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Adjustment Costs and the Flexible Accelerator

George A. Hay
Cornell Law School, george.hay@cornell.edu

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The flexible accelerator concept has provided the rationale for the regression equations used in several recent econometric studies of inventory behavior.[1] In such a model a desired (or equilibrium) level of inventories is defined, but because of costs involved in changing the level of stocks only a partial adjustment of inventories to their desired level is achieved in any time period. This leads to the familiar decision rule in which current inventory is a linear function of the previous period's stock plus a variable or set of variables representing current demand.

The purpose of this note is to question whether there are any significant costs specifically associated with changing the level of inventories *other than* those directly associated with changes in the level of production which may (or may not) be required to bring about the necessary stock adjustment.[2] If none exist, it is shown that a given time pattern of demand can lead a firm which acts according to the flexible accelerator to behave "irrationally" and incur unnecessary costs. Finally, an amended version of the flexibile accelerator is presented, and the resulting equation for inventories is compared with that derived from the original model.

The following is a simple version of the flexible accelerator model. Assume

$$(1.1) \qquad H^e_t = a + bS_t,$$

where S_t is sales and H^e_t is the equilibrium or desired level of inventories in period t.

The flexible accelerator concept implies

$$(1.2) \qquad \triangle H = d(H^e_t - H_{t-1}) \quad 0 < d < 1,$$

where H_{t-1} is the actual level of inventories in $t-1$. The extent to

1. For an example and further references, see P. Darling and M. Lovell, "Factors Influencing Investment in Inventories," pp. 131–61 in *The Brookings Quarterly Econometric Model of the United States*, J. Duesenberry *et al.*, editors, Rand-McNally & Co., Chicago, 1965.

2. The arguments in the present paper are primarily applicable to investment in finished goods inventories. To the extent that most of the work in the literature, including Darling and Lovell, *op. cit.*, involves total inventories, including goods-in-process and raw materials, the criticism is somewhat softened. However, those treatments are subject to a more basic criticism for using a single behavioral equation for what are essentially several different decisions.

which d is less than 1 is determined by the cost of adjusting the inventory stock in a single period.

Substituting the equation for desired inventory in 1.2 gives

$$\triangle H = da + dbS_t - dH_{t-1}, \tag{1.3}$$

where the effects of errors of anticipation with regard to S_t are ignored.

What, if any, are the costs of adjusting the level of inventories? Here the distinction between static and dynamic costs must be stressed. Obviously the firm incurs higher costs by maintaining a higher level of inventories. Storage costs are likely greater, and the total return foregone on the money tied up in stock is increased. However, although these costs will influence the desired level of inventories, they should not affect the speed at which the firms adjust the actual level to the desired. What is required to justify the flexible accelerator model is a cost which corresponds specifically to the rate of change of inventories as opposed to their level, similar to hiring and firing costs incurred in changing the level of employment.

In searching for the natural equivalent of hiring and firing costs for inventories, however, some difficulties are encountered. Presumably there could be some costs associated with negotiating for a greater or lesser amount of storage space, and some expense incurred in transferring the goods to or from storage, but these are unlikely to be significant. It seems reasonable that the principal deterrent to rapid adjustment of stocks is the cost directly associated with the changes in the level of production required to bring about the desired level of inventories. But if it is costs of adjusting production that are the determining influence, then these costs should be reflected in the model directly. Failure to do so can lead to predictions which reflect seemingly irrational behavior on the part of firms, as is demonstrated by the following simple example.

Assume that

$$a=0 \quad b=0.5 \quad d=0.5.$$

Furthermore, suppose that $H_o=0$ and the firm is faced with sales in the first period of 60 units. According to the flexible accelerator hypothesis we would have

$$\begin{aligned} S_1 &= 60 & H^e_1 &= 30 \\ \triangle H &= 0.5(30-0) = 15 & H_1 &= 0+15=15 \\ X_1 &= S_1 + \triangle H = 75, \end{aligned}$$

where X_1 is production in the first period. Now suppose that S_2

is 60. Then, following the flexible accelerator behavior, we would have

$$H^e_2 = 30$$
$$\triangle H = 0.5(30-15) = 7.5 \qquad H_2 = 15+7.5 = 22.5$$
$$X_2 = S_2 + \triangle H = 67.5.$$

This requires, since $X_1 = 75$, that $\triangle X = -7.5$.

It is questionable in this situation whether the rational firm would act in accordance with the predictions of the flexible accelerator hypothesis. Note that had the firm simply maintained its previous rate of production, i.e., $X_2 = X_1 = 75$, it could have achieved the desired level of inventories for the second period at no additional cost. Instead, as a result of the behavior implied by the original hypothesis, not only does the firm fail to achieve its desired inventory position but, even more, it incurs a cost for changing the rate of production.

The point of this example is that, if the only costs associated with the adjustment of inventories are those which result from any change in the rate of output which may be necessary to accomplish the inventory adjustment, then it is the rate of production which will be adjusted imperfectly, and any failure of inventories to achieve their desired level will be simply a byproduct of the inventory-sales-production relationship. This may be seen from the following model, which is formulated on the assumption that *production* in any period will be such as to close some positive fraction of the gap between the equilibrium or desired production for the current period and the actual production of the previous period.

We assume that

(1.4) $H^e_t = a + bS_t$ and

(1.5) $X^e_t = S_t + H^e_t - H_{t-1}$,

where X^e_t is desired production in period t.

Because of the costs associated with changes in the rate of output, only a fraction of the difference between desired production for the current period and last period's actual production will be made up in any period, i.e.,

(1.6) $\triangle X = g(X^e_t - X_{t-1}) \qquad 0 < g < 1.$

Therefore,

(1.7) $X_t = g(X^e_t - X_{t-1}) + X_{t-1}$,

and since $H_t = X_t - S_t + H_{t-1}$ we have

(1.8) $H_t = ag + [gb - (1-g)]S_t + (1-g)H_{t-1} + (1-g)X_{t-1}.$

Finally, subtracting H_{t-1} from both sides gives

(1.9) $\triangle H = ag + [gb - (1-g)]S_t - gH_{t-1} + (1-g)X_{t-1}.$

This equation for the rate of inventory investment can be compared with equation (1.3) above, and it will be noted that the behavior implied by the flexible inventory accelerator is, in general, different from that implied by our production adjustment model. Specifically, if one assumes $d=g$, $(1-g)S_t$ in the former is replaced by $(1-g)X_{t-1}$, thereby introducing lagged production as well as lagged inventories into the determination of inventory investment, and reducing the weight attached to current sales.

More generally, even the second model presented, where production is planned to close a fixed percentage of the gap between equilibrium output and actual output of the previous period, is an oversimplification of the decision processes of the firm. There are other costs to be considered as well as production adjustment costs — there is even the possibility at least of varying price to bring actual sales to a more "convenient" level. The point is that all variables must be determined simultaneously, and when this is done there is no reason to suspect that any simple relation between two variables will suffice to explain the decisions which result.[3]

In closing we note that the flexible accelerator concept has also been used to explain investment in plant and equipment. Here, however, our criticisms do not necessarily carry over. There is a basic asymmetry between investment in inventories and investment in fixed capital in that the former is merely a byproduct of the process of production, and any inflexibility in inventories is likely to be merely the result of inflexibility in production. Investment in fixed capital, however, is presumably not, except in special cases, a result of the production process, and in rapidly expanding fixed capital there may be difficulties which are completely separate from any problems of inflexible production within the firm. Where this is the case,[4] the flexible accelerator may still be a useful concept.[5]

YALE UNIVERSITY

3. For an example of such a "comprehensive" approach, see G. Hay, "Production, Price and Inventory Theory," *American Economic Review* (forthcoming).

4. Investment in inventories of raw materials may fit in this category. The situation with inventories of goods-in-process is somewhat ambiguous.

5. There is a growing literature which challenges the value of the flexible accelerator in explaining fixed investment. See J. Gould, "Adjustment Costs in the Theory of Investment of the Firm," *Review of Economic Studies*, XXXV (Jan. 1968), 47–56; and R. Lucas, "Optimal Investment Policy and the Flexible Accelerator," *International Economic Review*, VII (Feb. 1967), 78–85.