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Reactor Options for Disposition of Excess Weapon Plutonium: Selection Criteria and Decision Process for Assessment

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January 1994

Lawrence Livermore National Laboratory

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Executive Summary

DOE is currently considering a wide range of alternatives for disposition of excess weapon plutonium, including using plutonium in mixed oxide fuel for light water reactors (LWRs). Lawrence Livermore National Laboratory (LLNL) has been tasked to assist DOE in its efforts to develop a decision process and criteria for evaluating the technologies and reactor designs that have been proposed for the fission disposition alternative.

This report outlines an approach for establishing such a decision process and selection criteria. The approach includes the capability to address multiple, sometimes conflicting, objectives, and to incorporate the impact of uncertainty. The approach has a firm theoretical foundation and similar approaches have been used successfully by private industry, DOE, and other government agencies to support and document complex, high impact technology choice decisions. Because of their similarity and relatively simple technology, this report focuses on three light water reactors studied in Phase I of the DOE Plutonium Disposition Study. The decision process can be extended to allow evaluation of other reactor technologies and disposition options such as direct disposal and retrievable storage.

The model can be used to identify ranges of parameter values that lead to selection of a particular reactor. If desired, one output of the model could be a single figure of merit for each reactor that can be used for direct comparison of reactor technologies. However, as with all analytical tools, the most important results are qualitative rather than quantitative results. Furthermore, the model will indicate the relative sensitivity of a particular reactor technology choice to each of the input variables.

The approach has been implemented as an electronic spreadsheet model to illustrate the technique. The reader is encouraged to examine and experiment with the spreadsheet model in order to gain a better understanding of the assumptions underlying the computations. At the current stage, the model is intended to illustrate the approach, rather than to provide a final assessment. It is planned that the model will be refined and expanded to include other disposition options, selection criteria, cost and performance data, and sources of uncertainty.

1.0 Background

DOE is currently considering a wide range of alternatives for disposition of excess weapon plutonium, including use in mixed oxide (MOX) reactor fuel. Phase I of the DOE Plutonium Disposition Study considered three light water reactor (LWR) designs for possible implementation of the LWR disposition option: (1) a boiling water reactor designed by General Electric Corp. (GE-ABWR), (2) a pressurized water reactor designed by Westinghouse Corp. (W-AP600), and (3) a pressurized water reactor designed by Asea Brown Bovari/Combustion Engineering Corp. (ABB/CE-System 80+). Additional options that are being studied include a modular high temperature gas-cooled reactor (MHTGR) designed by General Atomics Corp., which uses PuO_2 spherical particle fuel, and an advanced liquid metal reactor (ALMR) designed by General Electric Corp., which recycles a U-Pu-Zr alloy fuel that is being developed by Argonne National Laboratory. The heavy water CANDU-type reactor is now also being considered as an option.

Lawrence Livermore National Laboratory (LLNL) has been tasked with assisting DOE in its efforts to develop a process and criteria for evaluating these reactor designs for the fission alternative. LLNL's task statement is as follows:

> ...assistance to DOE in defining and structuring alternatives, defining the objectives and criteria, assessing the tradeoffs, developing a method of combining attributes and assessing the impact of uncertainties, calculating the value of reducing uncertainties, and structuring the presentation of results. Nonproliferation, environment, safety and health, and economic and technical risks shall be included in these criteria. The resultant process should enable DOE to perform a comparative assessment of reactor disposition alternatives.

This report describes initial efforts to develop a decision process and selection criteria for reactor disposition of plutonium. A model that illustrates the techniques for assessing three LWR technologies is presented. This model accounts for uncertainty as well as the presence of multiple, sometimes conflicting, objectives. The model can be used to identify ranges of parameter values that lead to selection of a particular reactor. These ranges could be depicted graphically as shown below in Figure 1.1. Presentation of results in this form is particularly important in the absence of a single decision maker. It also reduces the need to identify specific values for parameters in the model. If desired, one output of the model could be a single figure of merit for each reactor that can be used for comparisons. As with all analytical tools, the most important results are qualitative rather than quantitative results.

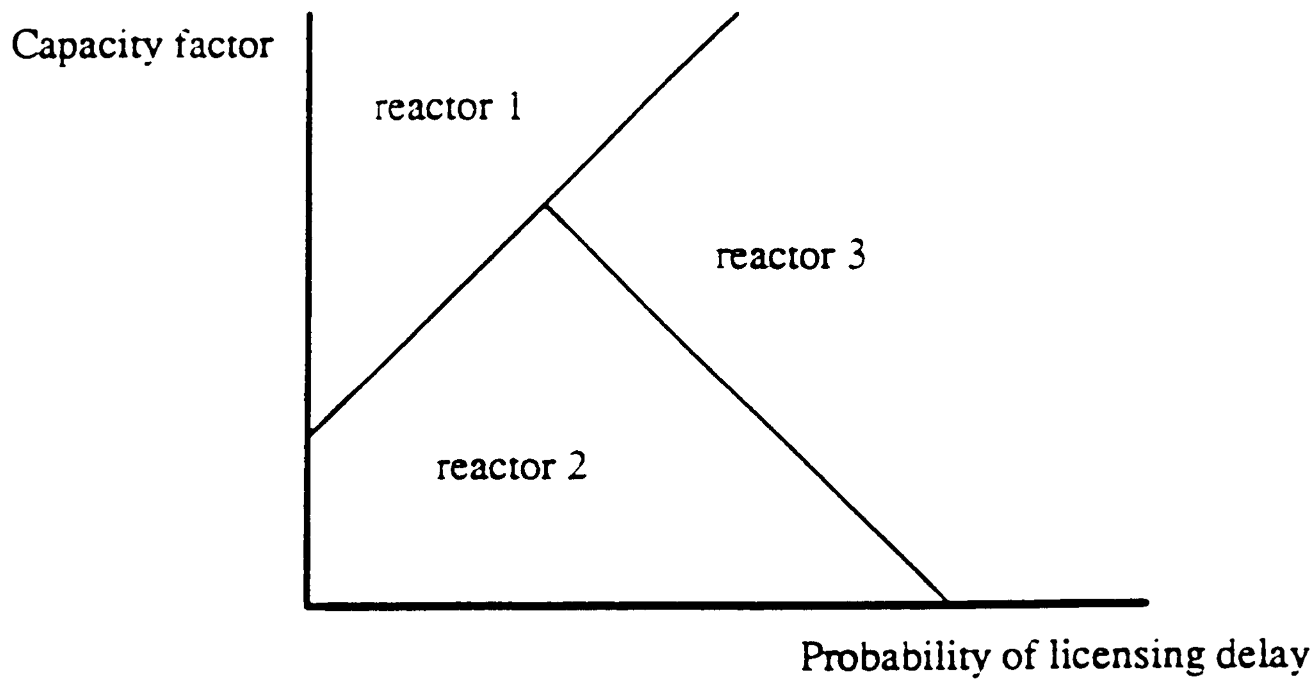

Figure 1.1 Examples of ranges of parameter values leading to different preferred reactors

The discussion and model are intended to be used for illustration of the methods, rather than to make a final assessment. The model will be expanded and refined to meet the needs of DOE, and to reflect additional input from national laboratories, reactor vendors, and other experts, where appropriate.

Section 2 describes the alternatives, uncertainties, objectives, and measures that have been included in a demonstration model developed to illustrate the approach. Section 3 presents a more detailed description of methods used to resolve conflicting objectives and to represent attitudes towards risk. Section 4 describes how to represent results in terms of ranges of parameters. Section 5 summarizes the analysis at the current stage. Appendix A provides a list of references describing applications of the techniques presented here. Appendix B describes equations used by the model to fit utility functions. Finally, Appendix C is a print out of the spreadsheet model.

2.0 Decision Model for Assessment of Reactor Alternatives

This section describes alternatives, uncertainties, objectives, and measures that have been included in a demonstration model for evaluation of plutonium disposition alternatives. An evaluation of three LWR disposition alternatives is used to illustrate the method, although the model could be expanded to include non-LWR and non-reactor disposition options.

The demonstration model has been implemented on a commercial electronic spreadsheet software package (Excel™, by Microsoft Corp.) to illustrate the calculations. The model and its associated data are not intended to represent a final assessment, but rather to demonstrate the approach. It is anticipated that the model and data will be revised substantially to meet the needs of DOE, and to reflect additional input from the national laboratories, vendors, and others as deemed appropriate by DOE. Portions of the spreadsheet model are shown in Appendices B and C.

The general structure of the model is depicted below in Figure 2.1. At the end of each branch in the figure, there are four measures of objectives. Alternatives, uncertainties, objectives, and measures are described in the following sections. In addition, the spreadsheet model includes extensive labeling and variable definitions to facilitate understanding by persons familiar with electronic spreadsheet packages. Cost and performance data currently in the model are for illustration purposes only.

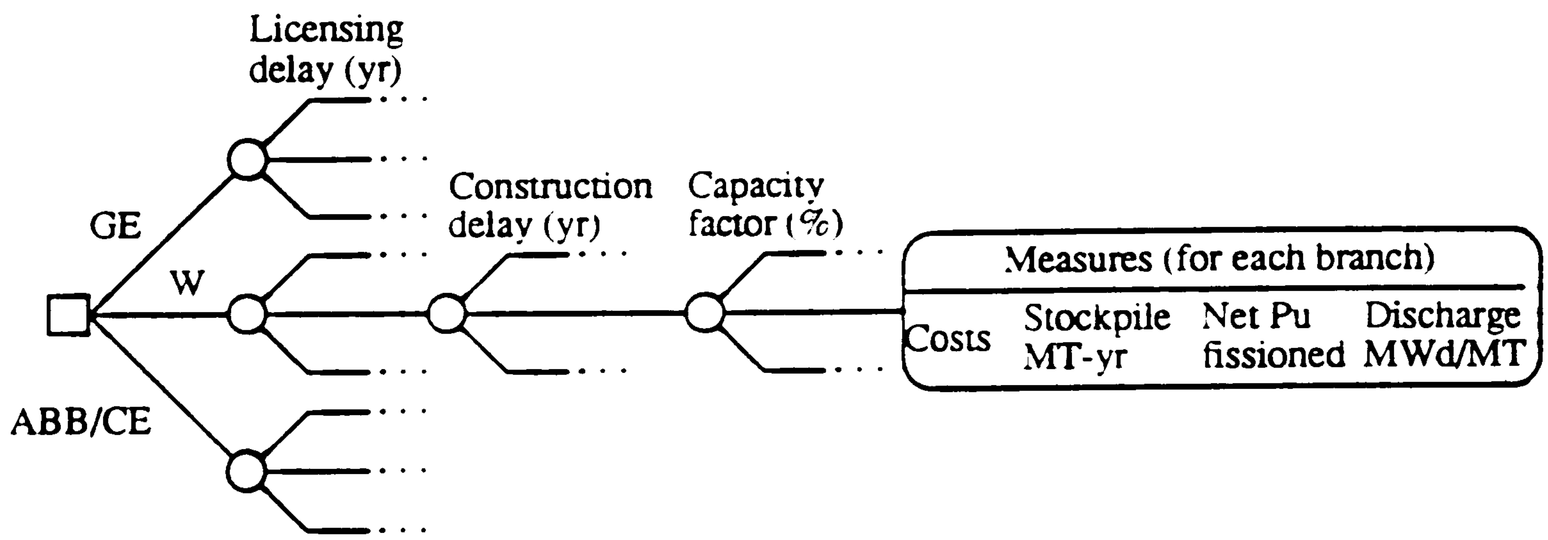

Figure 2.1 Alternatives, chance events, and measures in demonstration model

2.1 Alternatives

Decision makers select from among alternatives (e.g., a particular reactor design or deployment strategy) in a manner that furthers some objective, or set of objectives. As shown in Figure 2.1, three reactor options are included in this demonstration model: 1) GE-ABWR, 2) W-AP600, and 3) ABB/CE System 80+. Base case technical specifications for each reactor type include the net electrical output, years required to license, years required to construct, construction cost stream, operations and maintenance (O&M) cost stream, electricity revenues, base case plutonium processing capability in MT plutonium per year, and core inventory of plutonium. If these base case technical parameters are changed, the model automatically updates the calculations and generates a new result.

2.2 Uncertainties

Uncertainties regarding licensing delays, construction delays, and performance are included in the model. These uncertainties are indicated in Figure 2.1 as circles with three branches emanating to the right (an ellipse (...) at the end of a branch indicates that the tree structure is the same as that shown in the middle branch of the figure). For example, possible licensing delays are represented as three possible outcomes to approximate the continuous probability distribution associated with the actual delay. As shown in Figure 2.2 below, probabilities are assigned to each branch of a chance node (note that they must sum to one).

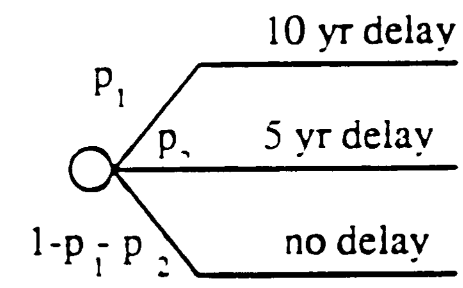

Figure 2.2 Example chance node for licensing delay

Additional branches may be added to reflect the desired level of precision, and associated probabilities can be specified independently for each reactor. The effect of a licensing delay is to delay all cash flows for that particular reactor alternative, and to delay processing of the stockpile.

Possible construction delays are also represented as three possible outcomes and associated probabilities that can be specified independently for each reactor. Construction delays are assumed to occur at the midpoint of the construction schedule. The effect of the construction delay is to defer cash flows after the delay point, and to delay plutonium disposition.

Variations in performance are represented as three possible reactor capacity factors (i.e., fraction of time that the reactor is operating at its design output) and associated probabilities. The capacity factor determines the rate at which the excess plutonium stockpile is depleted, as well as the amount of revenue generated by electricity sales each year of operation.

2.3 Objectives and measures

The approach compares disposition alternatives by evaluating the benefits they contribute to various objectives. Examples of objectives are to "minimize costs" or "minimize environmental risks." The objectives identified in the plutonium disposition problem include minimizing nonproliferation, safety, environmental, economic, and technical risks. Some nonproliferation, safety, and environmental risks might be represented as thresholds in such a way that a disposition alternative is not considered unless a certain threshold is met.

A set of *attributes* are developed to indicate the degree to which the different reactor types meet each objective. Example attributes may be cost or release to the environment. *Measures* are used to quantify the attributes, such as "net present cost in dollars" or "radioactive material released to the environment." In the plutonium disposition example analysis, there is a one-to-one correspondence between attributes and measures.

The four measures included in the demonstration model are as follows: 1) net present value of construction and net operating costs, 2) integral of excess weapon plutonium stockpile over time (metric ton-years of stockpile), 3) net percentage of plutonium from stockpile that is destroyed, and 4) radioactivity, measured by fuel exposure in megawatt days per metric ton (MT) of initial heavy metal (MWd/MTIHM).

The first measure has an economic basis, while the others are intended to primarily address nonproliferation, environmental, and safety impacts. The second measure, metric ton-years (MT-yr) of stockpile, is related to the general objective of processing the plutonium in the stockpile as rapidly as possible. The third attribute, fraction of plutonium fissioned, also addresses the nonproliferation objective in that a higher fraction fissioned reduces the amount of plutonium available for theft in the spent fuel, or plutonium that must be disposed of in a repository. Finally, the fuel exposure measure is a proxy for the radiation field generated by spent fuel elements. A higher radiation field would create greater safety and environmental risks, but may make it more difficult to steal. It may be possible to incorporate this consideration as a threshold instead.

The objectives and measures described above are intended to illustrate the process rather that to represent a final assessment. They will be expanded and modified as needed to meet the needs of DOE. Detailed calculations of these measures are described below.

2.3.1 Cost calculations

Construction cost streams, O&M costs, and electricity prices are inputs required by the model. The present value of the series of annual construction costs are computed as one component of the total cost. The present value of annual O&M costs are also computed. Finally, the plant capacity factor is used to compute the present value of revenues from electric power sales. The present value calculations for all cost streams are adjusted to account for licensing and construction delays. The discount rate used to calculate net present values is a variable that can be specified in the model. These three cost components are then summed to obtain total life cycle cost. This total cost is the first measure.

2.3.2 Stockpile MT-yr

Stockpile MT-yr is computed by integrating the stockpile profile over time. The stockpile profile is illustrated below in Figure 2.3.

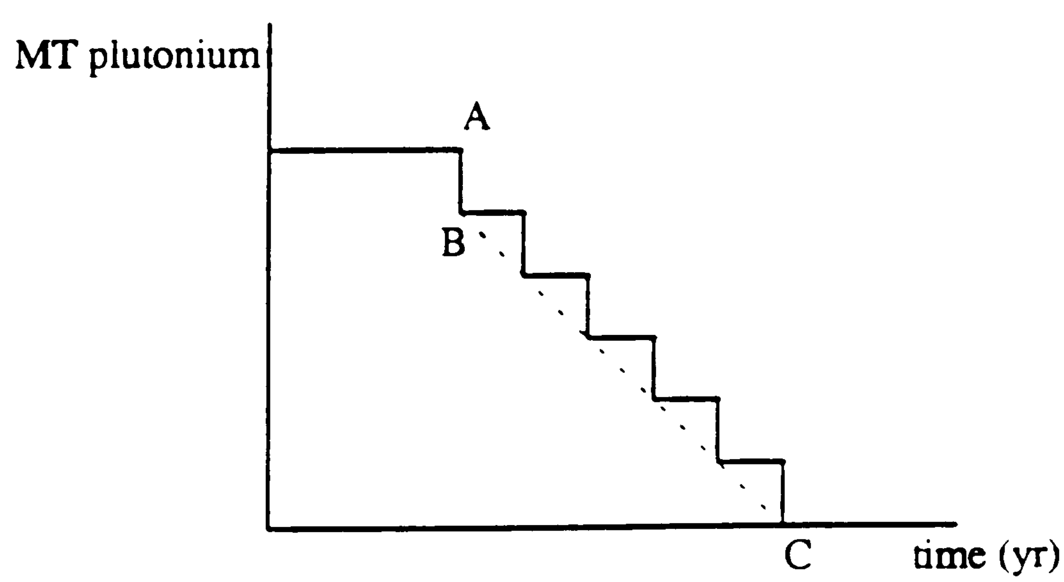

Figure 2.3 Stockpile as measured in MT-yr

As indicated in the figure, the stockpile of excess weapon plutonium remains at a constant level until point A, where the initial load of MOX fuel is charged to the reactor. The stockpile is reduced with each successive fuel load to generate the stair step pattern in the figure. The dashed line segment BC in the figure is used as an approximation in the model. The slope of the dashed line is determined by multiplying the base case plutonium processing rate by the ratio of the actual capacity factor to the base case capacity factor. If desired, additional detail can be included at a later date to model the effect of conversion of weapons pits to feed material for fuel element fabrication.

2.3.3 Net plutonium destruction and fuel exposure

The last two measures, net plutonium destruction and fuel exposure, are entered directly into the model. These values vary among reactor types and thus are means of discriminating among them.

2.4 Describing the consequences of selecting each alternative

Selection of a reactor alternative leads to consequences that are quantified in terms of each of the four measures described above. Because of the presence of chance events, the consequences of choosing a particular alternative may be uncertain. If a probability distribution over the possible outcomes of a chance event is specified (e.g., the probability of a licensing delay of n years), then a probability distribution over the consequences can be determined. Integration over the possible outcomes will yield the expected value of the consequence of selecting an alternative.

Calculation of the expected value of measures associated with a selection of a reactor alternative can be illustrated using the simplified example in Figure 2.4.

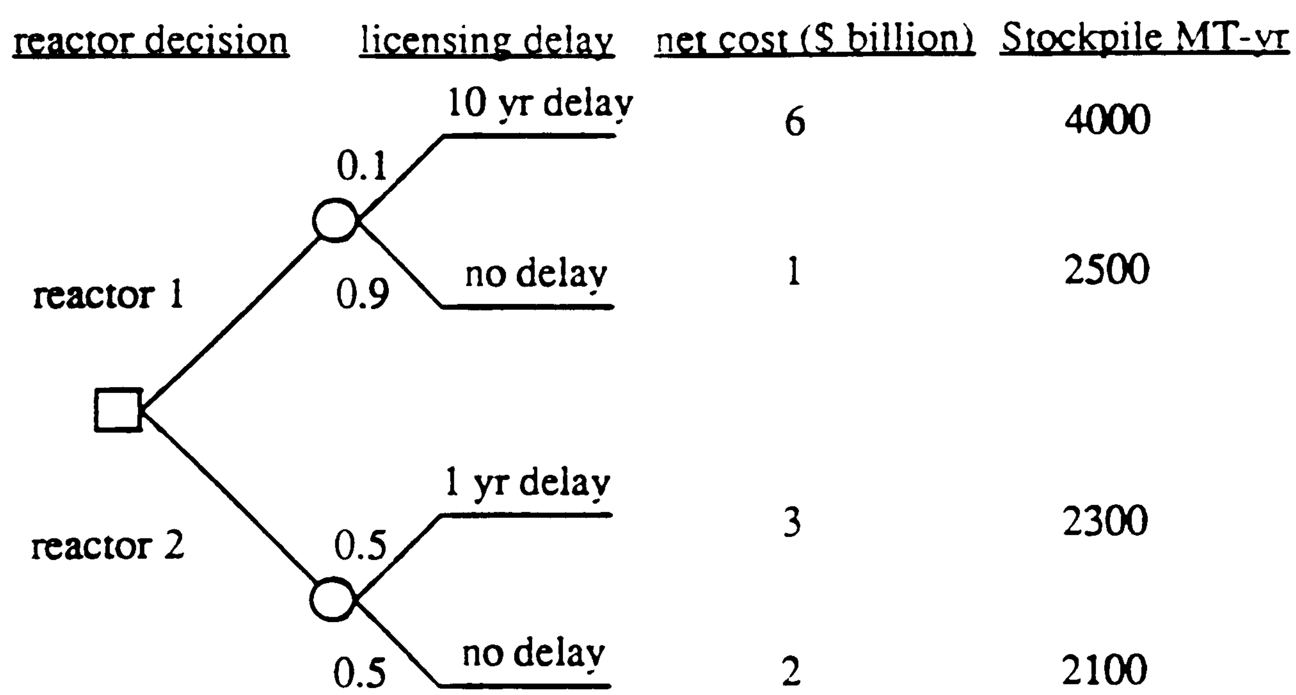

Figure 2.4 Graphical illustration of the decision problem

In Figure 2.4, the chance event is a licensing delay. As shown in the figure, the probability distribution over licensing delay depends upon which reactor is selected (different probability distributions over licensing delay may reflect different states of technology maturity). For example, if reactor 1 is chosen, then the first of the two possible outcomes of the licensing delay chance event is realized with probability 0.1 and the second of the two possible outcomes is realized with probability 0.9. The expected values of cost and MT-yr stockpile can be calculated for each reactor in the example, as shown below.

Reactor 1 expected value(net cost) = 0.1(6) + 0.9(1) = $1.5 billion
Reactor 2 expected value(net cost) = 0.5(3) + 0.5(2) = $2.5 billion

Reactor 1 expected value(MT-yr) = 0.1(4000) + 0.9(2500) = 2650 MT-yr
Reactor 2 expected value(MT-yr) = 0.5(2300) + 0.5(2100) = 2200 MT-yr

Expected values for all four measures are calculated in the same manner in the demonstration model.

Note that reactor 1 is preferred on the basis of expected cost, while reactor 2 is preferred on the basis of expected MT-yr of stockpile. If the overall preferred reactor is not obvious and if further refinement is desired, the techniques of utility theory may be employed to assist in the decision process. These techniques and their application are discussed in Section 3.

3.0 Attitudes Towards Risk and Conflicting Objectives

This section shows how a preference model can be developed that addresses the complexities introduced by uncertainty and by multiple attributes. The model includes two types of preference assessments: (1) preferences that reflect tradeoffs between attributes and (2) preferences for accepting risks. The resulting model can be used to obtain a final figure of merit for each reactor alternative. It can also be used to identify ranges of parameter values that lead to a particular reactor choice, as discussed in Section 4. This capability is particularly important in the absence of a decision maker.

3.1 Modeling tradeoffs between attributes

The first type of preference assessment is to determine the relative importance of different attributes. When more than one attribute is involved, it is not possible to directly compare outcomes unless one alternative is superior with respect to all measures. A means of combining measures into a single figure of merit is needed. This figure of merit, called a *multiattribute utility function*, can be derived from the answers to tradeoff questions regarding pairs of attributes. One example of such a tradeoff question is as follows:

Reactor 1		Reactor 2	
NPV cost	MT-yr stockpile	NPV cost	MT-yr stockpile
1.5	3,000	?	2,000

In this tradeoff question, a decision maker is presented with two hypothetical reactor options as shown above. Reactor 1 costs $1.5 billion and involves 3,000 MT-yr of stockpile. Reactor 2 involves only 2,000 MT-yr of stockpile, and is more desirable than Reactor 1 in this regard. The cost of Reactor 2 is not specified *a priori*. A decision maker would be asked to provide a cost for Reactor 2 such that he or she is indifferent between Reactors 1 and 2. For example, if the decision maker specified a value of $2 billion for Reactor 2, this would imply that the decision maker is willing to pay an additional $0.5 billion to reduce the MT-yr of stockpile from 3,000 to 2,000. Note that this *does not* imply that a 1,000 MT-yr stockpile decrease is worth $500 million in all circumstances because this tradeoff may not be valid for the entire range of an attribute.

Other forms of questions, such as lottery questions, can also be used to elicit the tradeoffs. The decision maker's responses to these tradeoff questions can be used to derive coefficients in an expression that combines the single attribute utility functions (described in Section 3.2) to form a multiattribute utility function. The simplest form of a multiattribute utility function is the additive form shown below:

$$\text{additive multiattribute utility function: } k_1u_1(m_1) + k_2u_2(m_2) + k_3u_3(m_3) \ldots$$

where the coefficients k_i are computed based upon the responses to the tradeoff questions, the functions u_i are the utility functions (see section 3.2), and the variables m_i are the measure levels of the objective. In its simplest implementation the utility function is equal to the measure level. The ratios of the coefficients (k_i) represent the decision maker's tradeoff between attributes.

The additive utility function assumes that the tradeoffs between attributes is constant over the entire range. This assumption may not hold in all cases. For example, two attributes may be complements (e.g., coffee and sugar, right and left shoes). In this case, having more of one attribute is not valuable unless there is a corresponding increase in the other. Nonadditive functional forms are needed to represent these types of preferences. For example, a product form (e.g., $u_j(m_j)u_k(m_k)$) is needed if attribute j is complementary to attribute k. The product form reflects complementary attributes because it is 1.0 only if both attribute j and attribute k are at their best levels, and is 0 if one of the attributes is at its worst level. A commonly used form for nonadditive multiattribute utility functions that allows modeling such preference patterns is shown below:

$$\text{multiplicative multiattribute utility function: } \frac{\prod_{i=1}^{n}[1 + kk_iu_i(m_i)] - 1}{k}$$

For discussion purposes, an additive form for the multiattribute utility function is assumed. The four coefficients, k_1 through k_4, are computed by solving four simultaneous linear equations constructed using responses to three tradeoff questions and a normalization condition. Their solution is illustrated in Appendix B.

3.2 Modeling attitudes toward risk

As discussed in Section 2, the outcomes from selecting a particular reactor alternative can be uncertain. In some cases, a decision maker may simply prefer the alternative with the greatest expected value. This is frequently true when there is little or no possibility of severely undesirable outcomes. But, one needs to recognize that in other cases one possible outcome represents such a severe consequence (or such a great opportunity) that a prudent decision maker would try to avoid it (or seek it out) even though it has a small probability of occurring. Such attitudes cannot be formally represented by a linear function of the measure (e.g., simply using the expected value of the measure). Fortunately, for a very broad range of situations, this behavior can be modeled by a simple function of the measure itself. These functions can reflect a number of different preference patterns.

In the presence of uncertainty, decision makers may exhibit risk averse, risk prone, or risk neutral preference patterns. These preference patterns can be illustrated by considering a chance node with two equally probable outcomes: one outcome results in 4,000 MT-yr of stockpile and the second outcome results in 2,000 MT-yr of stockpile. This chance node is show below in Figure 3.1. The chance node could be viewed as a lottery over outcomes.

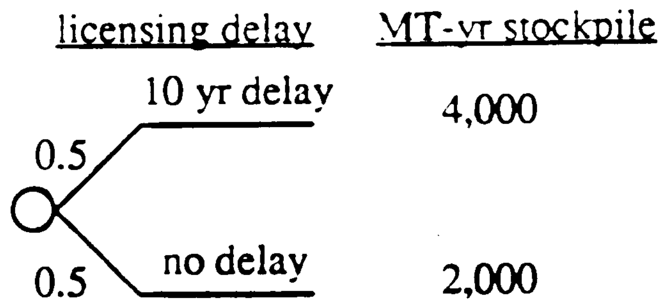

Figure 3.1 Chance node with two MT-yr stockpile outcomes

The expected value of the outcome of this lottery is 3,000 MT-yr of stockpile. Now, consider a decision maker who is given the choice of either accepting this uncertain lottery, or implementing an option with a known (i.e., certain) number of MT-yr of stockpile. In comparing the lottery and the known outcome, the decision maker could prefer the lottery, prefer the known outcome, or be indifferent between the lottery and known outcome. Three types of behavior are possible:

1) The decision maker prefers a known outcome greater than 3,000 MT-yr of stockpile over a lottery with an expected value of 3,000 MT-yr of stockpile. This is an example of risk averse behavior. The decision maker is willing to accept an outcome worse than the expected value of the lottery in order to avoid the risk of the 4,000 MT-yr bad outcome in the lottery.

2) The decision maker prefers a lottery with an expected value of 3,000 MT-yr of stockpile over a known outcome of less than 3,000 MT-yr of stockpile in the hope of achieving the 2,000 MT-yr outcome. This is an example of risk prone behavior, in which the decision maker prefers the lottery because it offers the possibility of an outcome which is better than the known outcome.

3) The decision maker is indifferent between a known outcome of 3,000 MT-yr of stockpile and a lottery with an expected value of 3,000 MT-yr of stockpile. This is an example of risk neutral behavior.

Such risk preferences can be calibrated by making a series of comparisons between lotteries and known outcomes. Using these comparisons, a function can be developed that essentially transforms the original measure to reflect the attitudes toward risk. These functions are called *utility functions*. When the utility function is properly calibrated, the alternative with the highest expected utility is preferred. (They are scaled so that the value corresponding to the most desirable outcome is 1.0, and the value corresponding to the least desirable outcome is 0.0.) Utility functions illustrating the three types of risk behavior described above are shown in Figures 3.2a, 3.2b, and 3.2c.

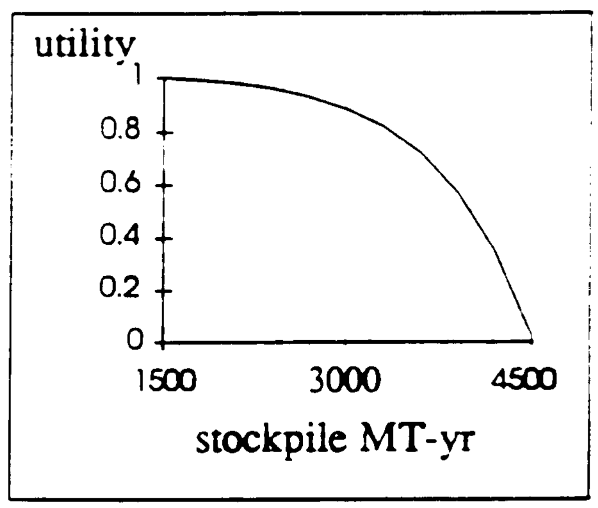

Figure 3.2a Utility function with risk averse behavior pattern

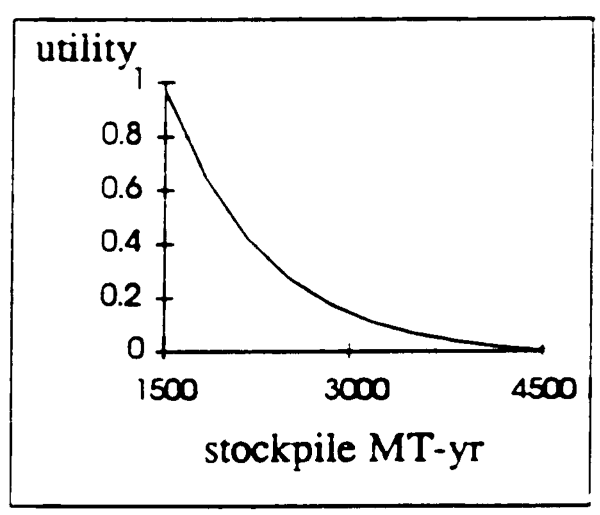

Figure 3.2b Utility function with risk prone behavior pattern

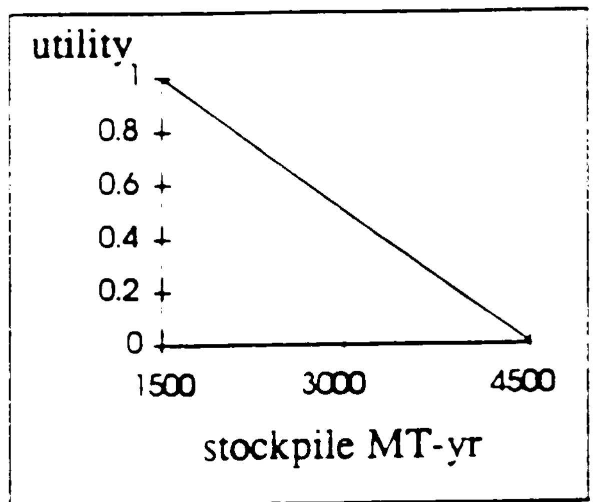

Figure 3.2c Utility function with risk neutral behavior pattern

Note that the risk averse behavior pattern, illustrated in Figure 3.2a, shows that the objective is satisfied to a large extent by relatively small improvements over the worst possible outcome. The opposite behavior is illustrated in the risk prone pattern illustrated in Figure 3.2b, whereas the risk neutrality pattern shown in Figure 3.2c indicates that each MT-yr of stockpile decrease provides the same amount of satisfaction.

To further illustrate the manner in which utility functions capture risk preferences, consider the MT-yr of stockpile measures for the reactor alternative shown in Figure 3.1. As shown previously, the expected value for the reactor alternative is 3000 MT-yr of stockpile. If decisions were made solely on the basis of this expected value, one would be indifferent between this reactor and a second reactor that results in 3000 MT-yr with certainty. However, the variation in outcomes is higher for the first reactor. Intuitively, one would expect that under a risk averse behavior pattern, reactor 2 would be preferred to reactor 1. The utility function depicted in Figure 3.2a models this behavior. Using this utility function, the expected utilities of the two reactor alternatives are calculated below.

Reactor 1 expected value(utility) = 0.5(0.4) + 0.5(0.98) = 0.69
Reactor 2 expected value(utility) = 1.0(0.91) = 0.91

This calculation illustrates that if a utility function with a risk averse behavior pattern is used for decision making, then the superior risk characteristics of the second reactor alternative is recognized by the model.

In this demonstration model, a utility function is included for each of the four attributes. The function can be used to fit responses reflecting any of the three types of preference patterns illustrated above, as well as sigmoid, or "s-shaped" preference patterns in which a decision maker is risk averse in one region of the curve and risk prone in another region. This functional form is needed to represent threshold effects, where little benefit is gained by increasing the measure until a threshold is crossed, at which point most of the benefit is realized.

These utility functions are derived from answers to three lottery questions regarding the outcomes for an attribute measure. A function is then fit to two of the responses (lottery questions and function fitting logic is included in the spreadsheet model). The third response provides a consistency check to the functional fit. Responses to lottery questions are entered into cells in the spreadsheet, and all results are updated automatically. In the absence of a single decision maker to specify responses to lottery questions, results can be presented as a range of responses that lead to selection of a particular alternative.

3.3 Ranking the alternatives

The multiattribute utility function provides an overall figure of merit for each combination of alternative selection and chance events. Using the probability distributions assigned to each chance event, the expected utility of the consequences of selecting a particular alternative can be computed. For example, the expected utility (EU) associated with reactor 1 in Figure 2.4 can be computed as follows:

$$EU(\text{reactor 1}) = 0.1*(k_1u_1(\$6)+k_2u_2(4000\text{ MT-yr})) + 0.9*(k_1u_1(\$1)+k_2u_2(2000\text{ MT-yr}))$$

Using the utility function shown in Figure 3.2a for MT-yr of stockpile, the utility u_2(4000 MT-yr) is equal to 0.4 and the utility u_2(2000 MT-yr) is equal to 0.98. Assuming that the utilities for cost are $u_1(\$6)=0.25$ and $u_1(\$1)=0.8$, and the tradeoff coefficients are $k_1=0.55$ and $k_2=0.45$, the expected utility can be computed as follows:

$$EU(\text{reactor 1}) = 0.1*(0.55*0.25+0.45*0.4) + 0.9*(0.55*0.8+0.45*0.98)=0.82$$

Expected utilities are computed in this manner for each alternative. The multiattribute utility function is defined so that the alternative with the highest expected utility is the preferred alternative for implementation.

The procedures described above have been developed, documented, and used for a wide range of decision problems, including a number of problems related to nuclear technology choice. The theoretical basis for multiattribute utility theory is described in Keeney and Raiffa (1976). A list of references describing applications is included in Appendix A.

In the demonstration model, tradeoff questions with respect to different attribute measures are used to establish the relative importance of achievement on different attributes. The tradeoff responses are entered into cells in the spreadsheet, coefficients are computed, and results are updated by the model. The overall attribute for each alternative and combination of chance events is computed using these coefficients and the four attribute measures. Calculation of the utility functions is discussed in more detail in Appendix B.

4.0 Sensitivity Analysis

Sensitivity analysis can be performed to determine which parameter variations have significant impact on the choice among alternatives. Sensitivity with respect to the tradeoff values assigned to measures, probabilities assigned to chance events, and basic technology data can be assessed. For example, the model could determine the sensitivity of the optimal reactor alternative to the relative magnitudes of the tradeoff coefficients k_1 and k_2. If cost were deemed to be less important, then the coefficients shown in the previous example might be changed to k_1=0.3 and k_2=0.7 and the expected utility of reactor 1 would change to 0.87. This illustrates that the expected utility of reactor 1 is relatively insensitive to the tradeoff coefficients. Other reactors may be more sensitive to these parameters, and their utility values could change enough to make one of them the preferred alternative.

Sensitivity analysis can also be used to identify ranges of parameter values that lead to a particular reactor choice. Results could be presented in a number of different formats. One possible graphical format for presentation is to display regions of parameter space that correspond to selection of particular alternatives. For example, the sensitivity with respect to the assessed tradeoffs for three attributes might be presented graphically as follows:

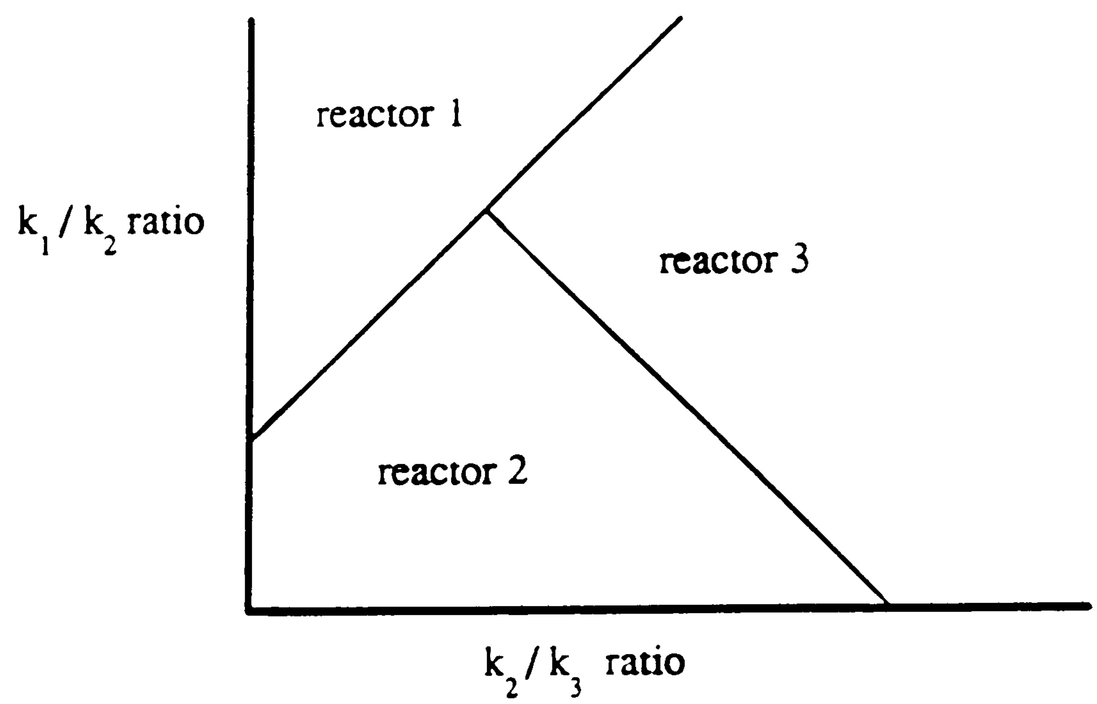

Figure 4.1 Sensitivity of reactor choice with respect to tradeoff coefficient ratios

As indicated in the figure, alternative 1 is preferred if the ratio of the coefficients k_1/k_2 is high and the ratio of the coefficients k_2/k_3 is low. In this region, these ratios imply that attribute 1 is more important than attribute 2 and attribute 2 is less important than attribute 3. Alternatives 2 or 3 are preferred if the two coefficient ratios are assume other values. Similar graphs could be constructed for chance event probabilities and technology data, as illustrated below in Figure 4.2.

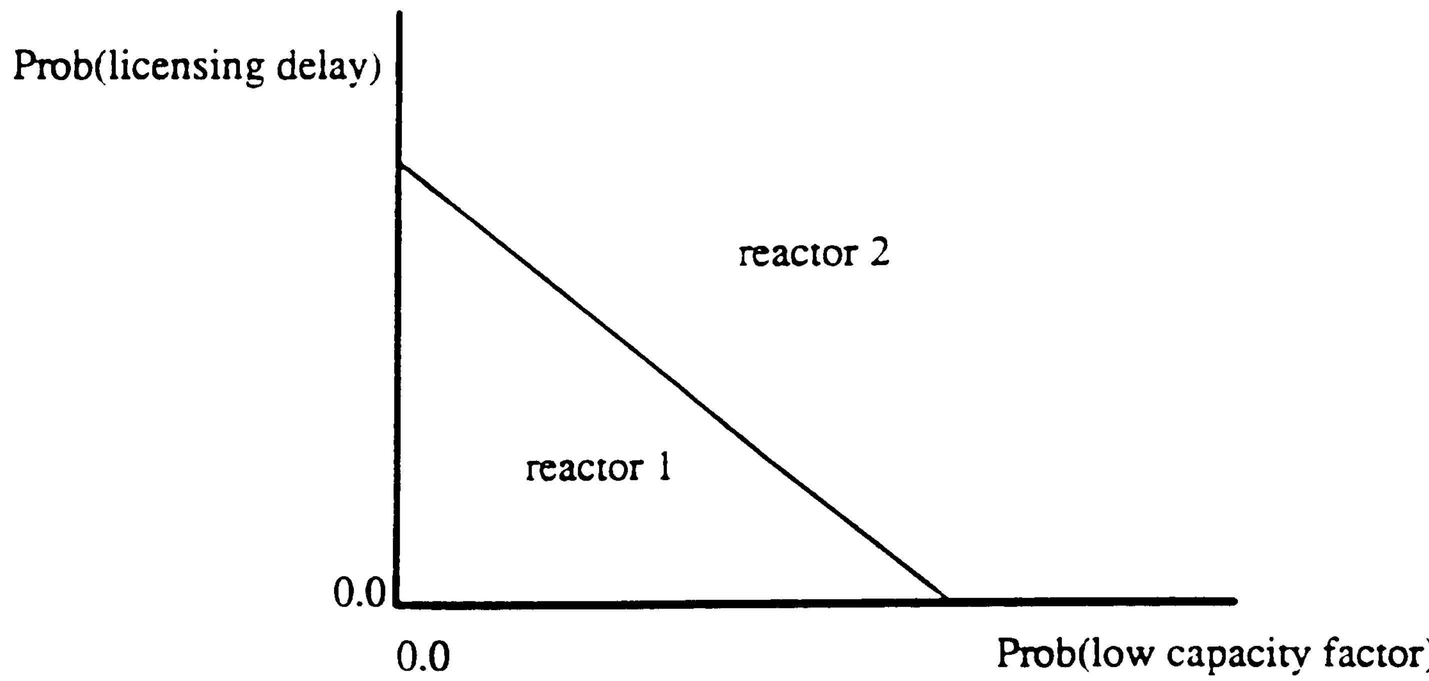

Figure 4.2 Range of licensing delay probabilities that lead to selection of reactor 1

As indicated in the figure, reactor 1 is preferred if the probabilities of a licensing delay and a low capacity factor for that reactor are low. The second-best reactor, Reactor 2, becomes the preferred alternative if either of these probabilities are high. A range analysis could then be performed on Reactor 2.

In addition, the value of further R&D efforts to reduce or remove uncertainty can be ascertained with this methodology. For example, the model can determine the value of conducting a detailed study on licensing delay that would remove uncertainty with regard to this issue. It can specifically answer questions about how much a study is worth and how perfect information about licensing delay would change the optimal reactor choice. This would place an upper bound on the value of a licensing study since the results of the study would likely be something less than perfect information that removed all uncertainty. The technique is illustrated in Raiffa [1968] and Winston [1987].

5.0 Summary and Conclusions

A model has been developed to illustrate an approach for establishing a decision process and selection criteria for evaluating reactor options for plutonium disposition. The model has been implemented on an electronic spreadsheet package so that the calculations can be readily observed. The reader is encouraged to examine and experiment with the spreadsheet model in order to gain a better understanding of the assumptions underlying the computations. The model is intended to illustrate the approach, rather than provide a final assessment. Additional data will be included in the model and updated as needed.

It is planned that the model will be refined and expanded to include additional disposition options, selection criteria, cost and performance data, and sources of uncertainty. The model provides a figure of merit that can be used for direct comparison of reactor technologies. However, the intention is to use the model to identify assumptions that would lead to selection of a particular technology. Thus, selection of a reactor technology could be justified when the assumptions are valid (i.e., model input values fall within assumed ranges). This allows use of the model in the absence of a decision maker. Furthermore, the model will indicate the relative sensitivity of a particular reactor technology choice to each of the input variables.

A more sophisticated commercial software package designed for this type of analysis will be required as significant expansion of the model is anticipated. However, the more advanced software packages employ the basic calculations illustrated in the simple spreadsheet model.

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Appendix A - Other Applications of Proposed Methodology

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Appendix B - Fitting Utility Functions

This appendix describes portions of the demonstration model that provide a user interface and logic to construct utility functions based upon responses to lottery and tradeoff questions.

B.1 Single attribute utility functions

The demonstration model includes an interface and logic for fitting a utility function for each of the four attributes. As an example, the lottery questions used to determine the utility function for the attribute "MT-yr stockpile" are shown in Figure B.1. The alphanumeric headings on the left and top identify the location of this portion of the spreadsheet model.

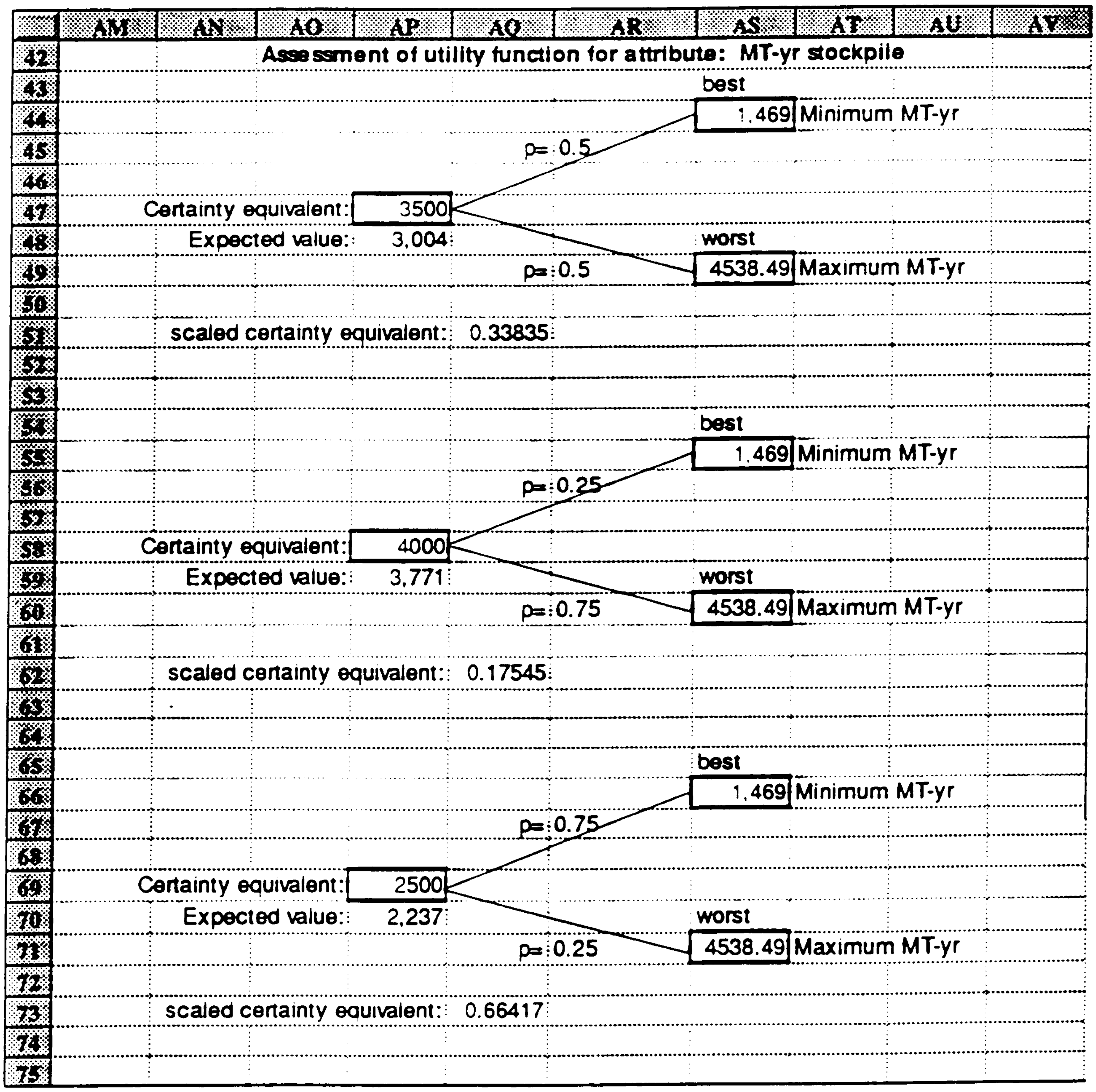


Figure B.1 Lottery questions for assessing a utility function

As indicated in Figure B.1, the responses to three lottery questions are used as input to fit the utility function. Each lottery includes the best and worst possible outcomes in terms of MT-yr of stockpile. The probabilities associated with each outcome are shown on the two branches of each lottery. In the cell labeled "certainty equivalent," the user provides the certain outcome to which the lottery is compared. A certainty equivalent value is placed in the cells associated with each lottery such that a hypothetical decision maker is indifferent between the certainty equivalent and the lottery.

The expected value of each lottery is also computed and displayed to provide a point of reference for input of the certainty equivalent value. The scaled certainty equivalent, the certainty equivalent value expressed as a fraction of the total range of the attribute, is also computed. This scaled certainty equivalent is used in the computations described below. The first two lotteries are used to fit the utility function and the third lottery is used as a consistency check.

The functional form of the utility functions used in the demonstration model is shown below. Here, x is the value of the attribute rescaled to the range [0,1] and the parameters b and c are derived from the responses to the lottery questions described above.

$$u(x) = \frac{x^b}{x^b + c(1-x)^b} \tag{B.1}$$

Other functional forms could be used as well. This function was selected because it can assume an "s" or sigmoid shape to represent risk averse and risk prone preference patterns in different ranges of the attribute. In addition, analytic expressions can be used to fit the function to two of the lottery question responses, as derived below.

First, equation B.1 can be written for each of the two lottery questions as follows:

$$u_1 = u(x_1) = \frac{1}{1 + c\left[\frac{1-x_1}{x_1}\right]^b} \tag{B.2}$$

$$u_2 = u(x_2) = \frac{1}{1 + c\left[\frac{1-x_2}{x_2}\right]^b} \tag{B.3}$$

where u_1 = expected utility of the first lottery
u_2 = expected utility of the second lottery
$u_1(x_1)$ = utility of the certainty equivalent for first lottery
$u_2(x_2)$ = utility of the certainty equivalent for second lottery
x_1 = scaled certainty equivalent value for first lottery
x_2 = scaled certainty equivalent value for second lottery

Equation B.2 expresses the equivalence between the expected utility of the first lottery and the utility of the specified certainty equivalent. Note that because the attribute values on the lower and upper branches of the lottery have utilities of 0.0 and 1.0, respectively, the expected value of the lottery is simply the probability associated with the upper branch. Hence, u_1=0.5 and u_2=0.25 in this example. The scaled certainty equivalent values are also know so the two equations can be solved for the parameters b and c.

To solve equations (B.2) and (B.3), it is useful to define the following auxiliary parameters.

$$r_1 = (1-x_1)/x_1$$
$$r_2 = (1-x_2)/x_2$$
$$q_1 = (1-u_1)/u_1$$
$$q_2 = (1-u_2)/u_2$$

Then equations (B.2) and (B.3) can be rewritten as follows:

$$u_1 = \frac{1}{1+cr_1^{\,b}} \tag{B.4}$$

$$u_2 = \frac{1}{1+cr_2^{\,b}} \tag{B.5}$$

$$q_1 = cr_1^b \tag{B.6}$$

$$q_2 = cr_2^b \tag{B.7}$$

Dividing equation (B.6) by (B.7) to eliminate the parameter c yields:

$$b = \ln(q_1/q_2)/\ln(r_1/r_2) \tag{B.8}$$

The parameter c is then computed as follows:

$$c = q_1 r_1^{-b} \tag{B.9}$$

The calculations for the lottery questions shown in Figure B.1 are displayed below in Figure B.2.

	AW	AX	AY	AZ	BA
41	**Single attribute utility function**				
42			x:	u(x):	MT-yr
43	x1:	0.33835	0	0	4538.49
44	x2:	0.17545	0.1	0.12868	4231.57
45	u1:	0.5	0.2	0.28974	3924.64
46	u2:	0.25	0.3	0.44489	3617.72
47	r1:	1.95548	0.4	0.5823	3310.79
48	r2:	4.69969	0.5	0.69851	3003.87
49	q1:	1	0.6	0.79384	2696.95
50	q2:	3	0.7	0.8701	2390.02
51	b:	1.25289	0.8	0.92937	2083.1
52	c:	0.43161	0.9	0.97322	1776.17
53			1	1	1469.25

Figure B.2 Fitting utility function to lottery question responses

Finally, the resulting utility function is graphed below in Figure B.3. Points corresponding to the first two lottery questions are shown as points on the function. The point corresponding to the third lottery question, the consistency check, is the point below the function. Although not exactly on the line, this point is in general agreement with the utility function. All three responses are shown as lighter points on the plot.

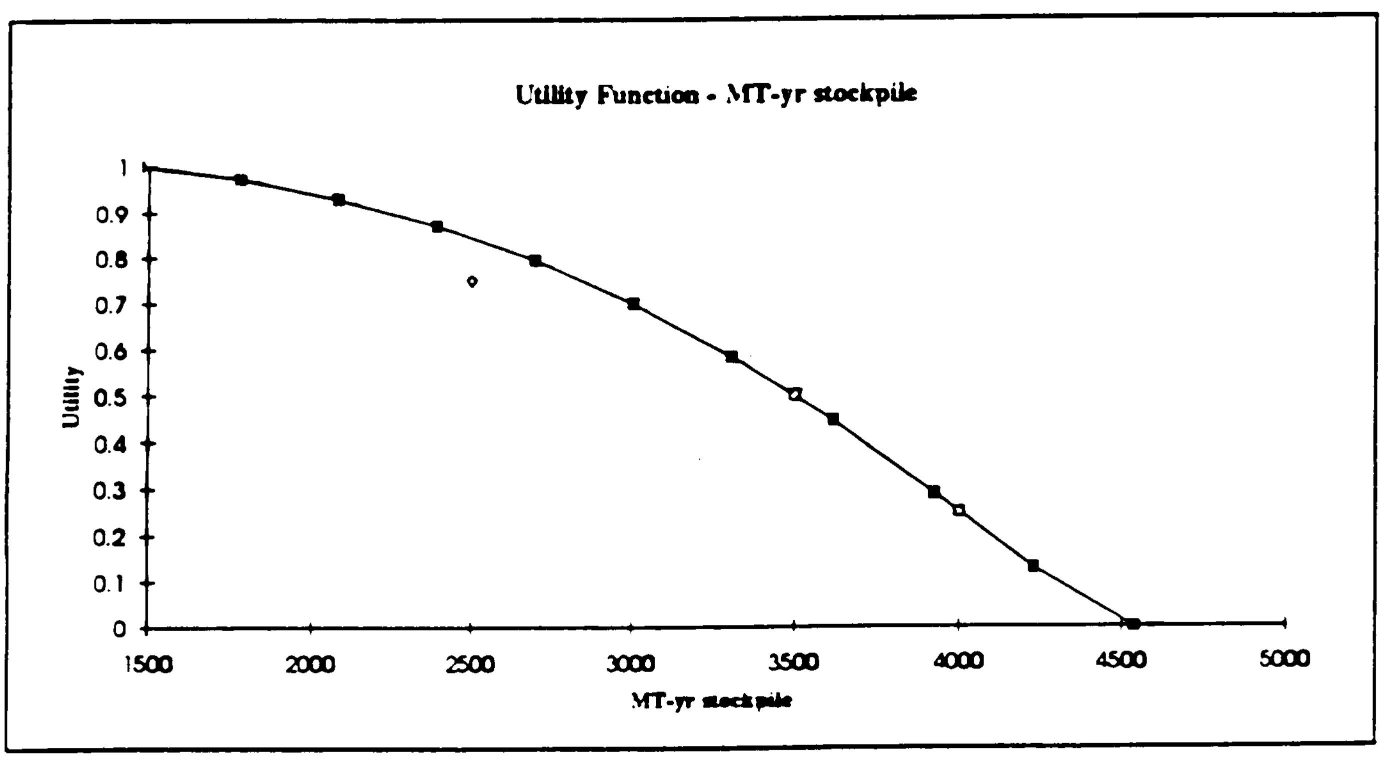


Figure B.3 Graph of utility function fitted to first two lottery questions

B.2 Coefficients for multiattribute utility function

The single attribute utility functions derived above are combined into a multiattribute utility function. The assumed form of the multiattribute utility function is shown below, where the subscripts a, b, c, and d refer to the four attributes NPV cost, MT-yr of stockpile, net plutonium destroyed, and discharge exposure of fuel.

$$U(u_a,u_b,u_c,u_d) = k_a u_a + k_b u_b + k_c u_c + k_d u_d \qquad \text{(B.10)}$$

The single attribute utility function coefficients k_a, k_b, k_c, and k_d are assessed by three pairwise comparison of reactor options. Each pair of options differ with respect to only two of the attributes. This makes it possible to measure the tradeoff of one attribute for another. The interface that provides for user input of the options for comparison is shown in Figure B.4 below.

	BF	BG	BH	BI	BJ	BK
21	Assessed measures for indifference between options A and B					
22	Option A			Option B		
23	NPV	MT-yr		NPV	MT-yr	
24	1500	2000		0	3000	
25	NPV	Pu destru		NPV	Pu destru	
26	2000	20%		0	10%	
27	NPV	disch. exp		NPV	disch. exp	
28	1000	30000		0	20000	

Figure B.4 Interface for pairwise comparison of options

Three pairwise comparison of options are specified in the figure. The first compares options, designated option A and option B (or reactor 1 and reactor 2), that differ with respect to the attributes NPV cost and MT-yr of stockpile. The second compares two options that differ with respect to NPV cost and fraction of plutonium destroyed. The third compares options that differ with respect to NPV cost and discharge exposure. The values for these attributes are adjusted until the user is indifferent between option A and option B in all three comparisons.

The spreadsheet model uses the utility functions derived in subsection B.1 to compute single attribute utility values for each of the attribute values in Figure B.4. Results of these computations are shown in Figure B.5 below.

	BL	BM	BN	BO	BP	BQ
22	Option A utilities			Option B utilities		
23	NPV	MT-yr		NPV	MT-yr	kb/ka
24	0.642499	0.942732		0.849284	0.699844	0.8513593
25	NPV	Pu destru		NPV	Pu destru	kc/ka
26	0.549569	0.88066		0.849284	0.396287	0.6187713
27	NPV	disch. exp		NPV	disch. exp	kd/ka
28	0.724274	0.769622		0.849284	0.5	0.4636502

Figure B.5 Single attribute utility values for pairwise comparison of options

Because options A and option B are specified such that one is indifferent between the two, the multiattribute utility function values for the two options must be the same in all three comparisons. Expression (B.10) then implies:

$$k_a u_a^A + k_b u_b^A + k_c u_c^A + k_d u_d^A = k_a u_a^B + k_b u_b^B + k_c u_c^B + k_d u_d^B \qquad \text{(B.11)}$$

where the superscripts A and B refer to options A and B. As an example, for the first comparison (line 24 in the figure), the utility function values for the attributes c and d are the same. So equation (B.11) for the first comparison is as follows:

$$k_a u_a^A + k_b u_b^A = k_a u_a^B + k_b u_b^B \qquad \text{(B.12)}$$

This equation can be solved for the ratio of the coefficients of attributes a and b.

$$\frac{k_b}{k_a} = -\frac{u_a^A - u_a^B}{u_b^A - u_b^B} \qquad \text{(B.13)}$$

The result of this calculation is shown in cell BQ24 of Figure B.5. The coefficient ratios k_c/k_a and k_d/k_a are determined in a similar manner using the option comparisons in rows 26 and 28 of Figure B.5, respectively. Given these three coefficient ratios, the normalization condition can be used to compute the coefficient k_a as follows:

$$k_a + k_b + k_c + k_d = 1.0 \qquad \text{(B.14)}$$

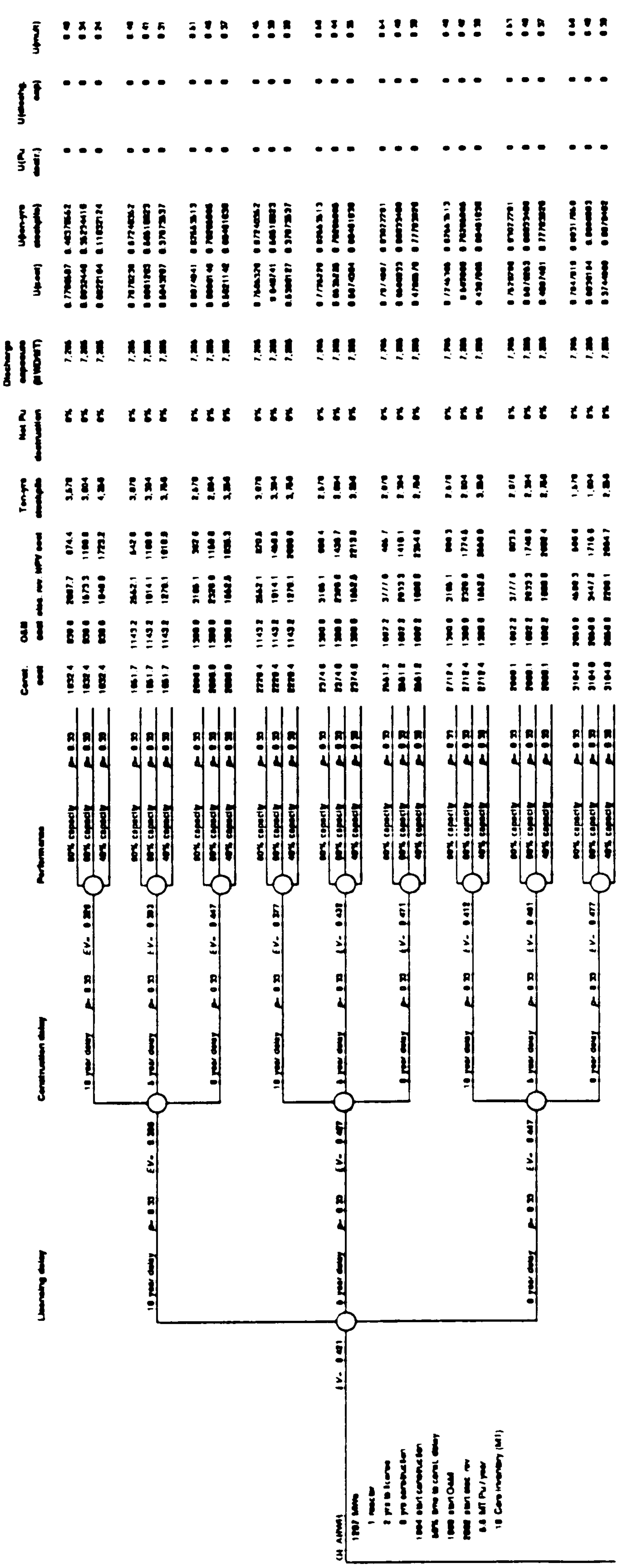
$$k_a = \frac{1}{1 + \frac{k_b}{k_a} + \frac{k_c}{k_a} + \frac{k_d}{k_a}} \qquad \text{(B.15)}$$

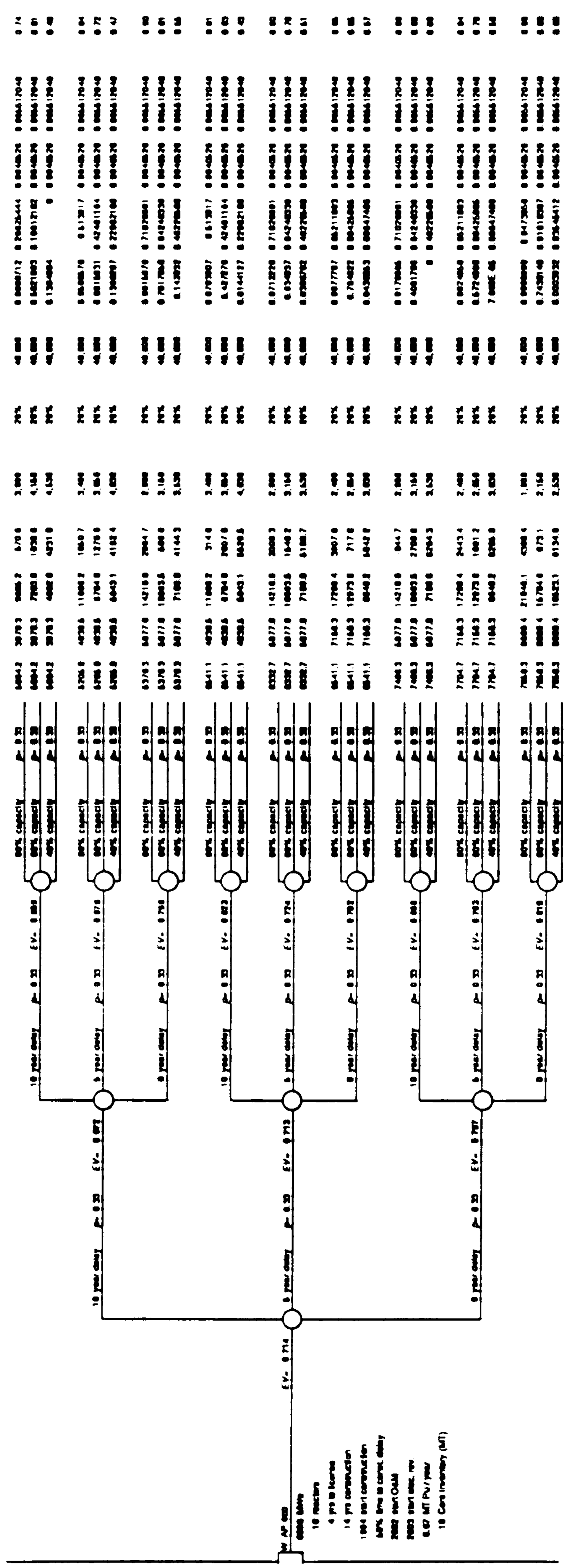
The remaining coefficients can then be computed using the coefficient k_a and the ratio of the remaining coefficient to k_a. The results of these calculations are shown below in Figure B.6.

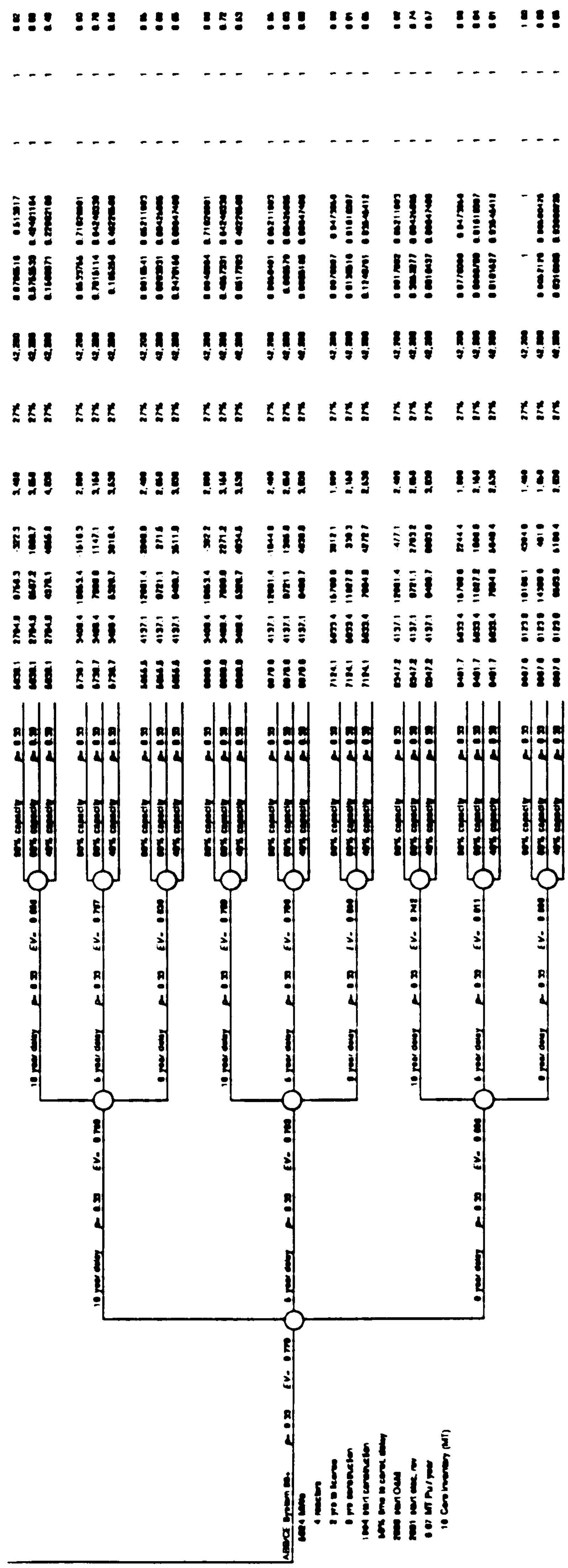
	BM	BN	BO	BP	BQ
32	Assume additive form: U = kaUa + kbUb + kcUc + kdUd				
33	ka=	0.340857	coefficient for NPV cost		
34	kb=	0.290192	coefficient for MT-yr stockpile		
35	kc=	0.210913	coefficient for fraction Pu fissioned		
36	kd=	0.158038	coefficient for discharge exposure		
37	sum:	1			

Figure B.6 Computation of coefficients in multiattribute utility function

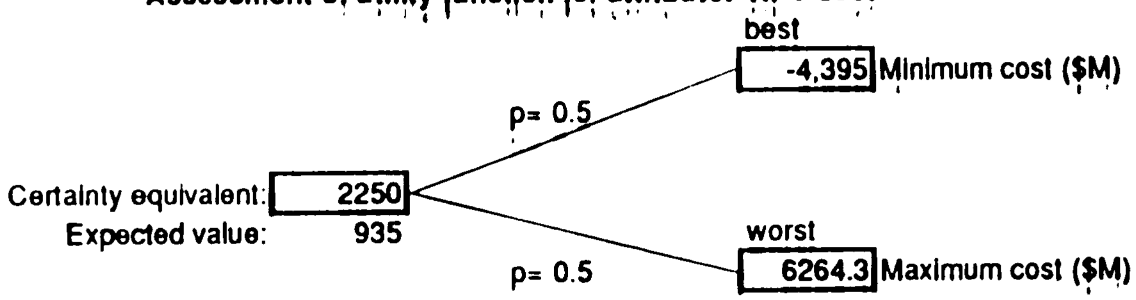
Appendix C Spreadsheet Model



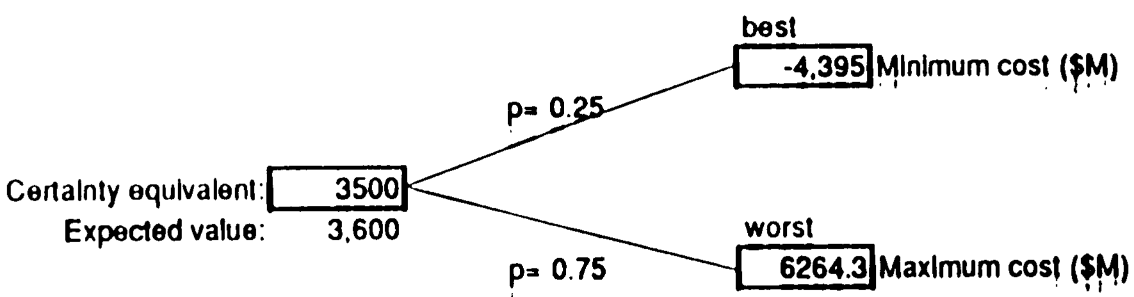




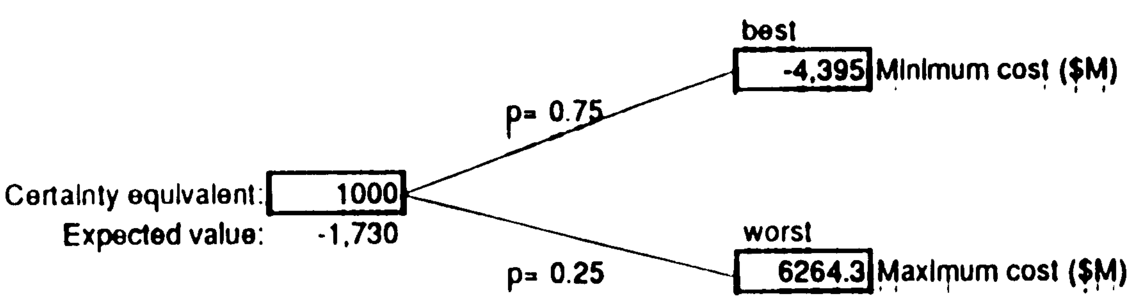
Assessment of utility function for attribute: NPV cost



scaled certainty equivalent: 0.376613



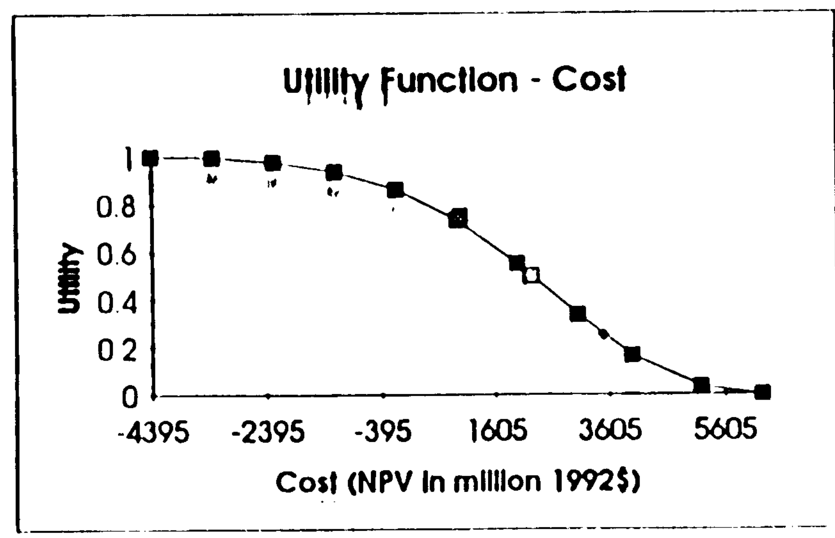
scaled certainty equivalent: 0.259341



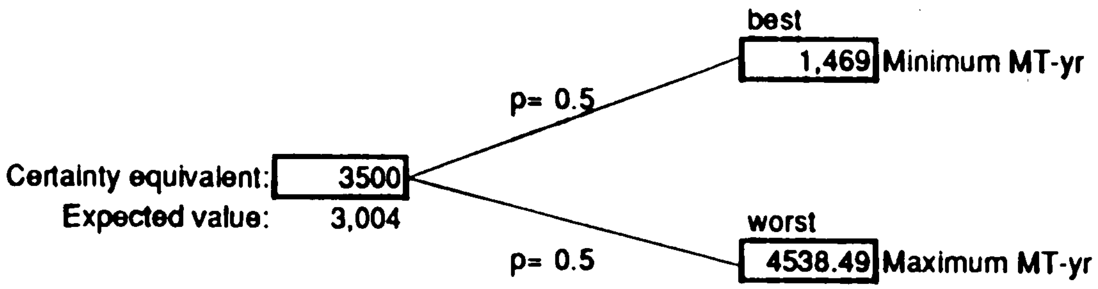
scaled certainty equivalent: 0.493886

Single attribute utility function

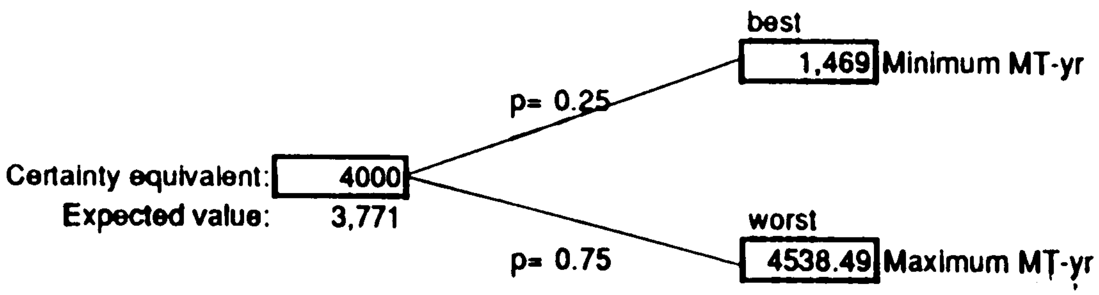
		x	u(x)	cost
x1:	0.37661	0	0	6264.3
x2:	0.25934	0.1	0.03197	5198.4
u1:	0.5	0.212	0.16463	4000
u2:	0.25	0.3	0.33369	3066.62
r1:	1.65524	0.4	0.54943	2000.72
r2:	2.85593	0.5	0.734	934.831
q1:	1	0.6	0.86196	-131.062
q2:	3	0.7	0.93829	-1196.96
b:	2.01414	0.8	0.97827	-2262.85
c:	0.36239	0.9	0.99568	-3328.74
		1	1	-4394.64



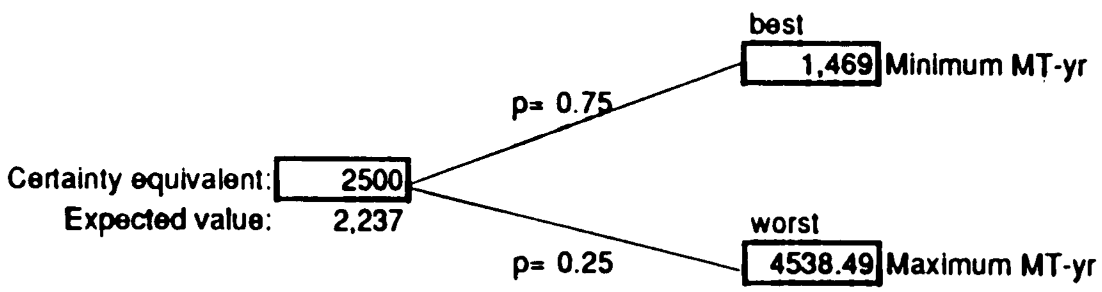
Assessment of utility function for attribute: MT-yr stockpile



scaled certainty equivalent: 0.338354



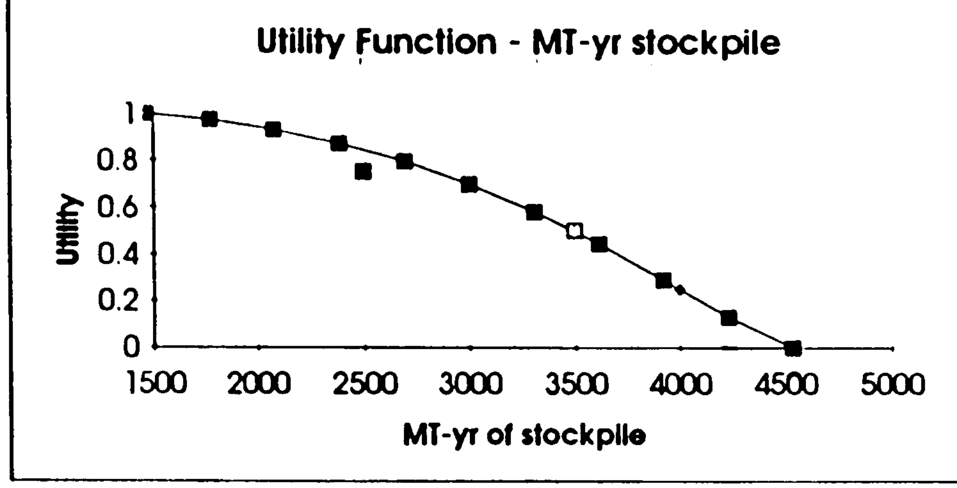
scaled certainty equivalent: 0.175448



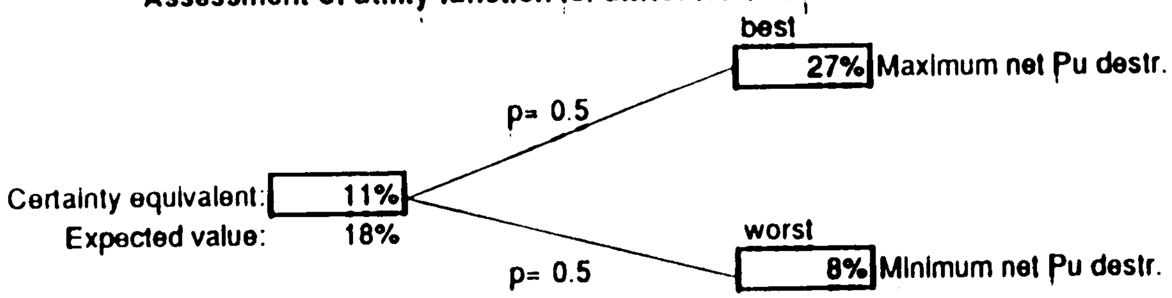
scaled certainty equivalent: 0.664167

Single attribute utility function

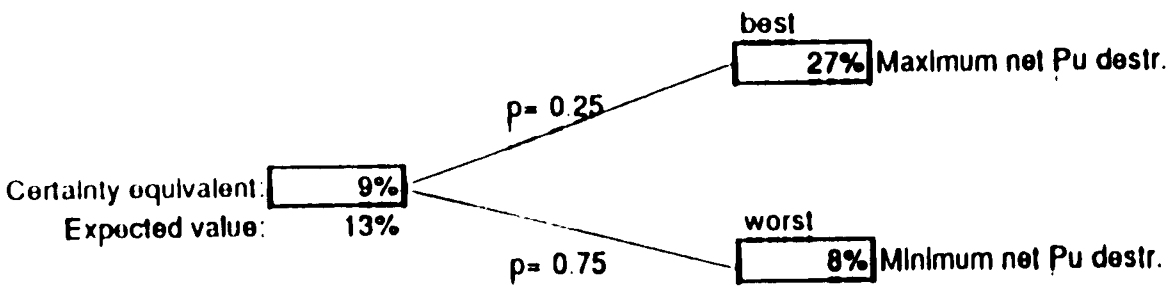
		x	u(x)	MT-yr
x1:	0.33835	0	0	4538.49
x2:	0.17545	0.1	0.12868	4231.57
u1:	0.5	0.2	0.28974	3924.64
u2:	0.25	0.3	0.44489	3617.72
r1:	1.95548	0.4	0.5823	3310.79
r2:	4.69969	0.5	0.69851	3003.87
q1:	1	0.6	0.79384	2696.95
q2:	3	0.7	0.8701	2390.02
b:	1.25289	0.8	0.92937	2083.1
c:	0.43161	0.9	0.97322	1776.17
		1	1	1469.25



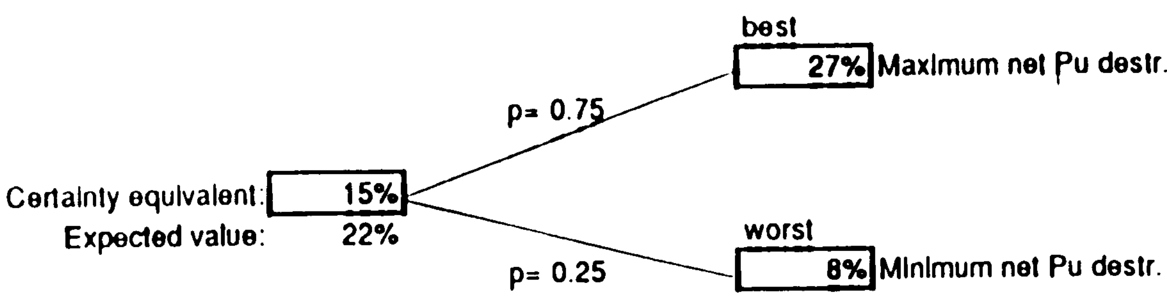
Assessment of utility function for attribute: Net Pu destruction



scaled certainty equivalent: 0.157895



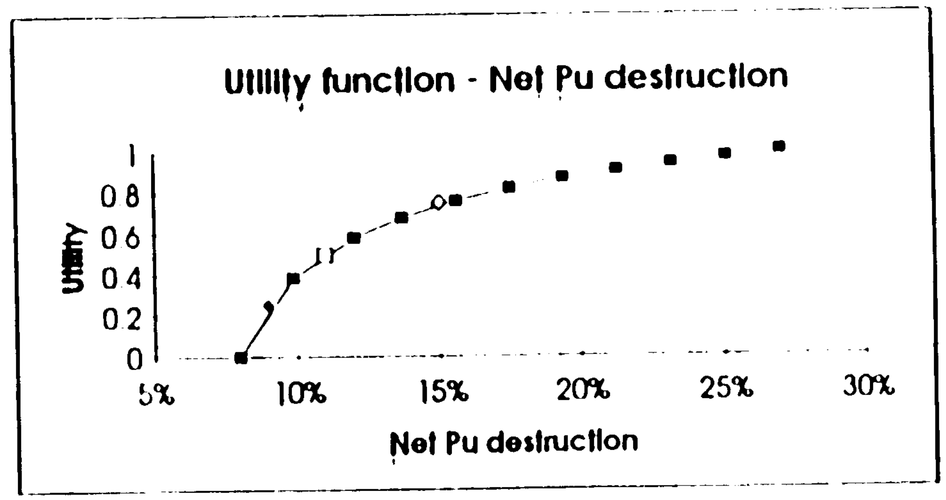
scaled certainty equivalent: 0.052632



scaled certainty equivalent: 0.368421

Single attribute utility function

		x	u(x)	Pu dest
x1:	0.15789	0	0	8%
x2:	0.05263	0.1	0.38401	10%
u1:	0.5	0.212	0.58138	12%
u2:	0.25	0.3	0.67844	14%
r1:	5.33333	0.4	0.75872	16%
r2:	18	0.5	0.81934	18%
q1:	1	0.6	0.86739	19%
q2:	3	0.7	0.90697	21%
b:	0.90317	0.8	0.9407	23%
c:	0.22049	0.9	0.97058	25%
		1	1	27%



Assessment of utility function for attribute: Discharge exposure

Certainty equivalent: 20,000
Expected value: 24,743

p= 0.5 — best: 42,200 Maximum dischg. exp.
p= 0.5 — worst: 7,285 Minimum dischg. exp.

scaled certainty equivalent: 0.36417

Certainty equivalent: 13,000
Expected value: 16,014

p= 0.25 — best: 42,200 Maximum dischg. exp.
p= 0.75 — worst: 7,285 Minimum dischg. exp.

scaled certainty equivalent: 0.163683

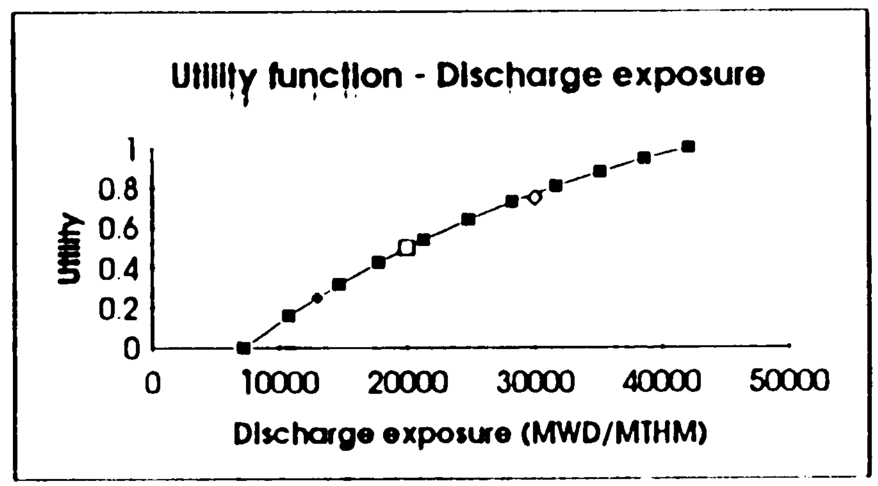
Certainty equivalent: 30,000
Expected value: 33,471

p= 0.75 — best: 42,200 Maximum dischg. exp.
p= 0.25 — worst: 7,285 Minimum dischg. exp.

scaled certainty equivalent: 0.65058

Single attribute utility function

		x	u(x)	disch. exp.
x1:	0.36417	0	0	7285
x2:	0.16368	0.1	0.15738	10776.5
u1:	0.5	0.212	0.31638	14702.1
u2:	0.25	0.3	0.42636	17759.5
r1:	1.74597	0.4	0.53876	21251
r2:	5.10936	0.5	0.63881	24742.5
q1:	1	0.6	0.72811	28234
q2:	3	0.7	0.80801	31725.5
b:	1.02314	0.8	0.87959	35217
c:	0.56541	0.9	0.94366	38708.5
		1	1	42200



Multiattribute utility function - assessment of tradeoffs between pairs of attributes

Assessed measures for indifference between options A and B

Option A		Option B		Option A utilities		Option B utilities		
NPV	MT-yr	NPV	MT-yr	NPV	MT-yr	NPV	MT-yr	kb/ka
1500	2000	0	3000	0.6424987	0.9427323	0.8492842	0.6998436	0.85135929
NPV	Pu destru	NPV	Pu destru	NPV	Pu destru	NPV	Pu destru	kc/ka
2000	20%	0	10%	0.5495686	0.8806596	0.8492842	0.3962874	0.61877133
NPV	disch. exp	NPV	disch. exp	NPV	disch. exp	NPV	disch. exp	kd/ka
1000	30000	0	20000	0.7242738	0.7696224	0.8492842	0.5	0.46365017

Option A(normalized measure)		Option B(normalized measure)	
NPV	MT-yr	NPV	MT-yr
0.446977	0.8270737	0.587704	0.5012609
NPV	Pu destru	NPV	Pu destru
0.400068	0.6315789	0.587704	0.1052632
NPV	disch. exp	NPV	disch. exp
0.493886	0.65058	0.587704	0.3641701

Assume additive form: U = kaUa + kbUb + kcUc + kdUd

ka=	0.3408571	coefficient for NPV cost
kb=	0.2901919	coefficient for MT-yr stockpile
kc=	0.2109126	coefficient for fraction Pu fissioned
kd=	0.1580384	coefficient for discharge exposure
sum:	1	

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