(t), \tag{16}$$

where

$$\begin{aligned}
Z(t) &= \sigma_e W_e(t) + \sum_{n=1}^{N_t} J_{n,t} - \phi\lambda t \\
\ln(1+J_{n,t}) &\sim N\left(\Phi, \delta^2\right) \text{ for } n = 1, 2, \ldots\ldots \\
\Phi &= \ln(1+\phi) - \frac{1}{2}\delta^2,
\end{aligned}$$

$S(t)$ denotes the domestic (dollar) spot price of the foreign currency, $M_i(0,t) = M_i(t)/M_i(0)$, σ_e is the parameter for the excess diffusion volatility, $W_e(t)$ is a standard Brownian motion, $J_{n,t}$ is the random percentage jump size, ϕ is the mean jump size, δ^2 is the variability of the jump size, and N_t is a Poisson random variable with jump intensity λ. The standard Brownian motion, $W_e(t)$, the Poisson-distributed variable, N_t, and the random jump size, $J_{n,t}$, are uncorrelated with each other and with the pricing kernels, $M_i(0,t)$. The jump and excess volatility risk in equation (16) are not priced since $Z(t)$ is a martingale that is uncorrelated to the pricing kernels.

We apply Itô's lemma to equation (16) and obtain the dynamics of the exchange-rate logarithm under the physical measure $\mathbb{P}$,

$$\begin{aligned} d\ln S(t) &= \left[r_D(t) - r_F(t) + \theta_s(Y(t), t) + \theta_c(X(t), t) - \frac{1}{2}\sigma_e^2 - \phi\lambda \right] dt \\ &\quad + \sigma_s(Y(t), t)\, dW(t) + \sigma_c(X(t), t)\, dW_c(t) + \sigma_e dW_e(t) \\ &\quad + \ln(1 + J(t))\, dN(t) \end{aligned} \tag{17}$$

where

$$\theta_s(Y(t), t) = \frac{1}{2}\left[\Lambda_D^2 - \Lambda_F^2\right]$$

$$\theta_c(X(t), t) = \frac{1}{2}\left[\Lambda_{c,D}^2 - \Lambda_{c,F}^2\right]$$

$$\sigma_s(Y(t), t) = \Lambda_D(t) - \Lambda_F(t)$$

$$\sigma_c(X(t), t) = \Lambda_{c,D}(t) - \Lambda_{c,F}(t).$$

Equation (17) implies that expected changes in exchange rates consists of several components, namely, the nominal interest-rate differential, $r_D(t) - r_F(t)$, the time-varying premium for interest-rate risk, $\theta_s(Y(t), t)$, and the premium for pure currency risk, $\theta_c(X(t), t)$. In other words, the expected currency returns compensate for the differences in interest rates and squared market price of risks across countries. The specification in equation (17) incorporates both stochastic volatility and jumps in the exchange rate process. In addition, it allows exchange rates to be determined by factors that are not in the interest-rate process. The quadratic variation of the currency returns can be induced by the risk factors of the pricing kernels and the market incompleteness. The pricing kernels' variance can be decomposed into "interest-rate" and pure currency variances, while the market incompleteness has a diffusion component and a jump component.

For given values of the state variables, the jump-induced conditional variance, skewness, and kurtosis of the currency returns are respectively given by

$$\sigma_j^2 = \lambda \left(\Phi^2 + \delta^2 \right) \tag{18}$$

$$\mathrm{Sk}_j \left(t \right) = \frac{\lambda \left(\Phi^3 + 3\Phi\delta^2 \right)}{\left[\mathrm{Var}_d \left(t \right) + \lambda\delta^2 + \lambda\Phi^2 \right]^{1.5}} \tag{19}$$

$$\mathrm{Ku}_j \left(t \right) = 3 + \frac{\lambda \left(\Phi^4 + 6\Phi^2\delta^2 + 3\delta^4 \right)}{\left[\mathrm{Var}_d \left(t \right) + \lambda\delta^2 + \lambda\Phi^2 \right]^2}, \tag{20}$$

where $\mathrm{Var}_d \left(t \right)$ is the conditional variance of the diffusion components,

$$\mathrm{Var}_d \left(t \right) = \sigma_s^2 \left(Y \left(t \right), t \right) + \sigma_c^2 \left(X \left(t \right), t \right) + \sigma_e^2. \tag{21}$$

The total conditional variation of the currency returns can therefore be decomposed into smooth, diffusion-driven variation, $\mathrm{Var}_d \left(t \right)$, and jump-induced variation, σ_j^2. The stochastic diffusion volatility of the currency returns can generate skewness and excess kurtosis independent of the jump component. However, as noted in Johannes (2004), multi-factor diffusion models can generate only a small fraction of the excess kurtosis. Ahn et al. (2002) find that the three-factor affine and quadratic diffusion models cannot fit the fat tails of the distribution.

1.4 The Forward Premium

The forward exchange rate, $F \left(t, \tau \right)$, is denoted as the time-t domestic price of a forward contract that delivers one unit of foreign currency at time $T = t + \tau$. Following Cox, Ingersoll, and Ross (1981), we note that the domestic price of a such a forward contract is

$$F \left(t, \tau \right) = S \left(t \right) \frac{P_F(t, \tau)}{P_D(t, \tau)}. \tag{22}$$

Rearranging terms and taking logarithms provide us with the term structure of forward premiums,

$$\begin{aligned} f(t,\tau) - s(t) &= [A_F(\tau) - A_D(\tau)] + [B_F(\tau) - B_D(\tau)]' Y(t) \\ &\quad + Y(t)' [C_F(\tau) - C_D(\tau)] Y(t), \end{aligned} \tag{23}$$

where $f(t,\tau) = \ln F(t,\tau)$, $s(t) = \ln S(t)$, and $A_i(\tau)$, $B_i(\tau)$, and $C_i(\tau)$ are the factor loadings from equation (13). We observe that the term structure of the forward premium in equation (23) is a quadratic function of the state variables. As shown in Ahn (2004), in an instantaneous setting, the covered interest-rate parity condition in continuous-time is given by

$$f(t) - s(t) = r_D(t) - r_F(t), \tag{24}$$

where $f(t)$ denotes the logarithm of the instantaneous forward exchange rate.

The linear projection of the expected changes in exchange rates in equation (17) onto the forward premium in equation (24) defines the model-implied slope coefficient, a_2, as

$$a_2 \equiv 1 + \frac{\mathrm{Cov}\left[r_D(t) - r_F(t), \theta_s(Y(t), t)\right]}{\mathrm{Var}\left[r_D(t) - r_F(t)\right]}. \tag{25}$$

For notational convenience we henceforth denote this linear projection on the premium as LPP. From equation (25), we see that a_2 is negative when the vector of squared market price of risk differentials is sufficiently negative correlated with the interest rate differential.

In accounting for the forward premium anomaly with a common-factor structure, at least one of these state variables must affect the pricing kernels in the two countries differently. As shown in Inci and Lu (2004), multi-currency quadratic term structure models with common factors have more flexibility than their affine counterparts to account for the anomaly. The common-factor quadratic models can easily satisfy the Fama conditions in a variety of ways because there are no sign restrictions on the parameters β_i, η_{0i}, and η_{1i} in equations (1) and

(7).[8] The cross products, $\eta_{0D}\eta_{1D}$ and $\eta_{0F}\eta_{1F}$, can generate sufficiently high variations in the interest-rate risk premium $\theta_s(Y(t), t)$ because they are completely independent of the factor volatilities. Moreover, this flexibility allows the quadratic models to account theoretically for the anomaly even with only country-specific factors.[9]

2 Empirical Analysis

In this section, we select four parametric special cases for the quadratic model and two for the affine models. For the feasibility of the econometric investigation, we assume three underlying interest-rate factors, $N = 3$, for all models. This section also describes the data and method used to estimate these models.

2.1 The Model Selection

2.1.1 A Canonical Form for the General IQ-JD

Empirically, it is not feasible to estimate all the parameters of the general IQ-JD model. In particular, as with the fully specified affine model in Dai and Singleton (2000) and the all-encompassing quadratic model in Ahn et al. (2002), some of the parameters of this model are econometrically unidentifiable because of the unobservability of the state variables, $Y(t)$. To address this issue, we introduce several parameter restrictions and normalizations.

First, similar to Ahn et al. (2002), we assume that $\beta_i = 0_N$ in equation (1). They argue that this assumption is necessary to have the long-term mean of the state variables, μ, identifiable. Given the positive semi-definiteness of Ψ_i and the assumption of a strictly positive α_i, the assumption $\beta_i = 0_N$ also guarantees the positivity of the nominal interest

[8] The admissibility condition, $\alpha_i - \frac{\beta_i^2}{4\Psi_i} \geq 0$, imposes a lower and upper bound on β_i, but does not preclude negative values for β_i.

[9] With the appropriate choice of parameters, it is easy to show the variety of ways that the quadratic models can account for the anomaly theoretically with both common and country-specific factors. We omit these illustrations to save space.

rates. Second, we assume that the submatrices of ξ are diagonal, and that Σ^i and Σ^C are lower triangular matrices. As pointed out in Ahn et al., the econometric identification requires that only one of ξ and Σ be fully specified because both matrices determine the covariance of the state variables.

Third, we introduce several normalization rules for Ψ_i, η_{0i}, and η_{1i}. Specifically, for the common-factor quadratic model ($k = 0$), we impose the following parameter restrictions:

$$\begin{aligned} \Psi_{D,jj} &= \eta_{0F,j} = \eta_{1F,j} = 1 \quad \text{for } j = 1, 3, 5, ... \\ \Psi_{F,jj} &= \eta_{0D,j} = \eta_{1D,j} = 1 \quad \text{for } j = 2, 4, 6, ... \end{aligned} \tag{26}$$

The parameter of the market price of pure foreign currency risk, $\sigma_{c,F}$, is also assumed to be equal to one. For the common-idiosyncratic-factor model ($1 \leq k \leq N/2$), the normalization rules are

$$\begin{aligned} \Psi_{D,jj} &= \eta_{0F,j} = \eta_{1F,j} = 1 \quad \text{for } j = 2k + h, \text{ and } h = 1, 3, 5, ... \\ \Psi_{F,jj} &= \eta_{0D,j} = \eta_{1D,j} = 1 \quad \text{for } j = 2k + h, \text{ and } h = 2, 4, 6, ... \end{aligned} \tag{27}$$

These parameter restrictions and normalizations provide us with a canonical jump-quadratic model, which we denote as the $IQ_k(N)$-JD model for notational convenience. The nested complete market quadratic diffusion model is labeled the $IQ_k(N)$ model. We observe that our canonical form for the IQ-JD model is not unique since there are a variety of alternative normalizations and transformations that can be applied to the general IQ-JD model.

2.1.2 Three-factor Models

To empirically investigate the impact of international factor structures on the LPP, we first consider three alternative subfamilies of the three-factor quadratic model that preclude jumps and pure currency risks: 1) the idiosyncratic-factor case with a symmetric common-factor,

$IQ_{1S}(3)$; 2) the common-idiosyncratic-factor model, $IQ_1(3)$; and 3) the common-factor model with orthogonal state variables, $IQ_{0U}(3)$. We then consider the full-fledged $IQ_0(3)$-JD model.

We note that, starting with the more restrictive $IQ_{1S}(3)$ model, the level of flexibility increases with each subsequent model. For the $IQ_{1S}(3)$ model, we are particularly interested in the LPP matching capabilities of the idiosyncratic factors. We therefore assume that the market price of the common-factor risks is equal across countries, that is, $\eta_{0D,3} = \eta_{0F,3}$ and $\eta_{1D,3} = \eta_{1F,3}$. This assumption implies that the common factor affects the domestic and foreign pricing kernels in the same way and, consequently, has no effect on the foreign exchange risk premium. The exchange rate movements are therefore only determined by idiosyncratic factors. This specification allows us to isolate the impact of the currency-specific factor on the LPP, while accommodating nonzero correlation among the cross-country interest rates. For convenience, we also assume that the price of the risk of the country-specific factors is equal across countries, that is, $\eta_{0D,1} = \eta_{0F,2} = \eta_0$ and $\eta_{1D,1} = \eta_{1F,2} = \eta_1$. The total number of free parameters in the $IQ_{1S}(3)$ model is therefore equal to 14.

The $IQ_1(3)$ model relaxes the assumption of equal market price of the common-factor risks across countries. As a result, we note that the $IQ_1(3)$ model admits both idiosyncratic and global factors in the determination of the risk premium and, hence, in the LPP matching. The additional flexibility of the $IQ_1(3)$ model relative to the $IQ_{1S}(3)$ model introduces only two extra parameters. A drawback of both the $IQ_{1S}(3)$ and $IQ_1(3)$ model is that the nominal interest rates are perfectly correlated across countries, which is not consistent with the correlation structure in Panel B of Table I.

The subfamily of $IQ_{0U}(3)$ models increases the flexibility of the $IQ_1(3)$ model in two related ways. First, it eases the restrictions on the international factor structure and allows all three factors to influence the nominal interest rates in each country.[10] Second, the three

[10] This is advantageous since the empirical literature suggests that three factors are required to describe the term structure in a single-country economy [e.g., Ahn et al. (2002), Dai and Singleton (2000), Dai and Singleton (2002), and Knez, Litterman, and Scheinkman (1996)].

common factors in the $IQ_{0U}(3)$ model can accommodate a richer correlation structure between the cross-country interest rates than the one common factor in the $IQ_1(3)$ model. The total number of parameters to be estimated for the $IQ_{0U}(3)$ model is 20.

The $IQ_0(3)$-JD model has maximal flexibility relative to the other three subfamilies because it allows for positive and negative correlations among the state variables, pure currency risk, jumps, and excess volatility. We are particularly interested in the role that these correlations play in explaining the forward premium anomaly. In addition, we can gauge whether market incompleteness, induced partly by jumps, can significantly improve the overall fit of the quadratic models. The $IQ_0(3)$-JD model has 19 free parameters.

To assess the empirical performance and LPP matching of these quadratic models relative to their affine counterparts, we also estimate two parametric special cases of the three-factor multi-currency affine model in Appendix C. We consider a common-idiosyncratic-factor affine structure, $IA_1(3)$, and a common-factor with orthogonal state variables, $IA_{0U}(3)$.

For our econometric implementation, we impose several parameter restrictions and normalizations on the three-factor affine models. In order to identify the long-term mean, θ, of the state variables, we constrain α_i in Appendix C to zero for both the $IA_1(3)$ and $IA_{0U}(3)$ models. We normalize the several elements of β_i, namely, $\beta_{D,1} = \beta_{D,3} = \beta_{F,2} = 1$. For the $IA_1(3)$ model, we set $\beta_{D,2}$ and $\beta_{F,1}$ equal to zero so that $Y_1(t)$ is the domestic factor and $Y_2(t)$ is the foreign factor. In addition, we assume again that the price of the risks of country-specific factors are equal across countries, that is, $\lambda_{D,1} = \lambda_{F,2} = \lambda$. The total number of parameters to be estimated for the $IA_1(3)$ and $IA_{0U}(3)$ models is 12 and 15, respectively.

2.2 Data

The weekly data for the United States (US), United Kingdom (UK), Germany, Canada, and Japan from October 3, 1980 through October 4, 2002 are obtained from Datastream. There are 1150 observations for each series. The data include London Euro-currency interest rates for the maturities of 7 days, 3 months, 6 months, and one year. These annualized interest rates are middle quotes for Euro-currency deposits at the close of the London market on Thursday, and are transformed into continuously compounded yields with the 7-day yield as the short-term nominal interest rate, $r_i(t)$. The foreign currencies are the British pound (£), Deutsche mark (DM), Canadian dollar (C$), and Japanese yen (¥), with the exchange rates in US dollars (US$) per unit of foreign currency.

In figures 1 and 2, we see that the early 1980s was a high-interest-rate regime, whereas the 1990s was a low-rate era for the Euro-currencies. We also observe relative stability and convergency among most of the Euro-currency interest rates during the 1990s. Table I describes the sample properties of Euro-currency interest rates and the spot exchange rates. In Panel A of Table I, the Euro-British pound interest rates have the highest mean, while the Euro-Canadian dollar has the highest standard deviation. The Euro-US dollar interest rates and the pound are respectively more skewed and leptokurtic than the other interest rates and foreign exchange rates. In Panel B of Table I, the correlation matrix shows that the US and Canada have the highest cross-country interest-rate correlation, and the mark and the yen exhibit the highest currency correlation.

2.3 Estimation Method

We start by estimating the sample slope coefficient, a_{2T}, in the forward premium regression

$$s_{t+1} - s_t = a_{1T} + a_{2T}(f_t - s_t) + \text{residual}, \tag{28}$$

where s_t is the log of the spot price of the foreign currency and f_t is the log of the one-period forward exchange rate. The results of this regression are reported in Table II and are used to test the LPP matching of the term structure models. The sample slope coefficients a_{2T} are negative and significantly different from unity, with their magnitudes varying from -2.82 for the UK to -1.165 for Canada. These results indicate strong rejection of uncovered interest-rate parity and are consistent with the empirical findings in the literature.

We use the efficient method of moments (EMM) of Gallant and Tauchen (1996) to estimate the parameters of the quadratic and affine models. This method accommodates for the unobservability of the state variables and avoids issues of discretization bias in these continuous-time term structure models. Gallant and Tauchen show that the EMM estimates are consistent and efficient. This method has been used recently by several authors, e.g., Andersen and Lund (1997) used it to estimate a stochastic volatility model for interest rates, Dai and Singleton (2000) to perform specification analysis of affine models, and Ahn et al. (2002) to estimate quadratic term structure models.

The EMM estimation procedure consists of two steps. The first step approximates the conditional density of the observable data, y_t, using a semi-nonparametric (SNP) approach. Let us denote this approximate conditional density as $\widehat{f}\left(y_t|x_{t-1},\gamma\right)$, where γ is the parameter vector and x_{t-1} are the lagged values of the observables. The procedure to obtain this SNP approximate density is outlined in Gallant and Tauchen (1989). It entails augmenting a Gaussian vector-autoregression (VAR) with the potential for ARCH innovations by a Hermite polynomial expansion, resulting in an auxiliary SNP model,

$$\widehat{f}\left(y_t|x_{t-1},\gamma\right) = c\left(x_{t-1}\right)\left[\epsilon_0 + \left[h\left(z_t|x_{t-1}\right)\right]^2\right]\varphi\left(z_t\right), \tag{29}$$

where $\varphi\left(.\right)$ is the density of the standard normal distribution, $h\left(z_t|x_{t-1}\right)$ is a Hermite polynomial in z_t, $c\left(x_{t-1}\right)$ is a normalization constant, ϵ_0 is an arbitrary small positive number, and z_t is the following demeaned transformation of y_t:

$$z_t = R_x^{-1}\left(y_t - \mu_x\right), \tag{30}$$

where μ_x is the mean vector of y_t and $R_x R_x'$ is the variance-covariance matrix of y_t. As suggested in Gallant and Tauchen, we search for the best $\widehat{f}\left(.\right)$ for the observable data by using an expansion strategy based on the Schwartz selection criterion (BIC).

Using this strategy, we fit SNP models to the vectors of observable data for the four country pairs. Each vector of observables consists of the 7-day domestic yields, the 7-day foreign yields, and the weekly currency returns. We find that the auxiliary SNP models for the UK, Germany, and Canada can be described as "Non-Gaussian, VAR(1), ARCH(4), Homogeneous Innovation."[11] In other words, the mean vector is a first-order vector autoregression, the innovations follow a fourth-order ARCH process, and the deviations from normality are best described by a fourth-order Hermite polynomial that has constant coefficients. This high order ARCH-like specification is quite similar to that of the SNP specifications in Andersen and Lund (1997) and Ahn et al. (2002).

In contrast, we find that the one-week yields on the 7-day Euro-yen and the weekly returns of the yen induce a low order ARCH-like specification. In particular, the SNP model for Japan can be described as "Non-Gaussian, VAR(1), ARCH(2), Homogeneous Innovation." In this specification, the lower order ARCH process for the innovations captures the slowdown in the economic activities in Japan during the 1990s. The total number of free parameters in this auxiliary SNP model is equal to 36, as compared to 42 in the SNP model for UK, Germany, and Canada.

The second step in the EMM technique involves simulating the data and estimating the parameters of the term structure models and the exchange rate dynamics by using the scores of the likelihood function from the auxiliary SNP model as the moment conditions, $m'(\rho, \widehat{\gamma})$. As in Andersen and Lund (1997), we use the Euler scheme to generate the data from the continuous-time models. For the simulation from the jump component, we employ the approximation and smoothing technique used in Andersen, Benzoni, and Lund (2002). Specifically, they use a Binomial distribution to approximate the Poisson counter, $dN(t)$, and then smooth the discontinuity of the Binomial random variable over an interval centered

[11] Since the US is the "domestic" country in each country pair, we only use the "foreign" country when referring to a country pair in the empirical analysis. For example, Canada represents the country pair US-Canada.

around $1-\lambda$.

The optimal GMM criterion function is the quadratic form

$$m^{'}\left(\rho, \widehat{\gamma}\right) W^{-1} m^{'}\left(\rho, \widehat{\gamma}\right), \tag{31}$$

where ρ is the parameter vector, $\widehat{\gamma}$ is the vector of the quasi-maximum likelihood estimates of the SNP model, and the weighting matrix W^{-1} is the quasi-information matrix. We then proceed with the goodness-of-fit tests of these models through the test statistic

$$T m^{'}\left(\rho, \widehat{\gamma}\right) W^{-1} m^{'}\left(\rho, \widehat{\gamma}\right) \sim \chi^2_{K-J}, \tag{32}$$

where T is the simulation length of the data, K denotes the number of scores, J is the dimension of ρ, and $K-J$ is the degrees of freedom. To adjust the χ^2 in equation (32) for the degrees of freedom, we employ the z-statistic, which is defined as $\frac{\chi^2-(K-J)}{\sqrt{2(K-J)}}$.

Next, treating the EMM estimated parameters ρ as the true parameters and using the analytical second moments in Appendix B, we compute the model-implied or population coefficients a_2. We then use simple t-tests to examine whether the sample slope coefficients, a_{2T}, in Table II differ significantly from the model-implied slope coefficients. This matching criterion is more demanding than generating negative values for a_2 to satisfy the Fama conditions.

3 The Results

3.1 Parameter Estimates and Interest-Rate Dynamics

For all models considered, the α_i estimates are positive, ensuring strictly positive nominal interest rates. The conditional volatility of the nominal interest rates implied by these quadratic models is the highest for the US rates. In Tables V and VI, the point estimates for Ψ_i, ξ, and $\Sigma\Sigma'$ in the $IQ_{0U}(3)$ and $IQ_0(3)$-JD models indicate that the correlation with the Euro-US dollar interest rates is conditionally the highest for the Euro-Canadian dollar rates and the lowest for the Euro-DM rates, which coincides with the statistical correlation matrix in Panel B of Table I. These parameter estimates are also key determinants of the international factor structure. For example, the estimates for Ψ_i in Tables V and VI suggest that $Y_1(t)$ and $Y_3(t)$ can be considered as primarily "domestic" interest-rate factors in the $IQ_{0U}(3)$ and $IQ_0(3)$-JD models since they have a much greater effect on the Euro-US dollar interest rates relative to other Euro-currency interest rates. For most pairs of countries, the estimates for $\Sigma\Sigma'$ in Table VI indicate that $Y_2(t)$ is negatively correlated with $Y_1(t)$ and $Y_3(t)$, whereas $Y_1(t)$ and $Y_3(t)$ are positively correlated.

Regarding the market price of interest-rate factor risks, the absolute values of the estimates for η_{0i} and η_{1i} vary considerably across the quadratic models. For the $IQ_{1S}(3)$ model, we see in Table III that these parameter estimates for the market price of currency-specific risks are relatively high and differ substantially across the four pairs of countries. For example, η_0 ranges from -2.476 for UK to -1.675 for Canada, which together with the estimates for η_1, implies that the market price of the foreign factor risk is the highest for the UK and the lowest for Canada. In Table IV, the absolute values of parameter estimates for η_0 and η_1 in the $IQ_1(3)$ models are lower than those in the $IQ_{1S}(3)$ models, implying less burden on the market price of the currency-specific factor risks.

We observe in Table VI that the flexibility in the correlations among the state variables further reduces the burden on the market price of interest-rate risks. For almost all pairs of countries, the absolute value of the cross products, $\eta_{0i}\eta_{1i}$, is lower for the $IQ_0(3)$-JD compared to the $IQ_{0U}(3)$ models. In addition, for given values of $Y(t)$, we find that the market price of risks, $\Lambda_i(t)$, evaluated at the estimated parameters η_{0i} and η_{1i}, are lower for most factors in the $IQ_0(3)$-JD models.

Consistent with Backus et al. (2001), we see in Panel B of Tables IV and V that accounting for the anomaly places a great demand on the market price of risks of the affine models. In most cases, we note that the market price of factor risks implied by the affine models is larger than that of the quadratic models. For example, for the $IA_1(3)$ model applied to UK and Japan, the respective values of -4.260 and -3.116 for λ_3 in Panel B of Table V are greater than those implied by the $IQ_1(3)$ models in Panel A. The multi-currency affine models overburden the market price of risks because of the lack of flexibility in the factor volatilities and correlations among the state variables.

3.2 Matching the LPP

The last rows of Tables III through VI report the implied slope coefficient a_2. We see that the quadratic models can empirically generate negative values for the implied slope coefficients and, therefore, are consistent with the forward premium anomaly. For example, the values of these coefficients for the most restrictive $IQ_{1S}(3)$ model are -0.915, -0.352, -0.105, and -0.730 for UK, Germany, Canada, and Japan, respectively. These results suggest that the quadratic term structure model can empirically account for the anomaly even when the foreign exchange risk premium is only determined by country-specific interest-rate factor risks.

In contrast, the positive values for the implied slope coefficients a_2 in Panel B of Table IV imply that the $IA_1(3)$ models cannot account for the forward premium anomaly. This failure can be attributed primarily to the presence of country-specific factors. As shown in Backus et al. (2001), under an completely affine structure, these country-specific factors cannot contribute in explaining the anomaly and, simultaneously, retain strictly positive interest rates. We see in Panel B of Table V that replacing them with common factors significantly improves the LPP matching capability of the affine model. All the common-factor $IA_{0U}(3)$ models generate negative values for a_2 and, therefore, are consistent with the Fama conditions.

With respect to the stronger criterion of matching the sample coefficients a_{2T} in Table

II, we find that the idiosyncratic factors in the $IQ_{1S}(3)$ model cannot generate sufficient variation in the foreign exchange risk premium for the British pound. The value of -2.276 for the t-statistic in the last row/first column of Table III shows that the $IQ_{1S}(3)$ model cannot reproduce the high sample slope coefficient of -2.82 for the UK data. The t-statistics in the last row of Table IV indicate that allowing common factor risks in the foreign exchange risk premium provides a better sample LPP matching for the quadratic models. The implied slope coefficients of the $IQ_1(3)$ models statistically match the sample coefficients a_{2T}. In the last rows of Tables V and VI, we see that adding more orthogonal or correlated common factors further improves the sample LPP matching of the quadratic models. For the affine models, the t-values in Panel B of Table IV and V imply that all these models fail the stronger criterion of matching the sample coefficients a_{2T}.

3.3 Currency Risk Premium

In Table VI, the estimates for the parameter of the market price of pure currency risk, $\sigma_{c,D}$, are statistically significant and range from 0.592 for Germany to 1.102 for Canada. We use the parameter estimates for the market price of risks and the fitted values for the state variables to obtain the components of the model-implied currency risk premium. These results are reported in Table VII. Consistent with Brandt and Santa-Clara (2002), we find that in most cases the pure currency risk premium dominates the interest-rate risk premium.

With a proportion of 70.45 percent, the dominance of the pure currency risk premium is the greatest for the Japanese yen. From a US investor perspective, this can be explained as follows. The relatively large positive differentials between the US and Japan interest rates are accompanied throughout the sample by a Japanese yen that is slowly appreciating vis-à-vis the US dollar. The Japanese yen requires therefore substantial negative premiums. However, the low conditional volatilities of the Japanese interest rates cannot generate sufficiently large (negative) interest-rate risk premiums.

We note, however, that in contrast to Brandt and Santa-Clara (2002), the contribution of the interest-rate risk premium is economically significant for all the currencies. For Canada, it constitutes 58.9 percent of the total premium. The higher contribution of interest-rate premium can be explained by the parametrization of the market prices of interest-rate risk in our model. In comparison to the completely affine specification used in Brandt and Santa-Clara, our specification allows for more flexibility and variability in the interest-rate risk premium.

3.4 Jumps

In Table VI, the parameter estimates for the jump intensity parameter, λ, vary considerably across the foreign currencies. The likelihood of jumps is the highest (0.961) for the Canadian dollar and the lowest (0.342) for the British pound. In contrast, the variability of the jump size, δ, is the highest for the pound and the lowest for the Canadian dollar. Consistent with the sample skewness in Table I, the estimates of the skewness parameter, ϕ, suggest negative skewness for the pound and Canadian dollar, and positive skewness for the yen. We observe, however, that the parameter estimates for ϕ are statistically insignificant for all the currencies, except the yen.

Table VI also presents the conditional kurtosis implied by the parameter estimates of the jump component. As expected, for all cases, the jump-induced conditional kurtosis is lower than the sample kurtosis in Table I. Except for the Japanese yen, the model-implied kurtosis seems to provide a good match for the sample counterpart. Overall, the parameter estimates of the jump component suggest that the foreign currencies deviate significantly from normality, which is consistent with the findings in the literature.

3.5 Variance Decomposition and Market Incompleteness

In Table VII, we use the parameter estimates and the fitted values of the state variables to compute the variances of log currency returns and their decomposition. We find that the total conditional variances implied by $IQ_0\,(3)$-JD model are high for all the currencies, except the Canadian dollar. The yen records the highest conditional variance of 0.690. In all cases, except the Canadian dollar, the model-implied conditional volatilities of the log currency returns are slightly lower than the sample volatilities in Table I.

Consistent with Brandt and Santa-Clara (2002), the variance decomposition shows that the interest-rate factors alone cannot generate the high variance of the log currency returns. For all currencies, except for the Canadian dollar, the interest-rate component can only account for a relatively small fraction of the total variance. We find that adding time-varying pure currency risk factors to the interest-rate component does not enable the domestic and foreign pricing kernels to reproduce the extremely high exchange rate volatilities. The pricing kernels can explain a substantial part (31.4 percent) of the total variance of the Canadian dollar. For all the other currencies, the contribution of the pricing kernels is lower than 10 percent of the total currency variance. With a 0.7 percent, it is the lowest for the Japanese yen.

For all currencies, this limitation of the domestic and foreign pricing kernels implies a high degree of market incompleteness. Except for the German mark, the results in Table VII suggest that the market incompleteness is generated primarily by the nonsystematic jumps. The dominance of the jump discontinuities is the highest for the Canadian dollar: 68.10 percent of the total variance and 99.48 percent of the market incompleteness. This dominance is driven by the high estimated arrival rate of jumps of 0.961 for the Canadian dollar. With proportions of 62.2 and 58.5 percent, jumps also explain most of the total variance of the British pound and the Japanese yen, respectively. Yet, jumps alone cannot account for the incompleteness of the currency markets. Except for the Canadian dollar, the volatility that is left unexplained amounts to more than 30 percent of the currency variance. This excess volatility is generated by diffusion-like innovations that are not priced in these currency markets.

3.6 Specification Tests

Table III presents the goodness-of-fit tests for the $IQ_{1S}(3)$ models as applied to the UK, Germany, Canada, and Japan. The $IQ_{1S}(3)$ models only allow country-specific interest-rate factors to determine the exchange rate movements and constrain η_{0i} and η_{1i} to be the same across countries. The z-statistics of these models vary from 9.57 for Canada to 35.54 for Japan, and reject the overidentifying restrictions implied by these models. These results suggest that the idiosyncratic factor structure and the symmetry restrictions on the market price of risks significantly hinder these models' ability to fit the cross-country interest rates and exchange rate comovements. The $IQ_1(3)$ models allow for nonzero correlation of interest rates across currencies without imposing a symmetric impact of the common state variable. The z-statistics in Table IV suggest that the $IQ_1(3)$ models slightly improve the fit relative to the symmetric $IQ_{1S}(3)$ models.

The $IQ_{0U}(3)$ models further improve the performance of the quadratic models by allowing all three orthogonal factors to influence the nominal interest rates in each country. However, the z-statistics in Table VI show that the overidentifying restrictions implied by the $IQ_{0U}(3)$ models are still rejected at conventional significance levels. This relatively poor performance of the $IQ_{0U}(3)$ models can be attributed to the lack of flexibility in the correlation structure of the interest rates and the inability of these models to capture the exchange rate dynamics. For Canada, we see a significant improvement in the z-statistic, which falls to 5.26 relative to 8.44 for the $IQ_1(3)$ model. This result can be explained by the fact that for the Canadian dollar the interest-rate factors could capture a significant portion of the currency premium and the currency volatility.

The goodness-of-fit tests in Panel B of Tables IV and V indicate that the quadratic models outperform their affine counterparts for all countries under consideration. The $IA_1(3)$ models in Table IV perform poorly relative to the $IQ_1(3)$ models. These affine models for UK, Germany, and Japan are strongly rejected in the data, with z-statistics of 22.82, 30.04, and 42.67, respectively. The z-statistics in Table V show that the $IA_{0U}(3)$ models outperform the $IA_1(3)$ models, but underperform the $IQ_{0U}(3)$ models. These results suggest that there are nonlinearities in the short-term yields and expected depreciation rates that cannot be captured by affine models. The presence of these nonlinearities is consistent with recent findings in the literature.[12]

Although the multi-currency quadratic models with complete markets considerably improve upon their affine counterparts, we note that they are all rejected by the goodness-of-fit tests. This result suggests that interest-rate factors alone cannot account for the joint dynamics of the interest rates and the exchange rates. Using monthly observations from January 1974 through December 1998 for the US and Germany, Inci and Lu (2004) find however that the quadratic model provides a good description of these joint dynamics. A possible explanation for this discrepancy is the fact that the volatility of the currency returns is substantially higher during our sample period. For example, the volatility of the annualized returns for the German mark is 81.70 percent in our sample as compared to 39.05 percent in their sample. In addition, the higher frequency data used in the current study exhibit more kurtosis than the monthly data employed in Inci and Lu.

Table VI shows that accommodating for jumps and excess volatility for the exchange rates dramatically improves the fit relative to the $IQ_{0U}(3)$ models.[13] Except for Japan, the

[12] Aït-Sahalia (1996), Conley et al. (1997), and Stanton (1997) document important nonlinearities in the short-term interest rates, and Bansal (1997) provides evidence of nonlinearities in the expected depreciation rates as well as the cross-country interest rate differentials.

[13] We also estimate complete market quadratic three-factor models with correlated state variables to examine whether the improvements are due to the introduction of positive and negative correlations among the state variables. We find that these correlations significantly reduce the burden on the market price of risks when dealing with the anomaly, but provide little improvement in the goodness-of-fit of the multi-currency quadratic models. We do not report this results to save space.

z-statistics indicate that the overidentifying restrictions of the $IQ_0\,(3)$-JD models are not rejected by the interest-rate and exchange-rate data. With a z-statistic of 0.97, the $IQ_0\,(3)$-JD model provide the best fit for the Canadian data. The $IQ_0\,(3)$-JD model substantially improve the fit for the Japan, but cannot capture all the puzzling characteristics of this country. Since Japan has the second highest forward premium anomaly (-2.489), we expect the low interest-rate yen to depreciate throughout the sample. Yet, we observe that on average it appreciates during the period under consideration. In addition, the Japanese yen has the highest variance relative to the other currencies, but the pricing kernels can explain less than 1.00 percent of the total variation. Clearly, the case of Japan requires an alternative modeling approach. For the UK, Germany, and Canada, the $IQ_0\,(3)$-JD model provides a rather good description of the joint interest-rate and exchange rate dynamics.

4 Conclusion

In this paper, we have developed a multi-currency quadratic term structure model with jumps and excess volatility in incomplete currency markets. The model accommodates both country-specific and global interest-rate factors; the time-varying currency premium compensates for interest-rate risk and pure currency risk; and the market incompleteness consists of unsystematic jumps and diffusion innovations. We examine whether our proposed model can simultaneously explain the forward premium anomaly, describe the jump behavior, and capture the excessive variation in the exchange rates.

Using the efficient method of moments (EMM) of Gallant and Tauchen (1996), we estimate our model and compare its performance with that of the complete markets models. The results indicate that our model outperforms the complete market quadratic and affine multi-currency models. Consistent with Inci and Lu (2004), we find that the multi-currency quadratic models can provide a great improvement in accounting for the forward premium anomaly with reasonable market price of risks. In particular, accommodating for positive and negative correlations among the state variables in the quadratic model significantly reduces the burden on the market price of risks. Furthermore, the quadratic models can explain the anomaly with only pure country-specific risk factors, while still maintaining strictly positive nominal interest rates.

For most currencies, we observe that the pure currency risk premium dominates the interest-rate risk premium. Yet, the contribution of the interest-rate risk premium is economically significant for all the currencies. This result can be attributed to specification of the market price of interest-rate risks in the quadratic models, which allows for more flexibility and variability in the interest-rate risk premium. Except for Canada, we find that the domestic and foreign pricing kernels can account for only a small portion of the high variance of the currency returns. In all cases, the uncertainty arising from the market incompleteness induces most of the variation in the exchange rate changes. This high degree of market incompleteness is primarily due to the presence of jump discontinuities. Overall, the quadratic model with jumps in the incomplete currency markets provides a great improvement in the description of the joint dynamics of the interest rates and the exchange rates.

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Appendix A. Distribution of the State Variables

As in Ahn et al. (2002), the transition densities for the state variables in equation (2) are multivariate Gaussian densities, which are given by

$$Y(t+\tau)\,|Y(t) \sim \mathrm{MVN}_N\left(E\left[Y(t+\tau)\,|Y(t)\right], \mathrm{var}\left[Y(t+\tau)\,|Y(t)\right]\right), \tag{33}$$

where

$$E\left[Y(t+\tau)\,|Y(t)\right] = U\Lambda^{-1}\left[I_N - \Phi(\tau)\right]U^{-1}\mu + U\Phi(\tau)\,U^{-1}Y(t),$$

$$\mathrm{var}\left[Y(t+\tau)\,|Y(t)\right] = U\left[\frac{\upsilon_{ij}\left(1-\exp\left(\left(-\delta_i-\delta_j\right)\tau\right)\right)}{\delta_i+\delta_j}\right]_{NN}U',$$

$$\Phi(\tau) \triangleq \mathrm{diag}\left[\exp\left(-\delta_i\tau\right)\right]_N,$$

$$\left[\upsilon_{ij}\right]_N = U^{-1}\Sigma\Sigma' U'^{-1},$$

$$\mathrm{diag}\left[\delta_i\right]_N \triangleq \Lambda = U^{-1}\xi U,$$

which implies that U represents the matrix of N eigenvectors and Λ is the diagonal matrix of eigenvalues. The diagonalizability of ξ guarantees the linear independence of the eigenvectors.

Appendix B. Analytical Second Moments

For $N = 1$, the general solution for the stochastic differential equation (2) for $Y(t)$ is

$$Y(t) = \mu_y + \int_0^\tau \Sigma e^{-\xi u}\,dW(u), \tag{34}$$

where

$$\mu_y = \mu + \left[Y(t') - \mu\right]e^{-\xi\tau},$$

and $\tau = t - t'$. Using this explicit expression, we obtain the following analytical conditional second moments:

$$\mathrm{Var}_{t'}\left[Y\left(t\right)\right] = \frac{\Sigma^2}{2\xi}\left(1 - e^{-2\xi\tau}\right) \tag{35}$$

$$\begin{aligned}\mathrm{Var}_{t'}\left[Y\left(t\right)^2\right] &= 2\mu_y^2\frac{\Sigma^2}{\xi}\left(1 - e^{-2\xi\tau}\right) \\ &\triangleq 4\mu_y^2\mathrm{Var}_{t'}\left[Y\left(t\right)\right]\end{aligned} \tag{36}$$

$$\begin{aligned}\mathrm{Cov}_{t'}\left[Y\left(t\right), Y\left(t\right)^2\right] &= \mu_y\frac{\Sigma^2}{\xi}\left(1 - e^{-2\xi\tau}\right) \\ &\triangleq 2\mu_y\mathrm{Var}_{t'}\left[Y\left(t\right)\right]\end{aligned} \tag{37}$$

Appendix C. Three-Factor Multi-Currency Affine Models

The domestic and nominal interest rates of the three-factor affine models in this article are given by

$$r_i\left(t\right) = \alpha_i + \beta_i^{'}Y\left(t\right), \quad \text{for } i = \{D, F\}, \tag{38}$$

where α_i is a strictly positive constant, β_i is a three-dimensional vector of constants, and $Y\left(t\right)$ is a three-dimensional vector of unobservable state variables. Under the $\mathbb{P}$-measure, we assume that $Y\left(t\right)$ follows a trivariate square-root diffusion process,

$$dY\left(t\right) = \left[\theta - \kappa Y\left(t\right)\right]dt + \Sigma\sqrt{Y\left(t\right)}dW\left(t\right), \tag{39}$$

where θ is a three-dimensional vector of nonnegative constants, κ and Σ are diagonal 3×3 matrices, and $W\left(t\right)$ is a three-dimensional vector of standard Brownian motions which are mutually independent. The market price of risks are given by

$$\Lambda_i\left(t\right) = \sqrt{Y\left(t\right)}\lambda_i, \quad \text{for } i = \{D, F\}, \tag{40}$$

where λ_i is a three-dimensional vector of constants.

As in Backus et al. (2001), the implied slope coefficient, a_2, of the linear projection of the expected changes in the exchange rates onto the forward premium of the affine model

$$a_2 = 1 + \frac{(\beta_D - \beta_F)' [\mathrm{Var}(Y)] \left(\lambda_D^2 - \lambda_F^2\right)}{2(\beta_D - \beta_F)' [\mathrm{Var}(Y)] (\beta_D - \beta_F)}. \tag{41}$$

From equation (41), we see that the forward premium anomaly, $a_2 < 0$, imposes several restrictions on the parameters of the interest rates and market price of risks across countries.

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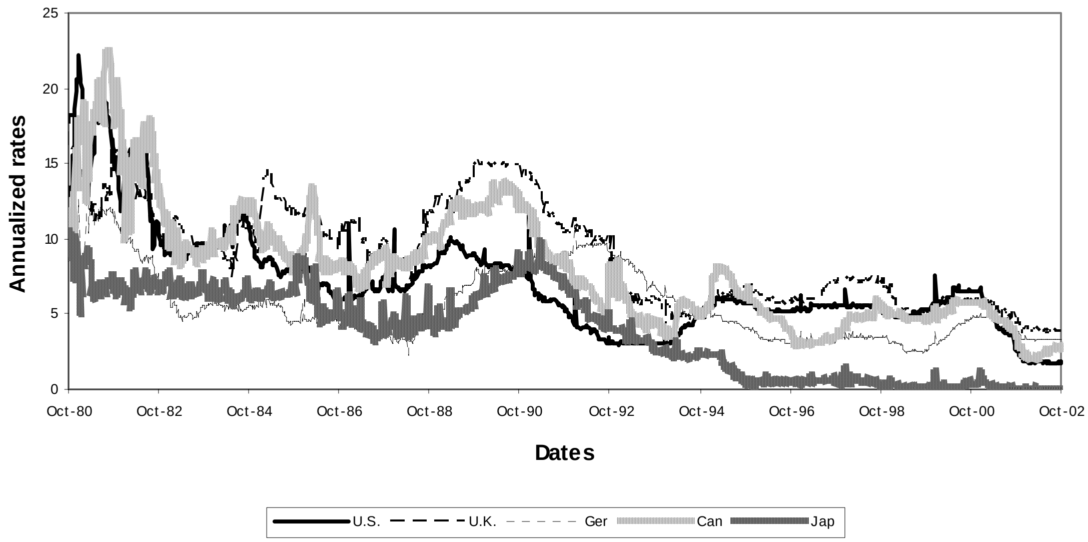

Figure 1. The 7-day Euro-currency interest rates. The sample period covers October 3, 1980 through October 4, 2002. The plots include the annualized 7-day interest rates for the Euro-dollar, the Euro-pound, the Euro-mark, the Euro-Canadian dollar, and the Euro-yen. The data are sampled at a weekly frequency.

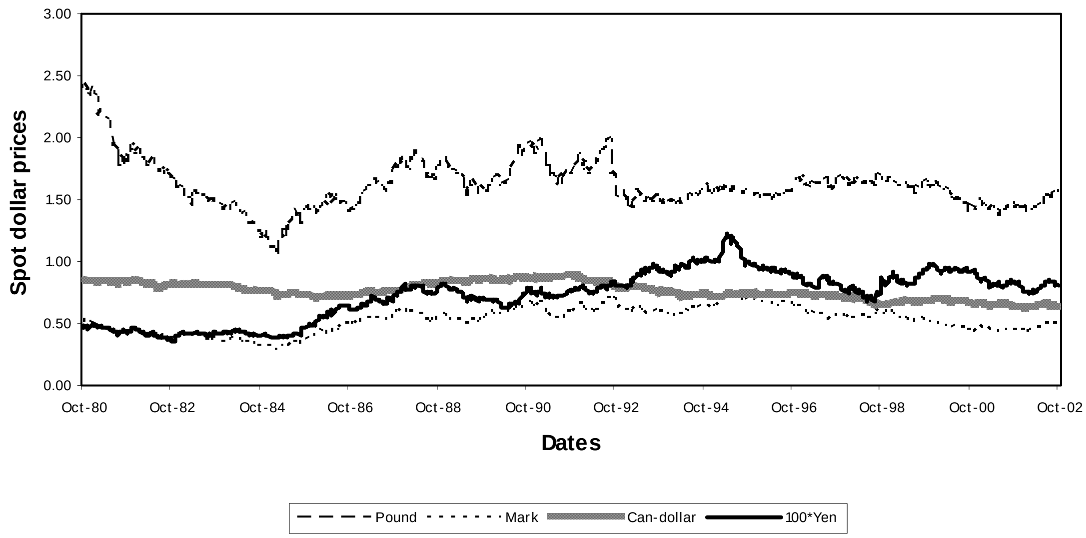

Figure 2. Time Series of Foreign Exchange Rates. This figure plots the weekly exchange rates for the British pound, the German mark, the Canadian dollar, and the Japanese Yen over the period October 3, 1980 through October 4, 2002. The exchange rates are denominated in U.S. dollars per unit of foreign currency. For expositional purposes, we multiply the exchange rates for the Japanese yen with 100.

Table I
Sample Statistics of Interest Rates and Currency Prices

The entries of Panel A are sample properties of annualized Euro-currency nominal interest rates and log returns on the dollar-denominated spot exchange rates. US, UK, Ger (Germany), Can (Canada), and Jap (Japan) represent the countries. 7d, 3m, 6m, and 12m refer to the 7-day, 3-month, 6-month, and 12-month Euro-currency annualized interest rates, respectively. The currencies are the British pound (£), the Deutsche mark (DM), the Canadian dollar (C$), and the Japanese yen (¥). We use weekly data for the interest rates and the foreign currencies. The sample period is from October 3, 1980 through October 4, 2002 (1150 observations). Panel B reports the correlations between the interest rates and the dollar-denominated spot exchange rates.

Panel A. Descriptive Statistics

Variable	Mean	Std	Max	Min	Skew	Kurt
I. Interest rates						
US-7d	7.05	3.52	22.25	1.73	1.43	5.64
US-3m	7.23	3.57	21.50	1.70	1.33	5.06
US-6m	7.34	3.54	19.63	1.69	1.21	4.53
US-12m	7.52	3.39	18.19	1.69	1.04	3.88
UK-7d	9.01	3.39	18.38	3.68	0.34	1.94
UK-3m	9.13	3.34	17.13	3.86	0.32	1.93
UK-6m	9.13	3.26	17.06	3.81	0.31	1.96
UK-12m	9.19	3.12	16.38	3.86	0.28	1.96
Ger-7d	5.66	2.39	14.00	2.25	0.86	2.83
Ger-3m	5.78	2.46	14.00	2.53	0.90	2.98
Ger-6m	5.83	2.44	13.69	2.53	0.88	2.95
Ger-12m	5.89	2.36	13.13	2.64	0.85	2.86
Can-7d	7.99	3.96	22.50	1.98	0.91	3.66
Can-3m	8.16	3.98	22.06	1.86	0.87	3.48
Can-6m	8.26	3.90	21.44	1.86	0.79	3.29
Can-12m	8.43	3.75	20.19	2.09	0.72	3.08
Jap-7d	3.78	2.86	10.50	0.031	0.022	1.58
Jap-3m	3.81	2.91	11.00	0.047	0.045	1.60
Jap-6m	3.81	2.89	10.88	0.047	0.046	1.59
Jap-12m	3.85	2.87	10.25	0.047	0.036	1.58
II. Currency Returns						
British pound	-4.55	76.31	317.73	-501.18	-0.29	6.03
German mark	-1.15	81.70	338.37	-291.77	0.14	3.79
Canadian dollar	-1.21	34.25	141.51	-232.67	-0.39	5.57
Japanese yen	2.37	84.35	640.15	-278.51	0.79	6.89

Panel B. Correlation Matrix

	US-7d	UK-7d	Ger-7d	Can-7d	Jap-7d	£	DM	C$	¥
US-7d	1.000								
UK-7d	0.653	1.000							
Ger-7d	0.471	0.623	1.000						
Can-7d	0.865	0.818	0.660	1.000					
Jap-7d	0.589	0.857	0.682	0.774	1.000				
£	0.409	0.372	0.469	0.369	0.267	1.000			
DM	-0.549	-0.335	-0.0802	-0.444	-0.406	0.336	1.000		
C$	0.409	0.719	0.740	0.615	0.781	0.499	0.0389	1.000	
¥	-0.702	-0.687	-0.379	-0.69	-0.746	-0.0177	0.808	-0.385	1.000

Table II
The Forward Premium Regressions

In this table, we present the regressions of the change in the log of the spot exchange rates on the forward premium (expressed in log form):

$$s_{t+1} - s_t = a_{1T} + a_{2T}\left(f_t - s_t\right) + \text{residual},$$

where s_t is the log of the spot price of the foreign currency at time t, f_t is the log of the one-period forward exchange rate at time t, and a_{2T} is sample slope coefficient of the linear projection. The regressor, f_t - s_t = $r_{D,t} - r_{F,t}$, is the forward premium, where $r_{D,t}$ and $r_{F,t}$ denote the continuously compounded one-week domestic and foreign yield, respectively. UK, Ger (Germany), Can (Canada), and Jap (Japan) represent the foreign countries. The numbers in parentheses are the estimated standard errors of a_{1T} and a_{2T}.

	UK	Ger	Can	Jap
a_{1T}	-0.145	0.0293	0.0487	0.204
	(0.0544)	(0.0521)	(0.0277)	(0.0758)
a_{2T}	-2.82	-1.412	-1.165	-2.489
	(0.839)	(0.790)	(0.5596)	(0.9022)
100*R^2	0.979	0.278	0.377	0.659

Table III
EMM Estimates for the $IQ_{1S}(3)$ models

Entries are EMM estimates of the parameters of the $IQ_{1S}(3)$ models described in Section 2.1.2. The parameter estimates are based on weekly Euro-currency interest rates and foreign exchange rates for the United States vis-à-vis the United Kingdom, Germany, Canada, and Japan. The *t*-ratios of these estimates are in parentheses. The sample period is from October 3, 1980 through October 4, 2002. This table also reports χ^2 statistics for the goodness-of-fit of the models, and a *z*-statistic that adjusts for degrees of freedom across the models and is distributed $N(0,1)$. In addition, in the last row, we present the model-implied values for the linear projection coefficients, a_2.

Parameter	Estimate (*t*-ratio)			
	UK	Ger	Can	Jap
α_D	0.122 (3.216)	0.121 (2.186)	0.119 (1.932)	0.125 (4.024)
α_F	0.151 (4.768)	0.0941 (1.431)	0.132 (2.554)	0.0672 (2.489)
$\psi_{F,33}$	0.254 (0.225)	0.251 (4.282)	0.409 (1.376)	0.183 (5.137)
μ_1	0.0568 (1.530)	0.0408 (3.795)	0.0593 (3.452)	0.0219 (1.078)
μ_2	0.0532 (0.987)	0.0279 (1.803)	0.0652 (2.311)	-0.0053 (-2.731)
μ_3	0.0649 (1.741)	0.0361 (2.560)	0.0399 (4.693)	0.0642 (0.506)
ξ_{11}	0.779 (1.836)	0.441 (0.621)	0.691 (1.106)	0.5175 (1.130)
ξ_{22}	0.625 (6.450)	0.357 (4.067)	1.812 (0.934)	1.0954 (9.842)
ξ_{33}	0.0618 (0.245)	0.240 (1.595)	0.596 (4.417)	0.2985 (3.006)
Σ_{11}	0.0095 (3.411)	0.0191 (2.602)	0.0016 (0.253)	0.0101 (1.754)
Σ_{22}	0.0038 (2.882)	0.0006 (1.273)	0.0001 (1.132)	0.0039 (2.308)
Σ_{33}	0.0304 (5.023)	0.0053 (3.009)	0.0143 (1.504)	0.0145 (2.642)
η_0	-2.476 (-2.917)	-2.208 (-1.856)	-1.675 (-3.629)	-2.2418 (-2.019)
η_1	0.685 (1.108)	0.894 (2.501)	0.774 (2.172)	0.463 (3.402)
χ^2	162.85	179.39	99.62	293.96
d.f.	28	28	28	22
z	18.02	20.23	9.57	35.54
a_2	-0.915 (-2.276)	-0.352 (-1.342)	-0.105 (-1.895)	-0.730 (-1.949)

Table IV
EMM Estimates for the $IQ_1(3)$ and $IA_1(3)$ models

Entries of Panel A and B are respectively EMM estimates of the parameters of the $IQ_1(3)$ and $IA_1(3)$ models described in Section 2.1.2. The columns display parameter estimates based on weekly Euro-currency interest rates and foreign exchange rates for the United States vis-à-vis the United Kingdom, Germany, Canada, and Japan. The sample period is from October 3, 1980 through October 4, 2002. The *t*-ratios of these estimates are in parentheses. This table also reports χ^2 statistics for the goodness-of-fit of the models and a *z*-statistic that adjusts for degrees of freedom across the models and is distributed $N(0,1)$. In addition, in the last row, we present the model-implied values for the linear projection coefficients, a_2.

Panel A: EMM Estimates for the $IQ_1(3)$ models

Parameter	Estimate (*t*-ratio)			
	UK	Ger	Can	Jap
α_D	0.123 (4.112)	0.120 (2.581)	0.121 (2.624)	0.124 (6.391)
α_F	0.146 (1.682)	0.0929 (5.934)	0.131 (3.502)	0.0704 (5.084)
$\psi_{F,33}$	0.670 (2.719)	0.298 (1.042)	0.852 (5.169)	0.0386 (3.421)
μ_1	0.0512 (5.303)	0.0592 (3.795)	0.0478 (2.742)	0.0692 (8.105)
μ_2	0.0586 (1.215)	0.0169 (1.357)	0.0614 (3.178)	0.0027 (1.147)
μ_3	0.0376 (3.041)	0.0227 (6.089)	0.0301 (1.005)	0.0243 (2.465)
ξ_{11}	1.095 (5.232)	0.714 (2.141)	0.462 (4.603)	0.639 (4.023)
ξ_{22}	0.683 (9.190)	1.073 (3.295)	0.995 (2.316)	1.079 (6.215)
ξ_{33}	0.713 (1.245)	1.1618 (2.007)	0.622 (1.728)	0.566 (2.029)
Σ_{11}	0.0125 (2.101)	0.0096 (4.108)	0.0028 (0.890)	0.0023 (7.426)
Σ_{22}	0.0089 (8.203)	0.0057 (1.679)	0.0019 (3.649)	0.0008 (3.182)
Σ_{33}	0.0162 (4.041)	0.0009 (0.423)	0.0087 (9.248)	0.0025 (3.012)
η_0	-1.201 (-4.182)	-1.047 (-1.856)	-0.359 (-0.907)	-2.110 (-5.975)
$\eta_{0D,3}$	0.273 (1.028)	0.264 (1.332)	0.196 (2.201)	0.248 (1.955)
η_1	0.0522 (3.001)	0.219 (2.501)	0.0026 (2.861)	0.394 (2.568)
$\eta_{1D,3}$	0.0452 (6.709)	0.127 (2.786)	0.0683 (1.933)	0.0294 (4.329)
χ^2	147.58	166.52	91.16	253.47
d.f.	26	26	26	20
z	15.98	18.51	8.44	30.13
a_2	-1.721 (-1.315)	-0.683 (-0.923)	-2.079 (1.633)	-2.731 (0.268)

Panel B: EMM Estimates for the $IA_1(3)$ models

Parameter	Estimate (*t*-ratio)			
	UK	Ger	Can	Jap
$\beta_{F,3}$	0.452 (3.728)	0.439 (2.451)	0.566 (4.073)	0.443 (2.369)
κ_{11}	0.902 (2.159)	0.934 (1.832)	0.942 (2155)	0.988 (3.148)
κ_{22}	1.062 (2.847)	1.108 (3.729)	1.096 (3.242)	1.075 (5.297)
κ_{33}	1.193 (4.568)	1.174 (2.510)	1.136 (2.968)	1.117 (2.004)
θ_1	0.0761 (2.640)	0.0749 (2.907)	0.0652 (1.893)	0.0776 (2.254)
θ_2	0.0619 (1.802)	0.0576 (3.632)	0.0750 (2.475)	0.0546 (1.986)
θ_3	0.0308 (3.127)	0.0234 (2.450)	0.0319 (4.132)	0.0382 (3.049)
Σ_{11}	0.0098 (4.564)	0.0062 (1.521)	0.0213 (0.719)	0.0063 (2.153)
Σ_{22}	0.0035 (0.993)	0.0024 (2.172)	0.0107 (2.065)	0.0019 (1.442)
Σ_{33}	0.0125 (2.811)	0.0021 (3.098)	0.0067 (3.488)	0.0086 (0.480)
λ	0.0574 (2.579)	0.0538 (4.812)	0.0496 (1.570)	0.0542 (2.606)
λ_3	-4.260 (3.604)	-2.621 (2.257)	-2.485 (3.492)	-3.116 (2.843)
χ^2	206.76	262.69	134.80	319.63
d.f.	30	30	30	24
z	22.82	30.04	13.53	42.67
a_2	0.265 (-3.683)	0.983 (-3.031)	1.006 (-3.879)	0.661 (-3.492)

Table V
EMM Estimates for the $IQ_{0U}(3)$ and IA_{0U} (3) models

Entries are EMM estimates of the parameters of the $IQ_{0U}(3)$ and IA_{0U} (3) models described in Section 2.1.2. The columns display parameter estimates based on weekly Euro-currency interest rates and foreign exchange rates for the United States vis-à-vis the United Kingdom, Germany, Canada, and Japan. The sample period is from October 3, 1980 through October 4, 2002. The *t*-ratios of these estimates are in parentheses. This table also reports χ^2 statistics for the goodness-of-fit of the models and a *z*-statistic that adjusts for degrees of freedom across the models and is distributed $N(0,1)$. In addition, in the last row, we present the model-implied values for the linear projection coefficients, a_2.

Panel A: EMM Estimates for the $IQ_{0U}(3)$ models

Parameter	Estimate (*t*-ratio)			
	UK	Ger	Can	Jap
α_D	0.125 (4.135)	0.123 (3.278)	0.127 (2.182)	0.122 (3.307)
α_F	0.144 (2.426)	0.0906 (2.522)	0.139 (3.646)	0.0743 (2.663)
$\psi_{D,22}$	1.067 (3.551)	0.745 (1.335)	0.9801 (1.728)	1.192 (1.960)
$\psi_{F,11}$	0.196 (5.137)	0.205 (4.711)	0.483 (2.146)	0.074 (3.386)
$\psi_{F,33}$	0.147 (2.314)	0.089 (3.173)	0.206 (1.948)	0.115 (4.767)
μ_1	0.0605 (3.629)	0.0573 (5.884)	0.0827 (2.577)	0.0601 (2.523)
μ_2	0.0237 (1.992)	0.008 (2.712)	0.0131 (4.519)	0.0064 (3.913)
μ_3	0.0307 (5.832)	0.0285 (3.829)	0.0412 (3.784)	0.0296 (2.615)
ξ_{11}	0.918 (2.504)	0.626 (1.713)	0.509 (2.295)	0.447 (1.823)
ξ_{22}	2.154 (4.671)	0.073 (4.917)	1.406 (5.826)	0.095 (6.471)
ξ_{33}	0.804 (2.930)	0.183 (2.595)	0.691 (3.808)	0.602 (3.192)
Σ_{11}	0.0083 (4.165)	0.004 (3.426)	0.0117 (2.519)	0.0051 (2.209)
Σ_{22}	0.0137 (6.003)	0.0046 (2.307)	0.0152 (3.402)	0.0037 (3.836)
Σ_{33}	0.0007 (3.114)	0.0001 (3.192)	0.0014 (2.691)	0.0002 (4.197)
$\eta_{0D,1}$	0.409 (1.962)	0.812 (1.781)	1.025 (3.238)	1.318 (2.350)
$\eta_{0D,3}$	1.025 (2.417)	0.691 (3.425)	0.832 (4.436)	0.455 (3.794)
$\eta_{0F,2}$	1.712 (3.651)	0.805 (1.612)	-1.499 (-5.937)	1.573 (1.012)
$\eta_{1D,1}$	1.319 (2.906)	0.901 (4.651)	0.794 (2.906)	0.356 (5.007)
$\eta_{1D,3}$	0.839 (4.771)	1.176 (2.379)	1.083 (3.490)	1.772 (2.843)
$\eta_{1F,2}$	0.924 (1.982)	1.225 (5.161)	1.053 (2.573)	0.638 (4.752)
χ^2	90.12	141.00	56.89	180.50
d.f.	22	22	22	16
z	10.27	17.94	5.26	29.08
a_2	-3.734 (1.084)	-1.702 (0.367)	-1.597 (0.773)	-1.879 (-0.676)

Panel B: EMM Estimates for the IA_{0U} (3) models

Parameter	Estimate (t-ratio)			
	UK	Ger	Can	Jap
$\beta_{D,2}$	0.341 (1.128)	0.275 (3.780)	0.368 (2.627)	0.409 (2.027)
$\beta_{F,1}$	0.0936 (3.402)	0.0481 (2.165)	0.245 (2.081)	0.0763 (0.481)
$\beta_{F,3}$	0.0715 (2.316)	0.142 (2.823)	0.268 (0.982)	0.169 (1.192)
κ_{11}	0.9534 (4.284)	1.0521 (1.298)	0.9559 (3.203)	0.9037 (2.673)
κ_{22}	1.0485 (2.705)	0.95312 (3.904)	1.0511 (1.836)	0.728 (3.506)
κ_{33}	0.8957 (1.693)	1.0948 (2.082)	0.9624 (2.195)	0.587 (2.251)
θ_1	0.0739 (3.554)	0.06274 (4.727)	0.0741 (6.772)	0.0582 (4.819)
θ_2	0.1812 (2.878)	0.0861 (3.156)	0.1581 (3.518)	0.0264 (3.053)
θ_3	0.0463 (2.033)	0.0528 (2.560)	0.0487 (5.420)	0.0313 (6.924)
Σ_{11}	0.0061 (0.416)	0.0150 (2.411)	0.0052 (2.089)	0.0133 (2.468)
Σ_{22}	0.0012 (2.657)	0.0008 (4.308)	0.0068 (3.852)	0.0006 (1.607)
Σ_{33}	0.0029 (3.906)	0.0105 (0.785)	0.0043 (1.331)	0.0116 (2.395)
λ_1	-3.106 (-2.282)	-1.492 (-2.294)	-1.987 (-4.145)	-2.628 (-0.926)
λ_2	-3.388 (-4.108)	-1.291 (-2.813)	-0.919 (-2.679)	-3.058 (-4.250)
λ_3	-3.992 (-2.352)	-3.107 (-3.029)	-2.135 (-3.208)	-2.623 (-2.874)
χ^2	151.78	214.46	87.40	286.13
d.f.	27	27	27	21
z	16.98	25.51	8.22	40.91
a_2	-0.809 (-2.402)	-0.050 (-1.724)	-0.0346 (-2.021)	-0.454 (-2.256)

Table VI
EMM Estimates for the $IQ_0(3)$-JD models

Entries are EMM estimates of the parameters of the $IQ_0(3)$-JD models described in Section 2.1.2. The columns display parameter estimates based on weekly Euro-currency interest rates and foreign exchange rates for the United States vis-à-vis the United Kingdom, Germany, Canada, and Japan. The sample period is from October 3, 1980 through October 4, 2002. The *t*-ratios of these estimates are in parentheses. This table also reports χ^2 statistics for the goodness-of-fit of the models and a *z*-statistic that adjusts for degrees of freedom across the models and is distributed $N(0,1)$. The implied kurtosis is the kurtosis induced by the jump component of the model. In addition, in the last row, we present the model-implied values for the linear projection coefficients, a_2.

Parameter	Estimate (*t*-ratio)			
	UK	Ger	Can	Jap
α_D	0.123 (3.216)	0.120 (2.186)	0.129 (1.932)	0.124 (4.024)
α_F	0.145 (4.768)	0.0939 (1.431)	0.132 (2.554)	0.0764 (2.489)
$\psi_{D,22}$	1.092 (2.354)	1.126 (3.515)	0.973 (1.826)	1.215 (5.603)
$\psi_{F,11}$	0.201 (6.106)	0.166 (4.924)	0.398 (2.712)	0.012 (3.270)
$\psi_{F,33}$	0.316 (1.984)	0.225 (5.808)	0.213 (3.694)	0.102 (2.753)
$(\Sigma\Sigma')_{11}$	0.846* (5.117)	0.162* (3.865)	1.368* (2.520)	0.261* (3.554)
$(\Sigma\Sigma')_{21}$	-1.231* (-4.236)	-0.0062* (-6.091)	0.624* (3.756)	-0.0157* (-4.380)
$(\Sigma\Sigma')_{31}$	0.0217* (3.147)	0.0002* (2.834)	0.107* (4.678)	0.0102* (5.235)
$(\Sigma\Sigma')_{22}$	5.198* (2.065)	0.212* (4.199)	2.314* (2.034)	0.137* (3.516)
$(\Sigma\Sigma')_{32}$	-0.0412* (-2.887)	-0.0032* (-3.623)	0.123* (2.642)	-0.0032* (-4.741)
$(\Sigma\Sigma')_{33}$	0.0361* (4.601)	0.0001* (2.719)	0.0196* (3.273)	0.0004* (2.235)
$\eta_{0D,1}$	1.072 (7.250)	1.081 (4.894)	1.168 (2.405)	1.029 (5.176)
$\eta_{0D,3}$	0.942 (5.876)	0.901 (2.163)	0.864 (2.618)	0.987 (3.301)
$\eta_{0F,2}$	1.005 (2.445)	0.981 (3.571)	1.012 (5.013)	0.97 (2.632)
$\eta_{1D,1}$	0.862 (3.602)	0.772 (6.523)	0.693 (3.995)	0.276 (4.284)
$\eta_{1D,3}$	0.785 (4.247)	0.804 (3.002)	0.692 (2.631)	0.841 (2.852)
$\eta_{1F,2}$	0.875 (1.982)	0.016 (4.173)	0.973 (2.264)	0.012 (3.345)
$\sigma_{c,D}$	0.683 (9.311)	0.592 (6.384)	1.102 (6.147)	0.864 (9.205)
σ_e	0.426 (8.012)	0.575 (6.214)	0.0224 (2.495)	0.529 (4.912)
λ	0.342 (2.629)	0.549 (2.927)	0.961 (3.162)	0.758 (3.087)
ϕ	-0.0714 (-0.869)	0.0918 (1.095)	-0.0306 (-1.747)	0.265 (3.824)
δ	0.881 (4.750)	0.690 (3.008)	0.265 (8.250)	0.734 (8.350)
χ^2	20.40	22.25	19.13	20.72
d.f.	19	19	19	13
z	1.21	1.56	0.97	3.18
Implied kurtosis	5.447	3.946	4.529	4.607
a_2	-2.087 (-0.877)	-1.981 (0.721)	-1.630 (0.830)	-3.072 (0.646)

$*\times 10^{-4}$

Table VII
Currency Premium and Variance Decomposition

This table reports the values of the model-implied currency premium, the variance of the exchange rates, and their components. θ_s, θ_c, and θ_T are respectively the interest-rate, pure currency, and total risk premium. σ_s, σ_c, σ_e, and σ_j respectively represent the currency volatility associated with the interest-rate factors, the volatility of the pure currency component in the pricing kernels, the excess volatility, and the jump volatility. σ_T is the conditional volatility of the currency returns.

	UK	Ger	Can	Jap
θ_s	-0.0128	-0.0092	-0.0082	-0.0026
θ_c	-0.0133	-0.0152	-0.0057	-0.0062
θ_T	-0.0261	-0.0244	-0.0139	-0.0088
σ_s^2/σ_T^2	0.0159	0.0253	0.292	0.0029
σ_c^2/σ_T^2	0.0281	0.0327	0.0218	0.0041
σ_e^2/σ_T^2	0.334	0.516	0.0052	0.408
σ_j^2/σ_T^2	0.622	0.426	0.681	0.585
σ_T^2	0.542	0.642	0.105	0.690