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Soil-Structure Interaction Effects for Laterally Excited Liquid-Tank System*

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Abstract

Following a brief review of the mechanical model for liquid-storage tanks which permits consideration of the effects of tank and ground flexibility, and lateral and rocking base excitations, the effects of both kinematic and inertia interaction effects on the response of the tank-liquid system are examined and elucidated. The free-field motion is defined by a power spectral density function and an incoherence function, which characterizes the spatial variability of the ground motion due to the vertically incident incoherence waves. The quantities examined are the ensemble means of the peak values of the response. The results are compared with those obtained for no soil-structure interaction and for kinematic interaction to elucidate the nature and relative importance of the two interactions. Only the impulsive actions are examined, the convective actions are for all practical purposes unaffected by both kinematic and inertia interactions. It is shown that the major reduction of the response is attributed to inertia interaction.

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Introduction

It is generally recognized that the responses to an earthquake of a structure with and without considering the flexibility of the supporting soil may differ significantly. This difference is known as the effect of soil-structure interaction. According to Veletsos [1], this interaction may be considered to consist of two effects, the effect of kinematic interaction and the effect of inertia interaction. The kinematic interaction is caused by the inability of the rigid foundation to conform to the generally nonuniform, spatially varying ground motion, and the inertia interaction is caused by the coupling between the vibrating structure, its foundation, and supporting soils. For convenience, the kinematic interaction is further divided into two effects. The first one is the result of the propagation of plane waves and is known as the wave passage effect. This effect, normally described deterministically, has been the subject of numerous previous studies [2-6] for building structures. The second effect is the consequence of wave incoherence which may be a result of the inhomogeneities along the path of the waves or from incidence of waves from different directions. This effect, however, is specified stochastically by a power spectral density function (psdf) and an incoherence function [7-10]. Comprehensive studies of the wave incoherence effect on the response of building structures were reported in recent papers by Veletsos and Prasad [11] and Veletsos and Tang [12]. These studies indicate that the effect of KI always reduces the structural response, and this reduction may be substantial depending on the frequency content of the ground motion and the properties of the supporting soil.

The response to ground shaking of liquid containing tanks has been the subject of numerous studies in recent years. The reader is referred to Veletsos [13] for an excellent state-of-the-art review. In early analyses of the problem the tank was presumed to be rigid and anchored to a rigid base, whereas subsequent analyses accounted for the flexibility of the tank

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wall but again considered the tank to be anchored to a nondeformable medium. In recent years, the effects of ground flexibility and of the associated interaction between the vibrating tank-liquid system and supporting medium have been studied for vertically excited systems [14] and for laterally excited systems [15] with the exclusion of the kinematic interaction effects. The construction in recent years of large capacity storage tanks, with diameters of the order of about 300 ft or more, has increased the importance of the consideration of the effects of the spatial variability of the ground motion, i.e., the kinematic interaction. As a result of this kinematic interaction, the tank foundation experienced not only the horizontal excitation, but also the torsional excitation. However, since the liquid is assumed to be inviscid, it is believed that the hydrodynamic pressure induced by this torsional motion is insignificant; therefore, the response due to the torsional motion is neglected in this study.

The objectives of this paper are to study the effects of kinematic and inertia interactions on the seismic response of liquid containing vertical cylindrical tanks subjected to a horizontal component of ground shaking, and to identify the relative importance of the two interactions. However, primary emphasis is placed on the kinematic interaction effects.

In this paper the free-field ground motion is specified stochastically by a power spectral density function and an incoherence function which characterizes the spatial variability of the motion. The quantities examined are the ensemble means of the peak values of the response; only the impulsive actions are considered. The analysis is based on the assumption that, in its fixed-base condition, the tank-liquid system responds in its fundamental mode of vibration as a single-degree-of-freedom oscillator. Accordingly, terms such as natural frequency, damping and modal mass refer to the fixed-base fundamental mode of vibration of the system. As a result of this assumption, the mechanical model presented in Ref. 15 can be adopted here to represent the tank-liquid system.

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System Considered and Assumptions

The system investigated is an upright, circular cylindrical tank of radius 'a', which is filled with liquid to a height H and is supported through a rigid circular mat of radius R at the surface of a homogeneous, elastic halfspace (R=a is assumed in this study). The tank wall is assumed to be of uniform thickness, h, and clamped to the base. The liquid is presumed to be incompressible, inviscid and free at its upper surface. Only linear actions are examined. The mass densities of the tank, liquid and soil are denoted by ρ, ρ_l and ρ_s, respectively; the modulus of elasticity and Poisson's ratio for the tank material are denoted by E and ν; and the shear modulus of elasticity and Poisson's ratio for supporting medium are denoted by G_s and ν_s, respectively. Points for the tank and the containing liquid are specified by the cylindrical coordinate system, r, θ, z, the original of which is taken at the center of the base.

The free-field ground motion for all points of the foundation-soil interface is considered to be a uni-directional excitation directed parallel to the horizontal $\theta=0$ axis, as shown in Fig. 1, with the detailed histories of the motions varying from point to point. Such motions may be induced by horizontally polarized incoherence shear waves propagating vertically or at an angle with the vertical, α_v. The intense portions of the motions are represented by a stationary random process of limited duration, t_o, and a space-invariant, local psdf, $S_g=S_g(\omega)$, in which ω=the circular frequency of the motions. The spatial variability of the motions is defined by a cross psdf, $S\left(\vec{r}_1, \vec{r}_2, \omega\right)$, in which $\vec{r}_1$ and $\vec{r}_2$ are the position vectors for two arbitrary points. This function is taken in the form suggested by Harichandran and Vanmarcke [16] as

$$S\left(\vec{r}_1, \vec{r}_2, \omega\right) = T\left(|\vec{r}_1 - \vec{r}_2|, \omega\right) \exp\left[-i\omega \frac{(d_1 - d_2)}{C}\right] S_g(\omega) \tag{1}$$

in which T, referred to as the incoherence function, is a dimensionless, decreasing function of

$|\vec{r}_1 - \vec{r}_2|$; $i = \sqrt{-1}$; d_1 and d_2 = the components of $\vec{r}_1$ and $\vec{r}_2$ in the direction of propagation of the wave front [see Fig. 1b]; and c = the apparent horizontal velocity of the front. The latter quantity is related to the angle of incidence of the waves, α_v, and shear wave velocity, V_s, by

$$C = \frac{V_s}{\sin\alpha_v} \tag{2}$$

The product of the exponential term in Eq. 1 and S_g represents the wave passage effect, whereas the product TS_g represents the effect of ground motion incoherence. The peak value of T is unity and occurs at $\vec{r}_1 = \vec{r}_2$.

Several different expressions have been suggested for the incoherence function [e.g., Refs. 7, 9, 16], and there is no general agreement at this time on the form that may be the most appropriate for realistic earthquakes. In this study, the single-parameter, second order function recommended by Mita and Luco [9] is used,

$$T\left(|\vec{r}_1 - \vec{r}_2|, \omega\right) = \exp\left[-\left(\frac{\gamma\omega|\vec{r}_1 - \vec{r}_2|}{V_s}\right)^2\right] \tag{3}$$

in which γ is a dimensionless factor, taken between zero and 0.5.

The response quantities examined include the hydrodynamic base shear and the bending moments at sections immediately above and below the tank base. The base moment in the tank, in combination with the ordinary beam theory, is normally used to evaluate the axial forces included in the tank wall, whereas the bending moment beneath the tank base is used in the design of the foundation.

Following the approach used in previous analyses of rigidly supported tanks, each response quantity is evaluated by the superposition of two components: (1) an impulsive component, which represents the effect of the part of the liquid that may be considered to move in unison with the tank wall as a rigidly attached mass, and (2) a convective component, which represents the action of the part of the liquid that experiences the sloshing motion. The impulsive component of the solution satisfies the actual boundary conditions along the lateral and bottom boundaries of the tank, and the condition of zero hydrodynamic pressure at the mean liquid surface level; as a result, it does not account for the effect of the surface waves associated with the sloshing action of the liquid. The convective component of the solution effectively corrects for the difference between the actual boundary conditions at the top and the one considered in the development of the impulsive solution.

Because it is associated with actions of significantly lower frequencies than the natural frequencies of the tank-liquid system and the dominant frequencies of the excitation, the convective part of the solution is unaffected by the flexibilities of the tank wall and supporting medium, and may, as demonstrated in [15], neglect the effect of inertia interaction. In addition, based on the study in Ref. 11 which shows that the kinematic interaction effect can be neglected for low frequency systems, one may conclude that the convective component is also unaffected by the kinematic interaction. As a result of this reasoning, the convective effects may be computed from the well established solution for rigid tanks (see, for example, Ref. 13).

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Mechanical Model

Based on the study presented in Ref. 15, the tank-liquid system subjected to a horizontal

base acceleration, denoted by $\ddot{x}(t)$, and a rocking base acceleration, denoted by $\ddot{\phi}(t)$, may be

represented by the model shown in Fig. 2. The mass m in this model is supported through a

flexible cantilever of height h' on a rigid horizontal member which has no mass, $m_b=0$, but

possesses a mass moment of inertia I_b about a centroidal axis normal to the plane of the paper.

The properties of the cantilever are considered to be such that the fixed-base natural frequency

of the model and the associated damping factor are equal to f_i the fundamental frequency of the

tank-liquid system, and ζ, the damping factor associated with the fundamental mode of vibration

of the tank-liquid system. Let u(t) be the deformation of the mass relative to its moving base;

then, it is easy to show that u(t) is related to the ground accelerations by the equation:

$$u(t) = \frac{-1}{\overline{\omega}_i} \int_0^t \left[\ddot{x}(\tau) + h'\ddot{\phi}(\tau)\right] \exp\left[-\zeta\omega_i(t-\tau)\right] \sin\left(\overline{\omega}_i(t-\tau)\right) d\tau \qquad (4)$$

in which $\omega_i=2\pi f_i$ and $\overline{\omega}_i=\omega_i\sqrt{1-\zeta^2}$.

Also, let A(t) be the pseudo-acceleration function defined by

$$A(t) = -\omega_i^2 u(t) \qquad (5)$$

than, it has been shown [12] that the hydrodynamic base shear, Q(t), can be evaluated by the

following equation:

$$Q(t) = mA(t) \qquad (6)$$

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and the bending moment induced at a section immediately above the tank base, M(t), is given by

$$M(t) = mhA(t) \qquad (7)$$

in which h may be interpreted as the height at which the base shear must be concentrated to yield the correct M(t). Finally, the foundation bending moment, M'(t), can be computed approximately by

$$M'(t) = mh'A(t) \qquad (8)$$

in which h' is the height of the model shown in Fig. 2.

Note that the response quantities given by Eqs. 6, 7 and 8 are controlled by the pseudo-acceleration A(t). The ensemble mean of the peak values of A(t) over a duration t_o will be denoted by $\overline{A}$. In this paper, the ratio of $\overline{A}$ to the mean of the maximum free-field ground motion, $\ddot{x}_g$, will be examined to reveal the effect of kinematic and inertia interactions on the response of the tank-liquid system.

The values of $m/m_l (m_l = \pi RH\rho_l =$ total liquid mass), h/H, h'/H, and $I_b/m_l R^2$ for steel tanks with the value of t/a taken as 0.001 filled with water (i.e., $v=0.3$ and $\rho_l/\rho=0.127$) and values of H/a in the range of 0.3 to 3.0 are listed in Table 1.

The values of coefficient C which is related to f_i by the equation

$$f_i = \frac{C}{2\pi} \sqrt{\frac{E}{\rho}} \cdot \frac{1}{H} \qquad (9)$$

are also listed in Table 1.

Analysis of System

The equations presented in this section are quoted either directly or with appropriate modifications [from Ref. 11] to fit the system considered herein. The reader is referred to that reference for detailed derivations of the equations.

The local psdf considered in this paper is taken in the form

$$
S_g = \begin{cases} \dfrac{f^4}{0.5+f^4}\left(1 - \dfrac{f^2}{f_c^2}\right) S_o & \text{for } f \leq f_c \\[2ex] 0 & \text{for } f > f_c \end{cases} \tag{10}
$$

in which S_o=a constant; f=the exciting frequency in cps; and f_c=the cut-out frequency, taken as 15 cps. The same form of psdf was used in the studies by Pais and Kausel [17] and Veletsos and Prasad [11].

Let $\ddot{x}_g$ be the mean of the absolute maximum peaks of the acceleration traces characterized by Eq. 8. This value can be evaluated from Der Kieureghian's empirical expressions [18] summarized in the appendix, considering the duration of the intense portion of the excitation to be t_o=20 sec. The resulting value is $\ddot{x}_g = 26.173 \ S_o$.

Let S_u be the psdf of u(t). It has been shown in Ref. 11 that S_u is related to S_g by the equation given by

$$
S_u = |H_u|^2 |T_f|^2 \ S_g \tag{11}
$$

for the effect of kinematic interaction, and by the equation

$$
S_u = |T_u|^2 |H_u|^2 |T_f|^2 \ S_g \tag{12}
$$

for the effect of soil-structure interaction. In these two equations, the vertical bars indicate the modulus of the enclosed quantity, and T_f is given by

$$T_f = \frac{1}{b_o} \sqrt{1-\exp\left(-2b_o^2\right)\left(I_o\left(2b_o^2\right) + I_1\left(2b_o^2\right)\right)} \qquad (13)$$

where $b_o = \gamma\omega R/V_s$, I_o and I_1 are modified Bessel functions of the first kind of the order 0 and 1, respectively. Equation 13 is for the case of $\gamma=0$; herefore, it represents the effect of wave incoherence. For the case of $\gamma=0$, i.e., the wave passage effect, the T_f given by Eq. 13 is replaced in the following equation:

$$T_f = 2\frac{J_1\left(C_o\right)}{C_o} \qquad (14)$$

where $C_o = \sin\left(\alpha_v\right)\cdot\frac{\omega R}{V_s}$. In Eqs. 13 and 14, T_f is obtained by averaging the incoherence function defined by Eq. 3 over the area of the soil-foundation interface.

In Eq. 10, H_u is the complex response function of a single-degree-of-freedom system, and function T_u represents the influence of the flexibility of the supporting medium. The expressions for H_u and T_u can be found in Ref. 11. In deriving the expression for T_u, the impedance functions for the disk foundation given by Veletsos and Verbic [14] were used. It is worthwhile to note that if T_f in Eq. 12 is set equal to one, one obtains the equation for considering the effect of inertia interaction.

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After S_u is obtained by either Eq. 9 or Eq. 10, the mean peak values of u(t), denoted by $\bar{u}$, are computed from Der Kiureghian approximation, and the desired quantity, $\bar{A}$, is calculated from the equation

$$\bar{A} = \omega^2 \bar{u} \tag{15}$$

Response of System

The data presented in this section are for steel tanks presented in Table 1; the radius of the foundation is considered to be equal to the radius of the tank and is taken as 100 ft; the foundation mass is presumed to be negligible; Poisson's ratio for the tank material and supporting soil are taken as 0.3 and 1/3, respectively; and the damping factor for the tank-liquid system in its fixed-base condition is taken as ζ=0.02.

Kinematic Interaction:

(a)   Wave Incoherence Effect: The values of $\bar{A}/\ddot{x}_g$ are plotted in Fig. 3 as a function of H/R for values of τ_l, the dimensional effective transient time defined by

$$\tau_l = \gamma \frac{R}{V_s} \tag{16}$$

equal to 0, 0.025 and 0.05. It is clear that the kinematic interaction effect reduces the magnitude of the response in whole range of values of H/a considered, and the reduction increases as the values of τ increases. It is also observed that the reduction is more pronounced for short, broad tanks than for tall, slender tanks, and the reduction may even be neglected for tall, slender tanks combined with low values of τ_l. This trend is not surprising and should be expected since it has

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been shown [11,12] that the kinematic interaction effect is important for high-frequency systems and inconsequential for low-frequency systems, and the data in Table 1 indicates that the tall tank is low-frequency system, whereas the short tank is a high-frequency system.

(b) Wave Passage Effect: Presented in Fig. 4 are the curves for three different values of τ_w defined by

$$\tau_w = \sin\alpha_v \cdot \frac{R}{V_s} \tag{17}$$

τ_w represents the time necessary for a wave train to pass through the foundation.

One observes that the general trends of the reduction revealed in Fig. 4 are similar to those in Fig. 3. This observation indicates that ground motion incoherence and wave passage effects may be interchangeable. This possibility has been suggested by Luco and Wong [8] and latter by Veletsos and Prasad [11] from the examination of the building structures.

Even though only one value of R (= 100 ft) is considered herein, it is felt that there is no need to examine the response for other values of R since the effect of changing R is the same as changing the frequency of the liquid-tank system. If R increases, then the frequency of liquid-tank system decreases, and more reduction on the response will have as the result of KI effect. The opposite argument applies for decrease of R.

Total Soil-Structure Interaction Effects: Figure 5 presents the values of $\overline{A}/\ddot{x}_g$ as a function of H/a for γ=0.2 and vs=1500 ft/sec. The solutions are displayed in this figure: (a) making no provision for soil-structure interaction, (b) providing only for the kinematic interaction effects, and (c) providing for both kinematic and inertia interaction effects as indicated as total SSI in the figures. The difference between solutions (b) and (c) is the result of inertia interaction effects. The following trends are observed in this figure:

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1.   Like kinematic interaction (KI), inertia interaction (II) reduces the response of the system in whole range of H/a considered. The reduction due to II is more pronounced for short, broad tanks than for tall, slender tanks; the reason of this trend is that the base rocking motion which has less radiation damping is more prominent for tall tanks.

2.   The II effects are generally more important than the KI effects for the cases considered herein.

Presented in Fig. 6 are the solutions for the effects of wave passage for the same three conditions considered in Fig. 5. Horizontally propagating waves ($\alpha_v=90°$) are considered in obtaining the solutions for this figure. One may observe that, in general, the trends in Fig. 6 are the same as those in Fig. 5 except that for short broad tanks in Fig. 6, KI seems to have more reduction than II effect does on the response of the liquid-tank system.

Conclusions

Kinematic and inertia interactions may affect significantly the seismic response of liquid containing, vertical cylindrical tanks. Both interactions reduce the impulsive components of the response, but has a negligible effect on the convective components. Both interaction effects are more pronounced for short, broad tanks than for tall, slender tanks. For low values of τ, the kinematic interaction effect may even be neglected.

Even though this study is based on the assumption that, in its fixed-base condition, the tank-liquid system responds in its fundamental mode of vibration as a single-degree-of-freedom system, it is believed that this study provides the general picture of the kinematic interaction effect on the seismic response of the tank-liquid system. As for the accuracy of this assumption on the effect of inertia interaction, the contribution of the higher modes of vibration to the response has been assessed in a recent paper by Veletsos and Tang [20]. It shows that this

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assumption gives excellent results for the tanks with H/a < 1.5. As for tanks with H/a > 1.5, the contributions of the higher modes are not particularly important, and may be evaluated without the consideration of the effects of soil-structure interaction.

The stochastical specification of the wave incoherence might scare away many engineers who are not familiar with the random vibration theory. There has been an attempt to assess this wave incoherence effect by a deterministic approach [12]. The data presented herein indicate that the effect of wave incoherence on the response of tank-liquid system may be assessed by an equivalent wave passage effect which can be analyzed by a deterministic approach. This finding offers a promising alternative to bypass the use of the stochastic approach. Currently this is the subject of ongoing research being carried out by the author.

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Table 1. Properties of Mechanical Model for Steel Tank with h/R = 0.001, ρ_ℓ/ρ_s = 0.127 and ν = 0.3

H/R	C	M/M_ℓ	h/H	h'/H	$I_b/M_\ell R^2$
0.3	0.0600	0.178	0.386	2.528	0.129
0.4	0.0666	0.241	0.385	1.841	0.106
0.5	0.0719	0.304	0.387	1.428	0.088
0.6	0.0762	0.365	0.392	1.163	0.075
0.7	0.0799	0.421	0.397	0.984	0.065
0.8	0.0829	0.472	0.403	0.859	0.057
0.9	0.0855	0.516	0.410	0.770	0.050
1.0	0.0875	0.554	0.417	0.706	0.045
1.2	0.0903	0.613	0.432	0.625	0.038
1.4	0.0916	0.652	0.448	0.581	0.033
1.6	0.0917	0.676	0.464	0.558	0.028
1.8	0.0910	0.691	0.478	0.547	0.025
2.0	0.0896	0.700	0.492	0.543	0.023
2.5	0.0849	0.706	0.523	0.549	0.018
3.0	0.0792	0.703	0.548	0.563	0.013
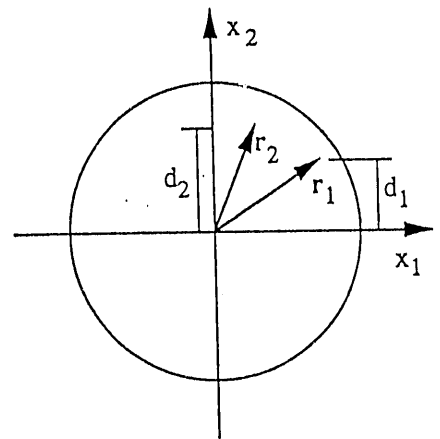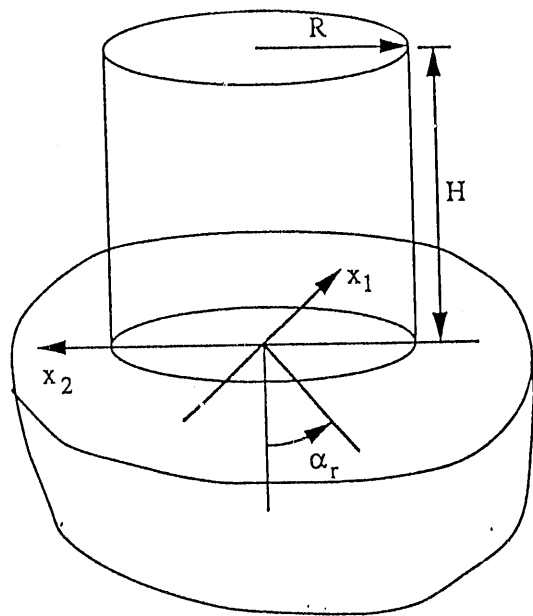
Fig. 1 System Considered

m

h

f_i, I_i

$m_b = 0, I_b$

$\ddot{x}_g(t)$

$\ddot{\varphi}(t)$
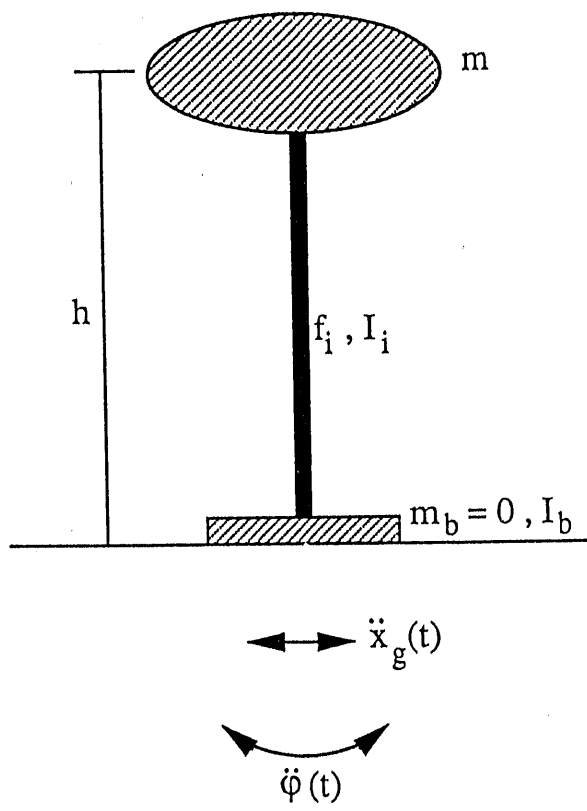
Fig. 2 Mechanical Model
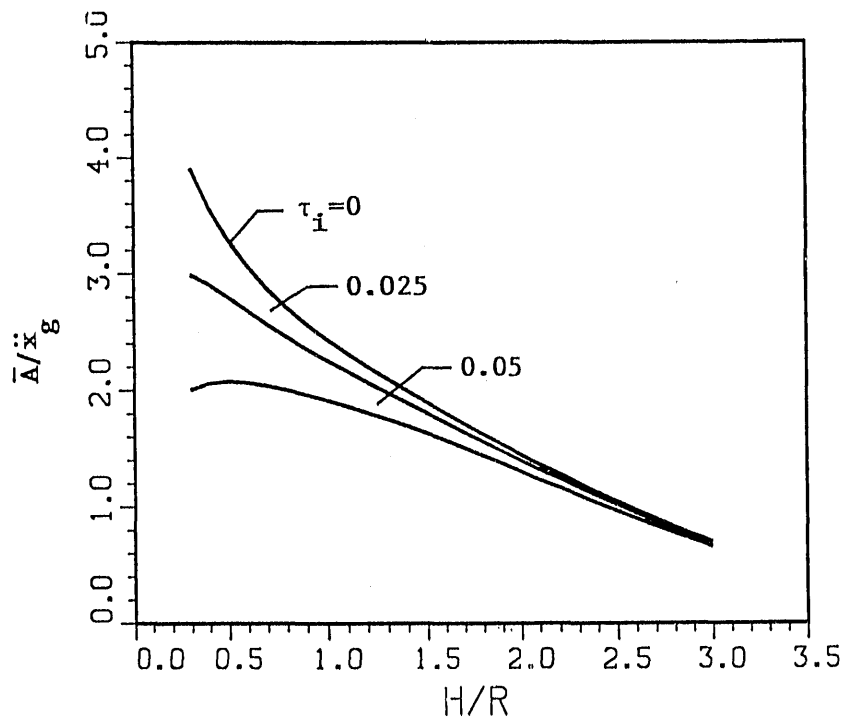
Fig. 3.   Wave Incoherence Effect
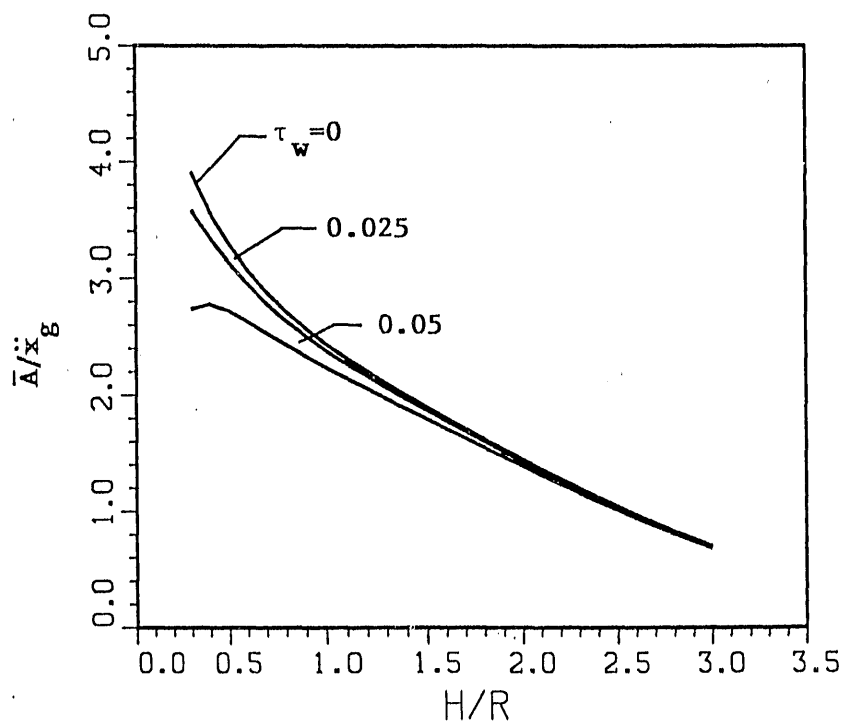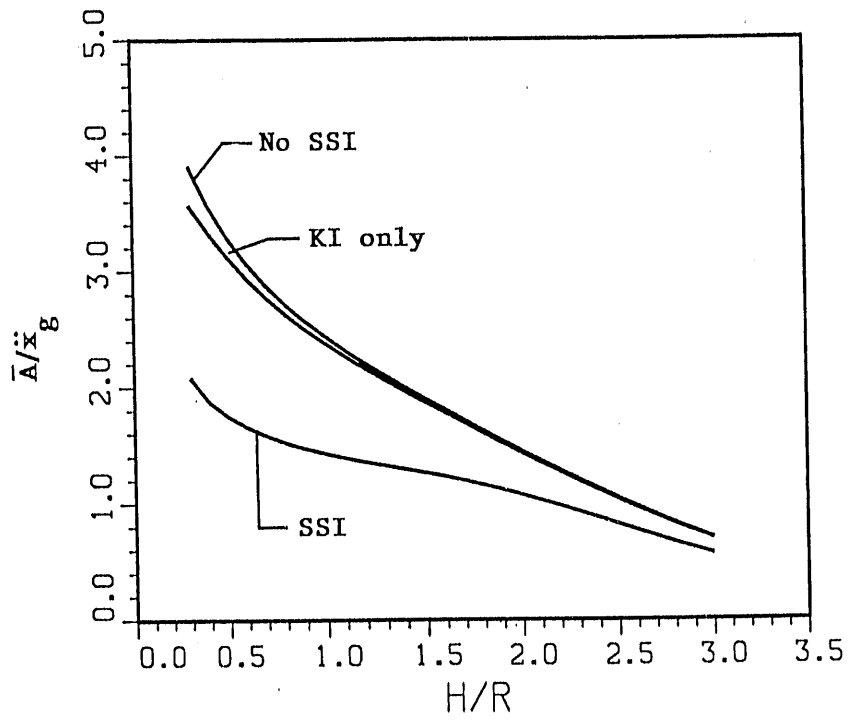
Fig. 4. Wave Passage Effect

Fig. 5. Comparison of Effects of Wave Incoherence
and Inertia Interaction

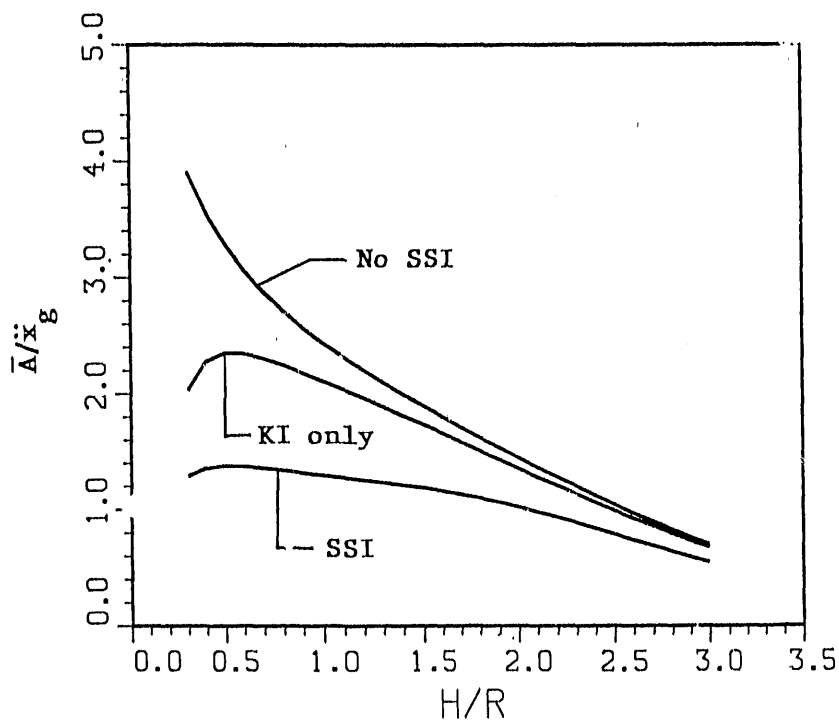Fig. 6. Comparison of Effects of Wave Passage
         and Inertia Interaction

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