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MAXIMIZING THE VALUE OF THERMALLY INTEGRATED HYDROELECTRIC GENERATING FACILITIES*

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ABSTRACT

This paper presents a demonstration of a phenomenon known as *hydro-shifting*, which relates to the provision of two sources of electric power (hydro and thermal) to customers. The extent that these resources are used in each period is shown to depend partly on the time of day in which the power is provided. Although the dispatcher could combine thermal resales with hydro power in equal proportions for its customers in both periods, a rationalization is presented that justifies the withholding of some water until the peak period, with the balance of off-peak (baseload) demand satisfied in larger proportions by resales of thermal power. Value is optimized for electrical generation, and environmental externalities are incorporated as minimum or maximum flow limitations. This paper also presents evidence that the optimal degree of hydro-shifting depends on the size of the reservoir adjacent to the dam, with this shifting behavior being more pronounced for larger reservoirs.

INTRODUCTION

The Western Area Power Administration (Western) schedules and sells hydroelectric energy from hydroelectric facilities in the western United States that provide power to the Bureau of Reclamation (BOR), electric utilities, and other customers. The electric power sold by Western is either generated from hydroelectric resources or is provided to its customers principally from thermal sources. In all instances the amount of water to be released is limited by generation capacity or hydrology. Only under extremely wet conditions is there enough water (fuel) for the generators to be baseloaded at full capacity. Hence the dispatching decision can be viewed as a dynamic resource allocation problem. One interesting feature of thermally integrated operations is that there are situations where hydro generation will occur more during peak times than during off-peak periods. In effect, water is stored overnight, and off-peak power is provided through the resale of purchased power. (Western owns no thermal generating resources, although it maintains long-term and short-term contractual agreements for other sources.) Water stored overnight is released in on-peak periods for power generation to maximize the value. This effect occurs, even when the entire contract commitment can be met safely with hydroelectric resources, as a means to increase net revenues via short-term and spot-market sales. Although it appears obvious that load-following deliveries would maximize the value of the resources provided to Western's customers, demonstrating this

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phenomenon as optimizing behavior requires several steps, as shown in this paper. The effect is important because the magnitude of flow fluctuations below the facilities increases as hydro-shifting increases under certain hydrological and market conditions. In turn, the magnitude of the fluctuations are believed to be related to some degree with the environmental impacts attributable to hydroelectric power generation. Although the capturing of these, and other, potential external effects is necessary to understand fully the economic and environmental implications of hydroelectric power generation, the work discussed in this paper focuses on the incentives to hydro-shift from the perspective of the Western dispatcher and the customers. As a result, this paper is presented as a first step in developing a more complete model that will characterize how Western sells hydroelectric power to its customers and the potential for environmental impacts of these activities.

ASSUMPTIONS AND MODEL STRUCTURE

The model presented is a daily model divided between two periods, on-peak and off-peak. The first period is a baseload period when the market price is usually lower. The second period is the on-peak period when the market price for electricity is usually higher. At any point in time, the dispatcher has two sources to meet the contractual obligation: (1) to purchase thermal power and resell to its customers or (2) to release water to generate hydro power and sell this power to customers; the third source is (3) some combination of these means. The dispatcher faces water release constraints, namely minimum flows set by the BOR, and maximum flows limited by generator capacity. In addition, Western is required to sell power in sufficient quantity to meet contractual obligations with its customers.

PRODUCTION

The quantity sold in any period is the sum of thermal power purchased for resale and the amount of hydro power generated by releasing water in the period. If we denote total power sold in period 1 (off-peak, base) as Q_1, and total power sold during period 2 (peak) as Q_2, the electric power provided by Western in each period is given by equations (1) and (2) below:

$$Q_1 = q_1^t + q_1^h(r_1) \tag{1}$$

$$Q_2 = q_2^t + q_2^h(r_2) \tag{2}$$

where q_j^t is the amount of thermal power purchased for resale in period j and $q_j^h(r_j)$ is the amount of hydro power generated and sold in period j, with r_j being the amount of water release required to generate the hydro power sold in period j. We assume that hydro power generation is a function of water release and reservoir capacity and that production is increasing in r_j. (We examine the implications of relaxing this assumption later in this paper.) Since only a limited quantity of water is available for release on any given day, the dispatcher faces a total water release (base release plus peak release) constraint that is expressed by the following weak inequality (where r is the total amount of water available for release for both periods):

$$r \geq r_1 + r_2 \tag{3}$$

Finally, the dispatcher is required to meed contractual obligations for power, so total sales plus purchases for resale must be sufficient to satisfy contract demand requirements in each period. These constraints are represented as follows:

$$Q_1^d \leq q_1^t + q_1^h(r_1) \quad (1')$$

$$Q_2^d \leq q_2^t + q_2^h(r_2) \quad (2')$$

REVENUE, PRODUCTION COSTS, GENERATION COSTS, AND TRANSMISSION COSTS

This version of the model will assume that Western is a price-taker during both periods (i.e., prices are exogenous and do not depend on Western's output). The price that Western charges for its power is an average charge per kilowatt-hour (kWh), limited to cost recovery, and the objective of Western is to maximize the value of its power marketing programs to its customers. As a result, to maximize value, Western behaves as if it is maximizing profits. In this case, "profits" are passed on to its customers as "savings" that result from power purchases or generation displaced by its marketing programs. The average price that Western receives from its customers for this power does not vary with time of day; that is, Western receives the base price (P_1) for power sold during both periods. Were the customers to purchase power on the open market during the peak time of day, they would pay a higher price (P_2) that would reflect expected market conditions. The customers therefore receive a benefit (surplus) from Western in the form of electric power that is cheaper to its customers than would be if the customers were to purchase this power from other sources. This version of the model will assume that peak power is more expensive than off-peak power and will represent this assumption as $P_1 < P_2$. In addition, the short time horizon (i.e., daily) of this model will preclude potential feedbacks that providing this power would have on the market price for electric power. Further, we shall assume that for each period, the demand for electric power is inelastic. These assumptions mean that we can express the revenues to Western (including the surplus conferred upon its customers) as $P_1Q_1 + P_1Q_2 + (P_2 - P_1)Q_2 = P_1Q_1 + P_2Q_2$.

The dispatcher faces costs for exercising both generation options. For thermal power, the dispatcher faces unit generation and transmission (G&T) costs that vary between periods. These costs are denoted by w_1^t for unit G&T costs for off-peak thermal power and w_2^t for unit G&T costs for peak thermal power. In addition, we assume that peak G&T costs are greater than off-peak G&T costs (i.e., that $w_2^t > w_1^t$). In essence, the dispatcher will buy power on the spot market at prevailing prices, and we will assume that the prevailing spot price during peak periods will exceed that for off-peak periods. Hydro G&T costs are given by w^h per unit of power and are assumed to be the same for both periods (i e., $w^h = w_1^h = w_2^h$). We also assume that hydro G&T costs are less than thermal G&T costs in both periods (i.e., $0 < w^h = w_1^h = w_2^h < w_1^t < w_2^t$).

OPTIMIZATION

The decision maker will maximize net revenues (revenue less costs, including the surplus to the customers) with respect to q_1^t, q_2^t, r_1, r_2, λ_1, λ_2, and λ_r, subject to the constraints given in equations (3), (1'), and (2') above. The objective is to maximize:

$$V = P_1(q_1^t + q_1^h(r_1)) + P_2(q_2^t + q_2^h(r_2)) - w_1^t q_1^t - w^h q_1^h(r_1) - w_2^t q_2^t - w^h q_2^h(r_2)$$

$$+ \lambda_1[Q_1^d - q_1^t - q_1^h(r_1)] + \lambda_2[Q_2^d - q_2^t - q_2^h(r_2)] + \lambda_r[r - r_1 - r_2] \tag{4}$$

The first-order Kuhn-Tucker conditions on the thermal power control variables are as follows:

$$\frac{\partial V}{\partial q_1^t} = P_1 - w_1^t - \lambda_1 \leq 0 \tag{5}$$

$$q_1^t \frac{\partial V}{\partial q_1^t} = q_1^t(P_1 - w_1^t - \lambda_1) = 0 \tag{6}$$

$$\frac{\partial V}{\partial q_2^t} = P_2 - w_2^t - \lambda_2 \leq 0 \tag{7}$$

$$q_2^t \frac{\partial V}{\partial q_2^t} = q_2^t(P_2 - w_2^t - \lambda_2) = 0 \tag{8}$$

The first-order conditions on the water release variables, r_1 and r_2, are as follows:

$$\frac{\partial V}{\partial r_1} = (P_1 - w^h - \lambda_1)\frac{\partial q_1^h}{\partial r_1} - \lambda_r \leq 0 \tag{9}$$

$$r_1 \frac{\partial V}{\partial r_1} = r_1[(P_1 - w^h - \lambda_1)\frac{\partial q_1^h}{\partial r_1} - \lambda_r] = 0 \tag{10}$$

$$\frac{\partial V}{\partial r_2} = (P_2 - w^h - \lambda_2)\frac{\partial q_2^h}{\partial r_2} - \lambda_r \leq 0 \tag{11}$$

$$r_2 \frac{\partial V}{\partial r_2} = r_2[(P_2 - w^h - \lambda_2)\frac{\partial q_2^h}{\partial r_2} - \lambda_r] = 0 \tag{12}$$

Finally, the first-order conditions on the constraint variables are as follows:

$$\frac{\partial V}{\partial \lambda_1} = Q_1^d - q_1^t - q_1^h(r_1) \leq 0 \qquad (13)$$

$$\lambda_1 \frac{\partial V}{\partial \lambda_1} = \lambda_1(Q_1^d - q_1^t - q_1^h(r_1)) = 0 \qquad (14)$$

$$\frac{\partial V}{\partial \lambda_2} = Q_2^d - q_2^t - q_2^h(r_2) \leq 0 \qquad (15)$$

$$\lambda_2 \frac{\partial V}{\partial \lambda_2} = \lambda_2(Q_2^d - q_2^t - q_2^h(r_2)) = 0 \qquad (16)$$

$$\frac{\partial V}{\partial \lambda_1} = r - r_1 - r_2 \geq 0 \qquad (17)$$

$$\lambda_r \frac{\partial V}{\partial \lambda_r} = \lambda_r(r - r_1 - r_2) = 0 \qquad (18)$$

FORMS OF SOLUTIONS

This type of analysis admits two possible solution forms. The first is an interior solution characterized by all endogenous variables having positive values at the optimum (i.e., the dispatcher relies on both thermal power resales and hydro generation in both periods). The second is a corner solution, in which at least one of the endogenous variables will take on a zero value at the optimum (e.g., no thermal power is sold in one of the periods). An interesting feature of this problem is that an interior solution requires that hydro generation be increasing in water release but at a decreasing rate; in other words, the $q_j^h(r_j)$ have positive first derivatives but negative second derivatives, with respect to r_j for $j = 1,2$. The latter part of this section will demonstrate that a unique interior solution does not exist to this problem when hydro power production is linear in the amount of water released.

The assumption of diminishing returns to the water release variable would be satisfied for dams that are adjacent to small reservoirs, where water release would not be compensated for by inlets into the reservoir that would serve to maintain a constant level of water pressure as water is passed through the generators. As a result, water release over a fairly short period of time will reduce the head of the dam, thereby reducing the amount of power that can be generated per unit of water release. For larger reservoirs that receive inflows of water on a continual basis (or where daily releases have a small impact on reservoir levels), an assumption of constant returns to water releases appears to be more reasonable. This section will examine both types of solutions, focusing on how the existence of each solution depends on what is assumed about hydro generation.

We shall first examine this problem under the assumption that a unique interior solution exists, with q_1^t and q_2^t both being strictly positive, which is implied by equations (5) and (7) holding exactly. Equations (5) and (7) can be rewritten as equations (5') and (7'), which provides for an interpretation of the λs.

$$P_1 - w_1^t = \lambda_1 \tag{5'}$$

$$P_2 - w_2^t = \lambda_2 \tag{7'}$$

These expressions simply state how much net revenues increase if the demand constraints are relaxed by one unit, expressed in terms of thermal sales. In terms of allocating thermal purchases between the two periods, these equations can be further manipulated to yield an additional first-order condition that indicates that the ratio of the net returns between off-peak and peak thermal purchases will correspond to the rate at which total power sales can be substituted between periods, in a manner that is consistent with profit maximization.

$$\frac{P_1 - w_1^t}{P_2 - w_2^t} = \frac{\lambda_1}{\lambda_2}$$

The first-order conditions on the water release variables, r_1 and r_2, as given by equations (9) through (12) below, can be interpreted in a fashion analogous to the interpretation of equations (5) through (8) above. Assuming an interior solution (i.e., $r_1 > 0$, $r_2 > 0$, $\lambda_r > 0$), equations (9) and (11) can be rearranged to form the following expression:

$$\frac{P_1 - w^h - \lambda_1}{P_2 - w^h - \lambda_2} = \frac{\frac{\partial q_2^h}{\partial r_2}}{\frac{\partial q_1^h}{\partial r_1}} \tag{19}$$

This expression defines the rate at which hydro-generated power will be substituted between peak and off-peak periods. The right-hand-side is analogous to the Marginal Rate of Technical Substitution (MRTS) between inputs, while the left-hand-side of this expression indicates the relative rates of return to water releases between the base and peak periods.

First, equations (5') and (7'), together with the assumption that $w_2^t > w_1^t$, imply that $P_2 - \lambda_2 > P_1 - \lambda_1$. This result, together with equation (19) above, will imply that:

$$\frac{\partial q_2^h}{\partial r_2} < \frac{\partial q_1^h}{\partial r_1} \tag{20}$$

A sufficient condition for this inequality being satisfied is that there are diminishing returns to hydro generation. This would then imply that $r_2 > r_1$, suggesting that at least some water will be withheld from release in period 1 for release in period 2.

In contrast to the interior solution just discussed, another possible outcome is a corner solution. For dams that are adjacent to large reservoirs, water release over the course of a day will have no significant impact on the size of the head, meaning that over these short periods, hydro power generation will be approximately linear in water release. This will mean that a unique interior solution to this problem will not exist, as will be demonstrated in the remainder of this section. Instead, linear hydro generation technology will result in a corner solution, a result that can be interpreted as a stronger form of hydro-shifting than exists when conditions are sufficient for an interior solution to exist.

To examine this in more detail, consider again our first-order conditions, along with the relationship that we derived from these conditions given in equation (19) above. Hydro generation that is linear in water release implies that for both $r_j > 0$, equation (20) will hold with equality, implying that both numerator and denominator of (19) be equal, or:

$$P_1 - w^h - \lambda_1 = P_2 - w^h - \lambda_2$$

Assuming that our optimum is characterized by both equations (5) and (7) holding with equality (i.e., that $q_j^t > 0$ in both periods), we obtain the following:

$$w_1^t - w^h = w_2^t - w^h$$

which implies further that:

$$w_1^t = w_2^t$$

a result that is contradicted by our assumption at the beginning of this paper that $w_2^t > w_1^t$. As a result, hydro generation that is linear in water release appears to be inconsistent with an interior solution to this problem. This result leads us to consider feasible corner solutions to this problem that are consistent with the theme of this paper — namely, the tendency for water release to be withheld until the peak period. One such solution is that characterized by no water being released in the first period ($r_1 = 0$) and no thermal power being sold in the second period ($q_2^t = 0$). Consider again the first-order conditions in thermal sales and water releases given in equations (5) through (12). No water being released in period 1 ($r_1 = 0$) means that equation (9) will not hold while equation (11) will hold with strict equality. In particular, we obtain:

$$(P_1 - w^h - \lambda_1)\frac{\partial q_1^h}{\partial r_1} < \lambda_r$$

$$(P_2 - w^h - \lambda_2)\frac{\partial q_2^h}{\partial r_2} = \lambda_r$$

With hydro power generation being linear in water release, the strict inequality and equation above can be rearranged and simplified to form the following expression:

$$P_1 - \lambda_1 < P_2 - \lambda_2$$

This condition simply tells us that the net value to hydro generation during the peak period exceeds that during the base period. The first-order conditions on the thermal sales control variables, from equations (5) and (7), with no thermal power sold in the second period, reduce to the following:

$$P_1 - w_1^t - \lambda_1 = 0 \Rightarrow P_1 - \lambda_1 = w_1^t$$

$$P_2 - w_2^t - \lambda_2 < 0 \Rightarrow P_2 - \lambda_2 < w_2^t$$

These conditions can then be further combined to form the following relationship between thermal generation and transmission costs in both periods and the relative returns to hydro generation (i.e., water releases) in the two periods:

$$w_1^t = P_1 - \lambda_1 < P_2 - \lambda_2 < w_2^t$$

This condition has the following interpretation. The higher cost of thermal generation and transmission in period 2, relative to the net return to hydro generation and transmission, induces the decision maker to sell only hydro power in the peak period. In addition, the requirement that sufficient power be sold during the base period, in order to satisfy the demand requirement, induces the decision maker to sell only thermal power during the first period. Together, these behaviors combine to represent a corner solution to this problem. Furthermore, they are consistent with observed behavior — namely, that to maximize the value of hydro resources, Western stores water during the first period for release during the second period, and it sells thermal power during the base period to satisfy the demand requirements of its customers.

CONCLUSIONS

This paper has presented a brief demonstration of behavior that has been observed — namely, that Western stores water during off-peak periods, satisfying baseload demand with thermal purchases. During peak periods, water is released, thereby generating sufficient electric power to satisfy the demand requirement during that period. That the nature of the solution depends on the technology of hydro generation does not seem too surprising. Linear generation of hydro power means that generation is constant cost; diminishing returns to water release, as found in the case that gave the interior solution, would suggest hydro generation to be an increasing cost phenomenon. Thus the opportunity cost of additional hydro generation does not increase with additional water releases, inducing the decision maker toward a more pronounced use of hydro generation to meet the demand requirement. Possible extensions to this model include expanding the time horizon of the model from daily to monthly, which would allow the optimization problem to incorporate preferences for water allocations across days rather than between peak and non-peak times during the course of a single day. A further extension to this model would be to relax the assumption that prices between peak and non-peak periods, as well as G&T costs for thermal power, are determined exogenously. This would allow a consideration of the direct impact that hydro sales have on the market price for power and of the indirect effect they have on the spread in prices between peak and non-peak periods.

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