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ELECTRICALLY-INDUCED STRESSES AND DEFLECTION IN MULTIPLE PLATES

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ABSTRACT

Thermohydraulic tests are being planned at the High Flux Beam Reactor of Brookhaven National Laboratory, in which direct electrical heating of metal plates will simulate decay heating in parallel plate-type fuel elements. The required currents are high if plates are made of metal with a low electrical resistance, such as aluminum. These high currents will induce either attractive or repulsive forces between adjacent current-carrying plates. Such forces, if strong enough, will cause the plates to deflect and so change the geometry of the coolant channel between the plates. Since this is undesirable, an analysis has been made to evaluate the magnitude of the deflection and related stresses.

In contrast to earlier publications in which either a concentrated or a uniform load was assumed, in this paper an exact force distribution on the plate is analytically solved and then used for stress and deflection calculations, assuming each plate to be a simply supported beam. Results indicate that due to superposition of the induced forces between plates in a multiple-and-parallel plate array, the maximum deflection and bending stress occur at the midpoint of the outermost plate. The maximum shear stress, which is inversely proportional to plate thickness, occurs at both ends of the outermost plate.

INTRODUCTION

Direct electrical heating of a metal test section is one of the methods which have been extensively used in tests to simulate decay heating in reactor fuel elements. The required currents are quite high if a low resistance material like aluminum is used in these tests. These high currents will result in an induced magnetic force between adjacent plates that will subject the plates to bending and shear stresses. The force is attractive when currents on different plates have the same direction, otherwise, it is repulsive [1]. For a practical situation where the plates are cooled by adjacent water flow, the induced stresses can alter the geometry of the coolant channels formed between the plates and thus result in an unpredictable heat transfer pattern. Since a significant alteration to the geometry is undesirable in these tests, it is important to accurately calculate the magnitude of this effect.

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CALCULATION AND ANALYSIS

Induced Magnetic Force

Based on Ampere's rule, magnetic field of a linear current 'i' is $B=k_1/d^2$, and magnetic force is $F=2(i/d)k_2$, where d is the distance, and k_1 and k_2 are constants depending on the units chosen [1]. In the case of a thin plate with breadth 'a' carrying current 'i' into the paper, the magnetic induction force per unit length has two components, $F_x=2(i/a)(\alpha_2-\alpha_1)k_2$ and $F_y=2(i/a)Ln(\rho_2/\rho_1)k_2$, where α and ρ are the angles and distances from the point in space to the edges of the plate. Now consider two parallel wires of equal length L (cm), at a distance b (cm) apart, carrying d.c. or single-phase a.c. i_1 and i_2 (ab-amperes) respectively [2], the total magnetic force (dyne/cm) on wire 2 per unit length is (see Figure 1.1)

$$\begin{aligned} F&=(\mu_m/\mu_v)(2*10^{-7}i_1i_2)[(1+(b/L)^2)^{1/2}/b-1/L] \\ &=(\mu_m/\mu_v)(2*10^{-7}i_1i_2)(1/b-1/L) \\ &=(\mu_m/\mu_v)(2*10^{-7}i_1i_2/b), \quad (\text{as } L\rightarrow\infty \text{ and } t\rightarrow 0) \end{aligned} \tag{1}$$

where μ represents magnetic permeability (cm-g/°C^2) of medium (m) or in vacuum (v).

Taking integration twice on equation (1) with respect to breadth 'a', the force exerted between two parallel and symmetrically placed thin plates with uniformly distributed currents i_1/a and i_2/a is (see Figure 1.2)

$$\begin{aligned} F&=(\mu_m/\mu_v)(2*10^{-7}i_1i_2/a^2)\int_0^a\int_0^a(b\,dz_1dz_2)/(b^2+(z_2-z_1)^2) \\ &=(\mu_m/\mu_v)(2*10^{-7}i_1i_2/a^2)\int_0^a[\tan^{-1}(z_2/b)+\tan^{-1}((a-z_2)/b)]dz_2 \\ &=\int_0^a(f)dz_2 \\ &=(\mu_m/\mu_v)(4*10^{-7}i_1i_2/a)[\tan^{-1}(a/b)-(b/2a)Ln(1+(a/b)^2)] \end{aligned} \tag{2}$$

where f is the unit force between these two plates.

Plate Deflection

If fuel plates are assembled by swaging, as shown in Figure 2.1, the simple beam and the fixed-end beam model bound the actual end conditions on the plate and permit calculation of plate stress and deflection. For conservatism, the simple beam model which yields a larger plate deflection and normal stress is chosen in this analysis (see Figure 2.2). The deflection equation is

$$\delta=\int\int(M/EI)(dx)^2 \tag{3}$$

where δ is the deflection (m), M the bending moment (N-m), E the modulus of elasticity and $E=10^{-7}psi=6.9*10^{-4}N/m^2$ for aluminum, I the moment of inertia and $I=t^3/12\ m^4$ per unit length along the direction of current, t the beam thickness (m), and x the distance (m) from beam end.

Assuming currents i_1 and i_2 (amperes) are uniformly distributed on two adjacent plates as shown in Figure 1.2, the bending moment at any point on plate 1 or 2 is

$$
\begin{aligned}
M= & (F/2)x-\int(fx)dx \\
= & (\mu_m/\mu_v)(2*10^{-7}i_1i_2/a^2)[(ax-(a^2-b^2)/2)\tan^{-1}(a/b)+(bx)Ln(b)+(a-x) \\
& (b/2)Ln(a^2+b^2)-((x^2+b^2)/2)\tan^{-1}(x/b)-((x^2-a^2+b^2)/2)(\tan^{-1}((a-x)/b))- \\
& (ab/2)Ln(x^2-2ax+a^2+b^2)] \qquad (4)
\end{aligned}
$$

Therefore, the exact solution for plate deflection based on the boundary conditions of $\delta=0$ at $x=0$ and at $x=a$ can be expressed as

$$
\begin{aligned}
\delta= & (4*10^{-7})(\mu_m/\mu_v)(i_1i_2/a)(1/EI)\{(1/12a)[((3b^4-6b^2x^2-x^4)/4)\tan^{-1}(x \\
& /b)+((3(a^4+b^4)-18a^2b^2+8a(3b^2-a^2)x+6(a^2-b^2)x^2-x^4)/4)(\tan^{-1}((a-x) \\
& /b)+((3ab(b^2-a^2)+2b(3a^2-b^2)x-3abx^2)/2)Ln(x^2-2ax+a^2+b^2)+(b^3x)Ln(x^2 \\
& +b^2)]+(1/12)[\tan^{-1}(a/b)-(b/2a)Ln(1+a^2/b^2)]x^3+(1/8a)[(b^2-a^2)\tan^{-1} \\
& (a/b)+(ab)Ln(a^2+b^2)+13ab/6]x^2+(1/48)[3(a^2-b^4/a^2)\tan^{-1}(a/b)-4(b^3/a \\
& +ab)Ln(a^2+b^2)-4(b^3/a+ab)Ln(b)-13ab]x+(1/16a)[(6a^2b^2-a^4-b^4)\tan^{-1}(a \\
& /b)+2ab(a^2-b^2)Ln(a^2+b^2)]\} \qquad (5)
\end{aligned}
$$

For reference, the maximum deflection in terms of the induced magnetic force between plates 1-2, 1-3, 1-4, 1-5, 1-7, and 1-10 at $\mu_m \approx \mu_v$, $i_1=i_2=4000$ amperes, a/t=6.096cm/0.127cm (=2.4"/0.05"), and $EI=11.77\ N\text{-}m^2$ are evaluated and listed in Table 1.

Bending and Shear Stresses

For a plate constructed of an elastic material with a linear stress-strain relationship, the bending (normal) stress for a plate with a rectangular cross section is

$$\sigma_x=M/S \qquad (6)$$

Here, σ_x and S represent, respectively, the plate bending stress (N/m^2) and section modulus (m^3) per unit length (m). For a rectangular plate, $S=1*t^2/6$.

The maximum bending stress according to equation (6) occurs at the midpoint of the plate (i.e. at x=a/2), where the maximum bending moment M can be calculated using equation (4).

As opposed to bending stress, the maximum shear stress appears at both ends of the beam (i.e. at x=0 & x=a), corresponding to the maximum local shear force V, where $V=F/2-\int(f)dx=F/2$. The general shear stress equation for a rectangular cross-sectional plate is $\tau=(V/IL)\int_0^{t/2}(z)dA$, at the neutral axis of z (i.e. z=t/2), from which the maximum shear stress can be derived as

$$\tau_{max}=3V/2A \qquad (7)$$

where τ_{max} is the maximum shear stress (N/m^2), V the maximum local shear force (N), and A=1*t the cross-sectional area (m^2) per unit length.

It is worth emphasizing that although each of the terms F, V, δ, σ, and τ is additive, that is, $F_1=F_{1-2}+F_{1-3}+.....+F_{1-n}$, etc., there is no linear change relative to distance b between plates, simply because of the appearance of transcendental functions of $\tan^{-1}(x)$ and Ln(x) after the integration of the induction force 'f' or bending moment 'fx'. For comparison, both maximum shear and bending stresses due to an induction force of 4000-amp current on each plate are calculated for values of $a=6.096*10^{-2}m$ (=2.4"), $t=1.27*10^{-3}m$ (=0.05"), $A=1.27*10^{-3}m^2$, $S=2.69*10^{-7}m^3$, and $I=1.71*10^{-10}m^4$, and the results are listed in Table 1.

CONCLUSION AND DISCUSSION

Analytical solutions of plate deflection and stresses due to the presence of induced magnetic force are obtained when electrical current is used to generate heat in a plate. Based on field theory which states that strength of the magnetic force among plates is additive, the deflection and stress on one plate can thus be evaluated in terms of forces between the specified plate and all the other plates. In a multiple-plate arrangement as shown in Figure 2.3, the inner plates always feel a smaller total force than do outer plates, due to the opposing forces exerted by plates on opposite side of the middle plate in question; therefore, the maximum deflection occurs at the two outermost plates. The reduction of inter-element spacing between the first and second plates can be described by

(Space Reduction)$_{1-2}$ = δ of plate 1 - δ of plate 2
= δ due to forces between plates 1-2 and 1-7
$\approx$ δ due to force between plates 1 and 2 only
= δ due to force between any 2 adjacent plates

Based on the same analysis, the reduction of the second inter-element spacing can be computed by

(Space Reduction)$_{2-3}$ = δ of plate 2 - δ of plate 3
= δ due to forces between plates 2-4 and 2-7
$\approx$ δ due to force between plates 2 and 4 only
= δ due to force between plates n-1 and n+1

Because of symmetry, the middle plate shows no deflection, as long as a constant and uniform current heats all plates.

From equation (6), the maximum bending stress relative to an ind-uced force resulting from a 4000-amp current on each of the 2nd through 5th plates is $1.77*10^7 N/m^2$, occuring in the middle of the 1st plate. Based on equation (7), the maximum shear stress on the same plate arrangement experiencing a 4000-amp current would be $2.87*10^5 N/m^2$, at either end of the 1st plate. In contrast to the stresses on all inner plates where the induced force can be partially or even totally (middle plate only) cancelled due to 'symmetry', maximum bending and shear stresses are evaluated respectively from the total bending moment and total shear force between plate 1 and all other plates, since induced magnetic force on the outermost plates is the summation of all individual forces when current flow on all plates is unidirectional.

REFERENCES

[1] Jackson J.D., Classical Electrodynamics, 2nd Ed., Chap. 1, 5 & 6, John Wiley & Son, Inc., 1975.

[2] Hague B., Electromagnetic Problems in Electric Engineering, Chap. 13, Oxford Univ. Press, 1929.

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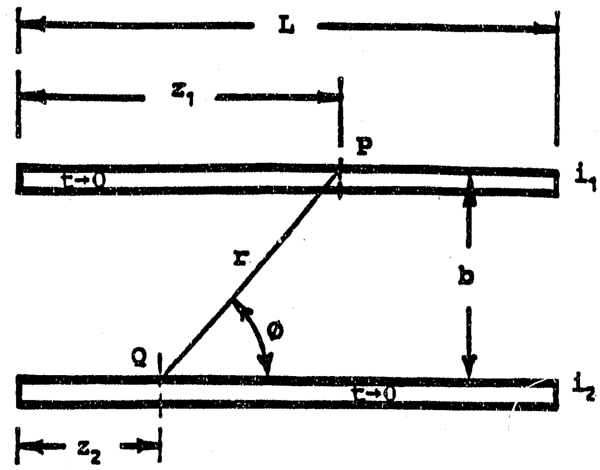
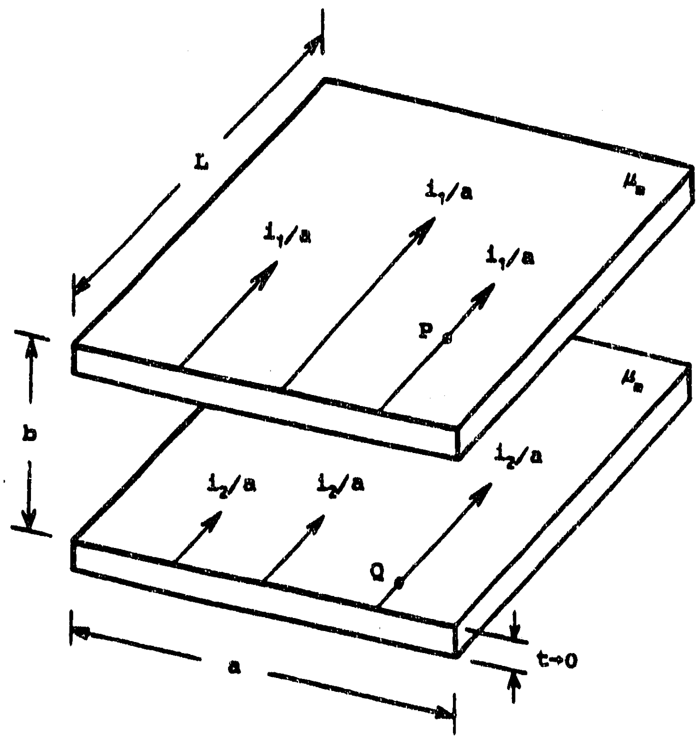
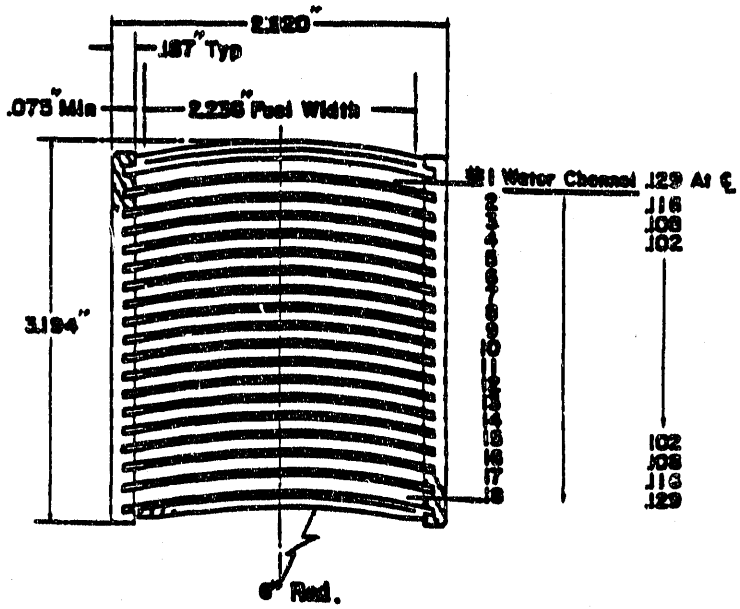


(1) Calculation of Force between Two Equal Parallel Wires

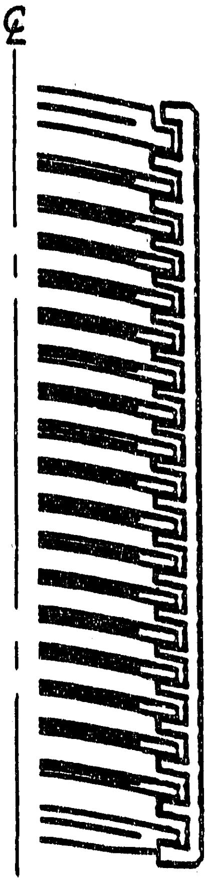


(2) Calculation of Force between Two Equal Parallel Plates with uniformly distributed Currents

Figure 1. 2-D and 3-D Coordinates used for Magnetic Induction Force Calculations



(1) Swaged Fuel-Plate Assembly



(2) Application of Simple Beam Model

(3) Multiple-and-Parallel Simple Plates used for Deflection and Stress Analysis

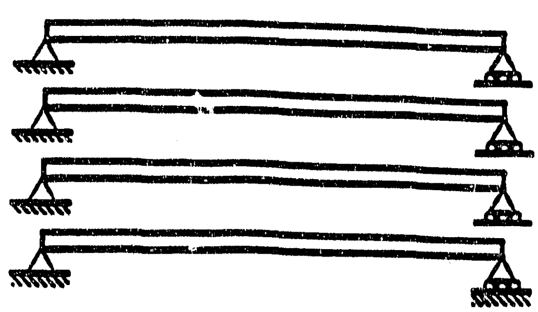


Figure 2. Simple Beam Model used for Plate Deflection and Stress Calculations

TABLE 1

Maximum Deflection and Stresses of the Outermost Plates

Plate Deflection = $\int\int(M/EI)(dx)^2$
Plate Bending Stress = MT/2I = M/S
Plate Shear Stress = 3V/2A = F/2

Inter-element Spacing (cm)	Maximum Deflection (cm)	Maximum Bending Stress (N/m^2)	Maximum Shear Stress (N/m^2)
b=0.254 (force 1-2)	$3.51*10^{-3}$	$4.89*10^{6}$	$8.66*10^{4}$
b=0.762 (force 1-3)	$2.77*10^{-3}$	$4.60*10^{6}$	$7.35*10^{4}$
b=1.143 (force 1-4)	$2.36*10^{-3}$	$4.28*10^{6}$	$6.62*10^{4}$
b=1.524 (force 1-5)	$2.01*10^{-3}$	$3.91*10^{6}$	$6.02*10^{4}$
b=2.286 (force 1-7)	$1.43*10^{-3}$	$3.26*10^{6}$	$5.08*10^{4}$
b=3.429 (force 1-10)	$8.86*10^{-4}$	$2.45*10^{6}$	$4.08*10^{4}$
location	x=a/2	x=a/2	x=0 and x=a

Note: $i_1=i_2=4000$ amp, a=6.096 cm, t=0.127 cm, $E=6.9*10^{-4}$ N/m^2, $A=1.27*10^{-3}$ m^2, $S=2.69*10^{-7}$ m^3, $I=1.71*10^{-10}$ m^4.

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