7-1-1984

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Recommended Citation

R. M. A. Azzam, "Inverting the ratio of the complex parallel and perpendicular reflection coefficients of an absorbing substrate using a transparent thin-film coating," J. Opt. Soc. Am. A 1, 699-702 (1984) http://www.opticsinfobase.org/josaa/abstract.cfm?URI=josaa-1-7-699 .

Inverting the ratio of the complex parallel and perpendicular reflection coefficients of an absorbing substrate using a transparent thin-film coating

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Received July 28, 1983; accepted March 7, 1984

An absorbing substrate can be coated with a transparent thin film of refractive index N_1 (within a certain range) and thickness d such that the ratio of complex reflection coefficients for the p and s polarizations of the film-covered substrate $\rho = R_p/R_s$ is the inverse of that of the film-free substrate $\bar{\rho} = \bar{R}_p/\bar{R}_s$ at an angle of incidence ϕ. A method to determine the relationship among ϕ, N_1, and d that inverts ρ (i.e., makes $\rho = 1/\bar{\rho}$) for a given substrate at a given wavelength is described and is applied to aluminum and silver substrates at 0.6328- and 10.6-μm wavelengths, respectively. Sensitivity of the inversion condition to incidence-angle and film-thickness errors is analyzed. ρ-inverting layers can be applied to *one* of the two metallic mirrors of a beam displacer or axicon to preserve the polarization state of incident monochromatic radiation.

1. INTRODUCTION

In this paper we show that it is possible to invert the ratio $\rho = R_p/R_s$ of the complex parallel (p) and perpendicular (s) reflection coefficients R_p and R_s of an absorbing substrate at oblique incidence using a transparent-film coating of properly selected refractive index and thickness. Such ρ-inverting layers, as we call them, can be applied to *only one* of the two metallic mirrors of a parallel-mirror beam displacer or axicon to preserve the state of polarization of incident monochromatic radiation. Likewise, they can be used to equalize the transverse-electric (TE ≡ s) and transverse-magnetic (TM ≡ p) complex eigenvalues of 90°-rooftop reflectors and waxicons.

2. CONDITIONS FOR INVERTING ρ

The ratio of complex p and s reflection coefficients of a monochromatic plane wave of light of wavelength λ at the interface between a transparent medium of real refractive index N_0 and an absorbing substrate of complex refractive index N_2 can be written as

$$\bar{\rho} = \bar{R}_p/\bar{R}_s = \bar{\rho}(\phi, N_0, N_2), \tag{1}$$

where ϕ is the angle of incidence. If the substrate is coated by a transparent film of thickness d and refractive index N_1, the ratio of complex reflection coefficients becomes[1]

$$\rho = R_p/R_s = \rho(\phi, d, \lambda, N_0, N_1, N_2). \tag{2}$$

The inversion condition for given ambient and substrate media at a given wavelength can be put in the form

$$\rho_n = \bar{\rho}(\phi)\rho(\phi, \zeta, N_1) = 1. \tag{3}$$

In Eq. (3) the fixed arguments N_0 and N_2 of the functions $\bar{\rho}$ and ρ have been dropped, and

$$\zeta = d/D_\phi \tag{4}$$

is the normalized film thickness, where

$$D_\phi = (\lambda/2)(N_1{}^2 - N_0{}^2 \sin^2 \phi)^{-1/2} \tag{5}$$

is the film-thickness period.

Equation (3) represents two constraints [e.g., $\mathrm{Re}(\bar{\rho}\rho) = 1$, $\mathrm{Im}(\bar{\rho}\rho) = 0$] on three parameters: ϕ, ζ, and N_1. To determine the relationship among ϕ, ζ, and N_1 that satisfies Eq. (3), and hence achieves inversion, we assigned values to one parameter (N_1) and solved for the other two (ϕ, ζ). (It is equally easy to set ϕ and solve for N_1 and ζ. The separation-of-variables method outlined below is applied by interchanging N_1 and ϕ.)

For a given N_1, we separate the determination of ϕ and ζ as follows.[2–4] We rewrite the inversion condition [Eq. (3)] in the form

$$\bar{\rho}\rho = \bar{\rho}(A + BX + CX^2)/(D + EX + FX^2) = 1, \tag{6}$$

where $\bar{\rho}$, A, B, C, D, E, and F are determined by the p and s Fresnel reflection coefficients at the ambient–substrate, ambient–film, and film–substrate interfaces,[1] and

$$X = \exp(-j2\pi\zeta). \tag{7}$$

We solve quadratic Eq. (6) for X:

$$X = f_\pm(\phi) \tag{8}$$

and require that

$$|X| = |f_\pm(\phi)| = 1 \tag{9}$$

for a transparent film. (The + and − correspond to the two roots of the quadratic equation.)

Equation (9) has ϕ as its only unknown; ϕ is readily found by direct numerical iteration. Once the solution, indicated by ϕ_{inv}, has been determined, complex X can be evaluated from Eq. (8). Next, ζ_{inv} is obtained from X as

$$\zeta_{\mathrm{inv}} = (-1/2\pi)\arg X. \tag{10}$$

All possible film thicknesses that produce inversion are given by

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$$d_{\text{inv}} = (\zeta_{\text{inv}} + m)D_{\phi_{\text{inv}}}, \tag{11}$$

where m is an integer. In what follows we will consider only the least normalized and actual film thicknesses: $0 < \zeta_{\text{inv}} < 1$ and $0 < d_{\text{inv}} < D_{\phi_{\text{inv}}}$.

3. ρ-INVERTING DIELECTRIC LAYERS ON AN ALUMINUM SUBSTRATE AT $\lambda = 0.6328\ \mu\text{m}$

As a specific example, the method outlined in Section 2 was applied to a system that consists of a transparent film of adjustable refractive index on an Al substrate of complex refractive index[5,6] $N_2 = 1.212 - j6.924$ at the He–Ne-laser wavelength $\lambda = 0.6328\ \mu\text{m}$. For an assumed value of $N_1 > 1$, a solution is sought for Eq. (9). Figure 1 shows $|X| - 1$ versus ϕ for $N_1 = 1.55$; this is characteristic of an Al_2O_3 (or Si_2O_3) film.[6] The two curves in Fig. 1 correspond to the two roots of the quadratic equation in X. Only one curve intersects the ϕ axis once at $\phi_{\text{inv}} = 75.8165°$. From Eq. (10) we determine that $\zeta_{\text{inv}} = 0.49164$. We also calculate $D_{\phi_{\text{inv}}} = 0.26163\ \mu\text{m}$. Thus a dielectric (oxide) layer of refractive index 1.55 and thickness 0.12863 μm, when coated onto an Al substrate, inverts its ratio of complex p and s reflection coefficients at an angle of incidence of 75.8165°.

As a check on the accuracy of the inversion, ρ_n is computed from Eq. (3) at ϕ_{inv} and ζ_{inv}, and we always make sure that $|\rho_n| - 1$ and $\Delta_n = \arg \rho_n$ are less than 10^{-6}.

To examine the sensitivity of the inversion condition to deviations of the angle of incidence ϕ from ϕ_{inv}, we plot, in Figs. 2 and 3, $|\rho_n| - 1$ and Δ_n, respectively, versus ϕ, as computed for an Al_2O_3 layer ($N_1 = 1.55$) of the correct ρ-inverting thickness ($d_{\text{inv}} = 0.12863\ \mu\text{m}$) on the Al substrate at $\lambda = 0.6328\ \mu\text{m}$. The results indicate reasonable insensitivity to errors of incidence angle.

Figure 4 shows the magnitude and phase errors $||\rho_n| - 1|$ and $|\Delta_n|$ that result from shifting the film thickness from the value required for inversion (d_{inv}) by as little as ±10 Å (1 nm) while keeping $\phi = \phi_{\text{inv}}$. It is apparent that the inversion of ρ is sensitively dependent on film thickness in this particular case.

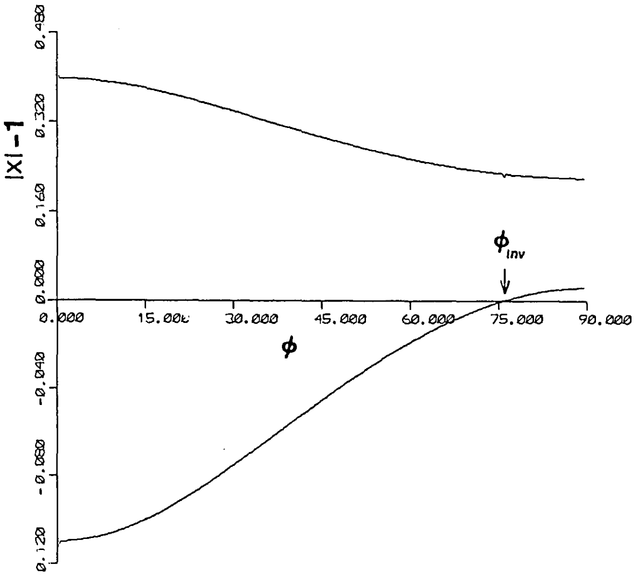

Fig. 1. Finding a solution for Eq. (9) for the angle of incidence ϕ_{inv} at which the ratio ρ of the complex p and s reflection coefficients of an Al substrate ($N_2 = 1.212 - j6.924$) can be inverted by using a transparent-film coating of refractive index 1.55 (e.g., Al_2O_3) at wavelength $\lambda = 0.6328\ \mu\text{m}$. Note that the vertical scale changes at zero.

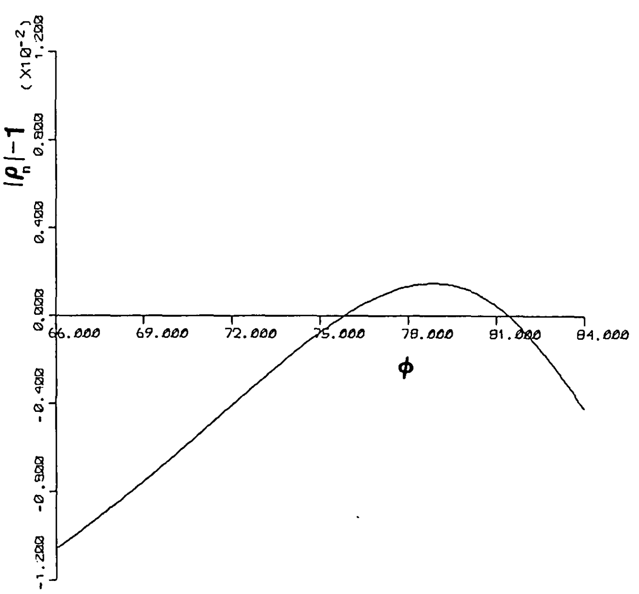

Fig. 2. Magnitude error $|\rho_n| - 1$ caused by shifting the angle of incidence ϕ from the value required for exact inversion $\phi_{\text{inv}} = 75.8165°$. The Al_2O_3–Al film–substrate system is assumed at $\lambda = 0.6328\ \mu\text{m}$ with the oxide-layer thickness set equal to the value required for inversion, $d_{\text{inv}} = 0.12863\ \mu\text{m}$.

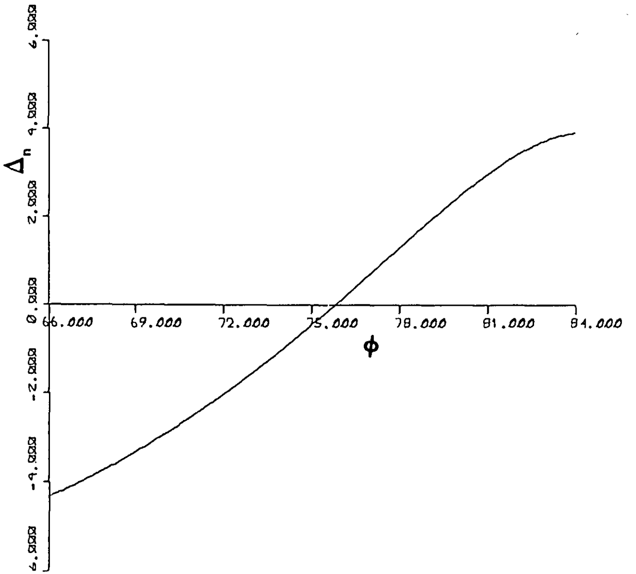

Fig. 3. Same as in Fig. 2, but for the phase error Δ_n. Both Δ_n and ϕ are in degrees.

To explore the ρ-inversion condition for all possible dielectric films on the Al substrate at $\lambda = 0.6328\ \mu\text{m}$, the procedure outlined in Section 2 was applied repeatedly for successively increasing discrete values of the film refractive index N_1 beginning with N_1 just slightly above 1. No inversion was found possible for $N_1 < \tilde{N}_1 \approx 1.5$ or $N_1 > \tilde{N}_1 \approx 2$. Furthermore, we find that, as N_1 increases from its lower limit to its upper limit, ϕ_{inv} decreases from 90° to 0, whereas d_{inv} decreases from ~0.14 to ~0.08 μm, both monotonically. The results are summarized in Table 1 and in Fig. 5. For reference, the p and s reflectances $\mathcal{R}_p$ and $\mathcal{R}_s$ of the film–substrate system, under the conditions of inversion, appear in Fig. 5.

As a second example, we also considered ρ-inverting layers on a Ag substrate ($N_2 = 9.5 - j73$)[7] at the infrared CO_2-laser wavelength of $\lambda = 10.6$ μm. The results are shown in Fig. 6. Notice, in particular, the extent to which ζ_{inv} remains near 0.5 ($\zeta_{inv} = 0.5$ for a perfect conductor) and the high reflectances (>95%) for both polarizations.

4. APPLICATIONS OF ρ-INVERTING LAYERS

An important direct application of ρ-inverting layers is the realization of the simplest possible polarization-preserving parallel-mirror beam displacers and biconical axicons. In these devices, light is reflected twice at the same angle. By leaving one metal surface bare and by coating the other with a ρ-inverting layer, polarization preservation is achieved upon double reflection. Coating only *one* mirror (or cone) with *one* layer is undoubtedly simpler than all other approaches[4,7,8] previously suggested for polarization-correcting parallel-

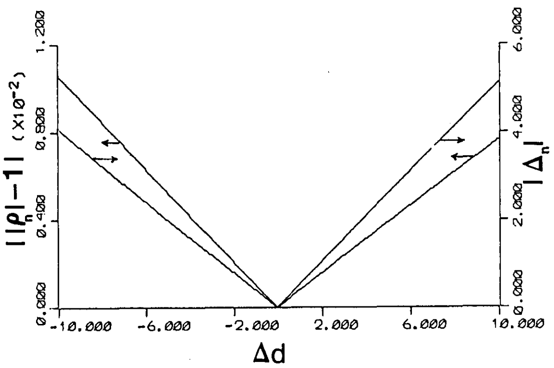

Fig. 4. Magnitude error $||\rho_n| - 1|$ and phase error in degrees $|\Delta_n|$ caused by shifting the thickness d of an Al_2O_3 film on an Al substrate from the value required for exact inversion, $d_{inv} = 0.12863$ μm. ϕ is kept fixed at $\phi_{inv} = 75.8165°$, and $\lambda = 0.6328$ μm. Δd is in angstroms.

Table 1. Characteristics[a] of ρ-Inverting Transparent Films on an Al Substrate ($N_2 = 1.212 - j6.924$) at $\lambda = 0.6328$ μm

N_1	ϕ_{inv}	ζ_{inv}	d_{inv} (μm)	$D_{\phi_{inv}}$ (μm)
1.95	8.945 38	0.481 474	0.078 372	0.162 775
1.92	19.250 00	0.482 478	0.080 708	0.167 276
1.89	26.611 34	0.483 197	0.083 263	0.172 317
1.86	32.234 59	0.484 034	0.085 948	0.177 565
1.83	37.175 77	0.484 857	0.088 811	0.183 170
1.80	41.250 00	0.485 920	0.091 794	0.188 908
1.77	45.500 00	0.486 721	0.095 065	0.195 317
1.74	49.500 00	0.487 541	0.098 564	0.202 166
1.71	53.920 17	0.487 971	0.102 454	0.209 960
1.68	57.788 51	0.488 702	0.106 536	0.217 998
1.65	61.250 00	0.489 697	0.110 846	0.226 355
1.62	65.583 33	0.490 105	0.115 734	0.236 141
1.59	69.250 00	0.491 063	0.120 825	0.246 047
1.55	75.816 53	0.491 638	0.128 626	0.261 627
1.53	79.577 73	0.492 054	0.132 836	0.269 962
1.51	85.069 33	0.492 461	0.137 321	0.278 846

[a] N_1 is the film refractive index; ϕ_{inv} is the angle of incidence (in degrees) at which inversion is possible; ζ_{inv} and d_{inv} are the least normalized and actual film thicknesses, respectively; and $D_{\phi_{inv}}$ is the film-thickness period evaluated at ϕ_{inv}. Retention of six decimal places is mathematically, not physically, meaningful.

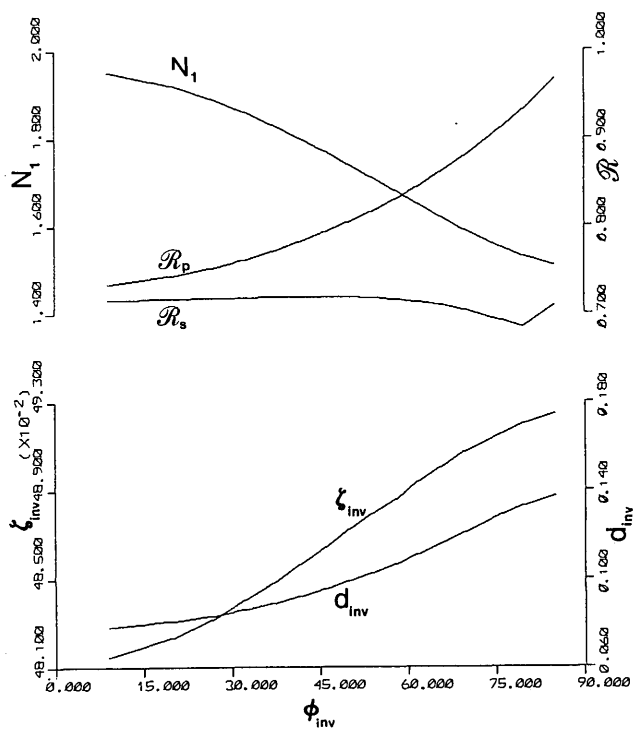

Fig. 5. Characteristics of all possible ρ-inverting transparent layers on an Al substrate ($N_2 = 1.212 - j6.924$) at $\lambda = 0.6328$ μm. N_1 is the film refractive index, and ζ_{inv} and d_{inv} are the normalized and actual (in micrometers) least film thicknesses, respectively, plotted versus the angle of incidence ϕ_{inv} (in degrees) at which inversion is accomplished. $\mathcal{R}_p$ and $\mathcal{R}_s$ are the reflectances of the film–substrate system for the p and s polarizations under the conditions of inversion. Notice that ϕ is unrestricted (hence it can be chosen anywhere between 0 and 90°), but N_1 is limited to the narrow range $1.5 \lesssim N_1 \lesssim 2$, which corresponds to several practical thin-film materials.

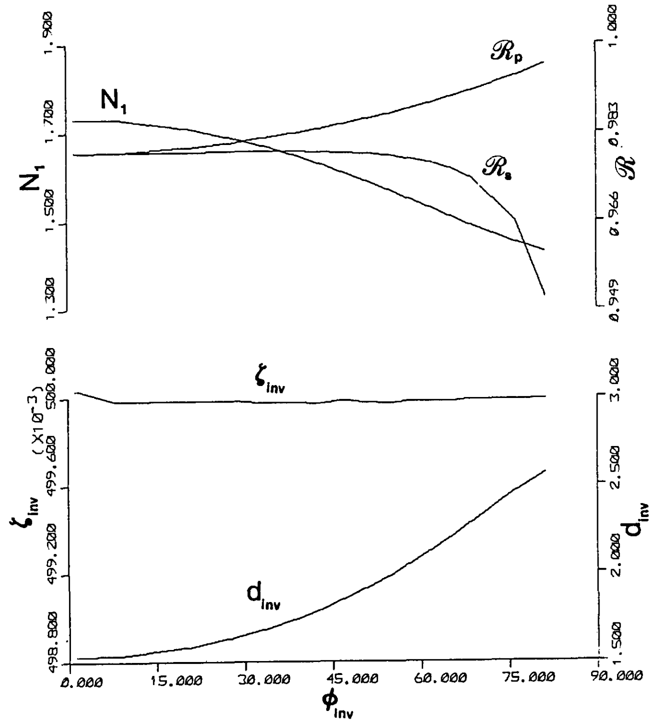

Fig. 6. Same as in Fig. 5, but for a Ag ($N_2 = 9.5 - j73$) substrate at $\lambda = 10.6$ μm.

mirror beam displacers and axicons.[9] Similarly, the application of a ρ-inverting layer to only one of the two mirrors of a biplanar 90°-rooftop reflector or biconial waxicon can equalize its TE (s) and TM (p) complex eigenvalues (net reflection coefficients).[10]

5. CONCLUDING REMARKS

The use of a transparent thin-film coating to invert the ratio of the complex p and s reflection coefficients of a substrate is fundamentally and practically significant. It establishes another facet of the historically and practically important analogy between thin-film and transmission-line theories.[11] In particular, it is well known that a quarter-wave section of a lossless transmission line terminated by a load impedence at one end inverts that impedence at the other.[12] The inversion of ρ of a substrate (load) by a transparent thin film (transmission line) of properly chosen refractive index and thickness represents an optical analog of this effect. It is interesting also to note that the required film thickness is *approximately* a quarter wavelength of light in the film medium ($d \simeq 0.5D_\phi \simeq \lambda/4N_1$, when $N_1 \gg N_0 \sin\phi$). If the incident light is linearly polarized at a 45° azimuth from the plane of incidence, ρ becomes a complex number that completely describes the reflected polarization state.[13] Such a complex polarization number is analogous to impedence[14] and is inverted by a ρ-inverting layer.[15]

ρ inversion at *different* angles of incidence can be pursued along lines similar to those developed here for inversion at the same angle. A change of one of the ϕ's in the inversion condition of Eq. (3) to ϕ', where $\phi' \neq \phi$, is required. This adds a degree of freedom that leads to more solutions. Perhaps ϕ and ϕ' can be constrained to satisfy a relation other than equality. For example, we may choose $\phi + \phi' = 90°$ to represent the important case of light reflection at an arbitrary angle ($\neq 45°$) from a 90°-rooftop reflector or from a waxicon with cones of any apex angle.

The analysis in this paper also applies, with minor modification, to the more general case of an inverting layer on top of a multilayer substructure. In this case, the proper reflection coefficients at the interface between the inverting layer and its underlying substructure must be used. For multilayer stacks, ρ inversion may be possible by proper design of an inner (embedded) layer, as this is possible with the outer (top) layer. Additional flexibility becomes available when the characteristics of more than one layer are adjusted to achieve inversion. This further generalization lies outside the scope of this paper.

ACKNOWLEDGMENTS

I am pleased to acknowledge support by the National Science Foundation and the assistance of M. E. R. Khan in carrying out the computations.

This paper was presented at the 1983 Annual Meeting of the Optical Society of America, New Orleans, Louisiana.

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