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Hesham M. Abdel-Rahman  
*University of New Orleans*

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## **Skill Distribution and Income Disparity in a North-South Trade Model**

Hesham M. Abdel-Rahman\*  
University of New Orleans  
Department of Economics and Finance  
Lakefront  
New Orleans, LA 70148  
FAX (504) 280-6397  
E-mail habdelra@uno.edu  
www.uno.edu/~habdelra

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### **Abstract**

What are the impacts of free trade agreement on the social welfare of different groups of labor force in a developed country? What is the impact of free trade on a developed country's income disparity? What is the effect of free trade on the skill distribution of a developed country? The objective of this paper is to address the above questions in a simple two-sector general-equilibrium North-South trade model in which both countries produce one final good and one high-tech intermediate input. Horizontally differentiated skilled workers produce the high-tech intermediate input. The final good is produced with the use of a high-tech intermediate input and unskilled labor. Each country is populated by a continuum of unskilled workers with differential potential ability. Workers in the North and South can acquire skill by investment in training or education. Thus, skill distribution in the North and South is determined endogenously in the model through a self-selection. I characterize two different types of equilibrium: a closed-economy equilibrium without trade and free trade equilibrium. Then, I investigate the impact of free trade, in the presence of training costs, on the skill distribution within each country, income disparity, and social welfare.

**Key words:** Training, Potential ability

**JEL classification:** D63; F10; J31

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## 1. Introduction

Several studies have indicated that income disparity in the U.S. as well as other developed nations has been rising during the past two decades.<sup>1</sup> This materialized in a dramatic decrease in the real wage of low skilled labor as well as an increase in the wage of highly skilled labor. Specifically, Juhn, Murphy, and Topel (1991) found for the U.S. that the 1990's wage of the lowest 20% of the American workforce had fallen by 25% below the 1973 real wage. Also Juhn, Murphy and Pierce (1993) found that between 1963 and 1989, a real average weekly wage for the least skilled workers decreased by 5%, where the same for skilled workers increased by 40%.<sup>2</sup> Furthermore, Machin (1996) showed that income inequality has been rising in the UK within group as well as between group during 1978-92. He suggested that within-group inequality in hourly income increased by 23% over the same period.<sup>3</sup> This rise in inequality has been the result of a relative wage growth in the top of the wage distribution and a fall in the bottom of the wage distribution. On the other hand, Cawley, Heckman, and Vytiacil (1998) showed that potential ability is correlated with the wage. This was shown especially in the case of white-collar workers. Furthermore, they showed that a significant portion of the rise in within-group inequality is unexplained by age and education. Hoxby and Terry (1999) decomposed the growing dispersion into three sources. The first is increasing diversity of people attending college. The second is an increasing return to aptitude. The third is the increasing correlation between the average aptitude of a college's student and its expenditure on education.

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<sup>1</sup> For general trends supporting this see Murphy and Welch (1993).

<sup>2</sup> See Richardson (1995) for a survey of the impact of trade on U.S. wage inequalities.

<sup>3</sup> Similar trends have been observed in Canada see Beaudry and Green (1999).

The growing number of free trade agreement, trade liberalization and increased globalization together with above trends in income disparities the raises some important questions concerning the their impacts on each country's labor market. A lively debate on whether free trade between developed and less-developed countries has detrimental effect on the welfare of the unskilled workers in a developed nation. Leamer (1993) and Wood (1995) suggested that the decreasing of the welfare of the unskilled workers in the developed nation is caused preliminary by the expansion of trade with the LDC. This is because of the link between the developed country's labor market and those of the LDC's, which resulted in a decline of the wage of the North's unskilled labor and a rise in the wage of the South's unskilled workers. Thus, leading to a change in the distribution of skill in both markets as well as income disparities. On the other hand, Lawrence and Slaughter (1993) among others, argue that trade is not a major factor in determining income disparity. In spite of this disagreement on the impact of free trade on income disparity there is a need for an approach that can analyze this issue in a general equilibrium model of a North-South trade in which the distribution of skill is endogenous. Furthermore, there is a need for a model that explains the opposition of free trade agreement by some groups in developed countries.<sup>4</sup>

One approach to analyze the impact of free trade on income disparities is within the framework of a two-sector Heckscher-Ohlin type model. In this case the developed country would have comparative advantage in the production of the skill-intensive good. Thus, the Stolper-Samuelson theorem would apply resulting in a reduction in the relative price of the less skill-intensive good and as a result of trade decreasing the return to the less skilled labor and the

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<sup>4</sup> Fisher and Serra (1996) presented a model that analyzes the opposition of developed country to free trade but with exogenous skill distribution.

rise in the skilled wage in the developed country.<sup>5</sup> However, in this framework, nothing can be said about within the skill group inequality and the impact of free trade on the distribution of skill in the economy. Abdel-Rahman, Norman, and Wang (2002) introduced a model that can explain the impacts of free trade on income disparities within-skill group and between skilled and unskilled labor. The model utilizes a matching between horizontally differentiated skilled worker in the North and the firm producing a high-tech good. The wage of the skilled labor is determined by a symmetric Nash bargain, while the wage of the unskilled labor is determined competitively. However, in that model, labor supply of both skilled and unskilled labor is fixed. Thus, the model does not take into consideration the impact of free trade on the distribution of skill in each country. Furthermore, it was assumed that the less developed county is only populated with unskilled labor. Thus, the high-tech good is produced only in the developed country, which is not a realistic assumption. As a result of that, the model predicted that free trade would result in a decrease in income disparities within the skilled labor force in the developed country, which is not consistent with most empirical finding.<sup>6</sup> Furthermore, no income disparity exists in the less developed country, since it is only populated with unskilled labor. Another related work is by Fisher and Serra (1996) where they presented a model that justifies the opposition of developed country to free trade by adopting the a majority voting approach in a model in which each agent is endowed with skill and unskilled labor and agent consume a differentiated product. However in their model skill distribution is given exogenously and therefore the impact of free trade on skill distribution cannot be analyzed. Furthermore due

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<sup>5</sup> Matusz (1996) analyzed the impact of free trade on a monopolistically competitive good market and in an efficiency wage labor market model. On the other hand, Abdel-Rahman, Norman, and Wang (2001) analyzed the impact of a free trade on a two-sector model where the labor market is characterized by a horizontal matching of skilled workers in the North using a symmetric Nash bargain model. However, in the Abdel-Rahman, Norman, and Wang model, the skill distribution is given exogenously, and thus the impact of free trade on the distribution of skill is ignored.

<sup>6</sup> See Jones and Neary for a survey (1984).

to the exogenous skill distribution income disparity is proportional to the initial skill distribution before and after trade.

Thus, there is a need for a model that can explain the impact of free trade on skill distribution, income disparities within-skill group as well as between skill and unskilled groups, and welfare of different group of labor in the developed as well as in the less developed country. This paper will extend previous work in four directions: first by introducing training and horizontal differentiation in potential ability into the model and thus making the choice of acquiring skill endogenous through a self-selection process, second, by making both the skilled labor market as well as the unskilled labor markets competitive, and third, by examining the impact of free trade on both the developed and the less developed country's labor markets, forth, by examining the impact of free trade agreement on varies groups of labor force in the developed country.

The objective of this paper is to address some of the issues related to a free trade agreement: skill distribution, income disparities, and welfare. More precisely, the paper addresses the following questions: What is the impact of free trade on the wage inequality of each country? What are the impact of free trade on the welfare of different groups of labor force in developed and developing countries? This paper addresses the above questions in a simple two-sector North-South trade general equilibrium model. The model investigates the impact of a free trade agreement on the skill distribution of a competitive labor market, in the presence of training costs. In this model the North and the South produce one final manufacturing good and one high-tech intermediate input. The final good is produced with the use of a high-tech intermediate input and unskilled labor, while horizontally differentiated skilled workers produce

the high-tech intermediate input.<sup>7</sup> Each country is populated by a continuum of unskilled workers with differential potential ability. Workers in the North and South can acquire skill by investment in training or education. This potential ability translates into productivity in the production of the high-tech intermediate input if workers acquire training. On the other hand, if workers do not acquire training, they have constant productivity regardless of their potential ability. Thus, potential ability causes labor to differ in productivity only when they acquire education and produce the high-tech intermediate input. In the context of the model, skill distribution in the North and South is determined endogenously in the model through a self-selection. Furthermore, through the determination of wages in competitive markets, income disparity is determined endogenously. The model characterizes equilibrium without trade and free trade equilibrium. The model generates income disparity within the skilled worker as well as between skilled and unskilled unlike versions of Heckscher-Ohlin model. Then, I investigated the impact of free trade, in the presence of training costs, on the skill distribution within each country, on income disparity and social welfare.

The main findings of the paper are as follows: First I show that free trade would lead to complete specialization in the South but may not lead to complete specialization in the North. This is unlike the outcome from Heckscher-Ohlin type model, where we have complete specialization. The reason for that is that if the North completely specializes in the production of the high-tech intermediate input workers with their lowest potential ability would end up with negative income after paying for training cost. But if the South completely specialized in the production of the final good all workers will have the same positive productivity. Second, the wage rate will be equalized between the North and the South in equilibrium for the unskilled workers but not for the skilled workers. This will result in a lower wage for the unskilled in the

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<sup>7</sup>For a review of models of trade based on differences in technology see Jones and Neary (1984).

North and higher unskilled wage in the South. As a result of that, I showed that free trade result in higher income disparity within the skilled group as well as between skilled and unskilled in the North. The opposite will happen in the South. Third, I showed that free trade would result in hire diversity within the skilled workers in the North, which is new to the literature.<sup>8</sup> This is because free trade would induce worker in the North to acquire training. Forth, I showed that free trade would increase the mass of the skilled worker in the North and would decrease it in the South. Finally, I sowed that free trade would decrease the welfare of the unskilled workers and will increase the welfare of the workers who are skilled before and after trade, as would be expected from H-O type model. While the welfare change of the workers who are unskilled before trade and acquire skill after is ambiguous, which is new to the literature.

The rest of the paper is organized as follows: Section 2 presents the assumptions and the specifications of the North-South trade model. Section 3 develops the equilibrium under autarky and establishes the existence of the equilibrium. Section 4 investigates the determinants of income disparities within group as well as between groups and its impact on social welfare. Section 5, characterizes a free trade equilibrium. Section 6 analyses the impact of free trade on the welfare of different types of workers. Finally, Section 7 contains the conclusions.

## **2. The Model**

We consider a world with two countries: the highly developed North and a less developed South, indexed by  $i = N, S$  respectively. Each country is populated with a fixed number of consumers/workers (to be described below) where each is endowed with one unit of labor. Furthermore, all consumers are identical in every aspect except for their potential ability and each has a utility as a function of the only consumption good  $X$  in the economy. Both countries

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<sup>8</sup> This trend has been documented by Hoxby and Terry (1999).



produce homogeneous final good,  $X$ , according to a constant-returns-to-scale Cobb Douglas production function given by

$$X_i = L_i^\beta Z_i^{1-\beta} \quad (1)$$

where  $L_i$  is the quantity of unskilled workers and  $Z_i$  is the quantity of a high-tech intermediate input, and  $\beta \in (0,1)$ . The behavior of a given firm in this industry is characterized by the following:

$$\left\{ \max_{L_i, Z_i} \pi_i = P_i X_i - W_i L_i - Z_i \mid (1) \right\}$$

where the price of  $Z$  is normalized to be one. The first order conditions of the above problem are given by

$$\beta P_i X_i = W_i L_i \quad (2)$$

$$(1 - \beta) P_i X_i = Z_i \quad (3)$$

From (2) and (3) we have

$$\frac{\beta}{(1 - \beta)} Z_i = W_i L_i \quad (4)$$

It is assumed that the total population of a given country  $i$  is given by  $M_i$ . Workers in both countries are identical in terms of productivity and each worker is endowed with one unit of labor. However, workers are heterogeneous in their potential ability if employed in the high-tech intermediate input industry,  $Z$ . This potential ability can be realized if labor acquires training or education. The productivity of a worker with potential ability  $\delta \in [0,1]$ , in country  $i \in \{N, S\}$  if he acquires training, in terms of good  $Z$  is given by

$$Z_i(\delta) = \alpha_i \delta \quad (5)$$

where  $\alpha_i$  is positive constant. Thus, (5) indicates that the high-tech intermediate input can be produced with a continuum of constant returns to scale technology. Furthermore, without loss of

generality it is assumed that  $\alpha_N > \alpha_S$ . Thus, each worker with potential ability  $\delta$  in the North is more productive in the high-tech intermediate input than the South. This can be further interpreted with the Ricardian view that trade will be a result of difference in technology between countries.<sup>9</sup> With this view, the North is having access to more productive technology than the South in the high-tech intermediate input. Workers of type  $\delta$  are uniformly distributed on a unit interval such that  $M_i(\delta) = M_i \forall \delta \in [0,1]$ . Figure 1 provides a graphical description for the potential ability profile. The potential ability of worker type  $\delta = 0$  is zero while the potential ability of worker type  $\delta = 1$  is  $\alpha$  as indicated by Figure 1. The mass of workers can be written as,  $M_i = \int_0^1 M_i(\delta) d\delta$ , where  $M_i(\delta) = M_i \forall \delta \in [0,1]$ . Labor can acquire training in a given country by investing  $t_i$  in terms of the intermediate input  $Z$ .<sup>10</sup> It can be seen from (5) that the worker with the highest productivity,  $\alpha_i$ , (after acquiring training) is the worker with the highest potential ability  $\delta = 1$ . This can be interpreted as if firms in industry  $Z$  have job requirements that perfectly match the potential ability of workers of type  $\delta = 1$ . On the other hand, a worker with potential ability  $\delta = 0$  has the poorest match with the skill requirements by industry  $Z$ . Thus, it is assumed that the worker with potential ability  $\delta = 0$  cannot produce the intermediate input even if he acquires training. Given the above specification, the total output of good  $Z_i$  is given as

$$Z_i = M_i \int_{\delta_{ui}}^1 \alpha_i \delta d\delta \quad (6)$$

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<sup>9</sup> The result of the model would not be affected if training were a function of the potential ability. However, a close form solution will not be possible (see Appendix A.2, for a simple training cost function).

<sup>10</sup> Another way of interpreting the potential ability equation and the training cost is given in Appendix A.3.

Where  $\delta_{Ui}$  is the measure of untrained labor, as depicted in Figure 1, to be determined below.

The marginal worker who will be indifferent to acquiring skill determines this measure of untrained labor or not. Thus, the marginal worker,  $\delta_{Ui}$ , is the one for which the wage offered to him by industry Z is equal to his opportunity cost,  $W_i^* + t_i$ , where  $W_i^*$  is the equilibrium wage of the untrained worker, to be determined below. Thus, the marginal worker can be determined as

$$\delta_{Ui} = [W_i^* + t_i] \alpha_i^{-1} \quad (7)$$

Consequently, the measure of trained labor in the intermediate good industry  $Z_i$  is  $(1 - \delta_{Ui})$ .<sup>11</sup> It is assumed that wages will be determined competitively for skilled and unskilled workers. Furthermore, without loss of generality it is assumed that  $t_S > t_N$ . This can be viewed also as a difference in the training technology. Thus, the potential ability in the North is higher than in the South while the training cost is lower.

### 3. Autarky Equilibrium

Under autarky, each country has to produce the high-tech intermediate good as well as the final good. Thus, given  $\{P_i^*, W_i^*, W(\delta)_i^*, U_i^*(\delta), \delta_{iU}^* ; > 0 \forall \delta \in (0,1), i \in \{N, S\}\}$  we say that an autarky equilibrium defined by  $\{P_i^*, W_i^*, W(\delta)_i^*, U_i^*(\delta), \delta_{iU}^* ; \delta \in (0,1), i \in \{N, S\}\}$  would result in the following conditions: profit maximization for firms in the final good and the intermediate good sectors; firms making zero profit in both sectors; equality of supply and demand in the labor markets and the intermediate good market; and the marginal worker will be indifferent to acquiring training or not. Now let us suppose that equilibrium exists. Then the aggregate supply of unskilled labor in country  $i \in \{N, S\}$  is given by

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<sup>11</sup> See the appendix for the proof.

$$L_{ix} = M_i \delta_{iU} = [M_i (W_i + t_i)] \alpha_i^{-1} \quad (8)$$

In addition, the aggregate supply of intermediate input in country  $i \in \{N, S\}$  is given as

$$Z_i = M_i \left[ \int_{\delta_{iU}}^1 \alpha_i \delta d\delta - t_i (1 - \delta_{iU}) \right] \quad (9)$$

where the first term between parentheses on the RHS of (9) is the total output and the second term on the RHS is the training cost. Integrating the above equation we have

$$Z_i = \frac{\alpha_i}{2} (1 - \delta_{iU}^2) - t_i (1 - \delta_{iU})$$

Substituting (7) into the above equation, we have

$$Z_i = \frac{M_i}{2\alpha_i} [(\alpha_i - t_i)^2 - W_i^2] \quad (10)$$

Equating the aggregate supply and demand by substituting (8) and (10) into (4) we have

$$W_i t_i = \frac{\beta}{2(1-\beta)} \left\{ (\alpha_i - t_i)^2 - \left( \frac{2-\beta}{\beta} \right) W_i^2 \right\} \quad (11)$$

The above quadratic equation has two roots; the relevant one (i.e., positive root) determines the equilibrium wage as<sup>12</sup>

$$W_i^* = \frac{-t_i(1-\beta) + \sqrt{t_i^2(1-\beta)^2 + (t_i - \alpha_i)^2 \beta(2-\beta)}}{(2-\beta)} \quad (12)$$

From the above equation, it can be seen that the  $W^*$  is positive as long as  $\alpha_i \neq t_i$ . Thus, we impose the following regularity condition

**Assumption 1.**  $\alpha_i > t_i$   $i \in \{S, N\}$

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<sup>12</sup> Katz and Autor (200) indicated that within-group inequality accounts for substantial fraction of overall earned income inequality.

The above assumption implies that, in each country, there exists at least some type of worker that will have positive wage in industry Z net of training cost. Note that if the above assumption is not satisfied in a given country,  $i$ , no worker will acquire training. Straightforward differentiation of the above equation would lead to the following result:

**Result 1** *Given Assumption 1, the equilibrium wage for the unskilled workers,  $W_i^*$ , is increasing in  $\alpha$  and  $\beta$  and decreasing in  $t$ .*

The intuition behind the above result is that an increase in  $\alpha$  would increase the supply of trained workers. This is because it will increase the incentive for workers to acquire training and consequently will decrease the supply of untrained workers. This would increase the wage of unskilled labor. On the other hand, an increase in  $t$  would lower the incentive to acquire training, which would increase the supply of unskilled labor. This would decrease the wage rate of unskilled workers. Finally, an increase in  $\beta$  would increase the demand for unskilled workers,  $L$ , and thus would increase the wage of the unskilled labor. Recall the assumption that  $\alpha_N > \alpha_S$  and  $t_S > t_N$ , we can conclude that the unskilled worker's equilibrium wage in the North will be higher than in the South. From the above specification of the model, we now obtain the following:

**Theorem 1.** *Given Assumption 1, there exist unique that result in autarky equilibrium  $\{P_i^*, W_i^*, W(\delta)_i^*, U_i^*(\delta), \delta_{iU}^*; \delta \in (0,1), i \in \{N, S\}\}$ , such that  $P_i^*, W_i^*, W(\delta)_i^*, U_i^*(\delta) > 0 \forall \delta_i \in (0,1), i \in \{N, S\}$ .*

**Proof.** First, I show that for equilibrium to exist we must have  $\alpha_i - t_i > W_i^*$ . This follows from the specification of the final good production function. It can be seen from Figure 1 that if  $\alpha_i - t_i \leq W_i^*$ , then no worker will acquire training. Thus, the high-tech intermediate input will not be produced and the final good production will not be feasible, as can be seen from (1).

Hence, utility will be zero, which contradicts the initial primes.<sup>13</sup> Thus, for autarky equilibrium to exist we must have  $\alpha_i - t_i > W_i^*$ . Second, by substituting (12) into this condition and rearranging terms, we have the above condition satisfied as long as  $\alpha_i > t_i$   $\square$

Observe that the equilibrium skill distribution in this model is socially optimal. This is obvious since; market failure doesn't exist in the specification above.<sup>14</sup> Now we can characterize the equilibrium parameter space under autarky as in Figure 2. The area above the line  $\alpha(t) = t$  is the parameter set where equilibrium will exist, given Assumption 1. Observe that for a given point like a in Figure 2, where the values of  $t_N$  and  $\beta_N$  would result in equilibrium in the North, we have an area, given by the triangle abc in figure 2, of parameter values in which equilibrium will exist in the South.

Next we examine the impact of the parameters on the equilibrium amount of untrained workers. Totally differentiating 8 with respect to  $\beta$ ,  $t$ , and  $\alpha$ , with the use of Result 1, would lead to the following:

**Result 3.** *The equilibrium amount of unskilled labor, under autarky*

(i) *is increasing in  $\beta$  and  $t$*

(ii) *is decreasing in  $\alpha$ .*

(iii) *is independent of the total population of the economy.*

The intuition behind this result is that higher  $\beta$  would result in higher demand for the unskilled labor and thus higher wage for the unskilled workers, which would result in a larger amount of untrained labor. Both  $\beta$  and  $t$  affect  $\delta_{U_i}$  directly and indirectly through  $W^*$  as can be seen from (7). However, the direct effect dominates the indirect effect in both cases. Next, a higher cost of

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<sup>13</sup> Note that equilibrium exists even though utility is zero, but we only restrict ourselves to positive equilibrium.

<sup>14</sup> See the Appendix, A.4, for the proof of this result.

training,  $t$ , would decrease the incentive for labor to train and thus increase the mass of untrained workers. On the other hand, higher  $\alpha$  leads to a higher wage for the trained labor but also a higher demand for unskilled labor and higher wages for the untrained labor. Thus, from Result 3 and given the assumption that  $\alpha_N > \alpha_S$  and  $t_S > t_N$ , we can conclude that *the mass of skilled workers in the North is larger than the mass of skilled workers in the South, which is intuitively appealing.*

Now substituting (12) into (10) we can derive the equilibrium amount of the intermediate good. Totally differentiating the equation and utilizing Result 1 would yield the following:

**Result 2.** *The equilibrium amounts of intermediate input  $Z_i^*$ , under autarky, is increasing in  $\alpha$  and decreasing in  $t$  and  $\beta$ .*

The intuition behind the above result is that an increase in the potential ability of workers or a decrease in the training cost would affect  $Z$  in two ways. It will affect  $Z$  directly or indirectly through  $\delta_{iU}^*$ . However, both the direct and the indirect effects of  $\alpha$  and  $t$  are in the same. First, an increase in the potential ability of workers would increase the productivity and consequently the output of industry  $Z$ . On the other hand, an increase in the training cost would decrease the mass of skilled workers and thus the equilibrium output of industry  $Z$ . Finally, an increase in  $\beta$  would increase the wage of unskilled workers leading to a decrease in the mass of skilled workers and thus the equilibrium output of industry  $Z$ .

Finally, the equilibrium price of good  $X$  in a given country can be derived by substituting (2) and (3) into (1) as

$$P_i^* = \beta^{-\beta} (1 - \beta)^{-(1-\beta)} (W_i^*)^\beta \quad (13)$$

Totally differentiating the above equation with the use of (12), we can conclude the following:

**Result 4.** *The price of the final good,  $X$ , under autarky is increasing in  $\alpha$  and decreasing in  $t$ .*

The reason for the above result is that  $P^*$  is increasing in  $W^*$ , which is increasing in  $\alpha$  and decreasing in  $t$ . Given the assumptions imposed on the parameters  $t$  and  $\alpha$ , the above result implies that the price of the final good is higher in the North,  $P_N > P_S$ . This result will be used in the following section to derive the pattern of trade under trade equilibrium.

#### 4. Income Inequalities and Social Welfare

In general, there are several ways to measure income disparities. In this paper I focus on earned wage inequality. Thus, I define two measures of income disparities; the first is a measure of disparity between skilled and unskilled workers in a given country, the second is a measure of income disparity within the skilled workers in a given country. Let us consider first income disparity within the skilled group. I defined this measure as the difference between the highest wage of the highest skilled worker and the wage of the lowest skilled worker, which is given by  $ID^W_i = \alpha_i(1 - \delta_{iU}^*)$ . But since in equilibrium the skill distribution is determined endogenously within the model by the equality of the wages of skilled and unskilled workers for the marginal worker, this measure will be given by

$$ID^W_i = \alpha_i - t_i - W_i^* \quad (14)$$

Observe that this measure defines the disparity within the skilled worker and could be used also as a measure of disparity between the skilled and unskilled workers in a given country. Totally differentiating the above equation with the use of (12) and Result 1, we can conclude the following:

**Result 5.** *Income disparity within-skilled group, in a given country, is decreasing in  $t$  and  $\beta$  and increasing in  $\alpha$ .*

The intuition behind the above result is that an increase in  $\alpha$  or a decrease in  $t$  would affect income disparity in two opposite directions. First, it would decrease income disparity indirectly,



through the equilibrium wage of the unskilled workers. Second, it would increase income disparity through the direct effect on ID. However, the direct effect dominates the indirect effect. Intuitively an increase in  $\alpha$  and a decrease in  $t$  would increase the net income of the most productive skilled worker, which increases income disparity. The impact of  $\alpha$  on the income inequality is only through the equilibrium wage. This is also intuitive since it would increase the wage of the unskilled labor, which results in lower income disparity. *Thus, given the assumption that  $\alpha_N > \alpha_S$  and  $t_S > t_N$ , disparity in the North is higher than in the South under autarky.*

Now let us consider a measure of income disparity between the skilled and unskilled which is defined as difference between the average income of skilled workers net of training cost  $\alpha$  and that of unskilled workers and the wage of unskilled worker. First we define the average income of the skilled workers as

$$I_{iAve} = (1 - \delta_i)^{-1} \left[ \int_{\delta_{iu}}^1 \alpha \delta_i d\delta_i - t_i (1 - \delta_i) \right]$$

Integrating the above equation and rearranging terms we have

$$I_{iAve} = \frac{\alpha_i + W_i^* - t_i}{2}$$

Observe that the average income of skilled worker is increasing in the equilibrium wage of the unskilled workers. This is intuitive since increasing the wage of the unskilled workers would decrease the incentive of unskilled worker to acquire skill and thus decreasing the supply of skilled workers and thus increase the average wage. The measure of income disparity is

$$ID^B_i = \frac{\alpha_i + W_i^* - t_i}{2W_i^*} \quad (15)$$

Totally differentiating the above equation with the use of (15) and Result 1, we can conclude the following:

**Result 6.** *Income disparity between groups, in a given country under autarky, is decreasing in  $t$  and increasing in  $\alpha$  and  $\beta$ .*

Observe that from Result 5 and 6 that both measures of income disparities behave the same in  $t$  and  $\alpha$  but not in  $\beta$ . The reason for that is that the average wage of the skilled group is increasing in  $\beta$ .

Let us define the social welfare function by uniformly aggregating all workers' equilibrium utility. It is assumed that the individual utility is linear in the only consumption good  $X$ . Thus, integrating the consumer income and deflating it with the price of the final good, we can derive the social welfare as

$$\omega_i = P_i^{*-1} M_i \left[ \delta_{iU}^* W_i^* + \int_{\delta_{Ui}}^1 \alpha_i \delta_i d\delta_i - t(1 - \delta_i) \right]$$

Integrating the above equation we can derive the social welfare as

$$\omega_i = BM_i W_i^{*(-\beta)} \left[ W_i^* \left( \frac{W_i^* + t_i}{\alpha_i} \right) + \left( \frac{(\alpha_i - t_i)^2 - W_i^{*2}}{2\alpha_i} \right) \right] \quad (16)$$

The first term between brackets is the income of the unskilled worker while the second term is the net income of the skilled worker. It can be seen that the welfare of the unskilled workers is increasing in  $W^*$  while the welfare of the skilled worker is decreasing in  $W^*$ . Furthermore, it is worth noting that the equilibrium in this model is a first best socially optimal. In other words, a central planner that maximizes the social welfare function would choose the mass of skilled worker identical to the equilibrium outcome. This should be obvious since there is no source of market failure in the model. Now from the first term in (16) we have the following result.

**Result 7.** *The Social welfare is increasing in  $\alpha$  and decreasing in  $t$ .*

It can be seen that both parameters affect social welfare directly and indirectly through the wage. But since as indicated above that the derivative  $\omega$  of with respect to  $W$  evaluated at  $\delta_{iU}^*$  is zero, the direct effect is the relevant one on  $\omega$ . Observe that an increase in  $\alpha$  would increase productivity and thus it is intuitive to increase social welfare. Also an increase in  $t$  would decrease resources devoted to non-consumption and consequently would decrease social welfare. Thus, the above result follows directly from differentiating  $\omega$  with respect to  $\alpha$  and  $t$ . Now let us characterize the relationship between income disparity and equilibrium utility of unskilled workers. From Result 6 and 7 we can conclude the following:

**Result 8.** *An increase*

- (i) *in  $\alpha$  increases income disparity and increases the social welfare.*
- (ii) *in  $t$  reduces income disparity and reduces the social welfare.*

Note that  $\alpha$  and  $t$  affect both types of income disparity in the same way. Furthermore, from the above Result and given the assumption on the parameters, it can be seen that the North will have higher income disparity and higher social welfare compared to the South under autarchy.

## 5. Trade Equilibrium

Now let us consider an integrated North-South world economy in which we have free trade and no labor mobility. In the absence of friction or transportation cost, trade will result in price parity for the final good  $X$  such that  $P_N = P_S = P$ . Given the parameters of the model, the price of the final good is higher in the North as can be seen from Result 7. Thus, the pattern of trade will be for the North to import the final good in return for exporting the high-tech intermediate input. As a result of that the unskilled worker wage will be equalized between both countries. Thus, wage disparity between unskilled labor in the North and South will vanish.

Given  $P_i^{**}, W_i^{**}, W(\delta)_i^{**}, U_i^{**}(\delta) > 0 \forall \delta_i$  we define a trade equilibrium by  $\{P^{**}, W^{**}, W(\delta)_i^{**}, U_i^{**}(\delta), \delta_{iU}^{**}; \delta \in (0,1), i \in \{N, S\}\}$  which results in the following conditions: profit maximization for firms in the final good and the intermediate good sectors; firms making zero profit in both sectors; equality of supply and demand in the labor markets for each country; and the demand of the intermediate good by both countries to supply. From the above specification of the model, we now obtain the following:

**Theorem 2.** *Given Assumption 1, there exists unique free trade equilibrium  $\{P^{**}, W^{**}, W(\delta)_i^{**}, U_i^{**}(\delta), \delta_{iU}^{**}; \delta \in (0,1), i \in \{N, S\}\}$ , such that  $P_i^{**}, W_i^{**}, W(\delta)_i^{**}, U_i^{**}(\delta) > 0 \forall \delta_i$ .*

**Proof.** The material balance condition is given by equating the aggregate supply and the aggregate demand in both countries as

$$\frac{(1-\beta)}{\beta} \sum W_i L_i = \sum_i Z_i \quad (17)$$

Substituting (8), (10), and (12) into (17) we have

$$\frac{(1-\beta)}{\beta} \sum_i W_i M_i \left( \frac{W_i + t_i}{\alpha_i} \right) = \sum_i \frac{M_i}{2\alpha_i} [(\alpha_i - t_i)^2 - W_i^2] \quad (18)$$

From (17) and without loss of generality assume that both countries are of the same size,  $M_S = M_N$  and that  $\alpha_S = \gamma\alpha_N = \gamma\alpha$  and  $t_N = \theta_N = \theta$  where  $\theta, \gamma \leq 1$  we have

$$\frac{(1-\beta)}{\beta} W_N \left( \frac{W_N + \theta}{\alpha} \right) + \frac{[W_i^2 - (\alpha - \theta)^2]}{2\alpha_N} = \frac{[(\gamma\alpha - t)^2 - W_S^2]}{2\gamma\alpha} - \frac{(1-\beta)}{\beta} W_S \left( \frac{W_S + t}{\gamma\alpha} \right) \quad (19)$$

Equation (19) is a quadratic with two roots. The relevant one (i.e., positive root) determines the equilibrium wage as

$$W^{**} = \frac{-(1-\beta)(1+\theta\gamma)t + \sqrt{(1-\beta)^2(1+\theta\gamma)^2 t^2 + 2(2-\beta)\beta(1+\gamma)[\alpha^2\gamma(1+\gamma) - 2t\alpha\gamma(1+\theta) + t^2(1+\theta\gamma)^2]}}{(2-\beta)(1+\gamma)}$$

Thus, there exists a unique equilibrium wage for the unskilled workers  $\square$

It can be seen that the wage rate will be equalized in equilibrium for the unskilled workers, Samuelson (1945). However, the wage will not be equalized for the skilled workers of a given type between both countries. The reason is that a worker of a given type in both countries has different potential ability. Thus, when they acquire training, they will end up with different productivity.

Now observe that if both countries are the same in every, then the wage of the unskilled workers after and before trade is the same. But if  $\gamma > 0$  and/or  $\theta > 0$  then the wage after trade of the unskilled worker in the North will be smaller than before trade. In other words, one of the above parameter is sufficient to represent a difference between both countries. Thus, smaller value of one of these parameters, indicates larger the difference between the North and South in terms of training technology and potential ability. In addition, smaller value of  $\theta$  or larger value of  $\gamma$  would result in smaller difference between before and after trade wage of the unskilled workers in the North. As a result of that, we can conclude as follows:

**Result 9.** *In a free trade equilibrium, there exist a set of parameters such that:*

(i)  $\delta_N^{**} \in (0,1)$  and  $\delta_S^{**} \in (0,1)$ . *The North and the South will produce both the high-tech and the final good.*

(ii)  $\delta_N^{**} \in (0,1)$  and  $\delta_S^{**} = 1$ . *The North will produce the final good as well as the high-tech intermediate input while the South will completely specialize in the production of the final good.*

The above result follows from equation (7) and from the fact that wage rate equalization of the untrained workers between the North and the South under free trade equilibrium would decrease the North's equilibrium wage and increase the South's equilibrium wage. The result indicates that free trade would increase the mass of skilled workers in the North, which has been documented by Machin (1996). Thus, free trade will induce acquisition of human capital in the

North. However, the opposite will accrue in the South. *In other words, skilled workers in the South will be better off working in the final good industry and earning unskilled wage than working in the production of the high-tech intermediate input.* It is interesting to note that *the South can completely specialize in the production of the final good if  $W^{**} = \gamma\alpha - t$* , where  $W^{**}$  is given by (19). However, the North will never completely specialize in the production of the high-tech intermediate input. The reason for that is the worker with the lowest potential ability will never have an incentive to acquire training. This result is new to this literature and is different from the conventional two-sector Heckscher-Ohlin model in which free trade would result in complete specialization in both countries. Turning to the impact of free trade on income disparity. Given both measures of income disparity within group (14) and between groups (15), we can conclude the following:

**Result 10.** *Free trade would result in higher income disparity within the skilled group and between skilled and unskilled groups in the North and the opposite will occur in the South.*

The above result follows from the wage rate equalization of the unskilled workers in both countries and from the fact that the wage of the skilled workers will not be affected by free trade. The reason that the skilled worker's wage will not be affected as a result of free trade is the different potential ability profile and the choice of numeraire. This implies that workers are different among countries, since each country has a different potential ability profile and, consequently, a different productivity profile. The above result is consistent with the finding of Hoxby and Terry (1999). The decline of income disparity in the South has been supported for some developing countries in East Asia, Wood (1995). On the other hand, widening income disparities between skilled and unskilled have been supported in the North by Wood (1995), Katz and Murphy (1992), and Bound and Johnson (1992) among others. The rise in income

disparities within the skilled group in the North has been documented by Machin (1996) and Katz and Autor (2000). This result is different from Abdel-Rahman, Norman, and Wang (2002) due to the fixed supply of skilled and unskilled workers in their model. We can also conclude from Result 9 and 10 we can conclude, that widening income disparity within the skilled group in the North is due to the increase in the diversity of skilled workers, which has been supported by Hoxby and Terry (1999).

## 6. Welfare and Free Trade

As we have seen that free trade will change the distribution of skill as well as income distribution in both countries. Now, let us examine the impact of free trade on the welfare of different types of workers in the North.<sup>15</sup> With the use of equation (16), let us define the change in welfare for a given worker of type  $\delta$  in the North as

$$\Delta\omega_N(\delta) \equiv \omega_N^{**}(\delta) - \omega_N^*(\delta) = \begin{cases} B\{W^{**1-\beta} - W_N^{*1-\beta}\} & \delta \leq \delta_U^{**} \\ B(\alpha\delta - \theta t)\{W^{**-\beta} - W_N^{*-\beta}\} & \delta > \delta_U^{**} \end{cases}$$

Observe that for  $\delta \leq \delta_U^{**}$  and given that since  $W^{**} \leq W^*$ , then  $\Delta\omega < 0$ . On the other hand, for  $\delta > \delta_U^{**}$  and given that  $W^{**} \leq W^*$ , thus we can conclude that  $\Delta\omega > 0$ . Thus we have the following result

**Result 11.** *The North's welfare of the workers*

(i) *who are unskilled before and after trade will decrease with free trade.*

(ii) *who are skilled before and after trade will increase with free trade.*

The intuition of the above result is as follows; free trade would affect the welfare of the untrained worker in the North in two ways; first it would decrease their wage and thus decrease utility and second it would decrease the price of the consumption good X which increases utility,

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<sup>15</sup> The impact of free trade on the South will be the opposite of the North.

however the first effect dominates the second, resulting in a decrease in their welfare. On the other hand, recall that due to the choice of the numeraire, the wage of the skilled workers will not be affected by free trade. Thus, the welfare of a trained worker in the North would improve under free trade because of the decline in the price of the consumption good X.

Now let us examine the impact of free trade on the welfare of the workers who are unskilled before trade but acquire training as a result of free trade. To do that let us define the change in the welfare of all workers who are unskilled before free trade but skilled after as

$$\Delta\omega_{CN} = W^{**-\beta} \int_{\delta_U^{**}}^{\delta_U^*} (\alpha\delta - \theta t) d\delta - (\delta_u^* - \delta_U^{**}) W^{*(1-\beta)}$$

Integrating the above equation and rearranging terms we have

$$\Delta\omega_{CN} = W^{**-\beta} \left( \frac{W^{*2} - W^{**2}}{2\alpha} \right) - W^{*(1-\beta)} \left( \frac{W^* - W^{**}}{\alpha} \right)$$

The first term on the RHS is the welfare after trade while the second term on the RHS is the welfare before trade. As it can be seen from the above equation, free trade will affect the welfare of worker who will switch from being unskilled to being skilled in two ways; first there will be a reduction in the wage of the unskilled workers as can be seen from figure 3, and second there will a reduction in the price of the final good X. Thus, there will be an income effect and a price effect. From the above equation, it can be shown that  $\Delta\omega_{CN} \begin{matrix} \geq \\ < \end{matrix} 0$  if the following condition is

satisfied

$$1 + \frac{W^{**}}{W^*} \begin{matrix} \geq \\ < \end{matrix} 2 \left( \frac{W^{**}}{W^*} \right)^\beta \quad (20)$$

Thus, the outcome of the above condition, the impact of free trade on this group, will depend on the relative wage before and after trade and on the parameter  $\beta$  associated with the unskilled



workers in the production function of the final good X. Given that if  $\frac{W^{**}}{W^*} = 1$  this implies that the North and the South have the same potential ability profile and training cost. On the other hand if both countries are different in either potential ability or training cost then  $\frac{W^{**}}{W^*} < 1$ . Observe that the larger the difference between the North and the South the smaller is this ratio. Let us now characterize condition (20) graphically. The by the line in Figure 4 represents the LHS of the condition (20) while the RHS is represented by one of the three curves depending each for different value of  $\beta$ . For each one of the three curves there exists a value of  $\frac{W^{**}}{W^*}$  such that the LHS is equal the RHS. Below that value the LHS is larger than the RHS and the opposite is true for values higher than  $\frac{W^{**}}{W^*}$ . The value  $\frac{W^{**}}{W^*}$  at which the LHS is equal to the RHS is larger for higher value of  $\beta$ . Thus we can conclude that the following

**Result 12.**

*(i) For a small value of  $\beta$ , if the North and the South are similar, then the welfare of the workers who are unskilled before trade and skilled after trade in the North will decrease as a result of free trade. The opposite will accrue if the North and South are dissimilar.*

*(ii) For a large value of  $\beta$ , if the North and the South are similar, then the welfare of the workers who are unskilled before trade and skilled after trade in the North will increase as a result of free trade.*

Recall that the similarities and differences between the North and the South are represented by the values the parameters  $\gamma$  and  $\theta$ . The intuition of the above result is that for a small value of  $\beta$  the price effect on welfare is weaker compared to the income effect. In other words, the

reduction in income due to the switch from unskilled to skilled would decrease the welfare by more than the increase in welfare due to the reduction in the price of the final good. On the other hand, for a large value of  $\beta$  the price effect is stronger than the income effect and the opposite impact on welfare will accrue. It is interesting to note that even though both countries may be similar the welfare of the worker that will switch jobs in the North may increase or decrease depending on the parameter of the production function, which is accessible to both countries.

This result sheds some lights on groups that may oppose free trade agreement in developed countries. As it can be seen all of the unskilled labor workers in the developed country will oppose free trade agreement with countries that are not similar to their own. While all workers who are skilled before and after free trade agreement will be for free trade. This result is similar to what can be expected from H-O type model. However, workers who will switch jobs may be the decisive group on whether or not a country may accept a free trade agreement. As we showed this group would favor free trade agreement with countries that are similar or not similar to their own country only if the technology is characterized by being highly unskilled labor intensive, high value of  $\beta$ . On the other hand, the same group will be oppose to free trade with countries that are similar to their own if the technology of the final good is characterized by being intermediate input intensive, low value of  $\beta$ .

Now let us consider a situation in which the decision on whether a rich country chooses free trade agreement or not is made by a majority vote approach like in Fisher and Serra (1996).<sup>16</sup> In this framework workers will either vote for or against free trade. Free trade will be chosen if the majority of worker's welfare increase as a result of free trade. Thus, the median voter in the North will determine the decision of whether to free trade will be accepted or not.

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<sup>16</sup> The majority voting approach has been first adapted to this trade decision by Baldwin (1982) and Mayer (1984).

**Theorem 3.** *The North will vote for free trade (against free trade) if  $\frac{\alpha}{\theta t} \geq \frac{4(1-2\beta)}{2-5\beta}$*

*(if  $\frac{\alpha}{\theta t} < \frac{4(1-2\beta)}{2-5\beta}$ ).*

**Proof.** Observe that if  $\frac{\alpha}{\theta t} \geq \frac{4(1-2\beta)}{2-5\beta}$  it implies from (7) and (12) that  $\frac{1}{2} \geq \delta_{NU}^*$ . But this implies

that the majority vote will be of workers who are skilled before and after trade. Thus, from Result 11, the North will vote for free trade  $\square$

**Theorem 4.** *Suppose that the only difference between the North and the South is potential ability*

*and that  $\frac{\alpha}{t} \leq \frac{2(1+\gamma-4\gamma\beta)}{(1+\gamma)(1-.5\beta-2\gamma\beta)}$  then the North will vote against free trade.*

**Proof.** Observe that if  $\frac{\alpha}{t} \leq \frac{2(1+\gamma-4\gamma\beta)}{(1+\gamma)(1-.5\beta-2\gamma\beta)}$  this implies from (7) and W\*\* that  $\delta_{NU}^{**} \geq \frac{1}{2}$ .

But this implies that the majority vote will be of workers who are unskilled before and after trade. Thus, from Result 11, the North will vote against free trade  $\square$

**Theorem 5.** *Suppose that the only difference between the North and the South is potential ability*

*and that  $\frac{4(1-2\beta)}{2-5\beta} > \frac{\alpha}{t} > \frac{2(1+\gamma-4\gamma\beta)}{(1+\gamma)(1-.5\beta-2\gamma\beta)}$  then the North's majority vote will be*

*undetermined.*

**Proof.** Observe that if  $\frac{4(1-2\beta)}{2-5\beta} > \frac{\alpha}{t} > \frac{2(1+\gamma-4\gamma\beta)}{(1+\gamma)(1-.5\beta-2\gamma\beta)}$  this implies from (7), (12), and

W\*\* that  $\delta_{NU}^* > \frac{1}{2} > \delta_{NU}^{**}$ . But this implies that the majority vote will be of workers who were

unskilled before and skilled after trade. Thus, from Result 12, the North's decision will depend on  $\Delta\omega$  of  $\delta_{NU}^* - \delta_{NU}^{**}$  which is undetermined  $\square$

Thus, we can conclude that there will be clear winners and clear losers under free trade. In addition there will be a group of workers who will acquire training as a result of free trade and their welfare is not determined. Furthermore, as a result of free trade the North will pay some training costs, which has been observed. The result provides in addition to the classical argument that only unskilled worker in developed country would oppose free trade agreement. The opposite will happen in the South. However, since the South can completely specialize in the production of the final good, the South welfare would increase in this case. In other words, there could be only winners if the South completely specializes. This would justify the willingness of a LDC to enter into a free trade agreement with a developed one.<sup>17</sup>

## **7. Conclusion**

This paper presented a North-South trade general-equilibrium model in which both countries produce a final good and a high-tech intermediate input. Each country is populated by a continuum of unskilled workers with differential potential ability. Workers in the North and South can acquire skill by investment in training or education. Thus, skill distribution in the North and South is determined endogenously in the model through a self-selection. The model analyzed the impact of a bilateral free trade agreement on income inequalities and social welfare of each country. Furthermore, the model examined the impact of free trade on the distribution of skilled labor in each country.

It has been shown, in the context of the model, that free trade would lead to complete specialization in the South but may not lead to complete specialization in the North. Second, the wage rate will be equalized in equilibrium for the to complete specialization. The wage rate will be equalized in equilibrium for the unskilled workers but not for the skilled workers. This would

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<sup>17</sup> See Fisher and Serra (1996) for a median vote with exogenous skill distribution model that generated similar result.

lead to lower wage for the unskilled in the North and higher wage for the unskilled in the South. As a result of that, it has been shown that free trade will result in higher income inequality within the skill group as well as between skill and unskilled in the North. Then, I showed that free trade would result in hire diversity within the skilled workers in the North, which is new to the literature. This is because free trade would induce worker in the North to acquire training. Thus, free trade would increase the mass of the skilled worker in the North and would decrease it in the South. Furthermore, free trade would result in higher welfare for the skilled workers in the North but lower welfare for the unskilled workers in the North. In addition, it has been shown that even though both countries may be similar, as a result of free trade the welfare of the worker that will switch jobs from being unskilled to skilled, in the North, may increase or decrease depending on the parameter of the production function, which is accessible to both countries. Thus, the welfare change on the workers who are unskilled before trade and acquire skill after is ambiguous, which is new to the literature. The opposite will happen in the South. .

## Appendix

**A.1.** Consider a closed economy producing two homogeneous goods (X and Z) with the use of labor. The total population of the economy is N. Good X represents a low-tech product that can be produced by skilled or unskilled workers, whereas good Z is a high-tech product that can be produced by skilled workers. The production function of sector X is a simple CRS given by  $X=L$ . Production efficiency implies that the firms in sector X will hire labor at the wage rate equal to their marginal product,  $P=W$ .

It is assumed that consumers/workers have an identical utility function. The behavior of some representative households is given as

$$\max_{x,z} \left\{ U = x^\beta z^{(1-\beta)} \mid W = Px + z \right\}$$

From the above behavior we can derive the aggregate demand for good X as

$$AD_z = (1 - \beta)P^{-1}M \left[ W\delta_U + \int_{\delta_U}^1 (\alpha\delta - t)d\delta \right]$$

Equating the aggregate demand and the aggregate supply,  $\int_{\delta_U}^1 \alpha\delta d\delta$ , and with the use of  $W=P$ , we can derive  $W^*$  as in (11).

**A.2.** Suppose that the training cost is given by a linear function of the potential ability as  $t_i = \tau_i - \delta$ . In this case the marginal worker, who will be indifferent between acquiring skill or not will be given by  $\delta_u = \frac{W_i^* + \tau}{\alpha_i + 1}$ . Observe that this is structurally the same as in equation (7), if

we define  $\alpha_i \equiv \alpha_i + 1$  and  $t_i \equiv \tau_i$ .

**A.3.** One can interpret the model in the following way: Given that each worker is endowed with one unit of time, suppose that the training cost is given in terms of time devoted to training,  $\tau_i$ .

The training cost is given by a linear function of the potential ability,  $t_i = \tau_i - \alpha_i \delta_i$ .

Furthermore, suppose that the production function of  $z$  for any skilled worker is  $Z=L$ , where  $L$  is the number of skilled workers. Thus, the productivity of any trained worker of any type is 1 unit

of good  $z$ . In this case, the marginal worker will be given as  $\delta_{ui} = \frac{W_i - (1 - \tau_i)}{\alpha_i}$ . Assuming that

$\tau_i > 1$  and defining  $t_i \equiv \tau_i - 1$ , we have equation (7). In this case, the worker with the highest potential ability would require the minimum amount of time on training to reach the constant productivity. The aggregate supply of  $z$  will be given by

$$Z_i = M_i \left[ \int_{\delta_{ui}}^1 (1 - \tau_i + \alpha \delta) d\delta \right] = M_i \left[ \int_{\delta_{ui}}^1 \alpha \delta d\delta - (1 - \delta_{ui}) t_i \right], \text{ which is the same as equation (9).}$$

**A.4.** To see the equilibrium is socially optimal; consider a centrally planning authority that maximizes the aggregate utility of population in the country.

$$\omega_i = (M_i \delta_{ui})^\beta \left[ M_i \int_{\delta_{ui}}^1 \alpha \delta d\delta - t_i (1 - \delta_i) \right]^{1-\beta}$$

Maximizing the above with respect to  $\delta_{ui}$ , we have

$$\frac{\partial \omega_i}{\partial \delta_{ui}} = \left\{ (1 - \beta) \delta_{ui} (t_i - \alpha_i \delta_{ui}) + \beta \left[ \frac{\alpha_i}{2} (1 - \delta_{ui}^2) - t_i (1 - \delta_{ui}) \right] \right\} = 0$$

Solving the above first order condition we have  $\delta_i^* = \frac{t_i + \sqrt{t_i^2 + \alpha_i (2t_i - \alpha_i) \beta (2 - \beta)}}{\alpha_i (2 - \beta)}$  Substituting

the above into equation (6) we get the equilibrium wage in equation (12).

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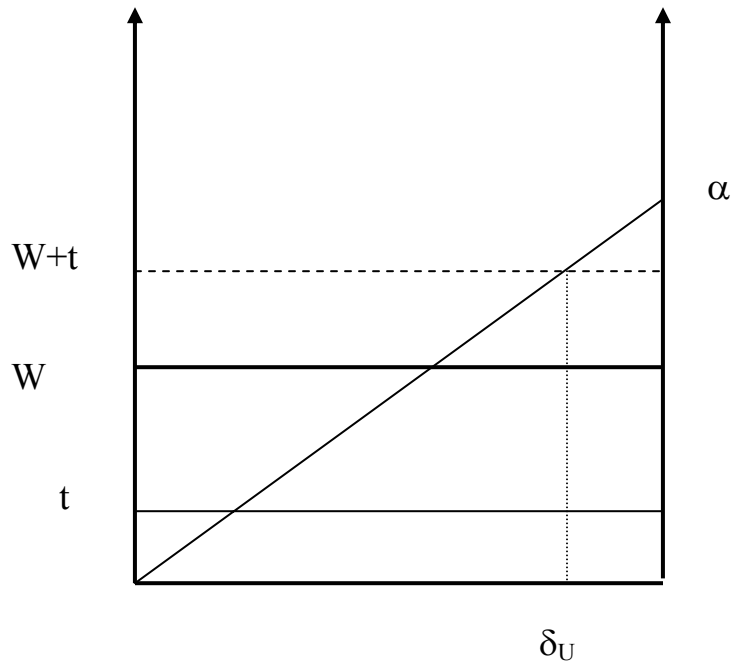
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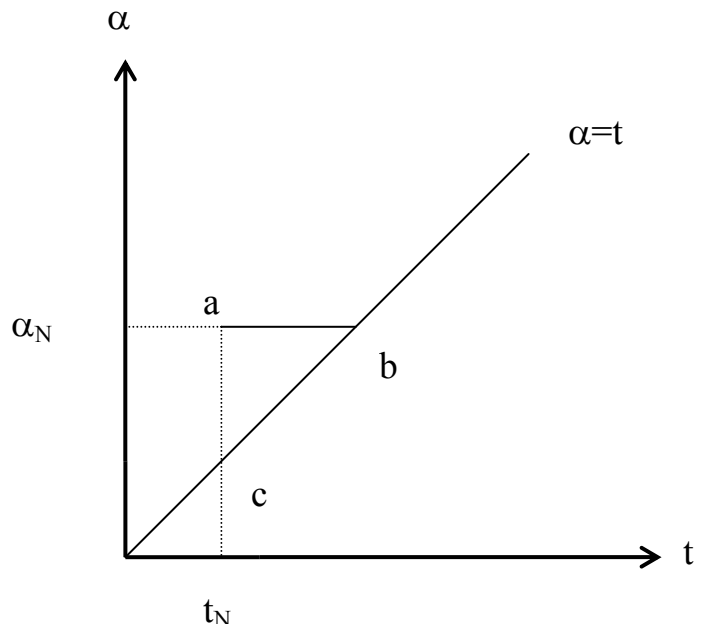
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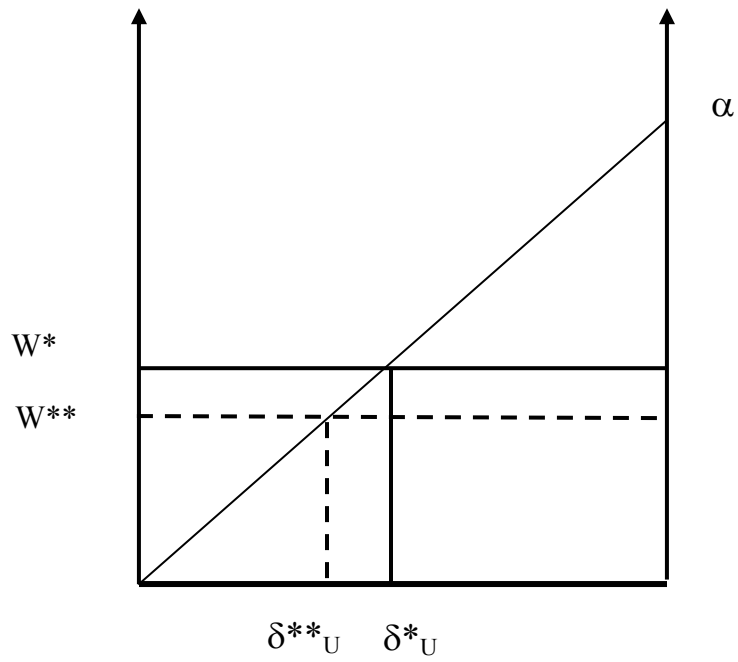
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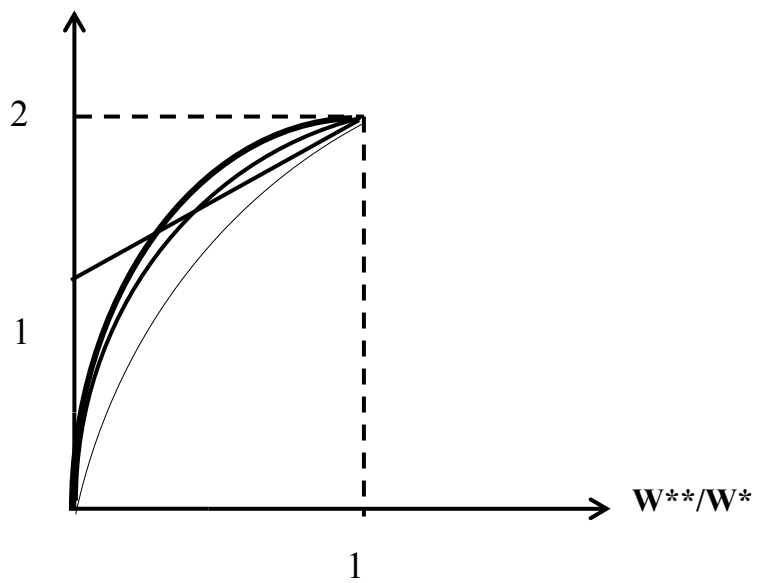
*Figure 1. Wage and productivity profiles*



*Figure 2. Equilibrium Parameter Space*



*Figure 3. The impact of free trade*



*Figure 4. Change in welfare*