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TWO-PHASE POWER-LAW MODELING OF PIPE FLOWS DISPLAYING SHEAR-THINNING PHENOMENA

by
Jianmin Ding, Robert W. Lyczkowski,* and William T. Sha
Argonne National Laboratory
Energy Technology Division
*Energy Systems Division
9700 South Cass Avenue
Argonne, IL 60439

ABSTRACT

This paper describes our work in modeling concentrated liquid-solids flows in pipes. COMMIX-M, a three-dimensional transient and steady-state computer program developed at Argonne National Laboratory, was used to compute velocities and concentrations. Based on our previous analyses, some concentrated liquid-solids suspension flows display shear-thinning rather than Newtonian phenomena. Therefore, we developed a two-phase non-Newtonian power-law model that includes the effect of solids concentration on solids viscosity. With this new two-phase power-law solids-viscosity model, and with constitutive relationships for interfacial drag, virtual mass effect, shear lift force, and solids partial-slip boundary condition at the pipe walls, COMMIX-M is capable of analyzing concentrated three-dimensional liquid-solids flows.

1. Introduction

The behavior of concentrated two-phase liquid-solids suspension flows has been a matter of interest for many years because the processing and transport of these flows are important operations in many industrial applications. This has called attention to the need for a fundamental understanding of the physical phenomena for macroscopic computer simulations of these flows. However, there is as yet no comprehensive theory that accounts for all of the effects in concentrated suspensions, and few computations have been performed for fully three-dimensional two-phase liquid-solids flow.

Argonne National Laboratory (ANL) has initiated research on concentrated two-phase suspension flows (Sekar et al., 1988; Lyczkowski and Wang, 1992; Ding et al., 1992; 1993). A coordinated methodology that involves theory (development of field and constitutive equations), experimentation, and computer modeling is being pursued. ANL anticipates that synergism will be the result when a conscious effort is made to coordinate the extension of this research program, which involves complex and interdisciplinary phenomena that require advances in both a theory and experimentation.

Earlier efforts to study concentrated suspension flow have been reviewed in our previous papers (Ding et al., 1992; 1993). The philosophy of ANL is to use a self-consistent methodology to link micro- and macro-fluid mechanical phenomena. This philosophy will ensure the internal consistency of the design and instrumentation of the experiments, the data acquisition and its processing for use in the development of field and constitutive equations, and computer code validation.

We use the COMMIX-M computer code to analyze the velocities and concentrations of concentrated liquid-solids flows in pipes. COMMIX-M, a three-dimensional transient and steady-state computer program developed at ANL, is capable of analyzing multiphase flow and heat transfer and uses the separated-phases model wherein each phase has its own mass and momentum equations. For a brief description of the COMMIX-M computer code, refer to our previous paper (Ding et al., 1992) and references therein. The COMMIX-M fluid velocities computed with the new two-phase power-law solids viscosity model, together with constitutive relationships for interfacial drag, virtual mass and shear lift forces, and solids partial-slip boundary conditions at the pipe walls, are compared with data obtained by Sinton and Chow (1991) with three-dimensional time-of-flight nuclear magnetic resonance (NMR) imaging techniques. Comparisons are presented in this paper for average solids volume fractions of 21%, 40%, and 52%.

2. Hydrodynamic models

A two-phase, three-dimensional, transient hydrodynamic approach was used to model concentrated liquid-solids flows in pipes. All of the solid particles with identical densities and diameters form a continuum, a particulate phase. Each phase has its own mass and momentum equations. These equations, along with constitutive relations for interfacial drag, were presented in our previous paper (Ding et al., 1992) and are in Table 1 as Eqs. T1-T5 and T8.

In the two-phase momentum equations (Eqs. T4-T8), the shear lift force per unit volume $\mathbf{F}_L$ was extended from Saffman's shear lift force expression (Saffman, 1965; 1968) for a single particle in a simple shear flow to a collection of particles in a general flow field. The expression for $\mathbf{F}_L$ is given by Eq. T6 in Table. 1. When a particle accelerates with respect to the fluid, a virtual mass force that is proportional to the relative acceleration is developed on the particle. The virtual mass force per unit volume $\mathbf{F}_V$ for a collection of particles is given by Eq. T7 in Table. 1. Detailed derivations for these two forces were given by Ding et al.

(1993).

The shear lift and virtual mass forces, represented by Eq. T6 and T7, respectively, were used to perform all the calculations in this paper. These two forces were found to be a secondary effect on the computed fluid velocities and fluid volume fractions because the relative velocity of two phases is small because the Reynolds number is low. Neglect of these two terms was found to produce results that agreed somewhat less with experimental data (Ding et al., 1993). Therefore, it is concluded that these two forces are probably generally necessary to properly describe the solids concentration and velocity fields.

The liquid phase is assumed to be a Newtonian fluid. For isothermal and laminar flow, the liquid viscosity μ_f is constant. For the solids phase, the model for the effective shear viscosity μ_s in two-phase flow is of major concern in this paper. Based on our previous analyses (Ding et al., 1993) and experimental rheological findings (see, for example, Sinton and Chow 1991; Sekar et al, 1988; Lyczkowski and Wang, 1992; Wildman et al; 1992), some concentrated liquid-solids suspension flows display shear-thinning rather than Newtonian phenomena. Such shear-thinning phenomena differ from shear-thickening behavior observed in some rapid gas-solids flow systems, for which a theoretically obtained expression for solids viscosity using a kinetic theory of granular approach was found to be quite successful (Ding and Gidaspow, 1990; Ding and Lyczkowski 1992; Sinclair and Jackson 1989; Pita and Sundaresan, 1991). In rapid gas-solids flow, particle-particle collisions are dominant for momentum transfer, i.e., the duration of two particle contacts is very short compared with a hydrodynamic time scale. Therefore, the kinetic-theory method can be used for rapid gas-solids flow and the solids viscosity obtained is proportional to the shear rate. In slow liquid-solids flows, such as we are studying, the duration of contact between two particles and the fluid lubrication between two particles may play important roles in the rheology. Under such conditions, the kinetic-theory approach may not be applied. When the shear rate of a liquid-solids mixture becomes high enough, the shear-thickening phenomena appear, as shown for example, in the experiments of Sekar et al. (1988).

For modeling the flow of liquid-solids in pipes, we implemented a modified two-phase non-Newtonian power-law model for solids viscosity to account for a resting (static) mixture viscosity. The relative mixture viscosity η can be expressed as

$$\eta = \frac{\epsilon_s \mu_s + \epsilon_f \mu_f}{\mu_f} = m[1 + (\lambda \dot{\gamma})^2]^{\frac{n-1}{2}}, \tag{1}$$

where m, λ, and n are parameters. Eq. 1 generalizes the model used by Lyczkowski and Wang (1992). We expect that parameters m, λ, and n depend on solids concentration, ratio of particle size to pipe diameter, and density ratio of fluid to solids. From Sinton and Chow's (1991) rheological data, as shown in Fig. 1, the solids concentration affects the rheological properties. At lower solids volume fractions, the flow is Newtonian. As the solids volume fraction increases, shear-thinning behavior becomes significant. For simplicity, we assumed the those parameters in Eq. 1 are a function of solids volume fraction only and used second order polynomial functions to correlate these parameters with one another based on Sinton and Chow's (1991) measured rheological data. We obtained

$$m = m(\epsilon_s) = 220.7 - 1636.4\epsilon_s + 3024.2\epsilon_s^2 \tag{2a}$$

$$n = n(\epsilon_s) = 1.088 - 0.378\epsilon_s - 0.194\epsilon_s^2 \tag{2b}$$

$$\lambda = \lambda(\epsilon_s) = 1026.3 - 7461.4\epsilon_s + 12257.4\epsilon_s^2 \tag{2c}$$

Equations 1 and 2 are plotted in Fig. 1, together with Sinton and Chow's (1991) data.

To solve the governing equations for fluid-solids flow given in Table 1, we need appropriate initial and boundary conditions for the two-phase velocities, fluid-phase pressure, and volume fraction. The initial conditions depend upon the problem under investigation. The inlet conditions are usually given. The boundary conditions at planes of symmetry demand zero normal gradient of all variables. The no-slip condition was used for the fluid-phase velocity at pipe walls. but this condition cannot always be applied to the solids phase because particles may slip along the wall. The mean solids slip velocity at a solid wall was proposed by Soo (1969) to be

$$v_2|_w = -\lambda_p \frac{\partial v_2}{\partial x_1}|_w, \tag{3}$$

where the x_1 direction is normal to the wall and the x_2 direction is tangential to the wall. The slip parameter λ_p is taken to be the mean distance between particles. Here we used the expression from the work of Ding and Lyczkowski (1992) for λ_p given by

$$\lambda_p = \frac{\sqrt{3\pi}}{24}\frac{d_p}{\epsilon_s g_0}, \tag{4}$$

where

$$g_0 = (1 - \frac{\epsilon_s}{0.65})^{-1.625}. \tag{5}$$

The new two-phase power-law solids-viscosity model, together with constitutive relationships for interfacial drag, virtual mass and shear lift forces, given in Table 1, Eqs. T6-T8, and solids partial-slip boundary condition at the pipe walls, given in Eq. 3, were used to model liquid-solids flow in pipes.

3. Comparison with experimental data

We have analyzed some of the steady-state, fully developed, and isothermal carrier-fluid velocity and solids concentration data of Altobelli et al. (1991) and Sinton and Chow (1991) which were obtained using three-dimensional time-of-flight nuclear magnetic resonance (NMR) imaging techniques (Ding et al., 1992; 1993). NMR imaging is a powerful technique to nonintrusively determine three-dimensional time-dependent velocity and concentration fields to assist development and validation of the constitutive models and the computer programs that describe concentrated suspensions. These experiments were carefully performed and probably represent the best available data of their kind in the open literature. In this paper, for the purpose of studying shear-thinning phenomena of neutrally buoyant dense suspensions in vertical pipes, we used the models presented in the previous section to analyze the data of Sinton and Chow (1991).

The Sinton and Chow (1991) experiments consisted of a suspension of neutrally buoyant, poly (methylmetkacrylate) spheres (Lucite 47G) with a median volume diameter of 0.131 mm, and a standard deviation of 0.051 mm, flowing in vertical pipes with diameters of 15.2, 25.4, and 50.8 mm and a 500-mm entrance length. Intensity and velocity data were collected over a range of 21 to 52 vol.% plastic spheres and Reynolds numbers ranging from 0.005 to 4.0. The carrier fluid was a mixture of polyether oil (Uncon oil, 75-H-90,000), water and sodium iodide to increase the fluid density to that of the solids having a density of 1190 kg/m^3.

NMR data were taken with a vertically oriented 4.7 T superconducting solenoid by techniques developed by Kose et al. (1985) and Majors et al. (1989). A positive-displacement Moyno pump was used.

Three runs were analyzed: (1) 21 vol.% solids, an average fluid velocity of 22.7 cm/s, and pipe diameter of 2.54 cm; (2) 40 vol.% solids, an average fluid velocity of 17.6 cm/s, and pipe diameter of 1.52 cm; and (3) 52 vol.% solids, an average fluid velocity of 17.5 cm/s, and pipe diameter of 1.52 cm. The pipes were modeled in two dimensions, assuming azimuthal symmetry. A total of 20 nodes were used in the radial direction and 25 in the axial direction.

Fluid velocities computed with both the current model and our Krieger's Newtonian-type solids-viscosity model were compared with the measured data. In our previous analyses (Ding et al 1992; 1993), the solids viscosity μ_s was obtained from Krieger's (1972) empirical expression for the relative viscosity given by

$$\eta = \frac{\epsilon_s \mu_s + \epsilon_f \mu_f}{\mu_f} = (1 - \frac{\epsilon_s}{0.68})^{-1.82}. \tag{6}$$

This expression was used by Phillips et al. (1992) in their analyses of concentrated suspension data. In Eq. 6, the solids viscosity is determined by the solids volume fraction only.

For the case of 21 vol.% solids, as shown in Fig. 1, the relative viscosity is nearly independent of shear rate. Hence, the computed fluid velocities with the new model and Krieger's model for solids viscosity are very close, as shown in Fig. 2. Reasonably good agreement exists between the model predictions and the data. This case exhibited basically Newtonian behavior.

Figure 3 shows the two model predictions and the NMR measured data for the case of 40 vol.% solids. Except near the pipe center, both models agree with the data very well. Because the rheological experiments show a slightly shear-thinning behavior, the new two-phase power-law model gives better agreement with the data near the pipe center.

As can be seen in Figs. 1 and 4, when the solids volume fraction reaches 52%, the shear-thinning phenomena become very prominent. For the case of 52 vol.%, the new two-phase power-law model agrees much better than the Krieger (1972) Newtonian-type model near the pipe center. The data clearly exhibit shear-thinning behavior with a blunted velocity profile. Therefore, the non-Newtonian shear-thinning model is strongly recommended for suspension flows at high solids volume fractions.

Our computer model predictions for these three cases did not show significant nonuniform distributions of solids in the radial direction. Slightly higher solids volume fractions were found in computational nodes away from the pipe walls. This results from particle migration toward the pipe center and is due to the shear lift effect at the wall. It should be also noted that these experiments did not report significant particle migration nor solids concentration distributions.

4. Conclusions

Based upon the agreement between the COMMIX-M computer code predictions and the experimental data, the models proposed in this paper appear to be reasonable and promising. Thus far, no adjustments in the literature model have been made. The parameters in the near empirical shear-thinning model are functions of the physical properties of the fluid and solids concentration. Rheological experiments can be performed to obtained these parameters as input to the computer model and to predict design of dense-suspension flow systems. The rheological properties of suspension flow should be further examined and studied to develop a shear-thinning model that not only includes the effect of solids concentration but also particle size and shape, density ratio of fluid to solids, and carrier fluid velocity and/or velocity gradients. Because some investigations have reported particle migration phenomena (Phillips et al., 1992; and references therein), particle migration mechanisms should also be studied.

Further improvements of the model should increase our confidence in predicting design and processing of concentrated suspension flow systems. Such improvements will be from additional comparisons with a wider data base of experimental measurements and COMMIX-M analyses, which will also serve to more critically evaluate the models.

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Nomenclature

C_d	Drag coefficient
d_p	Particle diameter
$\mathbf{F}_{\mathrm{L}}$	Shear lift force per unit volume
$\mathbf{F}_{\mathrm{V}}$	Virtual mass force per unit volume
$\mathbf{g}$	Acceleration due to gravity
g_0	Radial distribution function
$\bar{\bar{\mathbf{I}}}$	Unit tensor
m	Parameter defined in Eq. 1
n	Parameter defined in Eq. 1
p	Pressure
$\bar{\bar{\mathbf{S}}}$	Deformation rate tensor
t	Time
$\mathbf{u}$	Fluid phase velocity
$\mathbf{v}$	Solids phase velocity
Greek letters	
β	Two-phase drag force coefficient
ϵ	Volume fraction
η	Relative mixture viscosity
λ	Parameter defined in Eq. 1
λ_p	Mean distance between particles
μ	Viscosity
ρ	Density
$\bar{\bar{\tau}}$	Stress tensor
Subscripts	
f	Fluid phase
s	Solids phase
w	Wall

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Table 1. GOVERNING EQUATIONS FOR FLUID-SOLIDS SUSPENSION FLOW

CONTINUITY EQUATIONS

Fluid Phase

$$\frac{\partial}{\partial t}(\epsilon \rho_f) + \nabla \cdot (\epsilon \rho_f \mathbf{u}) = 0 \tag{T1}$$

Solids Phase

$$\frac{\partial}{\partial t}(\epsilon_s \rho_p) + \nabla \cdot (\epsilon_s \rho_p \mathbf{v}) = 0 \tag{T2}$$

$$\epsilon_f + \epsilon_s = 1 \tag{T3}$$

MOMENTUM EQUATIONS

Fluid Phase

$$\frac{\partial}{\partial t}(\epsilon_f \rho_f \mathbf{u}) + \nabla \cdot (\epsilon_f \rho_f \mathbf{uu}) = -\epsilon_f \nabla p_f + \epsilon_f \rho_f \mathbf{g} + \nabla \cdot \bar{\bar{\tau}}_{\mathbf{f}} + \beta(\mathbf{v} - \mathbf{u}) - \mathbf{F}_1 - \mathbf{F}_v, \tag{T4}$$

where

$$\bar{\bar{\tau}}_{\mathbf{f}} = 2\epsilon_f \mu_f \bar{\bar{\mathbf{S}}}_{\mathbf{f}} \tag{T4a}$$

$$\bar{\bar{\mathbf{S}}}_{\mathbf{f}} = \frac{1}{2}[\nabla \mathbf{u} + (\nabla \mathbf{u})^T] - \frac{1}{3} \nabla \cdot \mathbf{u} \bar{\bar{\mathbf{I}}} \tag{T4b}$$

Solids Phase

$$\frac{\partial}{\partial t}(\epsilon_s \rho_p \mathbf{v}) + \nabla \cdot (\epsilon_s \rho_p \mathbf{vv}) = -\epsilon_s \nabla p_f + \epsilon_s \rho_p \mathbf{g} + \nabla \cdot \bar{\bar{\tau}}_{\mathbf{s}} + \beta(\mathbf{u} - \mathbf{v}) + \mathbf{F}_1 + \mathbf{F}_v \tag{T5}$$

where

$$\bar{\bar{\tau}}_{\mathbf{s}} = 2\epsilon_s \mu_s \bar{\bar{\mathbf{S}}}_{\mathbf{s}} \tag{T5a}$$

$$\bar{\bar{\mathbf{S}}}_{\mathbf{s}} = \frac{1}{2}[\nabla \mathbf{v} + (\nabla \mathbf{v})^T] - \frac{1}{3} \nabla \cdot \mathbf{v} \bar{\bar{\mathbf{I}}} \tag{T5b}$$

Shear lift force $\mathbf{F}_1$

$$\mathbf{F}_1 = 6.17\epsilon_s(\rho_f \mu_f)^{1/2}(\mathbf{u} - \mathbf{v}) \cdot \bar{\bar{\mathbf{S}}}_{\mathbf{f}}(2\bar{\bar{\mathbf{S}}}_{\mathbf{f}} : \bar{\bar{\mathbf{S}}}_{\mathbf{f}})^{-1/4}/d_p \tag{T6}$$

Virtual mass force

$$\mathbf{F}_v = \frac{1}{2}\epsilon_s \rho_f (\frac{D\mathbf{u}}{Dt} - \frac{D\mathbf{v}}{Dt}) \tag{T7}$$

Fluid-solids drag coefficeints

For $\epsilon_f < 0.8$, (Ergun equation)

$$\beta = 150\frac{\epsilon_s^2 \mu_f}{\epsilon_f d_p^2} + 1.75\frac{\rho_f \epsilon_s |\mathbf{u} - \mathbf{v}|}{d_p} \tag{T8a}$$

For $\epsilon_f > 0.8$, (Wen and Yu's empirical correlation)

$$\beta = \frac{3}{4}C_d \frac{\epsilon_f \epsilon_s \rho_f |\mathbf{u} - \mathbf{v}|}{d_p} \epsilon_f^{-2.65}, \tag{T8b}$$

where

$$C_d = \frac{24}{Re_p}[1 + 0.15 Re_p^{0.687}], \quad For \quad Re_p < 1000 \tag{T8c}$$

$$C_d = 0.44, \quad for \quad Re_p \geq 1000 \tag{T8d}$$

$$Re_p = \frac{\epsilon_f \rho_f |\mathbf{u} - \mathbf{v}| d_p}{\mu_f} \tag{T8e}$$

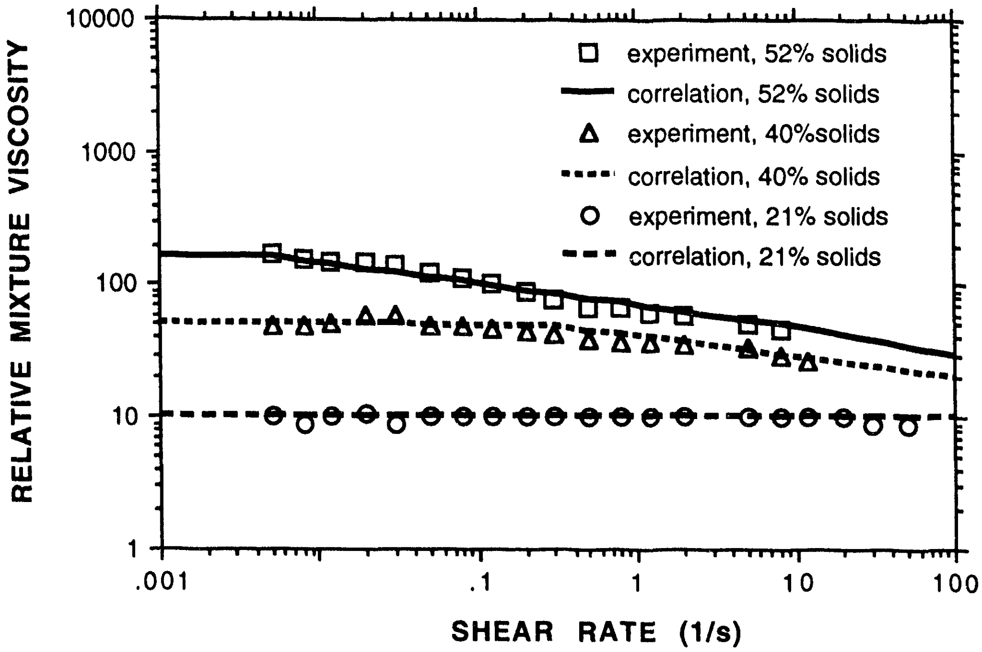

Figure 1. Relative mixture viscosity as a function of shear rate, as calculated by us and experimentally determined by Sinton and Chow (1991)

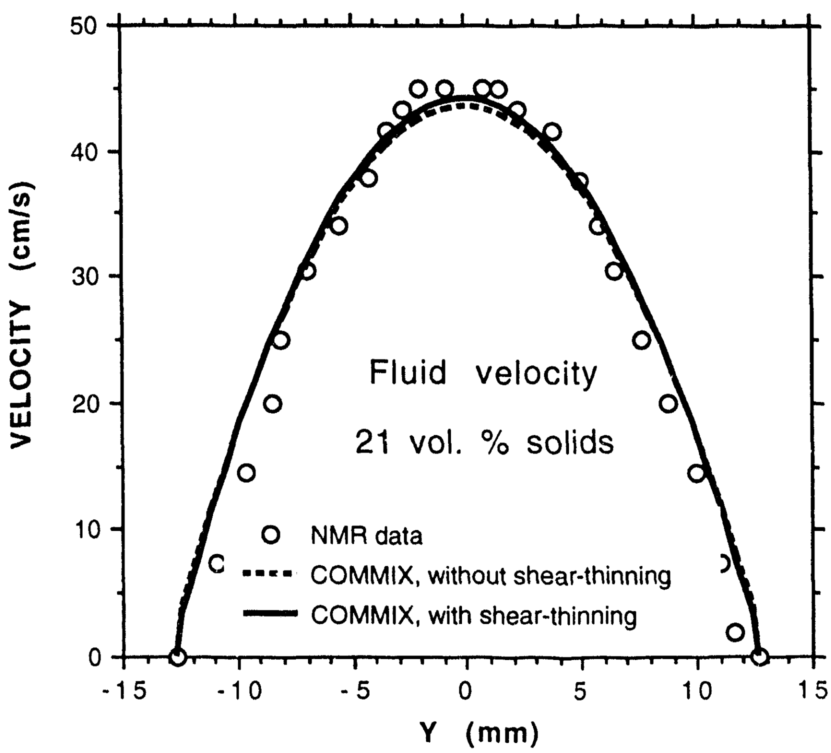

Figure 2. NMR fluid velocity data predicted by COMMIX-M and determined experimentally by Sinton and Chow (1991) for the case of 21 vol.% solids

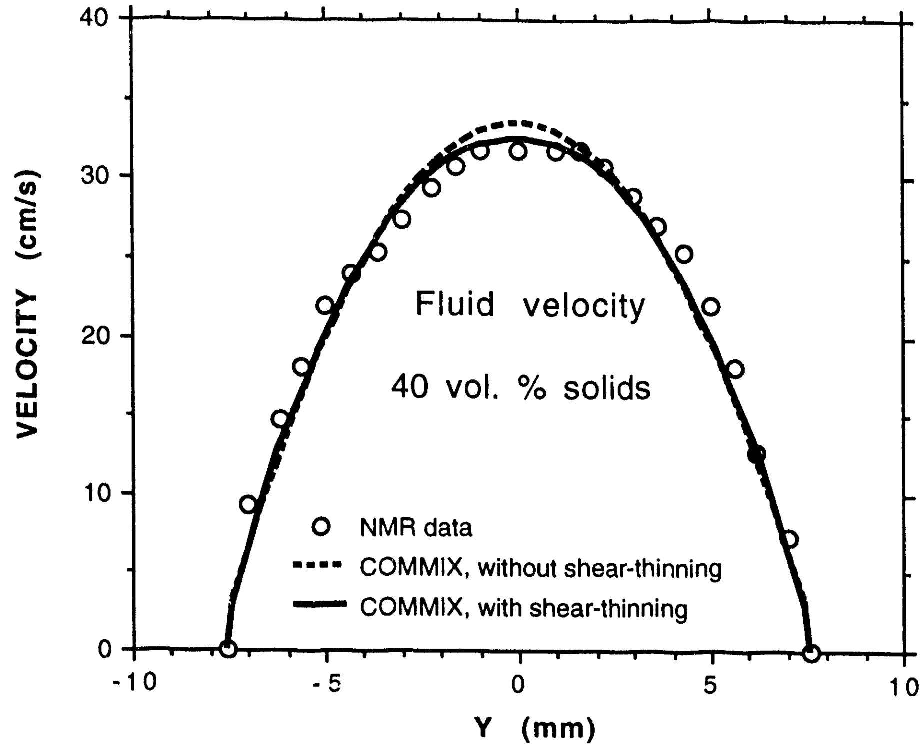

Figure 3. NMR fluid velocity data presented by COMMIX-M and determined experimentally by Sinton and Chow (1991) for the case of 40 vol.% solids

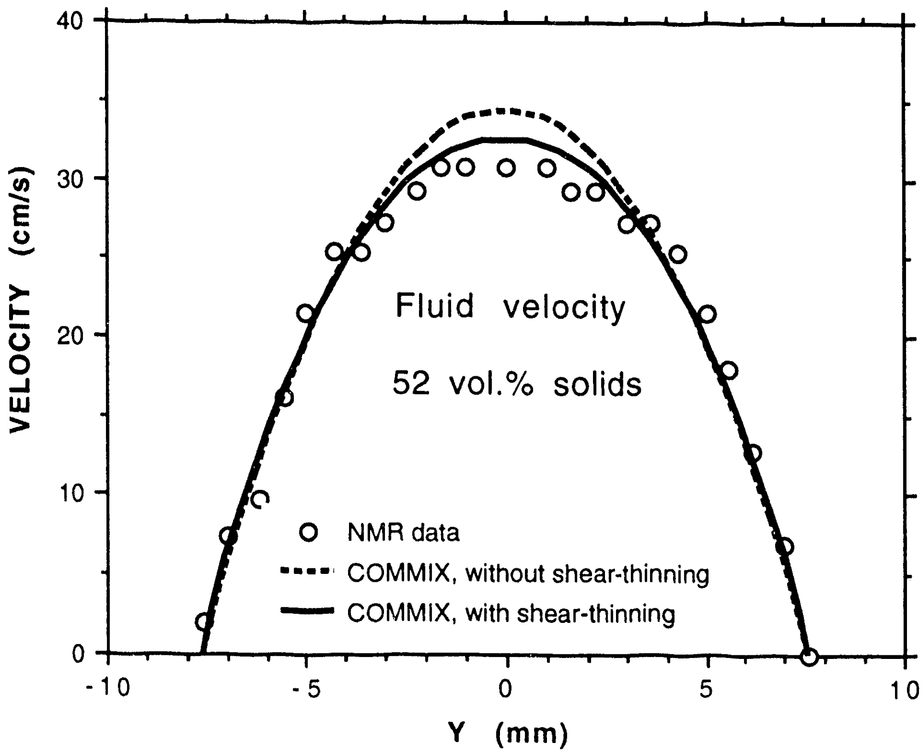

Figure 4. NMR fluid velocity data presented by COMMIX-M and determined experimentally by Sinton and Chow (1991) for the case of 52 vol.% solids

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