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**Coulomb Effects in Quasielastic  
Electron Scattering**

by

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**MASTER**

# Coulomb Effects in Quasielastic Elastic Electron Scattering

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Coulomb distortion plays an important role in interpreting both  $(e, e')$  and  $(e, e'p)$  reactions in the quasielastic region. A fully distorted partial wave calculation is presented, and the results are compared with the widely-used plane wave approximation and other distorted-wave calculations. The new calculation seems to give higher occupation numbers in the  $(e, e'p)$  reactions. The usefulness of the  $(e, e'p)$  reaction in studying different nuclear optical potentials is discussed. Also considered are the effects of electron Coulomb distortion in the separation of longitudinal and transverse structure functions in  $(e, e')$ .

I would like to report on some results of distorted-wave calculations of quasi-elastic electron scattering carried out at Ohio University by my colleagues, Yanhe Jin, Louis Wright, and myself.

The quasi-elastic peak occurs roughly where the momentum transfer  $\vec{q}$  and the energy transfer  $\omega$  are related by

$$\omega \approx \frac{\vec{q}^2}{2M} + E_B \quad (1)$$

where  $E_B$  is the (average) binding energy and  $M$  is the nucleon mass. This is the kinematic condition for striking a proton initially at rest; Fermi motion in the nucleus can then be said to account for the width of the peak.

If one observes the proton leaving the nucleus then it is possible to tie down its binding energy and the single-particle level within the nucleus. The difference between the proton momentum  $\vec{p}$  and the momentum transfer  $\vec{q}$  is often called the missing momentum:  $\vec{p}_m = \vec{p} - \vec{q}$ . If one were to ignore all final state interactions, this would be the momentum of the proton at the moment it is struck. The cross section reduced by dividing out an expression for the offshell electron-proton cross section  $\sigma_{ep}$ , is usually displayed as a function of  $p_m$ :

$$\rho_m(\vec{p}_m) = \frac{1}{pE\sigma_{ep}} \frac{d^5\sigma}{dk'_0 d\Omega_{k'} d\Omega_p} \quad (2)$$

Consider the elements which go into an  $(e, e'p)$  calculation. They can be divided into:

- those we can refine (more or less indefinitely):
  - 1) The electron wavefunctions
  - 2) The treatment of the electromagnetic interaction
- those on which we must take a chance
  - 1) Nucleon form factor (or expression for the nucleon current)
  - 2) Nuclear wavefunctions (no lack of candidates)
  - 3) Nucleon final state (is it described by the optical model?)

In our calculations we make a partial-wave decomposition of the electron wavefunctions with as many as 60 partial waves. The interaction includes all electromagnetic multipoles, EL and ML, a maximum  $L = 30$  is usually enough to ensure convergence. The interaction matrix elements are calculated numerically within a volume enclosing the nucleus, and analytically, using asymptotic expansions, in the exterior region. In this way we can set the accuracy required to 1 part in  $10^6$ . All of our tests indicate that this is sufficient to calculate the cross sections we need; we are able to increase the number of terms, for example, to get a precise treatment of a diffraction minimum if necessary.

For the nuclear parts of the calculation, we have used the free nucleon form factors, nuclear wavefunctions from a relativistic Hartree or Hartree-Fock code [1], and a standard relativistic 'global' optical model [2].

Some of our results are compared with experimental data from NIKHEF [3] in the following. First  $^{40}\text{Ca}$ , where Coulomb distortion is not likely to be serious.

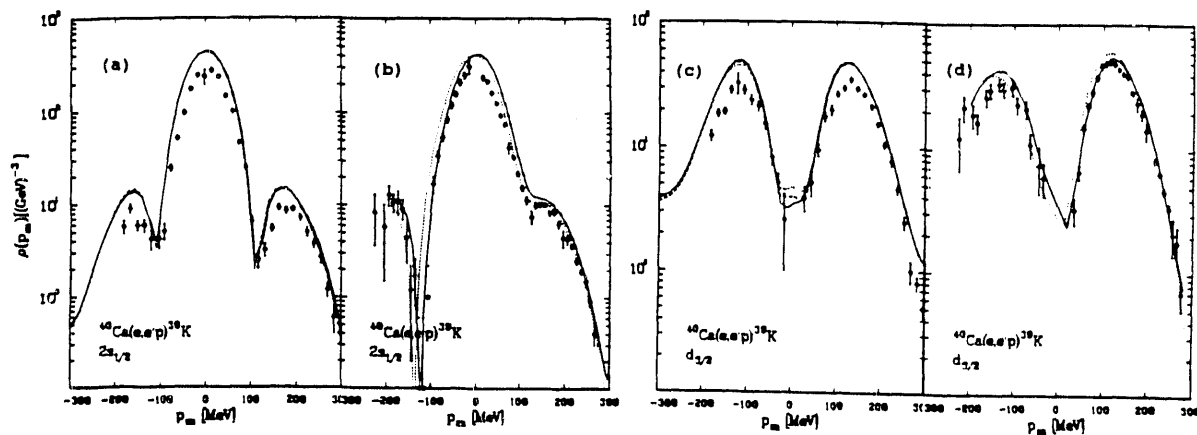


FIG. 1. Reduced cross section for  $^{40}\text{Ca}(e, e'p)$  for  $2s_{1/2}$  and  $1d_{3/2}$  with parallel (b,d) and  $\omega$ - $q$  constant (a,c) kinematics. The solid line is the DWBA calculation, and the dotted line the PWBA calculation. Data from Ref. 3.

The results (Fig. 1a,c) are for protons from the  $2s_{1/2}$  and  $1d_{3/2}$  orbitals assuming the states are 100% occupied, which is not realistic; if we reduce the occupations to 75% and 80% respectively we get a very satisfactory description of the data. These occupation fractions are easily comparable with those required by other reactions (pickup for example [4-5]). According to our Hartree code the overlap of the residual  $^{39}\text{K}$  state and the  $^{40}\text{Ca}$  + hole state the reaction leaves behind, is about 80%. Further reduction is expected by including other (correlated) components in the nuclear wavefunction not represented in the Hartree state.

In the data shown in Fig.(1a,c) the  $p_m$  is changed by varying the proton angle; this is called  $\omega$ - $q$  constant kinematics. In other experiments the proton is detected in the direction of the momentum transfer  $\vec{q}$ ; this is called parallel kinematics. With this arrangement, as shown in Fig.(1b,d), we again get a good description of the shape but we seem to require a smaller occupation fraction, which is very strange.

We have also looked at  $^{208}\text{Pb}$ . The Fig.2b shows the comparison with data; here we have already applied a reduction factor of 71% to give a best fit to the data. In this case that the inclusion of Coulomb distortion is essential. We also compare with rival calculations (Fig. 2a). McDermott's calculation [6](dotted line), like ours, uses relativistic wave functions and partial-wave analysis of the distorted electron waves. The other calculation (dashed line) is by Giusti and Pacati (Pavia). [7], and uses non-relativistic nuclear wavefunctions, and treats the electron distortion in a second-order eikonal approximation. Surprisingly we come closer to agreeing with the Pavia result. Turning off the electron distortion, so that these different treatments of the electron wavefunctions can be removed, we find we come even closer to the Pavia results. The use of relativistic as opposed to non-relativistic nuclear wavefunctions should not have a big effect in this process. The Pavia treatment of electron distortion appears to have the same effect as ours but exaggerates the magnitude. We are unable to explain our disagreement with McDermott.

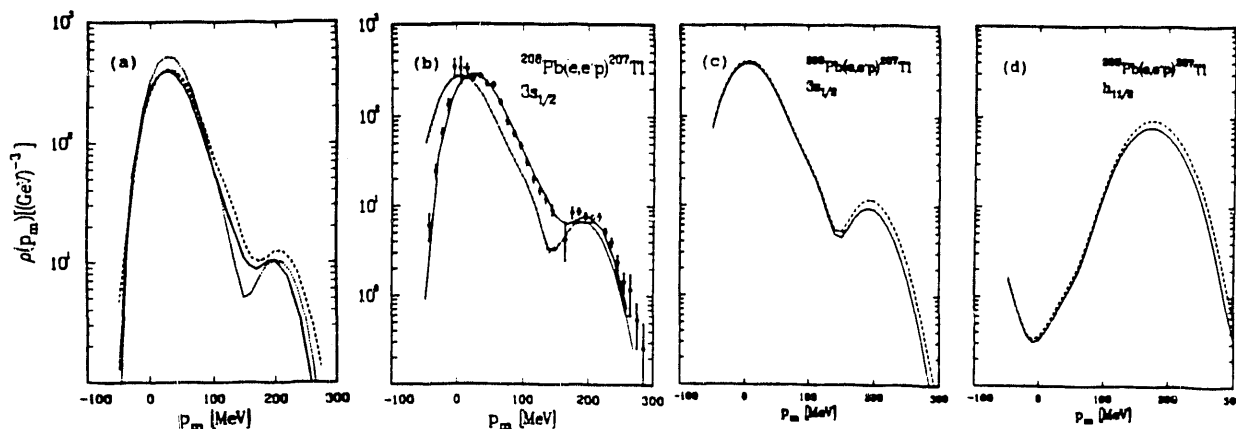


FIG. 2. Reduced cross section for  $^{208}\text{Pb}(e,e'p)$  with parallel kinematics; (a) the DWBA calculation (solid line) is compared to those of Ref. 6 (dotted) and Ref. 7 (dashed), (b) with experimental data [3] (the dashed line is PWBA). (c,d) show the effects from the use of SV (solid) and SVT optical potentials.

We have also looked at different proton distorted wavefunction for  $^{208}\text{Pb}$ . Whereas the relativistic optical model gives a good account of the proton cross section and spin observables, these are given entirely in terms of the phase shifts. But our problem should also be sensitive to the wavefunction in the nuclear interior. Using a phase-shift equivalent transformation given by B.C. Clark, *et al.* [8] we can consider a range of interior wavefunctions which share the same asymptotic behavior. Among these the standard potential is scalar + vector (SV), but another, the SVT [9], will give virtually the same result as an equivalent non-relativistic potential. Fig.(2c,d) shows that the cross section is affected very little in the neighborhood of the first peak but the different choices of wavefunction begin to show up at the second peak, this is more prominent for  $h_{11/2}$  (Fig.2d) than  $3s_{1/2}$  (Fig.2c).

Now we turn to the  $(e, e')$  reaction; since the energy of the proton is not observed, we have

to sum over all possible initial nucleon states, and indeed, neutrons as well as protons may now contribute. Since we no longer select nucleons which leave without initiating a further interaction, the optical model with its absorptive part is no longer appropriate. We deal with this rather simply. We discard the imaginary potential, and actually use the same real potential as for the bound states. While sacrificing some believability, we ensure orthogonality of initial and final states, also current conservation and gauge invariance - all potential problems with the calculation. The fits we obtain with this simple model are as good (or as bad) as with many more sophisticated treatments of the final state.

Indeed it is rare to find  $(e, e')$  cross sections published any more, instead we find the separated longitudinal and transverse response functions ( $S_L$  and  $S_T$ ), which are extracted from the cross sections by assuming that they are of the plane-wave form:

$$S(q, \theta) = S_L(q) + x(q, \theta)S_T(q) \quad (3)$$

Here  $S$  is the cross section reduced by dividing by a modified Mott cross section ( $\sigma_{Mott} q_\mu^4 / q^4$ ).  $S_L$  and  $S_T$  are functions of momentum transfer only, whereas  $x$  explicitly contains the electron scattering angle:

$$x = [\tan^2 \frac{\theta}{2} - \frac{q_\mu^2}{2q^2}] / \frac{q_\mu^4}{q^4} \quad (4)$$

Ideally, with the momentum transfer  $q$  fixed, the reduced cross section should appear as a linear function of  $x$ :  $S_L$  and  $S_T$  being the intercept and slope of the line. In practice, data are reduced to these same two functions by interpolating the measurements. The functions thus found do not correspond to theoretical expectations: the  $S_L$  extracted is usually too small by a factor of 20-50% - a dilemma known as the longitudinal suppression problem.

The suppression problem has, fairly obviously, little to do with Coulomb distortion. It is not correlated with the nuclear charge (it occurs in light nuclei for example). Nevertheless we can see the effects of distortion on  $S_L$  and  $S_T$ , by putting our calculated cross section into the equation for  $S$  above. We still get a straight line as a function of  $x$  (approximately), and so it is still possible to extract a slope and intercept, even though their meanings in physical terms are now unclear. The main effect is to shift the peak sideways, which is something we can take care of, to a large extent, by defining an effective momentum transfer.

Our fits to the  $(e, e')$  cross sections (we have used MIT data [10]) are not too bad, but when we look at the separated structure functions,  $S_L$  is poorly reproduced. In the Fig. 3a we look at the longitudinal structure function for  $^{40}\text{Ca}$  at  $q = 410$  MeV/c, with our theoretical curve. We deliberately extend the values of  $S_L$  to cover a range shown by the bold bars, while leaving  $S_T$  unchanged. Now look at the effect on the cross section: you see that the 'error' bars thus produced are comparable with experimental error at  $\theta = 90^\circ$ , and much smaller at  $\theta = 140^\circ$ . Evidently the cross sections are insensitive to changes in  $S_L$ . We should remark that the original publication does show a wide band (called the systematic error band) which is reproduced in our figure, but seldom shown in discussions of the suppression problem.

In conclusion, we look forward to using electron quasi-elastic scattering as a precision tool in the establishment of nuclear properties. The promise that the electrodynamics of the problem can be generated to any required degree of precision, still has a little way to go, however. What

could we use that is not now being measured? Data at more forward angles is important; polarisation data would be nice; also is  $(e, e'n)$  a possibility?

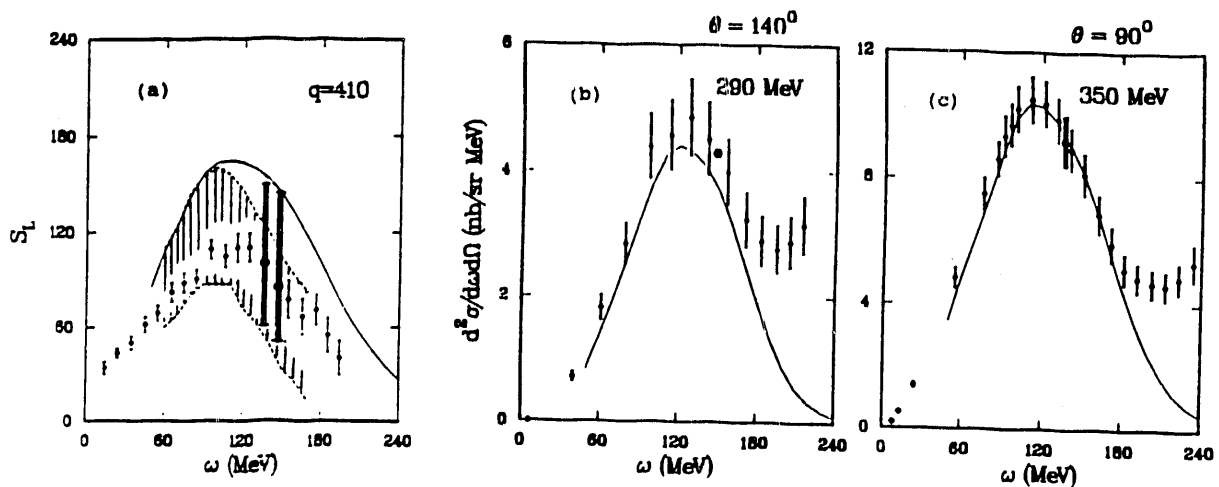


FIG. 3. Longitudinal structure function at  $q = 410$  MeV/c (a). The bold bars indicate the values used in the reconstruction of the experimental cross sections (bold bars shown in (b,c)). The shaded region corresponds to the systematic error band, and the solid line (b,c) are PWBA. The data are from Ref. 10.

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