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Decays rates for S- and P-wave bottomonium

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We use the Bodwin-Braaten-Lepage factorization scheme to separate the long- and short-distance factors that contribute to the decay rates of Υ, η_b (S-wave) and χ_b,h_b (P-wave). The long distance matrix elements are calculated on the lattice in the quenched approximation using a non-relativistic formulation of the b quark dynamics.

In heavy quarkonium decays that involve quark-antiquark ($Q\bar{Q}$) annihilation, this annihilation occurs at short distances ($\sim 1/M_q$). Bodwin, Braaten and Lepage [1] have shown that this enables one to factor such decay rates into a sum of products of a short-distance parton-level decay rate with a long-distance matrix element between quarkonium states. The short distance pieces are calculated perturbatively, while the long distance parts are accessible to lattice calculations. To lowest non-trivial order in v^2, the square of the quark velocity, ($v^2 \sim .1$ for bottomonium)

$$\begin{aligned}\Gamma(^{2s+1}S_J \to X) &= G_1(S)\hat{\Gamma}_1(Q\bar{Q}(^{2s+1}S_J) \to X)\\ \Gamma(^{2s+1}P_J \to X) &= H_1(P)\hat{\Gamma}_1(Q\bar{Q}(^{2s+1}P_J) \to X)\\ &\quad + H_8(P)\hat{\Gamma}_8(Q\bar{Q}(^{2s+1}S_J) \to X),\end{aligned} \quad (1)$$

where the X's represent states of light partons. The $\hat{\Gamma}$'s are the short-distance ($p \sim M_Q$) parton-level decay rates. G_1, H_1 and H_8 are the long-distance ($p \sim M_Q v$, $E \sim M_Q v^2$) matrix elements that we calculate on the lattice.

In our lattice calculations we have used 149 independent equilibrated quenched gauge configurations on a $16^3 \times 32$ lattice with $\beta = 6.0$. Heavy-quark, and hence quarkonium, propagators were calculated using the non-relativistic formulation of Lepage and collaborators [2]. We used the lattice version of the quark action that is based on the euclidean lagrangian

$$\mathcal{L}_Q = \psi^\dagger(D_t - \frac{\mathbf{D}^2}{2M_Q})\psi + \chi^\dagger(D_t + \frac{\mathbf{D}^2}{2M_Q})\chi, \quad (2)$$

which is valid to the lowest non-trivial order in v^2. We calculate the quark Green's function that obeys the evolution equation [2]

$$\begin{aligned}G(\mathbf{x},t+1) &= (1 - H_0/2n)^n U^\dagger_{\mathbf{x},t}(1 - H_0/2n)^n\\ &\quad \times G(\mathbf{x},t) + \delta_{\mathbf{x},0}\delta_{t+1,0}\,,\end{aligned} \quad (3)$$

with $G(\mathbf{x},t) = 0$ for $t < 0$, and $H_0 = -\Delta^{(2)}/2M_0 - h_0$. Here $\Delta^{(2)}$ is the gauge-covariant discrete laplacian, and M_0 the bare quark mass. $h_0 = 3(1 - u_0)/M_0$, where $u_0 = \langle 0|\frac{1}{3}\mathrm{Tr}U_{plaq}|0\rangle^{\frac{1}{4}}$.

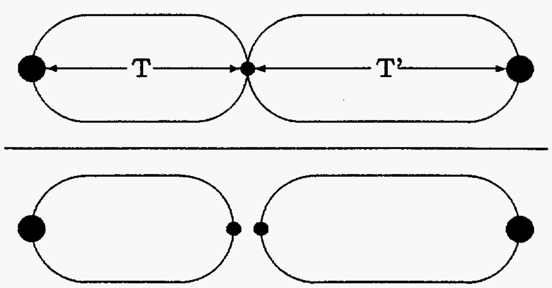
The matrix elements we calculate are defined as

$$G_1 = \langle ^1S|\psi^\dagger\chi\chi^\dagger\psi|^1S\rangle/M_Q^2 \quad (4)$$

$$H_1 = \langle ^1P|\psi^\dagger(i/2)\overleftrightarrow{\mathbf{D}}\chi\cdot\chi^\dagger(i/2)\overleftrightarrow{\mathbf{D}}\psi|^1P\rangle/M_Q^4 \quad (5)$$

$$H_8 = \langle ^1P|\psi^\dagger T^a\chi\chi^\dagger T^a\psi|^1P\rangle/M_Q^2\,, \quad (6)$$

On the lattice, we calculate the related quantities G_1^*, H_1^*, H_8^*, defined graphically below



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where the larger dots represent the "sources", the small dot in the numerator is the appropriate 4-fermi operator, and the small dots in the denominator represent point "sinks". For our calculations we generate the retarded (Eqn. (3)) and advanced quark propagators from noisy point and

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noisy extended sources on each of the 32 timeslices. (This differs from our preliminary calculations, in which the 4-fermi operator was used as a source.) Then as $T, T' \to \infty$

$$G_1^*(T,T') \to G_1 \frac{2\pi M_Q^2}{3|R_{1S}(0)|^2} = 1 + \mathcal{O}(v^4) \qquad (7)$$

$$H_1^*(T,T') \to H_1 \frac{2\pi M_Q^4}{9|R'_{1P}(0)|^2} = 1 + \mathcal{O}(v^4) \qquad (8)$$

$$H_8^*(T,T') \to \frac{H_8}{M_Q^2 H_1} + \mathcal{O}(v^4)\,, \qquad (9)$$

where R_{1S} is the radial wave function of the 1S state and R'_{1P} is the derivative of the radial wave function of the 1P state.

For bottomonium, we use input parameters determined by the NRQCD collaboration [3], which in our convention are: the bare b-quark mass, $M_{0b} = 1.5$, the inverse lattice spacing, $a^{-1} = 2.4\text{GeV}$, and the physical b-quark mass, $M_b = 2.06$. In Fig. 1 we show G_1^* as a function of T,T'. It is clearly very close to the vacuum saturation value of 1. In fact $G_1^* - 1 \approx 1.3 \times 10^{-3}$. H_1^* displays similar behaviour, but is more noisy. H_8^* is plotted in Fig. 2. We notice that it displays a fairly obvious plateau at small T, T', which degenerates into noise for larger values of T, T'. No improvement in H_8^* is obtained by using the extended source. Fitting the plateau, we obtain

$$H_8/H_1 \approx 0.06\,.$$

This is somewhat smaller than the value obtained from a simple perturbative estimate [1]. However, this estimate comes from assuming that H_8 becomes negligibly small when the momentum cutoff is Λ_{QCD}. If one assumes, instead, that H_8 becomes negligible at a cutoff closer to the bottomonium binding energy, then the perturbative estimate is closer to the lattice measurement. Of course, the lattice-regulated G_1, H_1 and H_8 differ from their continuum counterparts at $\mathcal{O}(\alpha_s)$, but since our methods are equivalent to using mean-field improved actions, these renormalizations are expected to be small.

We have also considered the S-wave decays through next-to-leading order in v^2. To this order, G_1 is no longer the same for Υ and η_b. However, we would need an improved action in order to calculate these corrections. In addition, there is a second term in Eqn. (1),

$$F_1(S)\hat{\Gamma}'_1(Q\bar{Q}(^{2s+1}S_J \to X)\,, \qquad (10)$$

where $\hat{\Gamma}'_1$ is another perturbative parton-level decay rate and F_1 can be calculated on the lattice using the Lagrangian of Eqn. (2). In the vacuum saturation approximation,

$$F_1(S) = \langle 0|\psi^\dagger\chi|0\rangle\langle 0|\psi^\dagger(\frac{-i}{2}\overleftrightarrow{\mathbf{D}})^2\chi|0\rangle/M_Q^4\,. \qquad (11)$$

On the lattice we measure F_1^*, defined as

$$F_1^* = \frac{M_Q^2 F_1}{G_1}\,. \qquad (12)$$

We find that

$$F_1^* = 1.3134(9) \text{ — non-covariant} \qquad (13)$$

$$F_1^* = 0.8519(6) \text{ — covariant}\,, \qquad (14)$$

where non-covariant and covariant refer to whether we use ordinary derivatives (in coulomb gauge) or gauge-covariant derivatives in Eqn. (11). As with G_1, H_1 and H_8, F_1^* requires renormalization. F_1 mixes with G_1. Since $F_1/G_1 \sim v^2$, this mixing can be significant. We have calculated these mixings to 1-loop order. Preliminary estimates of the F_1^*'s which take these mixings into account are

$$F_1^*(\mathit{renormalized}) = 0.76 \text{ — non-covariant} \qquad (15)$$

$$F_1^*(\mathit{renormalized}) = 0.62 \text{ — covariant}\,. \qquad (16)$$

Finally, in table 1 we present some mass and wavefunction calculations which were incidental to our calculations of matrix elements. Clearly our numbers are inferior to those obtained by the NRQCD collaboration [3], since we work only to lowest non-trivial order in v^2. However, they serve as a consistency check of our calculations.

We are now in the process of repeating these calculations for the charmonium system at $\beta = 5.7$ ($\beta = 6.0$ has too small a lattice spacing for NRQCD at the charmed-quark mass. Our earlier attempts [4] used charmed-quark masses which were too large.) The charmonium system affords the opportunity to confront our calculations with experiment, since there is already sufficient experimental data to allow extraction of H_8. In the

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Table 1
Properties of S- and P-wave bottomonium from our simulations. The lattice quantities include mean field renormalizations. The mass of the 1S state is obtained by using $M = 2(Z_M M_b - E_0) + E_n$ with Z_M and E_0 set at their mean field values.

	LATTICE	EXPERIMENT
M_{1S}	9.2766(9) GeV	$M_\Upsilon = 9.46037(21)$ GeV
$M_{1P} - M_{1S}$	0.434(9) GeV	$M_{\chi_b} - M_\Upsilon = 0.4398(7)$ GeV
$\|R_{1S}(0)\|^2$	4.33(2) GeV^3	7.2(2) GeV^3
$\|R'_{1P}(0)\|^2$	0.75(7) GeV^5	—

future, we hope to extend these calculations to next order in v^2 and a^2, and then to include the effects of light dynamical quarks. We are also calculating the complete renormalization matrix through $\mathcal{O}(\alpha_s)$ for the four operators discussed in this paper.

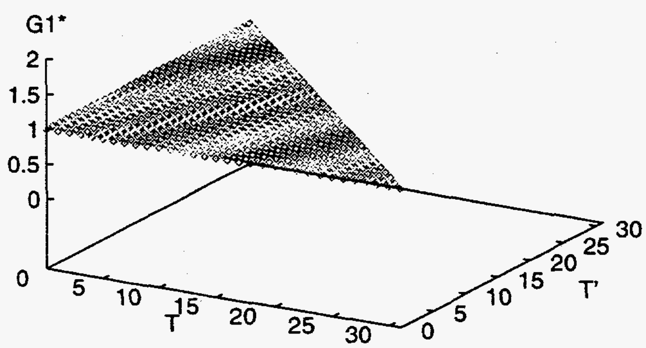

Figure 1. G_1^* as a function of T and T'.

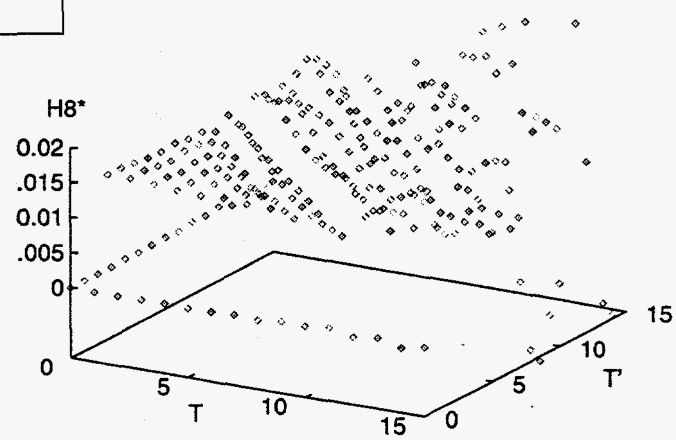

Figure 2. H_8^* as a function of T and T'.

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REFERENCES

1. G. T. Bodwin, E. Braaten and G. P. Lepage, Argonne Preprint ANL-HEP-PR-94-24 (1994) (Phys. Rev. D (to be published)) and references contained therein.
2. G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea and K. Hornbostel, Phys. Rev. D46, 4052, (1992); B. A. Thacker and G. P. Lepage, Phys. Rev. D43, 196 (1991)
3. C. T. H. Davies, K. Hornbostel, A. Langnau, G. P. Lepage, A. Lidsey, C. J. Morningstar, J. Shigemitsu and J. Sloan, Phys. Rev. Lett. 73, 2654 (1994); C. T. H. Davies *et al.*, Ohio State University preprint, OHSTPY-HEP-T-94-005 (1994), Phys. Rev. D (to be published) and references contained therein.
4. G. T. Bodwin, S. Kim and D. K. Sinclair, Nucl. Phys. B (Proc. Suppl.) 34, 434 (1994).