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INSTABILITY HEATING OF THE HDZP

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ABSTRACT

We present a model of dense Z-Pinch heating. For pinches of sufficiently small diameter and high current, direct ion heating by **m=0** instabilities becomes the principal channel for power input. This process is particularly important in the present generation of dense micro-pinches (e.g., **HDZP-II**) where instability growth times are much smaller than current risetimes, and a typical pinch diameter is several orders smaller than that of the chamber. Under these conditions, **m=0** formation is not disruptive: the large E_z field reconnects the instability cusps externally, after which the ingested magnetic flux decays into turbulent kinetic energy of the plasma. The continuous process is analogous to boiling of a heated fluid.

A simple analysis shows that an equivalent resistance

$$R_t = \frac{\ell}{4\sqrt{N m_i}} \left(\frac{\mu_0}{\pi}\right)^{3/2} \frac{I}{r}$$

appears in the driving circuit, where **I** is the pinch current, **N** is the line density, ℓ is the pinch length, m_i is the ion mass, and **r** is the pinch radius. A corresponding heating term has been added to the ion energy equation in a **0-D**, self-similar simulation, which had been written previously to estimate fusion yields and radial expansion of D_2 fiber pinches. The simulation results agree well with the experimental results from **HDZP-II**, where the assumption of only joule heating produced gross disagreement. Turbulent ion heating should be the dominant process in any simple pinch carrying meg-ampere current and having submillimeter radius.

INTRODUCTION

Initial experiments with **HDZP-II** have given some unexpected results. Among them are:

1. Expansion of the pinch appears to be extremely rapid. Shadowgrams and interferograms show the column expanded to approximately **1 mm** diameter in **20 to 30 ns** after the beginning of current flow, where theory that assumes joule heating and Spitzer resistivity would predict

only slight expansion. **Fig. 1** is an interferogram of the pinch taken **20 ns** after the onset of current. At this time, the pinch diameter appears to be slightly greater than **1 mm**.

2. Neutron yield is two orders of magnitude smaller than would have been expected for the parameters of the experiment.
3. Intense **m=0** instability appears from the earliest times at which images have been obtained; it does not appear to disrupt the pinch. However, **m=1** instability has not been observed.

In this paper, we make an estimate of the effect of direct plasma heating by **m=0** instability growth and show that the observed expansion and radiation behavior can plausibly arise from it.

Fig. 1. Interferogram (**6x6 mm**) taken **20 ns** into the current.

INSTABILITY HEATING, QUALITATIVE

It is assumed for this discussion that there is no **m=1** (kink) instability of the plasma column. This is in accord with observation, although we offer no explanation for its absence.

When **m=0** deformations occur, the plasma acquires a macroscopic fluid-kinetic energy associated with deformation of its original cylindrical shape. In the nonlinear limit of the growth of the **m=0** mode, circular cusps of plasma are ejected radially away from the pinch (**Fig. 2**). In early experiments with gas pinches, these cusps generally reached the discharge tube wall, so that full development of instability growth usually resulted in disruption and termination of the discharge. In the **HDZP-II**, radially ejected plasma is still very far from the wall, so no disruption occurs. Moreover, the applied electric field in the **HDZP-II** remains strong for hundreds of instability growth times, so that one should expect an external reconnection of the cusps, with the result that cells of $\mathbf{B}_\theta$ are isolated from the main circuit, and the main current is diverted to the region outside of the cusps. The total energy contained within the larger pinch now consists of ion and electron thermal energy, turbulent kinetic energy of ions, and isolated cells of magnetic energy.

A detached torus of flux within the plasma has no equilibrium state short of a collapse of its inner interface with the plasma to zero radius. In reaching this limit, it will have given up most of its magnetic energy to work against the plasma, and as a result the ions receive most of this energy.

This process then continually repeats itself, amounting to a virtual "boiling" of the pinch.

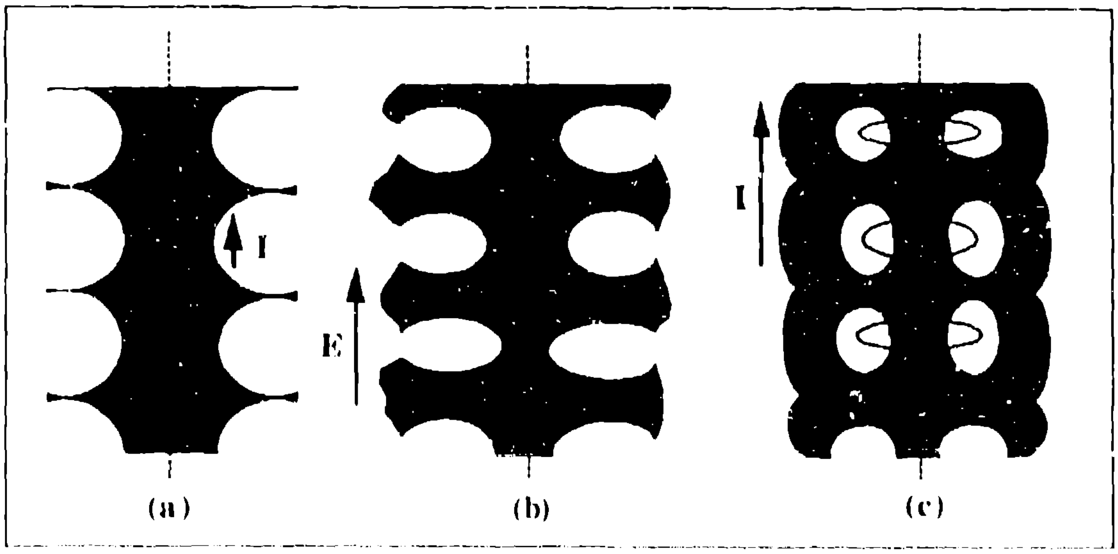

Fig. 2. The **m=0** turbulent heating cycle: (a) cusps eject plasma radially, (b) expanding cusps reconnect current, and (c) pinch current flows outside, trapped cells of **B**.

INSTABILITY HEATING, QUANTITATIVE

A convenient way to estimate energy transfer by an MHD instability is to describe its geometry as a time varying inductance, particularly if the plasma current is externally imposed. The total power input to an inductive circuit for which resistance is negligible is

$$P = IV = I\frac{d}{dt}(LI) = IL\frac{dI}{dt} + I^2\frac{d}{dt}L \quad . \qquad (1)$$

A slight rearrangement separates the terms functionally:

$$P = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) + \frac{I^2}{2}\frac{dL}{dt} \quad . \qquad (2)$$

The second term represents the power input to gross kinetic energy of the deformable conductor, while the first term is the rate of increase of magnetic energy.

The **m=0** instability raises the inductance of sections of the pinch through a shrinking of the inner coaxial conductor (plasma) diameter. The inductance of such a pinch section with radius **r**, length ℓ, and wall radius **b** is

$$L = \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{b}{r}\right) \quad , \qquad (3)$$

so that the rate of inductance change associated with pinch radius change is

$$\frac{dL}{dt} = -\frac{\mu_0 \ell}{2\pi r}\frac{dr}{dt} \quad . \qquad (4)$$

Equating the radial velocity of the pinch to the local Alfvén speed,

$$\frac{dr}{dt} = v_a = \frac{B_0}{\sqrt{\mu_0 \rho}} \quad , \qquad (5)$$

expressing B_θ in terms of current and radius,

$$B_0 = \frac{\mu_0 I}{2\pi r} \quad , \qquad (6)$$

and setting $\rho \pi r^2 = N m_i$, where **N** is the line density of the pinch, finally yields

$$\frac{dL}{dt} = \frac{\ell}{4\sqrt{N m_i}}\left(\frac{\mu_0}{\pi}\right)^{1/2}\frac{I}{r} \quad . \qquad (7)$$

This rate of inductance increase does not imply that the overall circuit inductance is continually increasing; multiplied by the current, Eq. (7) represents the rate at which magnetic flux is being detached from the circuit and ingested by the turbulent pinch. Multiplied by $I^2/2$, it gives the power input to macroscopic plasma motion, as mentioned earlier. This turbulence decays directly into ion thermal energy.

The inductance decrease associated with the outward moving cusps is small compared to the increase from the necks, if the motion is assumed to be approximately volume-conserving. The rate of inductance decrease associated with the overall expansion of the pinch is negligible, since the expansion goes at about 2% of the Alfvén speed.

The rate of loss of magnetic energy from the circuit through ingestion into the pinch is contained in the first right-hand term of **Eq.(2)**. Since, in the unstable motion, the relative growth rate of inductance is far greater than that of the current, it is reasonable to approximate **I** as constant over an instability growth time. In this case, the two right terms in **Eq.(2)** become identical, so the total power into ions is

$$P_i = I^2 \frac{dL}{dt} \tag{8}$$

PINCH SIMULATION

This heating term has been incorporated into a **0-D** pinch simulation, which was modified from one written previously to study the effects of alpha-particle retention at large burn rates. This code, **BURN4**, assumes self-similarity of the plasma profile during radius change. It incorporates joule heating, bremsstrahlung radiation, energy exchanges among various particle species, and gas dynamics that, as is usual under the self-similar assumption, assume the instantaneous transmission of pressure changes through the pinch. It also calculates **D-D**, or **D-T**, reaction rates. The relevant differential equations are given in **Appendix A**.

The new heating term is added to the ion energy equation. An arbitrary guess must be made of the fraction of the whole pinch length undergoing unstable radial motion at any instant. Based on analysis of a number of images showing **m=0** activity, we provisionally set this fraction to **0.1**.

SIMULATION RESULTS

For comparison, runs were made with and without the instability heating term. **Fig. 3** is a plot of radius, ion and electron temperatures, and current as a function of time, with the sinusoidal current waveform adjusted to the same risetime and peak current as those in the initial set of **HDZP-II** data taken through September, 1990. The turbulent heating is turned off. From an initial radius of **15 μm**, the pinch undergoes a rapid initial expansion and is subsequently recompressed to somewhat less than **100 μm**. Ion and electron temperatures remain nearly identical throughout the current rise and reach slightly over **2 keV**. **Fig. 4** shows calculated neutron output during the same interval. Without turbulent heating, the total yield up to **100 ns** is **7 x 10^{11}**, which is two orders more than what has been observed experimentally.

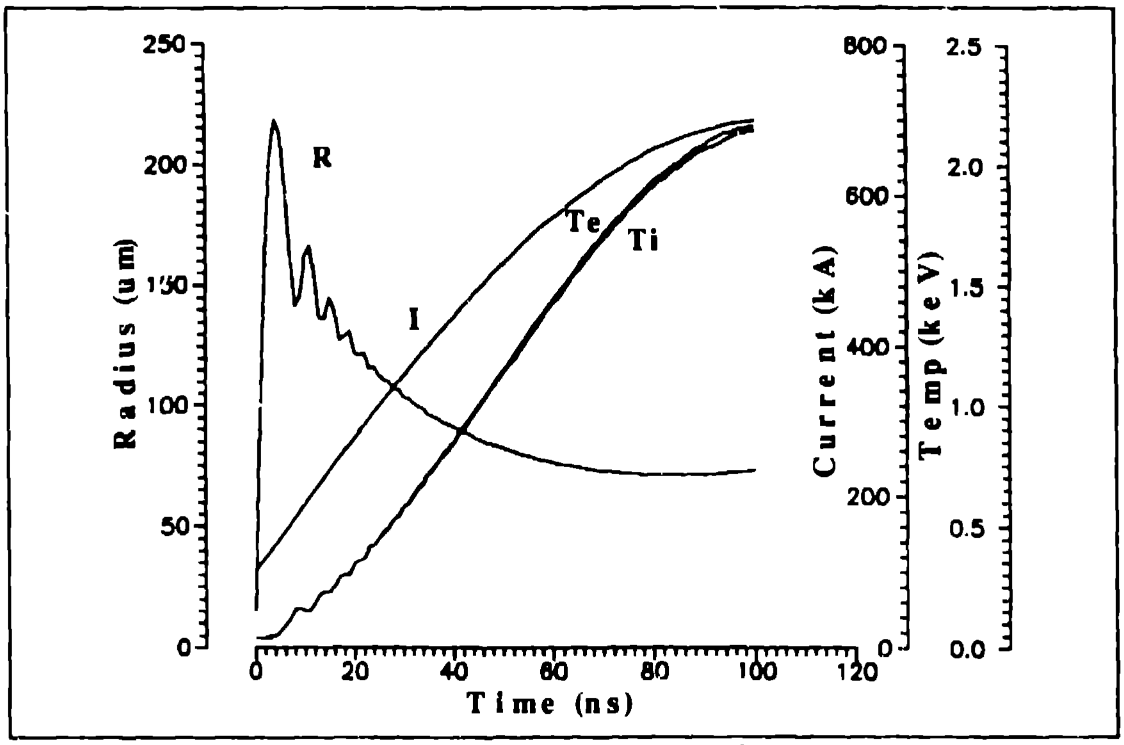

Fig. 3. Joule-heated **HDZP-II 0-D** simulation results.

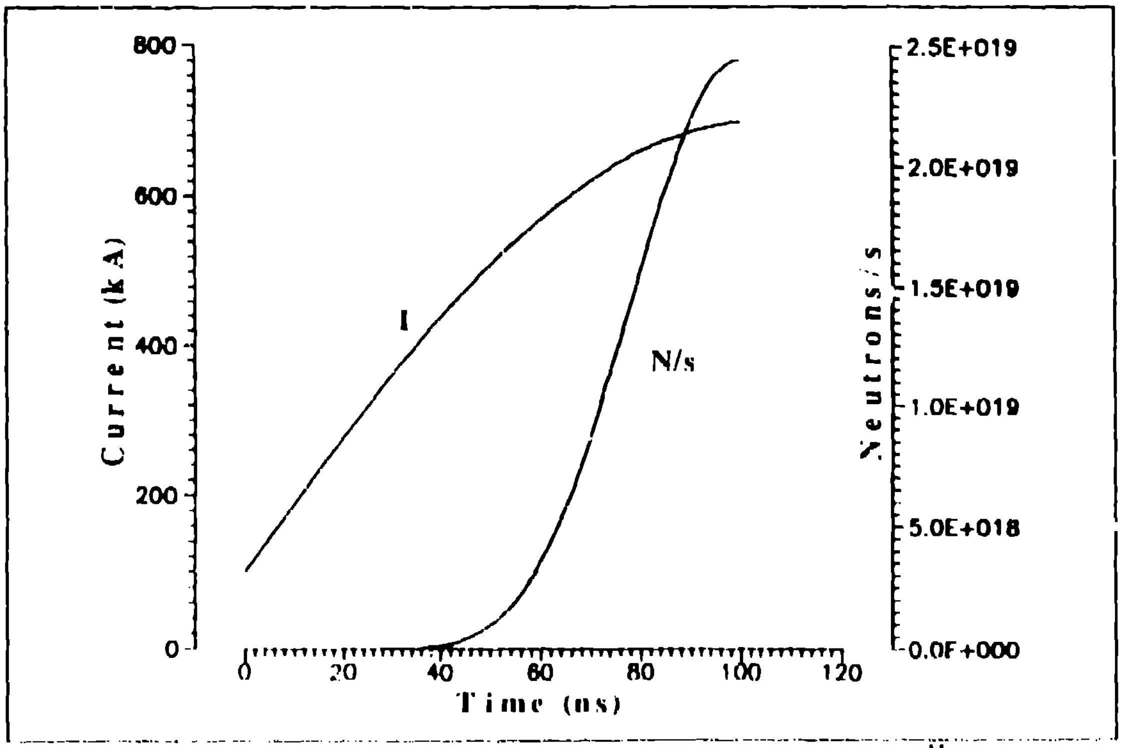

Fig. 4. Neutrons/sec for joule-heated **HDZP-II**, total yield 7×10^{11} neutrons.

In contrast, results of the simulation with turbulent heating turned on are remarkably consistent with observation. **Fig. 5** shows the radius increasing to nearly **3 mm** during the current rise. At **30 ns**, it has reached a radius of **500 μm**, which is its approximate value in the interferogram of **Fig. 1**. Ion temperature is nearly **4 keV** at **100 ns**, while the electrons remain fairly cold, not exceeding **700 eV**. The very rapid plasma expansion decouples ions and electrons, and the joule heating of electrons can barely supply the temperature lost to adiabatic decompression.

The neutron yield for this case is **8 x 10^9**, which is in the range of yields obtained experimentally.

The low electron temperature can explain some difficulty in obtaining unambiguous soft x-ray signals at low-keV energies.

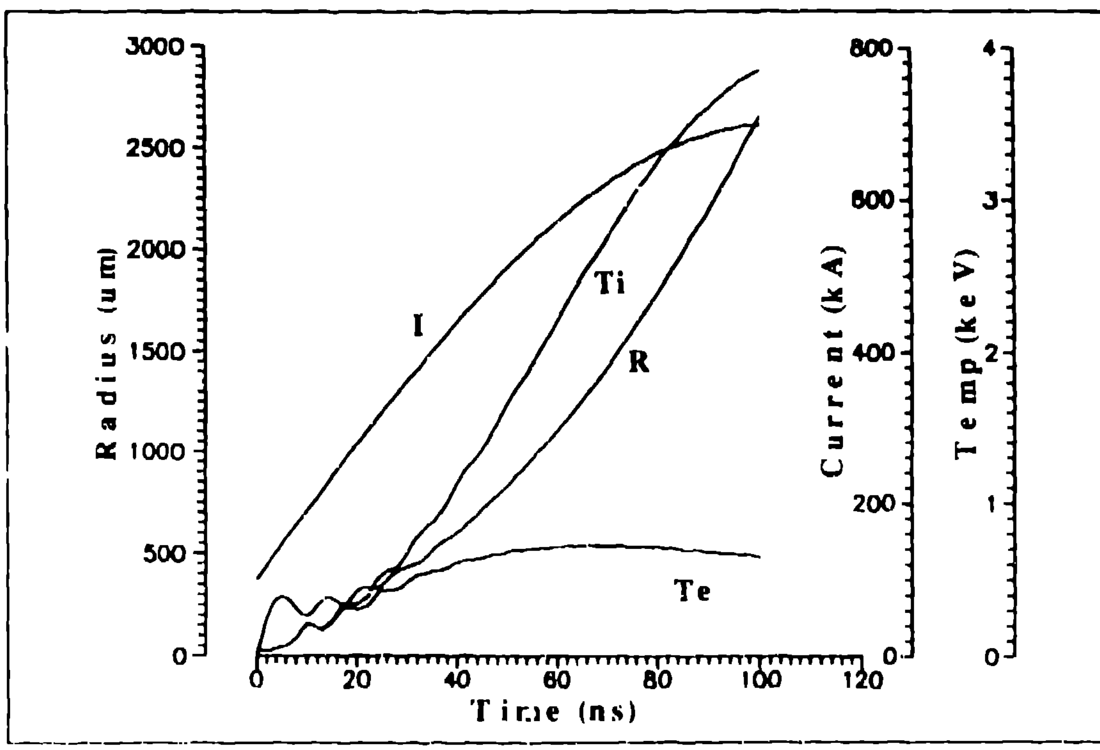

Fig. 5. The m=0 turbulent-heated **HDZP-II 0-D** simulation results.

IMPURITY ADMIXTURE

One experiment in the series that generated the data quoted above involved loading the **D_2** fiber with approximately 10% of neon. The motive was to produce significant early-time radiation loss that would inhibit the sudden initial expansion, and also to add inertia to the pinch. The neutron yield under these conditions was expected to decrease by a very large factor, if not to disappear altogether.

Contrary to expectation, the neutron output from this shot (**#193**) remained at the level of the pure D_2 shots. This result cannot be understood if one supposes that ion heating is through transfer from joule-heated electrons. However, it follows immediately from a turbulent-heating model, in that ions receive their energy directly from fluid kinetic energy, and are insensitive to electron temperature. T_e only needs to be high enough to keep the magnetic Reynolds number large, so that **B**-field energy goes mostly into fluid motion rather than resistive dissipation.

DISCUSSION

The occurrence of this kind of turbulent heating ought to be a universal property of simple (unstabilized) pinches. However, for several reasons one might not have expected it to be a dominant process for pinches formed by the traditional breaking down of a tenuous gas:

1. The explosive radial expansion resulting from turbulent heating would drive a "classical" gas pinch into its chamber walls almost immediately, since the wall/pinch radius ratio for these devices is usually not greater than about ten. It has been observed that such discharges usually disrupt shortly after instabilities appear. The **HDZP-II** is unique in being so far removed from the chamber wall (> 10^4 pinch radii) that no plasma-wall encounter can occur; turbulent heating can proceed throughout the current rise.
2. At the very low densities and much larger geometrical scale of gas pinches, the time required for turbulent plasma motion to decay into ion thermal energy is probably not much less than the lifetime of the pinch. As a consequence the process suggested above may not have time in which to achieve a steady state.
3. The effective turbulent resistance, dL/dt, varies as I/r. In the **HDZP-II**, this is probably three orders larger than in any previous pinch, so that here, uniquely in pinch research, turbulent heating is the dominant process.

The analysis presented above is very approximate in most of its details. However, one may be confident about some of its features. The radial expulsion of plasma by $m=0$ instabilities, with external circuit reconnection, and isolation and subsequent decay of cells of flux and turbulence, is a very likely mechanism. The expression for this effective power input to ions is subject to large uncertainties in its constant multipliers, but the scaling with current and radius is probably correct. The crude simulation employed here would ideally be followed by more sophisticated efforts; this may not be practical, however, because the next step should be a **2-D** code that would simulate instabilities in detail, and the cost and time needed to follow the system through hundreds of growth times and over hundreds of original pinch radii could be prohibitive.

To the extent that this model is correct, one may predict that rapid turbulent heating will be prominent in all of the currently planned **HDZP** experiments. In cases where the original fiber is significantly larger than the present **30 μm**, thus raising both **r** and **N**, and peak current smaller, the effect may be somewhat mitigated.

APPENDIX A: DIFFERENTIAL EQUATIONS

The set of coupled ODE's for the D_2 fiber simulation are for seven variables: ion density n_i (equal to the electron density n_e), their corresponding energy densities U_i, U_e, the pinch radius **r**, radial velocity **v**, integrated neutron yield N_n, and the total pinch current **I**. The deuterium of the pinch is assumed to be fully ionized at the beginning of the calculation, and to have a temperature of **10 eV**; the starting current is then set at a value that produces dynamic equilibrium at the initial pinch radius.

SI units are used, with the exception that the temperatures are in **keV**:

$$\frac{dn_i}{dt} = -\frac{2v}{r}n_i \qquad \textit{(expansion)} \tag{9}$$

$$\frac{dU_e}{dt} = \frac{3000en_e}{2}\frac{(T_i - T_e)}{\tau_{ei}} - \frac{2v\gamma}{r}U_e - \dot{U}_{Brems} + \dot{U}_{JouleHeat}$$

$$\dot{U}_{Brems} = 5.3\times10^{-37}n_e^2\sqrt{T_e} \qquad \dot{U}_{JouleHeat} = 3.3\times10^{-10}\frac{I^2\ln\Lambda}{r^4T_e^{3/2}} \tag{10}$$

$$\frac{dU_i}{dt} = \frac{3000en_i}{2}\frac{(T_e - T_i)}{\tau_{ei}} - \frac{2v\gamma}{r}U_i + \dot{U}_{TurbHeat}$$

$$\dot{U}_{TurbHeat} = 1.0\times10^{-12}\frac{I^3}{r^3\sqrt{m}} \tag{11}$$

$$\frac{dr}{dt} = v \tag{12}$$

$$\frac{dv}{dt} = \frac{2\pi r}{m}(P_{int} - P_{ext}) = \frac{2\pi r}{m}\left(\frac{2}{3}\sum U - \frac{\mu_0 I^2}{8\pi^2 r^2}\right) \tag{13}$$

$$\frac{dN_n}{dt} = \pi r^2 \ell_p n_i^2 \overline{\sigma v} \tag{14}$$

$$\frac{dI}{dt} = \frac{2\pi I_{max}}{\tau_I}\cos\frac{2\pi t}{\tau_I} \tag{15}$$

where ℓ_p is the pinch length, **5 cm** in the present experiment, and **m** in **Eqs. (11)** and **(13)** is the total mass/meter.

The **D-D** reaction rate and particle energy exchange time constant in the above equations are:

$$\overline{\sigma v}_{DD} = \frac{2.3 \times 10^{-20}}{T_i^{2/3}} \exp\left(- \frac{18.76}{T_i^{1/3}} \right) \quad \frac{m^3}{s} \ ,$$

$$\tau_{ei} = 2.5 \times 10^{19} \frac{T_3^{3/2}}{n_e \ln\Lambda} \quad .$$

In addition, the temperatures and energy densities are related as

$$T_{e,i} = 4.16 \times 10^{15} \frac{U_{e,i}}{n_{e,i}} \quad .$$

It is assumed that $\gamma = 5/3$.

Additional simplifying assumptions are made in **Eqs (9)-(15)** are:

- The plasma is assumed to retain a uniform distribution over **r**; the expansion terms in the particle and energy density equations reflect this assumption (**0-D** self-similarity).
- In **Eq. (13)**, the pinch mass is assumed to reside at the outer boundary.
- Since it is assumed in this calculation that the total burn fraction is too small to produce significant pressure from charged reaction products, equations for their number and energy conservation are not included.

Finally, a small viscosity term was included in **Eq. (13)** to damp oscillations in the solution near **t=0**. It appears to have had no significant effect on later progress of the calculation. Some residual oscillation is still apparent.