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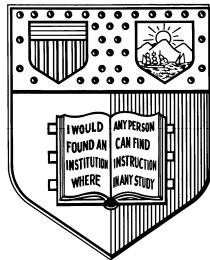
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Noncomparabilities & Nonstandard Logics

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ABSTRACT: Several theorists of justice have called attention to a vitiating contradiction besetting John Rawls's treatment of "primary goods" and their relation to "lifeplan"-satisfaction among persons. Rawls effectively claims, these critics observe, that (1) primary goods indices are interpersonally ordinally equivalent; (2) primary goods indices and degrees or levels of lifeplan-satisfaction are intrapersonally ordinally equivalent; and (3) degrees or levels of lifeplan-satisfaction are not interpersonally ordinally equivalent. These three claims are jointly inconsistent. John Roemer, in particular, shows them so via an elementary function-theoretic proof by *reductio ad absurdum*.

A natural objection that a Rawlsian might be tempted to offer to Roemer and other critics is that a standard mathematical (or any other) proof by *reductio* in the present context is illicit. For such proofs proceed implicitly on the basis of semantic bivalence and the Law of Excluded Middle. And a claim of interpersonal incomparability, such as (3), implicitly rejects at least one of these classical logical "laws" via its treatment of certain prospective comparisons as simply undefined, hence, semantically empty.

It is therefore a matter of some interest that a somewhat "softened" version of the Rawlsian contradiction is derivable in all of the familiar three-valued, supervaluational, intuitionist and modal logics that reject bivalence, Excluded Middle or both. As a number of formal proofs offered in the present article demonstrate, the tension in Rawls's

treatment of primary goods and their relation to the successful pursuit of lifeplans is “deep” and robust.

But it turns out that Rawls is spared contradiction, and his intentions to present a “political, not metaphysical” conception of justice best carried out, when, *pace* Rawls himself, both the second and third listed Rawlsian propositions are reformalized pursuant to a nonclassical logic. Such logics indeed appear, even for reasons apart from consistency, to constitute the most appropriate means by which to model Rawls’s politico-pragmatic understanding of the appropriate *distribuendum* – an index of primary goods – and its relation to citizens’ plural conceptions of the good in a liberal theory of justice. A broader lesson is that nonstandard logics offer the best means of modeling relations among propositions postulated by any similarly – epistemically or “politically” – modest, “pragmatic” conception of justice or political morality.

1. Introduction

In his remarkable survey of theories of distributive justice, John Roemer demonstrates a critical inconsistency in John Rawls’s treatment of “primary goods” and their relation to “lifeplan”-satisfaction among persons.¹ In a thoughtful draft article now folded into a separate piece co-authored with the present writer, Mathias Risse conditionally endorses Roemer’s finding, and argues that one of the three Rawlsian claims serving as premises of the derived contradiction both a) is the only one that might plausibly be dropped, and b) can indeed *safely* be dropped, without thereby vitiating Rawls’s basic understanding of primary goods and their role in a political/contractarian theory of justice.²

The three Rawlsian premises, as characterized (though not in all cases as worded) by Roemer and Risse, are that: 1) primary goods indices are interpersonally ordinally equivalent; 2) primary goods indices and degrees or levels of lifeplan-satisfaction are intrapersonally ordinally equivalent; and 3) degrees or levels of lifeplan-satisfaction are not interpersonally ordinally equivalent.

Intuitively, the inconsistency argument runs: If two people's primary goods indices must, *ceteris paribus*, either rise, fall or remain unchanged together when the two people hold identical ordered pairs of consecutively held bundles (1), and if each of these person's degree or level of lifeplan-satisfaction must, *ceteris paribus*, rise, fall or remain unchanged with his primary goods index (2), then by transitivity of the equivalence relation the two people's degrees or levels of lifeplan-satisfaction must themselves, *ceteris paribus*, surely rise, fall or remain unchanged together (with their primary goods indices) as well – which is inconsistent with (3). Formally, the derivation of the inconsistency found by Roemer and endorsed by Risse takes the character of an elementary function-theoretic proof by *reductio*.

Now a natural, if perhaps somewhat radical, objection that a Rawlsian might be tempted to offer to Roemer and Risse is that a standard mathematical proof by *reductio* in the present context is illicit. For such proofs proceed implicitly on the basis of semantic bivalence and the Law of Excluded Middle (or *tertium non datur*).³ And a claim of interpersonal incomparability, such as (3), implicitly *rejects* bivalence and/or Excluded Middle via its treatment of certain prospective comparisons as simply undefined – hence, of prospective claims *stating* such comparisons as semantically empty or indeterminate. (Interpersonal comparisons would lack standard semantic valuations – truth or falsity –

rather as expressions of the form “ $\forall/0$ ” lack numerical referents or fail to connote functions defined over certain salient domains, and thus as sentential units containing such expressions themselves lack standard semantic valuations.)

Another natural, if rather less radical, objection initially open to Rawlsians is that Rawls’s statement of the relation between primary goods and lifeplan-satisfaction – premise (2) – is misformalized in the Roemer and earlier Risse discussions. For as he states the relation, Rawls’s understanding of it looks to be not one of equivalence, but one of simple entailment – i.e., not of biconditionality, but of uniconditionality. The implicative relation runs from primary goods to satisfaction, but not from satisfaction to primary goods.⁴ (Of course, it therefore also would run, by contraposition, from the denial of satisfaction to the denial of primary goods; but that is not equivalence, and even contraposition might be jeopardized if bivalence and/or *tertium non datur* were denied.⁵)

It is therefore a matter of some interest that (a somewhat softened version of) the inconsistency derived by Roemer and endorsed by Risse also is derivable in sundry three-valued, supervaluational, intuitionist and modal logics – and this even when proposition (2) is weakened to a uni-, rather than a bi-, conditional characterization of the relation between primary goods indices and degrees or levels of lifeplan-satisfaction. I provide a number of representative such derivations in the present paper. Part 2 derives the contradiction in standard first-order quantificational calculus – the logical counterpart to Roemer’s function-theoretic argument. Part 3 derives the contradiction in several well-established 3-valued logics – those of Łukasiewicz, Bočvar, Smiley, Kleene and Woodruff. Part 4 indicates how the preceding proofs carry-over into, e.g., van Fraassen

supervaluational logic, Heyting intuitionist (or “epistemic”) logic, and the standard S4 modal logic.

Each of Parts 3 and 4 briefly explains the possible reasons that, and the senses in which, the particular formalisms employed might be thought suitably to express the Rawlsian intuition that degrees or levels of lifeplan-satisfaction are interpersonally incommensurable. In Part 5 I conclude that these results, far from constituting mere formal curiosities, bring into particularly sharp relief the deeply problematic nature both (a) of Rawls’s treatment of primary goods and their relation to lifeplan-satisfaction among persons, and (b) indeed of *any* claim simultaneously involving pessimism (or in Risse’s case, optimism) about the interpersonal tractability or salience, and optimism (or in Risse’s case, pessimism) about the intrapersonal tractability or salience, of the “subjective element” of well-being.⁶ For it turns out that the contradiction is avoided, and a coherent rationale for charitably *revising* the set of Rawlsian premises best articulated, only when the *second* Rawlsian premise is recast (not dropped, *pace* Risse) *with* the third, for essentially the same reasons, in effectively trivalent form.

In effect, the contradiction dealt with here is simply a case study of a more general point. The broader lesson here is that certain nonstandard logics offer a potentially useful means of modeling relations among propositions postulated by *any* epistemically or “politically” modest conception of justice or political morality.

2. Standard, Two-Valued First Order Formalization

The derivation of Roemer’s inconsistency in standard, two-valued first order quantificational logic is quite straightforward. The proof that follows makes use of the

following familiar sentential operators, connectives and quantifiers⁷: (The conversant reader may of course skip over or skim the remainder of this paragraph.) For “ \neg ” read “not,” or “it is not the case that.” (I thus use “ \neg ” only as a sentential operator, not as a direct modifier of predicate expressions or as immediately forming class-complements. The same holds for the next four operators.) For “ \wedge ” read “and.” For “ \vee ” read “or.” For “ \supset ” read “implies,” or “entails”; hence for “ $\forall \supset \exists$ ” read “ \forall implies/entails \exists ,” or “if \forall then \exists ,” or “ \forall only if \exists .” For “ $\forall \equiv \exists$ ” read “ \forall and \exists imply/entail one another,” or “ \forall if and only if \exists .” For “ $\forall x: \dots$,” read “for all x’s, ...”; hence, e.g., for “ $\forall x,y: xMy$ ” read “for all x’s and all y’s, xMy ,” or “all x’s and y’s stand in the M relation.” For “ $\exists x: \dots$,” read “There is an x such that ...”; hence, e.g., for “ $\exists x,y: xMy$ ” read “for some x and some y, xMy ,” or “some x and y stand in the M relation.” (I introduce two particular “M relations” below.) I generally separate “atomic” sentential components of longer, more complex “molecular” sentences by means of dots rather than repeated parentheses. More dots accompanying a connective indicate that the connective possesses a broader scope. Thus, for “ $\forall \supset \exists \equiv (\supset *)$,” read “ $(\forall \supset \exists) \equiv ((\supset *))$ ”; for “ $\forall \supset \exists \supset \supset \supset (\supset \vee : \vee : * \supset \supset \supset \wedge \supset \supset)$,” read “ $(\forall \supset \exists) \supset ((\vee (* \supset (\wedge \supset \supset)))$,” etc.

For ease of exposition and ready intuitability of what are, after all, some rather elementary formal statements, I also adopt two specialized relational operators (stand-in’s for “M” above) to characterize relations between 1) pairs of primary goods bundles hypothetically held by separate persons; 2) pairs of primary goods bundles hypothetically held by, and associated pairs of degrees or levels of lifeplan-satisfaction thereby yielded to, individual persons; and 3) pairs of degrees or levels of lifeplan-satisfaction

hypothetically achieved by separate persons: For “ $>_{o/i}$,” read “objectively equals or exceeds for person i.” For “ $>_{s/i}$,” read “subjectively equals or exceeds for person i.”

For “objectively” here, one might just as well read “interpersonally” or “intersubjectively”; and for “subjectively” one might just as well read “intrapersonally.” I intend no more metaphysically controversial distinction than that which I take to be implicit in Rawls’s treatment of primary goods and lifeplan-satisfaction, and which I take to underwrite Roemer’s (and Risse’s) use of the distinct functional expressions “ $:^i(x)$ ” and “ $u^i(x)$ ” respectively to represent the primary goods index number and degree or level of lifeplan-satisfaction associated with individual i’s holding the primary goods bundle x.⁸ By twice indexing the single relational term “ $>$ ” rather than once indexing each of two separate functional operators such as “ $:^i(x)$ ” and “ $u^i(x)$,” I hope (with perhaps a dash of heuristic optimism) to render the formal derivations that follow both easier on the eye and readily verbalized in simple English at no cost to critical detail.

The “readily verbalized” versions of Rawls’s three claims concerning primary goods and lifeplan-satisfaction can be named and informally stated as follows:

- 1) Primary Goods–to–Goods (“Objective”) Ordinal Equivalence: Any one primary goods bundle objectively equals or exceeds another such bundle for me if and only if it does so likewise for you – irrespective of who “you” and “I” are.
- 2) Primary Goods–to–Satisfaction (“Objective”/“Subjective”) Ordinal Quasi-Equivalence (i.e., Entailment): Any primary goods bundle subjectively equals or exceeds another such bundle for me (or for you) only if it objectively equals or exceeds that other bundle for me (or for you) – irrespective of who “you” are or “I” am.
- 3) Non Satisfaction–to–Satisfaction (“Subjective”) Ordinal Equivalence: It is not (necessarily) the case that any primary goods bundle subjectively equals or exceeds another such bundle for me if and only if it does so for you – irrespective of who “you” and “I” are.⁹

The three Rawlsian claims can be formalized, in the notation explicated *supra*, as follows. Let the variables x and y range over primary goods bundles, and the variables i , j and k over persons. Then:

$$1) \quad \forall x, y, i, j: x >_{o/i} y \equiv x >_{o/j} y.$$

$$2) \quad \forall x, y, k: x >_{o/k} y \supset x >_{s/k} y.$$

$$3) \quad \neg \forall x, y, i, j: x >_{s/i} y \equiv x >_{s/j} y.$$

Inconsistency among the set of propositions (1) through (3) is readily derived as follows: First, by assumption with a view to the familiar rule of universal-elimination,

$$4) \quad \text{let } x = m, y = n, k = a,$$

so that, by (2) and (4),

$$5) \quad m >_{o/a} n \supset m >_{s/a} n.$$

Again by assumption and universal-elimination,

$$6) \quad \text{let } k = b,$$

so that, by (2) and (6),

$$7) \quad m >_{o/b} n \supset m >_{s/b} n.$$

Once more by universal-elimination,

$$8) \quad \text{let } i = a, \text{ and } j = b,$$

so that, by (1), (4) and (8),

$$9) \quad m >_{o/a} n \equiv m >_{o/b} n.$$

Now assume:

$$10) \quad m >_{o/a} n.$$

Then by (9) and the definition of material equivalence as conjoined implications, and by conjunction-elimination and implication-elimination,

$$11) m >_{o/b} n.$$

But then by (7) and implication-elimination,

$$12) m >_{s/b} n.$$

Now, by (3) and by dint of familiar logical relations obtaining between the existential quantifier, the universal quantifier, and the sentential negation operator, and of course still assuming bivalence,

$$13) \exists x, y, i, j: \neg(x >_{s/i} y \equiv x >_{s/j} y).$$

Thus, by dint of familiar logical relations among the sentential negation, conjunction, and implication operators,

$$14) \exists x, y, i, j: x >_{s/i} y \wedge \neg x >_{s/j} y : \vee : \neg x >_{s/i} y \wedge x >_{s/j} y.$$

Now again by assumption, this time with a view to existential-elimination,

$$15) \text{let } x = m, y = n, i = a, j = b,$$

so that by (14) and (15),

$$16) m >_{s/a} n \wedge \neg m >_{s/b} n : \vee : \neg m >_{s/a} n \wedge m >_{s/b} n.$$

Now by (12) and the familiar rule of negation-elimination (assuming bivalence)), we know that

$$17) \neg\neg m >_{s/b} n.$$

So we know, by conjunction-elimination, that the first disjunct of (16) does not hold – that is, we know that

$$18) \neg(m >_{s/a} n \wedge \neg m >_{s/b} n).$$

So by (16), (18) and disjunction-elimination, we have

$$19) \neg m >_{s/a} n \wedge m >_{s/b} n.$$

And thus in particular, by conjunction-elimination, we have

20) $\neg m >_{s/a} n$.

But by (5) and (10) we have

21) $m >_{s/a} n$,

which of course contradicts (20).

Apart from rejecting bivalence or *tertium non datur*, the only specific objection to this derivation that I can envisage being offered by a Rawlsian is to the use of universal-elimination both to let $k = a$, then b , and to let $i = a$ and $j = b$. But this would not be a logical objection; and in fact it would amount to a denial of the Rawlsian premises themselves. Because premises (1) through (3) are universally quantified and their bound variables i , j and k taken to range over all persons, there simply is no formal reason for prohibiting any two – or even all three – of the variables i , j , and k from taking the same value. And assuming bivalence, to impose any restriction on any two such variables' taking the same value would be none other than to deny the universality of (at least) one of the premises – i.e., to deny (at least) one of those premises themselves. (We should have, that is, to say that the people for whom primary goods are interpersonally (“objectively”) ordinally equivalent cannot be the people for whom they and the lifeplan-satisfaction they yield are intrapersonally (“objectively/subjectively”) ordinally quasi-equivalent. I.e., we should have to say that premises (1) and (2) cannot be said of the same people.)¹⁰ The only conceivable Rawlsian way out, then, would appear to be to depart from classical bivalence or *tertium non datur*.

3. Three-Valued First Order Formalizations

Significantly, the “conceivable” way out proves not to be a felicitous way out. We can derive counterparts to the classical inconsistency just proved among Rawls's

three premises in all of the familiar, plausibly applicable three-valued logics, as well as in all other plausible non-classical candidates sketched in the next two Parts. In this Part I restrict myself to the trivalent logics. A brief word might first be in order on why one might be tempted to employ a third semantic value in modeling the three Rawlsian premises and their logical relations. (The reader is asked to skip over or skim the next subpart if well familiar with the standard reasons for rejecting classical logic.)

3.1. Canonical Reasons for Rejecting Bivalence

“Third” semantic values typically are proposed as means of formally recognizing the possibility that syntactically well-formed statements might be neither true nor false. (They also are offered as means of assigning unique semantic values to “meaningless” or “nonsensical” – i.e., to syntactically *non*-well-formed – sentences, when such sentences are dignified with the term “sentences” at all; and as “pragmatic” revisions to classical logic made with a view to accommodating challenges posed to the distributive laws by quantum theory. But these cases need not detain us; I can envisage no interesting Rawlsian analogue to either of them.¹¹) There are of course several well-known means by which a well-formed sentence may come to appear to lack a determinate classical truth-value:

1. *“Paradoxical” specifications and resultant sentences:* The specification “that barber who shaves all and only those people who do not shave themselves” seems connotative, in that its component words, both severally and jointly in the order in which they figure, are readily understood. And it seems connotative in that we can (at least vaguely) envisage sundry procedures by which to locate the nominative terms’ putative denotations and thus the denotation of the specification as a whole. (We look for barbers.

We look to whom they shave. We query who among the latter shave themselves. Etc.) But when we inquire of such a barber whether he shaves himself, it seems that if he does, he does not, and that if he does not, he does. It thus seems problematical to say of any sentence asserting that he does or doesn't that it is determinately true or false. And it therefore likewise seems problematical to assert that the specification, notwithstanding its connotation, bears a denotation. (More on denotationlessness in its own right in the following paragraph.) Similar remarks hold for sentences purporting to assert something of Russell's "class of all classes that are not members of themselves."¹² And related examples abound. (The Cretan "liar," "this sentence is false," etc.) But "paradox" here means more than simple contradiction. Drop the "not" from the second example and you have "the class of all classes that are members of themselves." Does that class belong to itself? How would we ever find out? There seems to be no procedure, even in principle, by which to determine an answer. Call the sentence "undecidable." We might conclude that such sentences are not syntactically well-formed after all. (And thus consign them to the "nonsense" heap.) Or we might assign them a new semantic value.¹³

2. *Missing term-referents more generally:* Expressions such as "my mother's cousin," "the largest prime," or "the King of America" are connotative, in that their component words both severally and jointly in the order that they figure are readily understood. And they are connotative in that we generally can (at least vaguely) envisage sundry procedures by which to locate the terms' putative denotations and then predicate additional descriptions of them. (One might learn that "my mother's cousin," for example, once one has traced the relevant branches of the family tree, also describes a well-known anthropologist, who now turns out to be my mother's cousin.) But such

expressions also, of course, notwithstanding their success in connoting, might fail to denote. There is no king of America, however surprisingly this might strike some; and it would seem as misleading either to affirm or deny any sentence formed by prefixing that specification to a sentential predicate as it would be to affirm or deny that a gentle soul had “stopped beating [her] spouse.”¹⁴ It would thus seem, at least for many purposes, misleading to ascribe either truth or falsehood to such sentences.¹⁵

3. *Vaguely determined predicate-extensions*: The claim that “I got into a car” is true if I have buckled myself into a sedan. It will generally be thought false if I have buckled myself into a helicopter. But to claim either that it is true, or that it is false, if I have buckled myself into a jeep, might in many contexts seem misleading. Likewise if I have dived through the opened window of the vehicle and my legs remain, kicking, outside. Such cases, as well as the better known Sorites puzzle and other “paradoxes of induction,” might seem to call for many-valued or “fuzzy” logics, of which three-valued logics are a special case.¹⁶

4. *Bivalence-entailed objectionable metaphysical (or epistemic) commitments*: Statements of the form “two thousand years from now there will be a sea battle,”¹⁷ or “it would have rained at Marathon had the clouds been seeded,” or “there is a greatest prime number which, when two is subtracted from it, yields another prime number” might seem to commit their utterers to metaphysical (or epistemological) views that are plausibly, if not readily, to be rejected. For it to be determinately true or false now that a sea battle will take place later, it might seem that there will have to be some feature of the *world* now in *virtue* of which the statement is, presently, true or false.¹⁸ But that would seem to imply that the future is now determined, and surely one may, perhaps even ought, reject

determinism.¹⁹ A similar objection might beset counterfactual claims.²⁰ And the claim that it is determinately true or false that the twin prime problem will, or can in principle, be solved might look similarly to entail metaphysical determinism, Platonism about mathematical objects (and the knowledge thereof), or both.²¹ Those who reject such doctrines might be tempted to reject the bivalence that seems to underwrite them, at least with respect to the suspect classes of sentences.

3.2. Rawlsian Reasons for Rejecting Bivalence?

I can see no applicability, to a prospective Rawlsian rejection of bivalence, of the argument from vagueness.²² I therefore provisionally dismiss that possibility for present purposes. Rawls also does not appear able to reject bivalence by appeal – at least by direct appeal – to the denotationlessness of any thus-far considered specifications or singular terms. After all, the “terms” here – “primary goods,” “fulfillment of life plans,” and the like – all are his own, and he surely purports to be writing about something.

But there is an *indirect* way in which Rawls might be taken to appeal implicitly to the possible denotationlessness of terms or specifications: He can be taken to argue that certain conceivable, salient, and indeed oft effectively *encountered* specifications would or do lack referents. Thus, for example, Rawls might seem by dint of some of his assertions concerning the “incommensurability of lifeplans” (or of the “conceptions of the good” expressed in them)²³ to be committed to the denotationlessness of some such expression as “the ratio of my degree of lifeplan-satisfaction to yours.” Expressions such as these might be readily analogized by Rawlsians to terms such as “7/0,” which fail to denote because the operations by reference to which the terms are partly formed – “/” in this case – are undefined over some domain an object of which – 0 in this case – also

figures, by name, into the term's formation. Ratio comparison over measures of separate lifeplan-satisfaction might then be thought simply "undefined" as is division by zero, even though the operations of ratio comparison and division (which seem indeed to amount to a single arithmetical operation, the one simply a particular application of the other) generally are defined over domains sufficiently broad as to grant them at least a *syntactically* well-formed status even in those few cases in which they are semantically undefined.

Yet this prospective Rawlsian argument from denotationlessness might seem a bit *ad hoc* and unconvincing as it stands thus far. A simple case of fiat. (Interpersonal comparisons are simply *deemed* undefined.) For if we can understand an expression such as "the *degree* of lifeplan-satisfaction" (i.e., of *one* person's such "degree") and *it* refers to something (something numerical), then why, and in what sense, can we not understand a successfully referring "ratio-*comparison* of (*two* persons') degrees [again, numerical things] of lifeplan-satisfaction"? (Do we actually know, prior to calculation, that such forthrightly numeric ratios stand on a less secure footing than the square-root of two?) And if the (glib) reply is that we do *not* understand (an unreffering) "degree of lifeplan-satisfaction," but only the veritably referring "intrapersonal *level* of lifeplan-satisfaction," then the obvious rejoinder is, why can we not understand a successfully referring "interpersonal *ordinal* comparison of *levels* of lifeplan-satisfaction"?²⁴ A peremptory Rawlsian brush-off of the form, say, "because life plans are incommensurable" clearly would not do. For, leaving to one side for the moment the obscurity of the notion of "incommensurable [even "mensurable"] lifeplans,"²⁵ it is not lifeplans that we purport to compare, only their degrees, or levels, of satisfaction – measures that Rawls seems to

believe exist (at least intrapersonally – recall premise (2)). And what is inherently incomparable about such seemingly objective, numeric things as (cardinal) degrees and (ordinal) levels?

Presumably, then, the Rawlsian would wish to argue against bivalence in matters of lifeplan-satisfaction-level-comparison not so much from the mere possibility or *ad hoc* stipulation of the unreferringness of terms or specifications alone, as from some paradox(es) or objectionable metaphysical or epistemic commitment(s) entailed by the very *prospect* of such terms' referring. What might such paradoxes or commitments be?²⁶ And how are they to be avoided? The proofs that follow will occasion skepticism as to whether any such means of avoidance that salvages either of premises (2) and (3) in their present forms can be coherently articulated. Indeed, paradox is what seems to afflict the supposition that it can. But I shall assume for the sake of argument – i.e., for the sake of the proofs themselves – that something *might* be so articulated. And in Part 5 I shall both a) speculate as to what leads Rawls (and Risse) astray both in supposing that there is such an articulation and in seeking to accommodate it, and b) show that the only plausible articulation requires a non-standard reinterpretation of *both* premises (2) and (3).

3.3. Implementing the Rejection of Bivalence

Three programmatic decisions attend the formulation of a logic that rejects bivalence. (The reader is again asked to skip over or simply skim the current subpart if well familiar with these preliminaries.) The first is not strictly necessary but generally desirable if the logic is to be “useful” or “applicable,” i.e., if it is to reflect motivation by anything beyond Hilbertian considerations: It is prospectively to interpret, and often thus appropriately to name, in some intuitively appreciable sense, the added semantic value,

values or gaps that will figure into the new logic. This decision of course follows directly, as a conceptual matter, upon considerations such as those just assayed with respect to why bivalence might be rejected in the first instance. Thus, for example, if the value designated “T” or “1” is to be taken to mean “true,” and “F” or “0” to mean “false,” then one will wish to take the third value, say “I,” “U,” “N,” “1/2,” a blank space or such-like to mean “indeterminate,” “unknown,” “unknowable,” “neither,” “both,” “in-between,” “paradoxical,” or the like. The choice of prospective interpretation often will guide – though of course it need not – the choice of valuational symbols themselves. Those mindful of possibly meaningful sentences with metaphysically indeterminate truth value might choose the valuation set {T, F, I}, “I” suggesting “indeterminate”; those more mindful of epistemic limitations and consequent unknowability the set {T, F, U}, “U” suggesting “unknowable,” etc.²⁷

The second programmatic decision is whether indeed to replace the standard, bivalent logic with a three-valued logic, or with a “gapped” or “supervaluational,” intuitionist or “epistemic,” modal or “contextual,” or some other non-truth-functional logic. The choice between these options does not implicate any issue that need long detain us here, particularly as I shall show that the proofs derivable in trivalent cases carry over to the plausible alternatives. Essentially, the choice between trivalence and some other plausible, non-truth-functional alternative reflects the “macro” choice between a) retention of finite semantic matrices – hence, retention of truth-functional semantics – at the cost of some lost classical tautologies (this is the three-value option); b) retention of all classical tautologies at the expense of accepting an infinite matrix and the consequent loss of truth-functionality (the supervaluational option); and c) sacrifice

either of finitary truth-functional semantics, of the classical tautologies, of complete axiomatizability, of interdefinable sentential connectives, or of some combination of these in the interest of some other – generally either metaphysically or epistemologically motivated – gain (the intuitionist or modal options). I shall say a bit more about these trade-offs logic-by-logic *infra*, Part IV, as their relevances emerge.

The third and final programmatic decision arises essentially only for veritably truth-functional, three-valued logics. It is to determine what effect valuation, with the third value, of a sentential part of a more complex sentential whole will have upon that whole. It is possible, for example, to take the complex proposition “ $\forall \vee \exists$,” when assigned the values “ $T \vee I$,” either to be true – because one of its components is true and this is all that is needed for the disjunctive whole to be true²⁸ – or to be indeterminate – perhaps because indeterminacy (or, more plausibly, “nonsensicality”) of some part of a sentence ought for some reason to be taken to vitiate the whole.²⁹ (“Green cheese yellow or I am happy” is perhaps as nonsensical as “Green cheese yellow” notwithstanding the sensicality of “I am happy.”) In effect, such choices can be read as assignments of meanings to the sentential connectives – “and,” “or,” etc.³⁰ Alternative ascriptions yield alternative logics. I shall briefly characterize each such set of ascriptions in introducing each of the standard three-valued logics below. What is more important for present purposes is that Rawls’s three claims – propositions (1) through (3) above – turn out to be jointly incompatible in all of them.

3.4. Proofs

In laying out the proofs I make use of two new formal devices additional to those laid out in Part II. The first is a new sentential operator: For “ \sim ” read an ascription of the

third semantic value, how ever we construe it, to the sentence that follows the operator. Hence “ $\sim\forall$ ” will read “It is indeterminate whether [or “unknowable whether,” “neither true nor false that,” etc.] \forall is the case.”

The second device I use is the familiar semantic matrix.³¹ By means of such tableaux I shall quickly characterize the meanings assigned by particular logics to the familiar sentential connectives. (The reader is, one final time, asked to skip over or skim the current paragraph if well familiar with this means of characterizing the sentential connectives.) That is to say that a matrix will reveal, at a glance, the content of what I just called the “third programmatic decision” implicit in each logic with respect to each sentential connective. A simple three-valued matrix for the connective “ \equiv ”, for example, might look like:

\equiv	1	.5	0
1	1	0	0
.5	0	1	0
0	0	0	1

This table shows that, in the logic in question, the operator “ \equiv ” veritably links two simpler sentences – i.e., that the more complex sentence formed by the connective is assigned a “1” – when and only when those simpler sentences are identically valued. The operator in this case is of course effectively bivalent, even if the sentences that it connects – which take values 0, .5, 1 – need not be. “Infection” of material equivalence by the third value, that is, is minimal. A policy of “maximal infection” of complex sentences by the third-valued parts would show itself in a modified matrix, namely, the preceding one save with its center row and center column of cells all taking the value “.5”:

\equiv	1	.5	0
1	1	.5	0
.5	.5	.5	.5
0	0	.5	1

And an intermediately infective policy might be to modify the last by changing the center value back to “1,” suggesting that indeterminates are indeterminately equivalent to determinates, but determinately equivalent to one another:

\equiv	1	.5	0
1	1	.5	0
.5	.5	1	.5
0	0	.5	1

Further such observations can await consideration of particular logics.

We derive the Roemerian contradiction from Rawls’s three premises in trivalent logics as follows. First, recall the first two premises:

$$1) \forall x, y, i, j: x >_{o/i} y \equiv x >_{o/j} y.$$

$$2) \forall k: x >_{o/k} y \supset x >_{s/k} y.$$

Next, we must decide how to recast premise (3) in keeping with our hypothetical

Rawlsian rejection of bivalence. Two revisory candidates suggest themselves:

$$3') \sim \forall x, y, i, j: x >_{s/i} y \equiv x >_{s/j} y.$$

$$3'') \forall x, y, i, j: \sim (x >_{s/i} y \equiv x >_{s/j} y).$$

The first connotes, in effect, that it is indeterminate/unknowable/neither true nor false to claim, etc., whether, for any two primary goods bundles and any two persons, one bundle subjectively equals or exceeds the other bundle for one person if and only if it does so for the other person. The second connotes, in effect, that for any two bundles and any two persons, it is (affirmatively) indeterminate/unknowable/neither true nor false to claim,

etc., whether one bundle subjectively equals or exceeds the other bundle for one person if and only if it does so for the other person.

Candidate (3') does not appear true to Rawlsian intentions. Rawls doesn't seem to hold that it is unknowable or indeterminate *whether* separate persons' ("subjective") satisfactions are interpersonally ordinally equivalent. He seems rather to hold, affirmatively, that those satisfactions definitely *are not* comparable, i.e., in the language of trivalent semantics, that prospective comparisons, and thus ordinal equivalence, are affirmatively, definitely, inavertibly, necessarily indeterminate, unknowable, etc. Proposition (3'') therefore seems to be the more faithful rendering of the Rawlsian intuition that we are exploring.

It is worth observing that we needn't actually *resolve* this question in the three-valued case, however, at least not apart from questions of interpretation.³² For the only expansion that I can envisage for (3') in *any* three-valued logic will itself be *included* in – it will be a proper *part* of – each of the expansions of (3'') warranted by those same trivalent logics. I explain: The only expansion that seems to make sense of (3') is:

$$4') \forall x, y, i, j: \sim x >_{s/i} y \vee x >_{s/j} y.$$

This comes of analogizing (3') to (3) and making the “~” in the former do the work of the “¬” in the latter (relying, of course, on the classical equivalence of “ $\forall \supset \exists$ ” and “ $\neg \forall \vee \exists$ ”). But the two disjuncts of (4') – call them “(4a'”) and “(4b'”) – we shall see, are among the four disjuncts that figure into the expansion of (3'') in Łukasiewicz's logic, and among the five disjuncts that figure into the expansion of (3'') in Bočvar's, Smiley's, Kleene's and Woodruff's logics. Since each disjunct is, in effect, a Rawlsian way out of the inconsistency discovered by Roemer, to show that all (four or five) (3'') avenues are

blocked is per force to show that all (both) (3') avenues are blocked as well. On, now, to those avenues:

3.4.1. Łukasiewicz

Łukasiewicz's trivalent logic is motivated by a concern over the earlier mentioned problematicity of future contingents ("there will be a sea battle ...").³³ Because ascribing a determinate truth-value to statements involving such contingents seemed to him to entail an objectionable commitment to determinism, Łukasiewicz recommended ascription of a third semantic value – "I," for "indeterminate," or "possible" – to them. Łukasiewiczian negation of a true or false sentence accordingly yields a false or true sentence as in classical, bivalent logic, but Łukasiewiczian negation of an indeterminate sentence yields another indeterminate sentence. Łukasiewicz's other logical operators are semantically characterized as follows:

\wedge	T	I	F
T	T	I	F
I	I	I	F
F	F	F	F

\vee	T	I	F
T	T	T	T
I	T	I	I
F	T	I	F

\supset	T	I	F
T	T	I	F
I	T	T	I
F	T	T	T

\equiv	T	I	F
T	T	I	F
I	I	T	I
F	F	I	T

Now we can expand (3'') Łukasiewicz-style simply by inspecting the semantic matrix associated with the Łukasiewiczian “ \equiv ” and determining which combinations of valuational assignments to two “atomic” formulae yield an “indeterminate” valuation to that “molecular” formula produced by connecting the two atomic formulae with the Łukasiewiczian “ \equiv ”. Since any such combination is sufficient to confer the “indeterminate” value upon a complex sentence involving the Łukasiewiczian “ \equiv ,” the disjunction of all such combinations will be equivalent to the original “ \equiv ”-statement. Treating “ \sim ” prefixed to a formula “ \forall ” as the syntactic indicator of semantic valuation with the “indeterminate” value, “ \neg ” of that with the “false” value, and “ \forall ” standing alone that of the “true” value, we see that (3'') accordingly is equivalent to:

$$4'') \forall x,y,i,j: x >_{s/i} y \wedge \sim x >_{s/j} y : \vee : \sim x >_{s/i} y \wedge x >_{s/j} y : \vee : \sim x >_{s/i} y \wedge \neg x >_{s/j} y : \vee : \neg x >_{s/i} y \wedge \sim x >_{s/j} y.$$

It will prove convenient to separate the component disjuncts of (4''). They are:

$$4a'') x >_{s/i} y \wedge \sim x >_{s/j} y.$$

$$4b'') \sim x >_{s/i} y \wedge x >_{s/j} y.$$

$$4c'') \sim x >_{s/i} y \wedge \neg x >_{s/j} y.$$

$$4d'') \neg x >_{s/i} y \wedge \sim x >_{s/j} y.$$

We can now derive the Roemer contradiction in the new, less restrictive logic. The object is to show that each of (4a'') through (4d'') is inconsistent with some entailment of premises (1) and (2). First, then, again let $x = m$, $y = n$, $i = a$, $j = b$, $k = a$, and $k = b$, as we did in Part II, so that all of the salient assumptions and results posited and derived in connection with premises (1) and (2) in that part carry over to the present part. In particular, then, we have (again),

4) let $x = m$, $y = n$, $k = a$,

6) let $k = b$,

8) let $i = a$, and $j = b$,

12) $m >_{s/b} n$.

And from Part II's (5), (10), and implication-elimination we have the previously underived:

22) $m >_{s/a} n$.

These results hold independently of any consequences we derive from (4a'') through (4d''). Now we derive some of those consequences. From (4), (4a'') and (8), we have

23) $m >_{s/a} n \wedge \sim m >_{s/b} n$.

This and conjunction-elimination yield:

24) $\sim m >_{s/b} n$,

which (softly) contradicts (12).³⁴ From (4), (4c''), and (8), we have:

25) $\sim m >_{s/a} n \wedge \neg m >_{s/b} n$.

This and conjunction-elimination yield:

26) $\neg m >_{s/b} n$,

which (firmly) contradicts (12). From (4), (4d''), and (8), we have:

27) $\neg m >_{s/a} n \wedge \sim m >_{s/b} n$.

This and conjunction-elimination yield:

28) $\sim m >_{s/b} n$,

which is the same result as (24), which as we noted before (softly) contradicts (12). Now note that (4), (4b'') and (8) yield:

29) $\sim m >_{s/a} n \wedge m >_{s/b} n$.

This and conjunction-elimination yield:

$$30) \sim m \supset_{s/a} n,$$

which (softly) contradicts (22).

We see, then, that in the Łukasiewicz system all four routes by which (4''), hence by which (3''), might avoid conflict with (1) and (2) are closed. The (revised) Rawlsian premises fare no better in our remaining trivalent logics.

3.4.2. Bočvar, Smiley

Bočvar's and Smiley's third semantic value is designated, like Łukasiewicz's, "I." In Bočvar's case, however, the intended reading of "I" is "paradoxical," or "senseless."³⁵ His concern is with properly valuing "barber," Russellian set, Cretan liar and like paradoxes as discussed in III.A. Smiley's "I" reads "undefined" or "truth-valueless," and is prompted by concern over sentences containing non-denoting terms, functions undefined over certain arguments, and the like – again as discussed earlier.³⁶ (Smiley's logic might therefore seem a natural vehicle through which to formalize a set of Rawlsian premises predicated on some form of incommensurability.) Again, as with Łukasiewicz, negation of a truth or falsehood yields a falsehood or truth, that of a "paradoxical," "meaningless," "undefined" or "truth-valueless" sentence another of the same semantic type. The other connectives are semantically characterized as follows:

\wedge	T	I	F
T	T	I	F
I	I	I	I
F	F	I	F

\vee	T	I	F
T	T	I	T
I	I	I	I
F	T	I	F

\supset	T	I	F
T	T	I	F
I	I	I	I
F	T	I	T

\equiv	T	I	F
T	T	I	F
I	I	I	I
F	F	I	T

Bočvar and Smiley also make use of an “assertion” operator (Smiley characterizes his as an “it is true that ...” operator) which yields a truth when prefixed to a truth, a falsehood when prefixed to a falsehood or to a “paradoxical,” “senseless,” “truth-valueless” or other such third-valued statement. Bivalence is thus restored at a second-order level, and a set of “external” connectives, each of them a counterpart to one of the just-characterized “internal” connectives, is defined on that basis. (Hence, a “ $\bar{}$ ” prefixed to a truth yields a falsehood, and to an indeterminate or false sentence a truth; an “ $\&$ ” yields a truth when connecting two truths, a falsehood otherwise; etc.) We need not dwell upon these “metasentential” matters, however, because any contradiction derivable in a “weaker,” three-valued logic of the “first order” can be expected likewise to turn up (in “firm” form) in the “stronger,” two-valued logic of the “second order” just as it does in the case of a bivalent first-order logic.

Roemer’s contradiction is again readily derivable in the Bočvar/Smiley logic. First, inspection of the semantic matrix for the Bočvar/Smiley “ \equiv ” reveals that (3”) will again entail the above disjunction (4”) of the four disjuncts (4a”) through (4d”), but now also will entail a fifth:

$$4e'') \sim x >_{s/i} y \text{ .}\wedge. \sim x >_{s/j} y.$$

This formula disjoined with (4a'') through (4d'') will yield a new (4'') – call it “(5'’)” – materially equivalent to (3'') in the new logic. Since we have just shown the hypothetical Rawlsian “ways out” represented by (4a'') through (4d'') to be blocked, all that remains to show a Rawlsian escape from Roemer’s contradiction foreclosed in the Bočvar/Smiley logic is to show that (4e''), too, is blocked. But this is readily done: By (4), (4e''), and (8), we have:

$$31) \sim m >_{s/a} n \text{ .}\wedge. \sim m >_{s/b} n.$$

This and conjunction-elimination yield both:

$$32) \sim m >_{s/a} n,$$

which is our earlier-derived (30), and

$$33) \sim m >_{s/b} n,$$

which is our earlier-derived (24) (and 28)) again. But (32) (softly) contradicts (22), while (33) (softly) contradicts (12). So (4e'') is (“twice-softly”) foreclosed, and (3'') thus once again fails to cohere with (1) and (2) in a (now more permissive) trivalent logic. Only two standard logics of that latter sort remain for Rawlsians to attempt.

3.4.3. Kleene, Woodruff

Kleene’s concern is with indefinite partial recursive functions (relations in particular) and undecidable propositions à la Church and Gödel.³⁷ His third semantic value – “U” – is accordingly to be read “undetermined,” “indefinite,” or “undecidable whether true or false.” (My earlier-mentioned question as to the self-membership of the “set of all sets that are self-members” comes to mind.) Woodruff’s “U” is to be read “undefined,” or “truth-valueless,” and is prompted by the same concerns as prompted

Smiley's logic – viz., missing term-referents.³⁸ Again, negation of a true or false sentence yields a false or true sentence, of an “undecidable,” “undefined” or “truth-valueless” sentence another of the same semantic type. The other sentential operators are semantically characterized as follows:

\wedge	T	U	F
T	T	U	F
U	U	U	F
F	F	F	F

\vee	T	U	F
T	T	T	T
U	T	U	U
F	T	U	F

\supset	T	U	F
T	T	U	F
U	T	U	U
F	T	T	T

\equiv	T	U	F
T	T	U	F
U	U	U	U
F	F	U	T

In Kleene's case, the tables just provided characterize what he calls “strong” connectives. Kleene's “weak” connectives are as Bočvar's previously tabulated “internal,” and Smiley's “primary,” connectives. (Kleene is thus thoroughgoingly – as it were “twice” – trivalent.) Woodruff for his part takes the connectives as just characterized for “weak” or “internal” connectives à la Bočvar and Smiley, then provides “stronger” bivalent valuations as do the latter two logicians for sentences prefixed with an assertion operator which he symbolizes “T.” Woodruff also makes use of several additional operators which further strengthen his second-order sentences but need not detain us here.

Fixing attention on the matrix for “ \equiv ” in particular, we see that it is isomorphic to Bočvar’s and Smiley’s. Accordingly, the Rawlsian premise (3”) will again, in the Kleene/Woodruff system, be equivalent to the five-termed disjunction we described and named “(5”)” above, and which we found to be inconsistent with (1) and (2). Roemer’s contradiction accordingly turns up in the (first order of) the Kleene/Woodruff logic in precisely the way that it does in the Bočvar/Smiley logic.

There remains one avenue to try. How might a Rawlsian fare at the Kleenean “second order,” with Kleene’s “weak” connectives, after having come (“softly”) a cropper with the “strong” ones? That is, what value will the Kleenean “weak” connectives assign to the derived contradictions themselves? To answer this query, we simply conjoin the derived mutually contradictory formulae and assign those conjunctions semantic values pursuant to the Kleenean “weak” connective matrices, which as we noted are those that Bočvar and Smiley provide respectively for their “internal” and “primary” connectives. The contradictions are:

C1): (24) & (12);

C2): (26) & (12);

C3): (27) & (12);

C4): (29) & (22);

C5): (32) & (22);

C6): (33) & (12).

Owing to the equivalence of (24), (28) and (33), and of (30) and (32), these six contradictions reduce to (C1), (C2), and (C4). Spelled out, they are:

C1) $\sim m \supset_{s/b} n \wedge m \supset_{s/b} n$;

$$C2) \neg m \succ_{s/b} n \wedge m \succ_{s/b} n;$$

$$C4) \sim m \succ_{s/a} n \wedge m \succ_{s/a} n.$$

Assigning semantic values to each conjunct pursuant to the aforementioned scheme whereby the “ \neg ” falsifies a formula, the “ \sim ” “indeterminates” it and the absence of any prefix amounts to asserting the truth of the formula, and then valuing the conjunctions themselves pursuant to the rules characterizing use of the Bočvar/Smiley “ \wedge ,” we arrive at:

$$C1) I \wedge T = I;$$

$$C2) F \wedge T = F;$$

$$C4) I \wedge T = I.$$

Since no “T”’s appear, we have no way out. There is in the case of each disjunct of (5”) either a hard or a soft contradiction ((C1) and (C4) are soft, (C2) hard), and so Rawls fails via premises (1) through (3) to speak univocally even in the most permissive of trivalent logics, Kleene’s.

4. Briefly on Some Other Logics

While three- (and other many-) valued logics are the most frequently encountered means by which formally to accommodate the objections to classical logic discussed at 3.1, and to examine the implications of doing so, they are not the only such means. In this part I briefly consider three other alternative logics that might be thought plausible means of salvaging Rawls’s treatment of primary goods and lifeplan-satisfaction. And I indicate how the proofs derived in the previous section carry-over into these alternatives.

4.1. Supervaluation

So-called “supervaluational” logic is an alternative means of accommodating the concerns that motivate many-valued logics without actually positing a “third,” or “*n*th,” semantic value.³⁹ Rather than an “I,” a “U” or a “1/2,” say, in the standard semantic tableaux, suspect formulae receive a “gap” or blank cell – that is, no semantic valuation at all. As a formal matter, this does not come to quite the same thing as a valuation with the “third” or “*n*th” value, a gap simply amounting to a third or *n*th symbol. For supervaluation also involves a critical second operational stage following the initial assignment of semantic values and gaps.

In that second stage, all constitutive “atomic” propositional formulae of a longer, “molecular” formula, what ever their *de dicto* or *de facto* valuations (or non-valuations) in the first stage – “T” or “1,” “F” or “0,” or blank – are hypothetically assigned, ensemble, all possible classical valuational combinations – that is, all possible combinations of “T”’s and “F”’s or “1”’s and “0”’s; and any complex formula which ends with a unique invariant valuation through all such combinations is supervaluated with that semantic value. By this means all classical tautologies (and, by parity, all classical contradictions, which are simply negated tautologies) derivable in the standard bivalent logic are preserved in the new logic.

The price, of course, is loss of truth-functionality – and consequent loss of the usability of finite matrices in semantic analysis – on the part of complex formulae. That is, the semantic value of a complex, “molecular” formula no longer is mechanically derivable from the semantic valuations of its constitutive “atomic” formulae. For while a formula such as “ $\forall \vee \neg \forall$ ” would receive a supervaluation of “T” no matter what value were assigned “ \forall ” in the first and second stages, and thus amount to a supervaluational

tautology, a formula such as “ $\forall \vee \exists$ ” would not; either of \forall ’s or \exists ’s being semantically gapped (i.e., truth-valueless, semantically empty) at the first stage would result in its – and the disjunction’s – remaining gapped at the end of the second stage. That is, “ $\forall \vee \exists$,” unlike “ $\forall \vee \neg\forall$,” would be neither valuated nor supervaluated, for “ $\forall \vee \exists$,” unlike “ $\forall \vee \neg\forall$,” is not a classical tautology or contradiction. In a many-valued logic, by contrast, both of these formulae would be valuated, and the semantics of the connective “ \vee ” thus fully truth-functional; but the tautological status of “ $\forall \vee \neg\forall$ ” might be lost, as inspection of the matrices for “ \vee ” laid out in III.D for all trivalent logics we have considered readily demonstrates.

In light of the foregoing, it should be clear that supervenient logic offers no help to Rawlsians in avoiding the Roemer contradiction. Supervenient logics differ from trivalent logics only in their preservation of classical tautologies in (“second stage”) semantic analysis. Motivated by the same philosophical concerns as those logics, they do not independently affect the syntax of those logics or the application of those logics to particular problems. We can see this perhaps most directly simply by systematically reinterpreting the tilde throughout the derivations carried out in Part 3.4. Rather than reading it as ascribing the third semantic value to the sentence following it, read it as ascribing a semantically gapped status to the same. The “hard” and “soft” contradictions derived in 3.4 are formally unaffected.

4.2. Intuitionist Logic

Intuitionist logic is prompted by concern to avoid certain objectionable metaphysical commitments of the sort discussed at Part 3.1 – or, perhaps, their epistemic counterparts.⁴⁰ At the outset, that concern which most troubled intuitionists was classical

mathematics’ – and its associated classical logic’s – apparent commitment to Platonism and/or some epistemic analogue to Platonism. Classical mathematicians and logicians, intuitionists believed, had misunderstood the nature of mathematic activity. The latter, by intuitionist lights, was not the exploration of an independent realm to which mental access was incomplete, but the active construction of mental objects pursuant to certain “principles of thought” bearing a distinctly Kantian coloration.

The intuitionist reinterpretation of mathematics lends itself to a reinterpretation of classical semantics, whereby “truth” comes to mean, more or less, “provability” or “constructability,” while “falsity” comes to mean something like “provability-not” or “nonconstructability” – the latter determined by the derivability of a (firm) contradiction from the assumption that a particular statement or object is in fact provable or constructible. This in turn lends itself to a reinterpretation of classical syntax, in which certain classical laws – notably *tertium non datur* – no longer hold and in which many classical theorems accordingly are no longer derivable, the classical sentential operators therefore effectively redefined. (The negation operator, for example, seems no longer simply to deny or contradict, but affirmatively to assert contrariness – i.e., to amount not to mere “reversal” of the truth-value of the proposition to which it is prefixed, but to independently warranted assertion that the proposition is affirmatively contraindicated.⁴¹ The classical tautology “ $\neg\neg\forall \supset \forall$ ” is thus lost, the rule of negation-elimination effectively relinquished. As are many others.)

Intuitionist principles have been extended to non-mathematical discourse by so-called “antirealists,” as well as, to a somewhat lesser extent, by certain “internal realists” and “irrealists,” who note analogies between the intuitionist view of mathematics on the one

hand and what it is to learn, understand and use any language on the other.⁴² Because to learn the meaning of and understand an indicative sentence is to learn and understand the conditions under which it is assertible, they reason, and because the conditions under which the familiar word “truth” is ascribable to such a sentence are themselves just those under which the sentence is assertible, the learning, use, and meaningfulness of language itself – which, after all, includes among its terms the learned word “truth” – amount to grist for a manner of Kantian “transcendental argument” to the effect that truth is none but assertibility.⁴³ And since assertability is in effect a generalization of the notion of mathematical constructability, antirealists hold that the same logic advocated by intuitionists in the latter context ought to be applied – now, in some cases, under the name “epistemic logic” – in other linguistic contexts as well. That is, intuitionist logic is, as a consequence of the workings of language, language-acquisition, language-understanding and thus language-meaning themselves, *the appropriate* logic. And its metaphysical and epistemic commitments (or non-commitments) constitute the appropriate metaphysical and epistemic stance. Not only future contingents and counterfactuals, but even certain statements about the past – if there be no way in principle to ascertain their truth or falsity – might then be thought to be of indeterminate truth value.⁴⁴

The intuitionist denial of *tertium non datur*, and the conceptual tie between non-ascertainability of truth-value (what we might, in honor of the neo-intuitionist antirealists, call “antireality”), on the one hand, and indeterminacy or unknowability on the other, suggest that intuitionist logic might amount to yet another means by which to formalize the Rawlsian premises (1) through (3) in conformity to Rawlsian intuitions regarding

commensurability. And indeed, we can readily carry out this formalization. The trouble for Rawlsians is that it does not help. Here is why:

Let us reinterpret the tilde utilized in Part 3.4 as an ascription, per intuitionist principles, of the non-ascertainability of the truth-value of the proposition which follows it. Since the first two Rawlsian premises do not involve that operator, they do not implicate any logical rules peculiar to, or applications objectionable to, intuitionist logic, and all results derived from them in Parts 2 and 3 carry over to this Part. The third Rawlsian premise, which does involve the tilde, would again then be formalized by the string that we labeled “(3”),” but would simply bear a (marginally) different intuitive meaning. Call that intuitionist rendition of the third Rawlsian premise “(3I)”:

$$3I) \forall x, y, i, j: \sim (x >_{s/i} y \equiv x >_{s/j} y).$$

Unpacking (3I) pursuant to the standard Heyting intuitionist axiomatics,⁴⁵ we derive:

$$4I) \forall x, y, i, j: \sim (x >_{s/i} y \supset x >_{s/j} y) \cdot \vee \cdot \sim (x >_{s/j} y \supset x >_{s/i} y).$$

Appealing once more to universal-elimination and setting $x = m$, $y = n$, $i = a$, $j = b$, we have:

$$5I) \sim (m >_{s/a} n \supset m >_{s/b} n) \cdot \vee \cdot \sim (m >_{s/b} n \supset m >_{s/a} n).$$

This formula and the intuitionist interpretation of implication in terms of disjunction and unascertainability yield:

$$6I) m >_{s/a} n \cdot \wedge \cdot \sim m >_{s/b} n : \vee : \sim m >_{s/a} n \cdot \wedge \cdot m >_{s/b} n.$$

But now note that the first disjunct of this formula is in essence just that which we labeled “(4a)” in the cases of those trivalent logics utilized in Part 3.4. And the second disjunct is just that which we labeled “(4b)” in the same. Both of these disjuncts, we saw, at least softly contradicted some formula derived from the first two Rawlsian

premises. It appears, then, that intuitionist logic – which *vis a vis* Rawls amounts to a strengthening of the logics considered in Part III (because it offers only two, rather than four (Łukasiewicz), five (Bočvar, Smiley, Woodruff), or six (Kleene) *prima facie* “ways out”) – offers no escape from Roemer’s contradiction.

4.3. Modal Logic

There are conceptual affinities among certain terms-of-art used by advocates of trivalent and intuitionist logics on the one hand, certain common “modal” expressions on the other. Thus Łukasiewicz, in semantically characterizing those future-contingent (“sea battle”) statements which he thought incapable of truth or falsity, sometimes used the word “possible.” (He also objected to the ascription of determinate truth values to such statements because he believed that to be determinately true or false they would have to be “necessarily” so.⁴⁶) And intuitionists, who generally have spoken in terms of “provability” and “provability-not” or “constructability” and “non-constructability,” sometimes in parallel fashion use such terms as “possibility” and “impossibility.” (A “constructible” object is, of course, an object that it is possible to construct.) The question thus arises whether some rationale that might render a trivalent or intuitionist logic attractive to a Rawlsian might also lend some attraction to a modal logic, and whether such a logic might offer Rawls a way out of the Roemer contradiction.

I think that we can dismiss this prospect for two reasons, one interpretive, the other formal. The interpretive problem is that it is difficult to envisage much sense that might attach to a reading of the Rawlsian premises along modal lines that hasn’t already been assayed in formalizing them trivalently, supervaluationally, or intuitionistically. Is the modal reading of (3) that it is “*merely* possible” that degrees or levels of lifeplan-

satisfaction be interpersonally ordinally equivalent? Is it that it is “*impossible*” that they be so? Or is it that it is impossible to *know*? What difference between any of these readings and the earlier-considered reinterpretations of (3) might we expect a modal logic possessed of the necessity operator “ \Box ” and the possibility operator “ \Diamond ” to capture while the earlier-considered logics did not? The last possibility – the epistemic (“impossible to *know*”) – seems to be the only one with any potential Rawlsian intuitive traction, yet this “epistemic” understanding already has been captured by the intuitionist logic.⁴⁷

The formal problem tracks the interpretive problem. For, as we might expect in view of the conceptual ties between intuitionist “-abilities” and epistemics on the one hand, the modal notions of possibility and contextuality on the other, it happens that the standard modal logic, S4, is formally isomorphic to the standard Heyting intuitionist logic.⁴⁸ That is, it has been metalogically proved that any well-formed formula is valid in Heyting logic if and only if its translation is valid in S4.⁴⁹ (It has also been proved, not surprisingly, that standard modal logics, like intuitionist and supervaluational logics, lack finite characteristic semantic matrices.⁵⁰) There thus seems to be little hope of salvaging all three Rawlsian premises through reformalization in either classical or any plausible – trivalent, supervaluational, intuitionist or modal – non-classical logic. Must we, then, abandon one of the premises, or might we reformulate more than one of them?

5. A Plausible – and Intuitively Attractive – Way Out

I wish to close with a suggestion as to how best to salvage Rawls’s treatment of primary goods and lifeplan-satisfaction in a manner that is faithful to the Rawlsian intuition of epistemically, politically, or politico-epistemically necessitated interpersonal subjective incomparability. (I do not here endorse that intuition, I only suggest how to

carry it out more satisfactorily.) Perhaps the problem captured by Rawls's intuition stems not from its comparability aspect, but from its subjectivity aspect – i.e., not from interpersonal comparison as such, but from would-be reliably observable, somehow measurable, and politically cognizable private or subjective magnitudes more generally. It is difficult, when one pauses to reflect upon the problem, to imagine just what sort of actual observation, measurement or moral-political problem not endemic to subjective phenomena *tout court* would be (uniquely) endemic to *interpersonal comparisons* of subjective phenomena. Is the Rawlsian idea that there is some irreducible something that is not sufficiently “public,” “political,” observable or detachable from the person to allow for comparison of its “possession” or “attainment” by or quantitative attributability to separate persons, but that is, somehow, sufficiently well-understood, isolable, public and legitimately politically salient nonetheless to allow for such attributability to individual persons at different times?⁵¹ Would such an idea be practically or operationally coherent? Its seeming near-inarticulability (as distinct from its formalizability) affords some grounds for skepticism, as do the formal results derived throughout this paper. I want to suggest that the idea is not coherent, and that focusing upon would-be detachable, reliably observable and measurable subjective magnitudes as culprit offers a means of recasting the Rawlsian premises in a manner that is both intuitively intelligible and formally consistent.

I think that the real Rawlsian intuition is that (“subjective”) lifeplan-satisfaction in some epistemic and/or normative-political manner defies, or at least somehow challenges, (even ordinal) quantification. That intuition apparently engages, in Rawls's case, at the point where satisfactions between persons would be compared. But all, if anything, that

renders such *comparison* pragmatically or politically problematic, it would seem upon reflection, is the difficulty likely to attend *any* interesting form of satisfaction-quantification. It is the operational intractability, and/or perhaps the normative political irrelevance or inadmissibility, of consistently reliable observation, consideration, and measurement of ineluctably “private” or “subjective” phenomena.⁵²

I have in mind here three specific possibilities, any combination of which, I believe, might appeal to Rawlsian concerns associated with a “political not metaphysical” conception of justice. First, the public inaccessibility of such privately privileged information as lifeplan-satisfaction assimilates it to mystical revelations and related phenomena associated with “comprehensive doctrines,” phenomena that Rawlsians believe do not belong in “the public sphere” or behind the “veil of ignorance.” Second and relatedly, the inescapable “vagueness” or “imprecision” of quantification in regard to such phenomena also renders them impracticable *distribuenda* for purposes of implementable, political justice. Finally, even were such epistemic and operational constraints not applicable, the familiar liberal conferral of “consumer sovereignty” and cognate statuses upon its subjects – treating each person as the sole authority as to the degree to which her lifeplan or desires are satisfied – presumably would bracket such information from public cognizance notwithstanding any hypothetical accessibility. So both the “public” (“objective”) and “private” (“subjective”) sides of what we might call Rawls’s “wall of separation” would give reason to treat, and formalize, private matters differently than public in the realm of politics.

Now of course there are sundry strategies, increasingly common since the turn of the last century, by which to attempt to get round the strictly epistemic aspect of the problem

– there are proposed incomplete (Paretian) social orderings, ordinal individual utilities ascribed on the basis of persons’ revealed preferences among goods vectors, “extended sympathies” from such preferences to the welfare of others, identification of “happiness” with theoretically measurable physical correlates such as C-fibres or endorphins, etc. – but none of these would seem in any practically appreciable way to validate Rawls’s second premise without calling into question his third (which, recall, denies even *ordinal* comparability of separate persons’ lifeplan-satisfactions). And, of course, it is perfectly possible in a strictly conceptual, non-operational sense to distinguish, formally, between intrapersonal and interpersonal measurement: There are, after all, those two formal premises themselves (premises (2) and (3)). And there are well-known information- (or group-) theoretic means by which to give mathematical expression, and precision, to the idea that interpersonal comparison is one thing, intertemporal intrapersonal comparison another.⁵³ But none of these strategies, how ever successful, would touch the “consumer sovereignty” or “liberal” aspect of Rawlsian political justice. And again the *point* here is, in any event, that it is difficult, as an empirical, operational or even as a political-theoretic matter, to discern what fact or facts about satisfaction or subjectivity would render it the case that satisfaction is or ought to be treated as, e.g., *intrapersonally* cardinally measurable but not *interpersonally* comparable, or *intrapersonally* but *ordinally* measurable yet *interpersonally* fully comparable.⁵⁴ Not all formally representable conceptual distinctions are epistemically, practically or political-theoretically significant – or even sustainable – distinctions. So perhaps what a Rawlsian (or indeed any similarly oriented “political, not metaphysical” theorist of justice) ought to do, in order to retain both formal consistency and politico-pragmatic intelligibility, is either to abandon that

third premise altogether, or, more plausibly, to recast the *second* premise along *with* the third.⁵⁵

A reformulation of Rawls's second premise in a manner consistent with the only rationale that might warrant his third – viz., an alleged politico-epistemic problematicity, perhaps a full-blown indeterminacy or unknowability, afflicting any quantitative or quasi-quantitative claim (cardinal or ordinal) concerning such “private” or “subjective” phenomena as satisfactions – might look like this:

$$2'') \forall x, y, k: \sim (x >_{o/k} y \supset x >_{s/k} y).$$

Appealing to our earlier applications of universal-elimination and preserving their results, we have (again):

$$4) \text{ let } x = m, y = n, k = a,$$

and:

$$6) \text{ let } k = b.$$

These in conjunction with (2'') yield:

$$2a'') \sim (m >_{o/a} n \supset m >_{s/a} n);$$

and:

$$2b'') \sim (m >_{o/b} y \supset m >_{s/b} n).$$

Unpacking these propositions via the semantic tableaux method employed in Part 3.4, we find that in the Łukasiewicz logic (2a'') is equivalent to:

$$3a'') m >_{o/a} n \wedge \sim m >_{s/a} n : \vee : \sim m >_{o/a} n \wedge \neg m >_{s/a} n,$$

while (2b'') is equivalent to:

$$3b'') m >_{o/b} n \wedge \sim m >_{s/b} n : \vee : \sim m >_{o/b} n \wedge \neg m >_{s/b} n.$$

Parsing-out all four disjuncts, each of which represents a Rawlsian way out, as we did in connection with the proposition (4'') in Part 3, we have:

$$3a1'') m >_{o/a} n \wedge \sim m >_{s/a} n.$$

$$3a2'') \sim m >_{o/a} n \wedge \neg m >_{s/a} n.$$

$$3b1'') m >_{o/b} n \wedge \sim m >_{s/b} n.$$

$$3b2'') \sim m >_{o/b} n \wedge \neg m >_{s/b} n.$$

Unpacking (2a'') pursuant to the Kleene and Woodruff logics simply adds another disjunct, i.e., another possible Rawlsian way out. Let us call it:

$$3a3'') \sim m >_{o/a} n \wedge \sim m >_{s/a} n.$$

Unpacking (2b'') in the same way yields the analogue:

$$3b3'') \sim m >_{o/b} n \wedge \sim m >_{s/b} n.$$

Unpacking (2a'') pursuant to the Bočvar/Smiley logic affords two more disjuncts, two more Rawlsian ways out. They are:

$$3a4'') \sim m >_{o/a} n \wedge m >_{s/a} n;$$

and:

$$3a5'') \neg m >_{o/a} n \wedge \sim m >_{s/a} n.$$

The analogues in the case of (2b'') are:

$$3b4'') \sim m >_{o/b} n \wedge m >_{s/b} n;$$

and:

$$3b5'') \neg m >_{o/b} n \wedge \sim m >_{s/b} n.$$

It is perhaps worth noting in passing that the first three and the fifth “a” disjuncts contradict the original Rawlsian premises (1) and (3) via our earlier derived (20), while the fourth “a” disjunct contradicts those original Rawlsian propositions via our earlier

derived (22). Parallel reasoning would show the same to hold true of the “b” disjuncts. We cannot avoid Rawlsian inconsistency, then, by revising premise (2) alone. Can we avoid inconsistency by revising both (2) and (3) pursuant to the revision of (3) proposed in Part 3 – (3’’) – and to that of (2) proposed here – (2’’) ? Indeed we can.

While a full-blown metalogical proof of the joint consistency of (1), (2’’) and (3’’) might be marginally more reassuring, it should suffice here to argue intuitively. First, as a general matter, note that all of the difficulties that Rawls encountered in Parts 2 and 3 decisively involved (2). That is, on each occasion that some entailment of (3) or (3’’) contradicted some other derived formula, that other derived formula required (2) as one of its premises. A suitable revision of (2), this suggests, might well be compatible with (3’’), even though we have observed in the previous paragraph that it would not be compatible with (3).

Next, to move to particulars, note that each disjunct of (4’’) and (5’’) discussed in Part III represented a prospective Rawlsian way out of the Roemer contradiction, a route that turned out to be closed by some entailment of (2). But since our new (2) – (2’’) – now also turns out to be disjunctive, it too offers a number of prospective Rawlsian ways out – two such in the Łukasiewiczian logic, three such (which properly include the Łukasiewiczian two) in the Kleene/Woodruff logic, and five such (which properly include the Kleene/Woodruffian three) in the Bočvar/Smiley logic. If either of the two (2’’) ways out are compatible with any of the four (3’’) ways out in the Łukasiewiczian logic, then there is a Rawlsian way out in that logic once the original premises (2) and (3) have been revised to (2’’) and (3’’) . And since all of those ways out constitute proper subsets of the (more inclusive) sets of prospective ways out offered by the other, more

permissive trivalent logics, to show that (revised) Rawls passes through Łukasiewicz will be to show that he passes through any standard trivalent (or equally permissive, e.g. supervaluational, intuitionist or S4 modal) logic as well.

Let us recall those earlier four prospective Rawlsian “ways out” via the Łukasiewiczian logic considered in Part 3.4, but let us fill in their free variables pursuant to (4) and (8) – which set $x = m$, $y = n$, $i = a$, $j = b$. Suitably filled, the prospective “ways out” are:

$$4a'') m \succ_{s/a} n \wedge \sim m \succ_{s/b} n.$$

$$4b'') \sim m \succ_{s/a} n \wedge m \succ_{s/b} n.$$

$$4c'') \sim m \succ_{s/a} n \wedge \neg m \succ_{s/b} n.$$

$$4d'') \neg m \succ_{s/a} n \wedge \sim m \succ_{s/b} n.$$

Now note that (3a1''), above, conflicts with (4a'') and (4d''), but is compatible with (4b'') and (4c''). Either of the latter two, conjoined with (3a1''), constitutes not just a prospective, but indeed a veritable Rawlsian way out of the Roemer contradiction. Next note that (3a2''), above, conflicts with (4a''), (4b''), and (4c''), but is compatible with (4d''): Another Rawlsian way out. Now note that (3b1''), above, conflicts with (4b'') and (4c''), but is compatible with (4a'') and (4d''), while (3b2'') conflicts with (4a''), (4b'') and (4d''), but is compatible with (4c''). Whether, pursuant to (2''), we set $k = a$ or $k = b$, then, we have three veritable Rawlsian-Łukasiewiczian ways out of the Roemer contradiction. It follows that we have at least three ways out in any other alternative logic thus far discussed as well, for the Łukasiewiczian is the most restrictive of them.

For the sake of completeness, however, we may as well take passing note of the other possible routes offered by the other trivalent logics: Note, then, that (3a3'') is

compatible with (4b'') and (4c''), though not with (4a'') and (4d''), while (3b3'') is compatible with the latter two but not with the former two. Whether $k = a$ or $k = b$, then, the Kleene/Woodruff logic offers two additional Rawlsian routes of escape from the Roemer inconsistency. That's five ways out and still counting. Recalling that Kleene and Woodruff (along with Bočvar and Smiley) allowed for a fifth disjunct in connection with (4''), viz. (suitably filled like (4a'') through (4d'')),

$$4e'') \sim m \succ_{s/a} n \text{ .} \wedge \text{ .} \sim m \succ_{s/b} n,$$

we note that yet another prospective Rawlsian way out in connection with (3'') is compatible with the following prospective Rawlsian ways out in connection with (2''): (3a1'') and (3a3''); parallel remarks hold for the (3b'')'s. Seven ways out and still counting.

Finally, note that (3a4'') is compatible with (4a'') but not with (4b'') through (4e''), while (3a5'') is compatible with (4b''), (4c'') and (4e'') but not the others. Parallel remarks holding for (3b4'') and (3b5''), we find that there are four more Rawlsian ways out offered by the Bočvar/Smiley logic. That's eleven ways out in total, for $k = a$ or $k = b$. One begins to suspect that, with our revised Rawlsian second premise – (2'') – we might even move to use of the full equivalence (biconditional) operator “ \equiv ” from the mere entailment (uniconditional) operator “ \supset ” and still preserve consistency, even though this constitutes something of a strengthening (within the greater weakening presented by the move to trivalence) of the relation that Rawls posits between primary goods and lifeplan-satisfaction. (Indeed, since in the Bočvar/Smiley logic “I”'s are assigned to exactly the same valuational combinations under “ \equiv ” as they are under “ \supset ”,

we *know* that we can move to full material equivalence in that case without sacrificing consistency.)

It might be well to interpret a few samples from among these “Rawlsian ways out” pursuant to the neo-Rawlsian intuition underwriting the revised versions (2”) and (3”) of premises (2) and (3). First consider the complete set of premises themselves. Premise (1) still holds that primary good indices are “objectively” interpersonally ordinally equivalent – that is, that one bundle “objectively” equals or exceeds another for me if and only if it does so for you. Premise (2”) in turn holds that primary goods indices and degrees or levels of lifeplan-satisfaction are in some sense “indeterminately,” or “not fully knowably,” or “not legitimately politically-*cognizably*,” “‘objectively’/ ‘subjectively’ intrapersonally quasi-ordinally related” – that is, perhaps, that one bundle’s “objectively” equaling or exceeding another bundle for me generally can be expected to, but cannot or should not (for purposes of a political conception of justice) fully knowably, result in its yielding equal or greater “private” or “subjective” lifeplan-satisfaction to me. (The relation is one of unconditional implication as we have interpreted it, though as suggested in the previous paragraph the weakening posed by the move to trivalence will allow us to preserve consistency even if we “strengthen” the “not fully knowable” relation to full ordinal equivalence.) Premise (3”) holds that degrees or levels of lifeplan-satisfaction yielded by primary goods are in some sense “indeterminately,” “not fully knowably,” or “not legitimately politically-congnizably,” “‘subjectively’ interpersonally ordinally equivalent” – that is, perhaps, that one primary goods bundle’s yielding equal or greater “private” or “subjective” lifeplan-satisfaction than another to me generally will, but again cannot or should not (for purposes of a

political conception of justice) be thought fully knowably, jointly to entail its doing the same for you.

What fuller sense are we to make of these three claims respecting primary goods and lifeplan-satisfaction? I propose the following: We know that primary goods are “objective” things exterior to the person and open to reliable public inspection, and that they therefore present no insuperable epistemic or political difficulties in principle as regards verifiable and politically cognizable observation and measurement; your and my holding the same bundle amounts, reliably and cognizably, to your and my holding the same “amount,” at least in an ordinal if not in a cardinal sense (premise (1)).⁵⁶ We also know that it is reasonable to suppose (or, as Rawls does, definitionally to stipulate) that more of these goods are better (more lifeplan-satisfying) than less in view of their usefulness in the pursuit of lifeplans – they are, from the point of view of lifeplan-satisfaction, “normal” goods – but, owing to difficulties attending the would-be consistently reliable observation and (cardinal or even ordinal) measurement of such “subjective” imponderables as lifeplan-satisfactions, in addition to the “publicity constraint”⁵⁷ placed by a political conception of justice upon the legitimate *cognizability* of ineluctably “private” or “subjective” imponderables, we cannot and need not ever act as though we *knew* with full confidence that more primary goods in all cases determinately succeed, *ceteris paribus*, in bringing about more lifeplan-satisfaction; it is only a reasonable supposition or “working hypothesis” – a matter of “common sense” or “background knowledge” behind the veil of ignorance, perhaps – in characterizing and implementing justice, which latter tasks include selecting an appropriate distribuendum (premise (2’)). Finally, we know it also reasonable to suppose, in view of primary

goods' multiple uses, instrumental fungibility (Rawls calls them "all-purpose means"⁵⁸) and "normality" from the point of view of lifeplan-satisfaction, as well as in view of their "objective" invariance in interpersonal holdings (premise (1)), that there will be in theory some correlation between one primary goods bundle's lifeplan-superiority over another for me and a similar lifeplan-superiority for you, but again owing to those aforementioned difficulties and related political constraints attending the would-be consistently reliable observation, measurement and anything but "background" salience of such ineluctably "private" or "subjective" imponderables as lifeplan-satisfaction, we cannot ever know or maintain with full confidence or legitimacy that this relation holds either; it too, like (2"), is a reasonable bit of "common sense," "background knowledge," general supposition or "working hypothesis" in characterizing and implementing justice, which includes selecting an appropriate distribuendum (premise (3")). It seems to me that this represents a fully coherent articulation of the ("pragmatic," "political") Rawlsian view of primary goods and lifeplan-satisfaction that not only avoids the inconsistency found by Roemer in Rawls's own articulation, but that also is truer – because more thoroughgoing in its faithfulness – to Rawls's own intuitions than is the literal Rawlsian articulation itself (as quoted by Roemer) or Risse's proposal simply to *drop* (2)).⁵⁹

One *particular* "Rawlsian way out" noted above seems to cohere especially attractively with this re-reading of the three Rawlsian premises. We observed, several pages up, that (3a1") and (4e") were compatible. A ready intuitive interpretation of these propositions pursuant to the general understanding of Rawls here proposed would run: "Primary goods bundle *m* objectively equals or exceeds primary goods bundle *n* for person *a*, but we cannot know or act as if we knew with certainty, even though it is safe

as a matter of common sense to assume as “background knowledge” out from behind the veil of ignorance, that it subjectively equals or exceeds that bundle for her; the latter is simply a reasonable background supposition for the purposes of political justice” (3a1”). And: “Not only can we not know or act as if we knew with certainty that primary goods bundle m subjectively equals or exceeds bundle n for person a , we cannot know or act as if we knew with certainty whether it does so for person b either; again, *this* is simply a reasonable general supposition for the purposes of political justice” (4e”). Again, it seems to me that these are entirely reasonable things to say, that they are consistent one with the other, and that they are fully in conformity with the spirit of the intuitive reading of the revised Rawlsian premises respecting primary goods and lifeplan-satisfaction offered in the previous paragraph – a reading, and revisions, which again appear more fully faithful to Rawlsian intuitions, as well as more internally harmonious, than are Rawls’s articulated premises themselves. Other Rawlsian ways out assayed above are perhaps more difficult to square with the proposed intuitive reading of the revised premises. But we need but one set, and perhaps it will be well for now to leave things at that.

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P. Woodruff, "Logic and Truth-Value Gaps," in K. Lambert, ed., *Philosophical Logic* (Dordrecht: Reidel, 1970), pp. 121-42

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¹ John E. Roemer, *Theories of Distributive Justice* (Cambridge: Harvard, 1996), pp. 167-72; John Rawls, *A Theory of Justice* (Cambridge: Harvard, 1971), pp. 92-95, 548-54.

² Mathias Risse, "Rawls on Responsibility and Primary Goods," available at ome.harvard.edu/~mrisse.academic.ksg/Papers/Papers%20-%20Philosophy/RawlsAugust02.pdf, pp. 13-22 (last visited March 3, 2003), now incorporated into Robert Hockett & Mathias Risse, "Primary Goods Revisited," in progress.

³ There is, technically, a critical distinction between bivalence and *tertium non datur*. The former constitutes a semantic characterization of any formal system in which all well-formed formulae possess one or the other of two semantic valuations (generally truth or falsity). *Tertium non datur*, on the other hand, constitutes a syntactic characterization of any formal system in which " $\forall \vee \neg \forall$ " is derivable as a theorem. Most formal systems, including many considered in the present paper, conform to or depart from both characterizations together. But it is well to bear in mind that the two characterizations can part company in some cases – e.g., in Kleene's system, considered *infra*, Part 3, which at least in its intended interpretation is bivalent but not subject to *tertium non datur* (the "third" value is not intended to supplement truth and falsity, but to express undecidability as between the two), and in van Fraassen's system, considered *infra*, Part 4.1, in which " $\forall \vee \neg \forall$ " remains a theorem (and *tertium non datur* therefore holds), but in which not all well-formed formulae are valuated. Reasons for choosing one such system or another are addressed in both Parts 3 and 4.

⁴ Roemer, note 1, p. 168, says that "[t]he converse ... must also be true, by reference to" a simple implicative claim made by Rawls. If by "converse" Roemer means contrapositive, he is of course correct, but not very interestingly so. If, by contrast, he means that from "if \forall then \exists " one may infer "if \exists then \forall ," he is invalidly "affirming the consequent." The force of this observation is not diminished by the distinction between material (i.e., extensional) equivalence and ordinal equivalence, particularly if bivalence be rejected. Ordinal equivalence, as defined *infra*, is simply a special case of material equivalence, also defined *infra*.

⁵ We could imagine a three-valued logic, for example, in which “ $\forall \supset \exists$ ” is not equivalent to “ $\neg \exists \supset \neg \forall$ ”. If \forall were assigned the third semantic value, construed, say, as “indeterminacy,” and \exists were assigned the value “true,” the first formula might take the value “true” while the second took the value “indeterminate” in a logic with weak material implication in which the truth of a consequent sufficed to confer truth on the conditional, and the falsity of an antecedent sufficed to confer the (non-“false”) value of the consequent on the full conditional. (Assuming, of course, that this logic also bore a negation operator “ \neg ” which “reversed” the valuation of a true or false sentential formula and left that of a third-valued formula invariant.) And in intuitionist logic, in which *tertium non datur* does not hold, i.e. in which “ $\forall \vee \neg \forall$ ” is not tautologous, “ $\neg \exists \supset \neg \forall$ ” does not entail “ $\forall \supset \exists$ ”. More on these matters *infra*, Parts 3 and 4.2.

⁶ The expression is Risse’s. Note 2, p. 16.

⁷ For those conversant with the symbolism, I follow Whitehead-Russellian practice with three exceptions: I replace the W-R tilde (which I put to different use in Part III) with “ \neg ,” the W-R conjunctive “ \cdot ” with “ \wedge ” (by analogy to “ \vee ” and to avoid confusion with dots put to another use), and the W-R universal quantifier “ $(x) Mx$ ” with “ $\forall x: Mx$ ” (for parallelism with the existential quantifier “ $\exists x: Mx$ ”). I also replace parentheses with dots in a manner slightly departing from theirs, again in order to avoid unnecessary confusion. See generally A. N. Whitehead & B. Russell, *Principia Mathematica* (Cambridge: University Press, 1910; Cambridge Mathematical Library Edition to *56, 1997), pp. 4f.

⁸ See, e.g., Rawls, note 1, p. 95: “It is worth noting that [the interpretation of expectations in terms of primary goods possession] represents, in effect, an agreement to compare men’s situations solely by reference to things which it is assumed that all prefer more of. This seems the most feasible way to establish a publicly recognized objective measure, that is, a common measure that reasonable persons can accept. Whereas there cannot be a similar agreement on how to estimate happiness as defined, say, by men’s success in executing their rational plans, much less on the intrinsic value of those plans.” Also John Rawls, *Justice as Fairness: A Restatement* (Cambridge: Belknap/Harvard, 2001), p. 60: Primary goods “belong to a partial conception of the good that citizens, who affirm a plurality of conflicting comprehensive doctrines, can agree upon for the purpose of making interpersonal comparisons required for workable political principles.” And Risse, note 2, pp. 8-10, on primary goods’ relation to Rawls’s “publicity condition” or “—constraint.”

⁹ I deliberately leave an ambiguity in the claim as stated. It might mean that it is not the case that for all “you” and “I,” any primary goods bundle subjectively equals or exceeds another such bundle for me if and only if it does so for you. (This is the rendering I have settled on and formalized below.) Or it might mean that for all “you” and “I,” it is not the case that any primary goods bundle subjectively equals or exceeds another such bundle for me if and only if it does so for you. (A formal rendering would be “ $\forall x, y, i, j: \neg (x >_{s/i} y \equiv x >_{s/j} y)$.”) In a bivalent logic, the former rendition amounts to a claim that for *at least one* pair of persons, one bundle will subjectively equal or exceed a second bundle for one of those persons while the second bundle exceeds the first for the other person; whereas the second rendering would be equivalent to the claim that for *any* two people, one bundle will subjectively equal or exceed another bundle for one of those persons only when the other bundle exceeds it for the other person. The latter seems surely to be a stronger claim than Rawls would wish to make. (Note, though, that his *incommensurability* intuition would appear to stand in some tension with *either* – another reason to consider departing from standard logics in Parts 3 and 4.) I leave the ambiguity in the informally stated premise, however, because I believe Rawls implicitly to be relying on the possibility of trivalence or some syntactic counterpart, in which case the two interpretations of (3) do not part ways so radically, and in which case both seem worth considering. I treat the logical consequences of each interpretation under conditions of trivalence – showing that one such set of consequences indeed properly includes the other – in Part 3.

¹⁰ These observations also seem to rule out any useful Rawlsian recourse to so-called “free logics” or “Meinongian semantics,” which amount to classical logics with restricted quantificational domains. See generally Ermano Bencivenga, “Free Logics,” in D. Gabbay & F. Guentner, eds., *Handbook of Philosophical Logic, Vol. III: Alternatives to Classical Logic* (Dordrecht: Reidel, 1985). Because *these* “alternative logics” do *not* appear to hold any Rawlsian promise, I do not treat of them here.

¹¹ In so far as “nonsense” is assimilated to “paradox,” and logics meant to accommodate the latter (misleadingly) named “nonsense logics,” I do treat of them. Please see next paragraph, and exposition of the systems of Bočvar, Kleene, and their followers *infra*, Part 3.4. On three-valued “quantum logic,” its justification and utility, see, e.g., G. Birkhoff & John von Neumann, “The Logic of Quantum Mechanics,” *Annals of Mathematics* 37 (1936): 823-43; P. Destouches-Février, “Les Relations d’Incertitude de Heisenberg et la Logique,” *Comptes Rendus de L’Académie des Seances* 204 (1937): 481-831; Hans Reichenbach, *Philosophical Foundations of Quantum Mechanics* (Berkeley: U. Cal., 1944); Patrick Suppes, ed., *Logic and Probability in Quantum Mechanics* (Dordrecht: Reidel, 1976); Hilary Putnam, “How to Think Quantum-Logically,” *Synthese* 29 (1974): 55-61; Hilary Putnam, “The Logic of Quantum Mechanics,” in *Philosophical Papers, Vol. I: Mathematics, Matter and Method* (Cambridge: University Press, 1979), pp. 174-97; E. Beltrametti & Bass van Fraassen, *Current Issues in Quantum Logic* (New York: Plenum, 1981); and M. L. Dalla Chiara, “Quantum Logic,” in Gabbay & Guentner, note 16, pp. 427-70.

¹² Whitehead & Russell, note 13, pp. 37f., 161f. Also Frank P. Ramsey, *Philosophical Papers*, D.H. Mellor, ed. (Cambridge: University Press, 1990), pp. 183f.

¹³ “Nonsense”-consignment is, of course, the Whitehead & Russell strategy. Cf. remarks on the “vicious circle principle” and rectificatory “theory of types” in Whitehead & Russell, note 13, pp. 37f., 161f. Cognate or analogous stratification strategies turn up in the canonical work of a number of set-theorists, notably Bernays, Fraenkel, von Neumann, Quine, Skolem and Zermelo, and are famously employed by A. Tarski, “Der Wahrheitsbegriff in den formalisierten Sprachen,” *Studia Philosophia*, I (1935): 261-405, reprinted in English as “The Concept of Truth in Formalized Languages,” in Alfred Tarski, *Logic, Semantics, Metamathematics*, J. H. Woodger, ed., (Indianapolis: Hackett, 2d ed. 1983); and S. Kripke, “Outline of a Theory of Truth,” *Journal of Philosophy* 72 (1975): 690-716. Another means of handling paradox without positing a new semantic value is via so-called “paraconsistent logics,” which avert difficulties by withdrawal of the classical sanction of inference to any proposition what ever from a contradiction. I see no interest in these logics for Rawlsians. See generally Diderik Batens et al., eds., *Frontiers of Paraconsistent Logic* (Baldock: Research Studies Press, 2000); Graham Priest et al., *Paraconsistent Logic: Essays on the Inconsistent* (Munich: Philosophia Verlag, 1989). Ronald Dworkin illuminatingly considers the relations between bivalence, *tertium non datur* and putatively undecidable legal questions in “Is There Really No Right Answer in Hard Cases?,” *New York University Law Review* 53 (1978), reprinted in Ronald Dworkin, *A Matter of Principle* (Cambridge: Harvard, 1985), pp. 119-45. See also Ronald Dworkin, “No Right Answer,” in P. M. S. Hacker & Joseph Raz, eds., *Law, Morality and Society: Essays in Honour of H. L. A. Hart* (Oxford: Clarendon, 1977); and Ronald Dworkin, *Taking Rights Seriously* (Cambridge: Harvard, 1977), Chapter 13.

¹⁴ One might argue that “the largest prime” is of a different sort than “my mother’s [perhaps non-existent] cousin,” in that the former, unlike the latter, cannot just “happen” to lack a bearer. Three replies: 1) If, then, the term is deemed nonsensical, we can simply drop it as a prototypical example here. 2) “The largest prime” certainly would not seem to be nonsensical in the way, say, that “the prime largest” would be. If anything, it would seem more “paradoxical” than “meaningless,” in which case I simply shift it to the previous category. Finally 3) Some intuitionists and constructivists would likely find the expression in fact to possess a denotation – that singled out by a particular connotation that they would ascribe to it, viz., “the largest prime thus far constructed.” More on that view of things *infra*, Part 4.2.

¹⁵ Though not, of course, in the view of Whitehead & Russell, who analyze such propositions as incorporating (false) existential claims. Whitehead & Russell, note 13, pp. 66f., 173f.; B. Russell, “On Denoting,” *Mind* 14 (1905): 479-93, reprinted in Russell (ed. R.C. Marsh), *Logic and Knowledge: Essays*

1901-1950 (London: George Allen & Unwin, 1956), pp. 41-56. The truth-valueless analysis, of course, is associated with Frege and with Strawson. Gottlob Frege, "Über Sinn und Bedeutung," *Zeitschrift für Philosophie und Philosophische Kritik* 100 (1894), translated into English as "On Sinn and Bedeutung," in A. Beaney, ed., *The Frege Reader* (Oxford: Blackwell, 1997), pp. 346-61; Peter Strawson, "On Referring," *Mind* 59 (1950): 320-44, reprinted in Strawson, *Logico-Linguistic Papers* (London: Methuen, 1971), pp. 1-27. See also Gareth Evans, *The Varieties of Reference* (Oxford: Clarendon, 1983), for state-of-the-art treatment of the Russellian position; and M. A. E. Dummett, *Frege: Philosophy of Language* (London: Duckworth, 1973), for state-of-the-art treatment of the Fregean. The theory of denotation has of course undergone a great deal of ferment in the last several decades, largely in connection with the so-called "new theory of reference". See, e.g., in addition to works just cited and sources cited therein, Keith Donnellan, "Reference and Definite Descriptions," *Philosophical Review* 75 (1966); Keith Donnellan, "Proper Names and Identifying Descriptions," in Donald Davidson & Gilbert Harman, eds. *Semantics of Natural Languages* (Dordrecht: Reidel, 1972); Keith Donnellan, "Speaking of Nothing," *Philosophical Review* 83 (1974); David Kaplan, "Demonstratives," in Joseph Almog et al., eds., *Themes from Kaplan* (Oxford: University Press, 1989); David Kaplan, "On the Logic of Demonstratives," in Peter French et al., eds. *Contemporary Perspectives in the Philosophy of Language* (Minneapolis: University of Minnesota Press, 1979); David Kaplan, "Quantifying In," in Donald Davidson & Jaakko Hintikka, eds., *Words and Objections: Essays on the Work of W.V. Quine* (Dordrecht: Reidel, 1969); Saul Kripke, *Naming and Necessity* (Oxford: Blackwell, 1980), and in Davidson & Harman, *op. cit.*, pp. 253-355; John McDowell, "The Sense and Reference of a Proper Name," *Mind* 86 (1977): 159-85, reprinted in his *Meaning, Knowledge and Reality* (Cambridge: Harvard, 1998), pp. 171-98; J. M. McDowell, "Truth-Value Gaps," in L. J. Cohen et al., eds., *Logic, Methodology and Philosophy of Science* (Amsterdam: North-Holland, 1982), pp. 299-314, reprinted in McDowell, *Meaning, Knowledge and Reality*, pp. 199-213; Putnam, *Reason, Truth and History*, and *Philosophical Papers, Vol. 3: Realism and Reason*, note 48, *passim.*; and of course the seminal W.V.O. Quine, *From a Logical Point of View: Nine Logico-Philosophical Essays* (Cambridge: Harvard, 1959); and W.V.O. Quine, *Word and Object* (Cambridge: MIT, 1960).

¹⁶ See, e.g., L. A. Zadeh, "Fuzzy Sets," *Information and Control* 8 (1965): 338-53; L. A. Zadeh, "Fuzzy Logic and Approximate Reasoning," *Synthese* 30 (1975): 407-25; J. Pavelka, "On Fuzzy Logic. I-III," *Zeitschrift für mathematische Logik und den Grundlagen der Mathematik* 25 (1979): 45-52, 119-34, 447-64; P. Hájek, *Metamathematics of Fuzzy Logic* (Dordrecht: Kluwer, 1998); V. Novák, *Fuzzy Sets and Their Applications* (Bristol: Adam Hilger, 1989); Gottwald, note 37, Chapters 18, 19; Bolc & Borowik, note 37, Chapters 8, 9. Also B. Urquhart, "Many-Valued Logic," in Gabbay & Guenther, note 16, pp. 71-116; M. A. E. Dummett, "Wang's Paradox," *Synthese* 30 (1975): 301-24, reprinted in Dummett, *Truth and Other Enigmas*, note 48; and Hilary Putnam, "Vagueness and Alternative Logic," *Erkenntnis* 19 (1983): 297-314, reprinted in Putnam, *Philosophical Papers, Vol. 3: Realism and Reason*, note 48, pp. 271-86. Precursors to "fuzzy logics" in the form of many-valued logics construed or construable as logics of "partial truth" or "verisimilitude" are assayed in E. Post, "Introduction to the General Theory of Elementary Propositions," *American Journal of Mathematics* 43 (1921): 163-85; and K. M. Popper, *Objective Knowledge* (Oxford: Clarendon, 1972). Many-valued logic construed as "probability logic" is investigated by Jan Łukasiewicz, note 36, whose work is further discussed *infra*. Professor Dworkin briefly considers vagueness as a possible reason for denying the decidability of legal questions in Dworkin, note 19.

¹⁷ Herewith a modified version of a well-known Aristotelian hypothetical. Aristotle, *De Interpretatione*, Book ix, trans. J. L. Ackrill (Oxford: University Press, 1963).

¹⁸ I thus think it mistaken to argue, as some have done, that the Aristotelian objection to determinately truth-valued future contingents involves a modal fallacy, viz., an illicit elision from " $\Box (\forall \supset \exists)$ " to " $\forall \supset \Box \exists$ ". See S. Cahn, *Fate, Logic and Time* (New Haven: Yale, 1967). Also A. N. Prior, "Three-Valued Logic and Future Contingents," *Philosophical Quarterly* 3 (1983): 317-26; A. N. Prior, *Time and Modality* (Oxford: Clarendon, 1957). It is not necessary for Aristotle (or Łukasiewicz, discussed *infra*, Part III.D, who endorses him) to argue to the necessity of a consequent from the necessity of the implication involving that consequent. He need only argue that the present conferral of truth-value upon statements involving the future must, to be intelligible, be warranted by some current state of the world; then that for

any such state to meet the appointed task it must unalterably determine that future state. Cf. similar observations in connection with “antirealism” about the past and about counterfactuals, *infra* Part 4.2.

¹⁹ John Lucas has famously – and controversially – argued that some of the very results derived by Gödel, some recursive relatives of which one three-valued logic considered *infra* – Kleene’s – is designed to accommodate, also supply a refutation of physical determinism. J.R. Lucas, *The Freedom of the Will* (Oxford: Clarendon, 1970), pp. 114f. Also J. R. Lucas, “Minds, Machines and Gödel,” *Philosophy* 36 (1961): 112-27; J. R. Lucas, “Mechanism: A Rejoinder,” *Philosophy* 45 (1970): 149-51. Contra: David Lewis, “Lucas Against Mechanism,” *Philosophy* 44 (1969): 231-33; David Lewis, “Lucas Against Mechanism II,” *Canadian Journal of Philosophy* 9 (1979): 373-76.

²⁰ Of course, David Lewis, *Counterfactuals*. (Oxford: Blackwell, 1973). Lewis is an unapologetic realist with respect to the possible worlds in terms of which counterfactualist semantics are understood; but he essays objections throughout his treatise. See also Dummett, note 48.

²¹ Cf. remarks on intuitionism at Part 4.2, *infra*.

²² John Broome apparently also would see none. See his “Incommensurable Values,” in Roger Crisp & Brad Hooker, eds., *Well-Being and Morality: Essays in Honour of James Griffin* (Oxford: University Press, 1999), pp. 21-38, reprinted in John Broome, *Ethics Out of Economics* (Cambridge: University Press, 1999), pp. 145-61, at 154: “I take the question of vagueness as a red herring when we are interested in the importance of incommensurability.” See also citations to Dworkin, note 19. “Fuzzy” sets and logics have, however, been found useful by some economists in tackling certain measurement problems. E.g., K. Basu, “Axioms for Fuzzy Measures of Inequality,” *Mathematical Social Science*, 14 (1987); E. Chiappero Martinetti, “A New Approach to Evaluation of Well-being and Poverty by Fuzzy Set Theory,” *Giornale degli Economisti*, 53 (1994); E. Ok, “Fuzzy Measurement of Income Inequality: A Class of Fuzzy Inequality Measures,” *Social Choice and Welfare*, 12 (1995); and other sources cited in A. K. Sen, *On Economic Inequality* (Oxford: Clarendon, 1973; “Expanded Edition,” 1997), pp. 121, 200.

²³ I find occasion to puzzle over the putative *sense* of this expression *infra*, Part 5, esp. note 57.

²⁴ Even treating “7/0” as simply “undefined” might seem, at least from one point of view, troublingly *ad hoc*. Assigning it the value “ ∞ ” would appear at least as plausible in view of the latter’s role as endpoint of the limit operator in such terms as “ $\lim (n \rightarrow \infty) 1/n$.” Since $1/\infty$, in effect, plays a role very much like 0, there is a certain principled symmetry in the notion that $1/0$ is ∞ .

²⁵ Please see note 29.

²⁶ Surely not that level-comparison entails (some manner of “incommensurable”) lifeplan-comparison. Why would, and how could, that be? How would such an entailment argument run? More on this at Part 5.

²⁷ Those more concerned with the elegant arithmetic or algebraic formulatability of the logic’s transformation rules might well prefer a set such as $\{1, \frac{1}{2}, 0\}$, in which case the effects of the classical sentential operators are readily characterized, e.g., as follows: $|\neg \forall| = 1 - |\forall|$; $|\forall \vee \exists| = \max |\forall|, |\exists|$; $|\forall \wedge \exists| = \min |\forall|, |\exists|$; etc. Such a system (or some analogue) is readily extended to the n -valued, even the infinitely many-valued, case. But such logics are not applicable to any circumstance that need concern us here.

²⁸ This is the “ $\max |\forall|, |\exists|$ ” option, note 33.

²⁹ This would entail a “ $\min |\forall|, |\exists|$ ” characterization, note 33.

³⁰ So argue Hilary Putnam, “Three-Valued Logic,” *Philosophical Studies* 8 (1957): 73-80, reprinted in *Philosophical Papers, Vol. 1: Mathematics, Matter and Method*, note 17, pp. 166-73; and Reichenbach, note 17.

³¹ See generally Siegfried Gottwald, *A Treatise on Many-Valued Logics* (Baldock: Research Studies Press, 2001), pp. 385-92 & *passim*.; Leonard Bolc & Piotr Borowik, *Many-Valued Logics I: Theoretical Foundations* (Berlin: Springer, 1992), pp. 63-77 & *passim*.; Susan Haack, *Deviant Logic, Fuzzy Logic* (Chicago: U. Chi., 1996), pp. 168-77. Also the venerable J. B. Rosser & A. R. Turquette, *Many-Valued Logics* (Amsterdam: North-Holland, 1952); and Nicholas Rescher, *Many-Valued Logic* (New York: McGraw Hill, 1969).

³² Please recall that it was necessary to resolve the question’s analogue in the bivalent case. Note 14.

³³ Jan Łukasiewicz, “O Logice Trówartościowej,” *Ruch Filozoficzny* 5 (1929): 170-71, translated into English as “On Three-Valued Logic,” in S. McCall, ed., *Polish Logic* (Oxford: University Press, 1967), pp. 16-18; Jan Łukasiewicz, “Philosophische Bemerkungen zu Mehrwertigen Systemen des Aussagenkalküls,” *Comptes Rendus Séances Société des Sciences et Lettres Varsovie*, cl. 3, 23 (1930): 51-77, translated into English as “Philosophical Remarks on Many-Valued Systems of Propositional Logic,” in McCall, *op. cit.*, pp. 40-65; Jan Łukasiewicz & Alfred Tarski, “Untersuchungen über den Aussagenkalkül,” *Comptes Rendus Séances Société des Sciences et Lettres Varsovie*, cl. III, 23 (1930): 30-50, reprinted in English as “Investigations into the Sentential Calculus” in *Logic, Semantics, Metamathematics*, note 19, pp. 38-59. Łukasiewicz also generalizes his Ł3 logic to the four- and many-valued cases, construing the former as a modal and the latter as a “probability logic.” His and other such systems with this interpretation are surveyed in Bolc & Borowik, note 37, Chapter 10. An extension of Ł3 to functionally complete axiomatization can be found in J. Śłupecki, “Der volle dreiwertige Aussagenkalkül,” *Comptes Rendus Séances Société des Sciences et Lettres Varsovie*, cl. III, 29 (1936): 9-11. Similar work in the many-valued case appears in B. Sobociński, “Axiomatization of Certain Many-Valued Systems of the Theory of Deduction,” *Roczniki prac naukowcy z zezeszenia asystentów Uniwrsytetu Józefa Pitsudskiego Warszawa* 1 (1936): 399-419.

³⁴ I call an “ $\forall \wedge \neg \forall$ ” conjunction a “firm” or “hard” contradiction, an “ $\forall \wedge \sim \forall$ ” or “ $\sim \forall \wedge \neg \forall$ ” conjunction a “soft” contradiction.

³⁵ D. Bočvar, “Ob odnom trechznačnom isčis lenii I ego primenii k analizaparadoksov klassičeskogo rašširenno go funkcional’nogo isčislenija,” *Matématčeskij Sbornik* 4 (1938): 287-308, translated into English as “On a Three-Valued Calculus and Its Application to the Analysis of the Paradoxes of the Classical Extended Functional Calculus,” *History & Philosophy of Logic* 2 (1939): 87-112; D. Bočvar, “K voprosu o neprotivorečivosti odnogo trechzanačnogo isčislenija,” *Matématčeskij Sbornik* 12 (1943): 353-69. Bočvar, a bit misleadingly in light of English connotation, characterized his as a “nonsense logic.” His work is carried on by a colleague with whom he coauthored a number of papers later in his career. See, e.g., V. K. Finn, *An Axiomatization of Some Propositional Calculii and Their Algebras* (Moscow: Vsiechšiznyi Institut Naučeskoi Informacii Akademii Nauk SSR, 1972). Other later work taking its cue from Bočvar includes S. Halldén, “The Logic of Nonsense,” *Uppsala Universitets arsskrift* 9 (1949): 132-46; L. Åqvist, “Reflections on the Logic of Nonsense,” *Theoria* 28 (1962): 138-58; K. Segerberg, “A Contribution to Nonsense Logic,” *Theoria* 31 (1965): 199-217; and K. Piróg-Rzepecka, *Systemy Nonsense-Logics* (Warszawa: Wrocław, 1977).

³⁶ T. J. Smiley, “Sense Without Denotation,” *Analysis* 20 (1960): 125-35. Work along similar lines, by a theoretical linguist, can be found in U. Blau, *Die dreiwertige Logik der Sprache: ihre Syntax, Semantik und Anwendung in der Sprachanalyse* (Berlin: de Gruyter, 1978).

³⁷ S. C. Kleene, “On Notation for Ordinal Numbers,” *Journal of Symbolic Logic*, 3 (1938): 150-55; S. C. Kleene, *Introduction to Metamathematics* (Amsterdam: North-Holland, 1952). Cf. Kurt Gödel, “Über formal unentscheidbare Sätze der Principia Mathematica und verwandter System, I,” *Monatschafte für*

Mathematik und Physik 38 (1931): 173-98, reprinted with English translation *en face* in Kurt Gödel, *Collected Works, Vol. I: Publications 1929-36*, Solomon Feferman et al., eds. (Oxford: University Press, 1986), pp. 145-95; Kurt Gödel, "Zum Entscheidungsproblem des logische Funktionenkalküls," *Monatshefte für Mathematik und Physik*, 40 (1933): 433-43, reprinted with English translation *en face* in *Collected Works, op. cit.*, pp. 307-36; Alonzo Church, "An Unsolvability Problem of Elementary Number Theory," *American Journal of Mathematics* 58 (1936): 345-63; Alonzo Church, "A Note on the Entscheidungsproblem," *Journal of Symbolic Logic* 1 (1936): 40-41. Generally Martin Davis, *Computability and Unsolvability* (New York: Dover, 1983).

³⁸ P. Woodruff, "Logic and Truth-Value Gaps," in K. Lambert, ed., *Philosophical Logic* (Dordrecht: Reidel, 1970), pp. 121-42.

³⁹ Bass C. van Fraassen, "Singular Terms, Truth-Value Gaps, and Free Logic," *Journal of Philosophy* 63 (1966): 481-95; Bass C. van Fraassen, "Presupposition, Implication and Self-Reference," *Journal of Philosophy* 65 (1968): 136-52; Bass C. van Fraassen, "Presuppositions, Supervaluations and Free Logic," in K. Lambert, ed., *The Logical Way of Doing Things* (New Haven: Yale, 1969), pp. 67-91. See also McDowell, "Truth Value Gaps," note 21, and Woodruff, note 44. The applicability of supervaluation to problems of vagueness is treated in Kit Fine, "Vagueness, Truth and Logic," *Synthese* 30 (1975): 265-300. Professor Broome briefly considers supervaluations in connection with vagueness in the piece cited at note 28, and in "Is Incommensurability Vagueness?," in Ruth Chang, ed., *Incommensurability, Incomparability and Practical Reason* (Cambridge: Harvard, 1998), pp. 67-89, reprinted in Broome, *Ethics Out of Economics*, note 28, pp. 123-44.

⁴⁰ See L. E. J. Brouwer, "Consciousness, Philosophy and Mathematics," *Proceedings of the Tenth International Congress of Philosophy* (Amsterdam: North-Holland, 1949), pp. 1235-49; Arend Heyting, *Intuitionism* (Amsterdam: North-Holland, 1966); M. A. E. Dummett, *Elements of Intuitionism* (Oxford: Clarendon, 1977).

⁴¹ Frege in effect raised long ago a potential pitfall standing in the way of this reinterpretation of classical negation; it appears to leave the content of "contrariness" or "contraindication" mysterious. Contrariness, that is, seems as though it might be parasitic upon simple contradiction. Gottlob Frege, "Die Verneinung. Eine logische Untersuchung," *Beiträge zur Philosophie des deutschen Idealismus* I (1918): 143-57, translated into English as "Negation," in Beaney, note 21, pp. 346-61. But we need not resolve this conundrum here. An excellent survey of efforts to do so (and more), by a linguist, is Laurence R. Horn, *A Natural History of Negation* (Chicago: CSLI, 2000).

⁴² Dummett, note 46. Also M. A. E. Dummett, *Truth and Other Enigmas* (London: Duckworth, 1978); M. A. E. Dummett, *The Logical Basis of Metaphysics* (London: Duckworth, 1991); M. A. E. Dummett, *The Seas of Language* (Oxford: Oxford, 1993); Crispin Wright, *Truth and Objectivity* (Cambridge: Harvard, 1992); Crispin Wright, *Realism, Meaning and Truth* (Oxford: Blackwell, 1993). Hilary Putnam in the past two decades also has shown sympathy, via his "internal realism," with antirealist views, as has Nelson Goodman via his "irrealism". Putnam's point of departure is the "new theory of reference" referenced at note 21, Goodman's the oft-unobserved indeterminacy of predicate expressions and the world-constitutive consequences of conceptualization. Quine, note 21, from whose "radical indeterminacy of translation" both Putnam and Goodman seem to have drawn far-reaching conclusions, has not been entirely comfortable with the extensions drawn by his erstwhile pupils. See, e.g., Hilary Putnam, *Philosophical Papers, Vol. 2: Mind, Language and Reality* (Cambridge: University Press, 1979); Hilary Putnam, *Reason, Truth and History* (Cambridge: University Press, 1981); Hilary Putnam, *Philosophical Papers, Vol. 3: Realism and Reason* (Cambridge: University Press, 1985); Hilary Putnam, *The Many Faces of Realism* (LaSalle: Open Court, 1988); Hilary Putnam, *Realism with a Human Face* (Cambridge: Harvard, 1992); Nelson Goodman, *Fact, Fiction and Forecast* (Cambridge: Harvard, 4th ed., 1983); Nelson Goodman, *Ways of World-Making* (Indianapolis: Hackett, 1983). We seem to have traveled some distance since the day when the formidable likes of Ramsey characterized intuitionists and finitists as posing a "Bolshevik menace" to logic and mathematics. See Ramsey, note 18, p. 219. Dworkin, note 19, cites Dummett early in his discussion of bivalence and the "no right answer" thesis in connection with "hard

cases” at law. He also takes note of the possible distinction, albeit in different terminology, between negation and contraindication.

⁴³ The “Kantian” argument would take the form: “Language-understanding and –meaning are possible only because people can be corrected in their usages, and they are corrected by ascription of the forthrightly normative words “true” and “false,” which amount to ascriptions of rightness or wrongness to assertions by reference to observable states of the world. Haack, note 37, pp. 105-08, argues that it is no more difficult to account for the meaningfulness of sentences understood in terms of (perhaps sometimes unascertainable or even non-existent) truth-conditions than it is to account for that of sentences understood in terms of assertibility-conditions. (An Ockamite reply might be that Haack has assumed the burden of proof to lie on the wrong side of the divide, given that, in order to explain language-acquisition, language-understanding and language-meaning, no conception of truth beyond assertibility is necessary.) Herewith the tip of a vast argumentative iceberg that can safely be passed over here. Cf. the polemics of Dummett and Putnam, note 48, in particular, which often engage (hostile) “realists” by name; also McDowell, *Meaning, Knowledge and Reality*, note 21, pp. 395-365; and J. McDowell, “Truth Conditions, Bivalence and Verification,” in G. Evans & J. McDowell, eds., *Truth and Meaning: Essays in Semantics* (Oxford: Clarendon, 1976) pp. 138-61, reprinted in McDowell, *Meaning, Knowledge and Reality*, note 21, pp. 3-28, who is sophisticatedly skeptical of antirealist and cognate claims; and Simon Blackburn, *Essays in Quasi-Realism* (Oxford: University Press, 1993), same. Even those who disagree with Dummett’s arguments about sentential meaning and understanding can, and often do, concede the possibility of truth-valueless meaningful statements, and that intuitionist logic might usefully model such phenomena.

⁴⁴ See in particular Dummett’s writing on McTaggart and “The Reality of the Past” in his *Truth and Other Enigmas*, note 48. Also his essay “Truth,” in the same collection: Example of person who dies young without ever having found occasion to manifest bravery or cowardice, with consequence that it would seem peculiar to hold it determinately true or false that this person was “brave.”

⁴⁵ A. Heyting, “Die Formalen Regeln der intuitionistischen Logik,” *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin, Physikal.-Mathemat. Klasse II* (1930): 42-56. Also Heyting, note 46; and I. Johanssen, “Der Minimalkalkül, ein Reduzierte Intuitionistische Formalismus,” *Compositio Mathematica* 4 (1936): 119-36, which drops Heyting’s tenth axiom but does not affect the results derived here. See generally M. C. Fitting, *Proof Methods for Modal and Intuitionistic Logics* (Dordrecht: Reidel 1983); and Bolc & Borowik, note 37, Chapter 5. It should be noted in passing that it has been proved that there does not exist any single many-valued logic whose set of tautologies is identical to the set of tautologies derivable in axiomatized intuitionist logic. See Kurt Gödel, “Zum intuitionistische Aussagenkalkül,” *Anzeiger Akademik der Wissenschaften Wien, Math.-naturwisse. Klass* 69 (1932): 65-66, reprinted with English translation *en face* in *Collected Works*, note 43, pp. 223-25; and S. Jaśkowski, “Recherches sur le système de la logique intuitioniste,” *Actes du Congrès Internationale de Philosophie Scientifique*, 6 (1936): 58-61.

⁴⁶ Łukasiewicz also gave an explicitly modal reading to his four-valued logic in Jan Łukasiewicz, “Philosophische Bemerkungen,” note 39. See also Jan Łukasiewicz, “A System of Modal Logic,” *Journal of Computing Systems* 1 (1953): 111-49; and Jan Łukasiewicz, *Aristotle’s Syllogistic* (Oxford: University Press, 1957).

⁴⁷ Significantly, modal logics are sometimes characterized as “contextual” logics, “context” being understood in distinctly *epistemic* terms. (The “context” in question is that in which the reasoner, in all of her epistemic finitude, finds herself.)

⁴⁸ S4 is the modal logic resulting from addition of \Box -elimination (i.e., $\Box\forall \supset \forall$) and \Box -repetition (i.e., $\Box\forall \supset \Box\Box\forall$) to the “basic contextual logic” known as “■”. The latter is simply standard first-order prepositional logic with the addition of the “ \Box ” and “ \Diamond ” operators and their rules (e.g., $\Box\neg \forall \supset \neg\Diamond\forall$, $\Diamond\forall \supset \neg\Box\neg\forall$, etc.). See generally Fitting, note 51. Also R. B. Marcus, “Modalities and Intensional Logics,” *Synthese* 13 (1961): 303-22, reprinted in her *Modalities: Philosophical Essays* (Oxford: University Press, 1993), pp. 3-30; C. I. Lewis, “Alternative Systems of Logic,” *Monist* 42 (1932): 481-507.

⁴⁹ J. C. C. McKinsey & Alfred Tarski, “Some Theorems About the Sentential Calculi of Lewis and Heyting,” *Journal of Symbolic Logic* 13 (1948): 1-15. Also Gödel, note 51; Saul Kripke, “Semantical Analysis of Intuitionistic Logic I,” in J. Crossley & M. A. E. Dummett, eds., *Formal Systems and Recursive Functions* (Amsterdam: North-Holland, 1963), pp. 92-129; Fitting, note 51.

⁵⁰ J. Dugundji, “Note on a Property of Matrices for Lewis’ and Langford’s Calculi of Propositions,” *Journal of Symbolic Logic* 5 (1940): 150-51.

⁵¹ Rawls appears to hold that lifeplan-satisfactions are incommensurable because lifeplans – or at any rate the “conceptions of the good” that are expressed in lifeplans – are incommensurable. See John Rawls, “Social Unity and Primary Goods,” in Amartya Sen & Bernard Williams, *Utilitarianism and Beyond* (Cambridge: University Press, 1982), pp. 159-85, at 161, reprinted in John Rawls, *Collected Papers* (Cambridge: Harvard, 1999), pp. 359-87, at 361. But it is difficult to understand what “mensurability” or “incommensurability” even might mean, let alone whether they might be ascribable, in connection with such non-magnitudinal entities as “lifeplans” or “conceptions of the good”. (It is thus difficult to understand what those words might connote in connection not only with *interpersonal* lifeplan comparabilities (premise (3)), but also in connection with *intrapersonal primary goods-to-lifeplan* comparabilities (premise (2)). The notion of “incommensurable -,” “mensurable -,” or “measurable lifeplans” or “— conceptions of the good” is reminiscent of such would-be notions as “discomfited polygons” and “exponentiated hairsprays”.) Whereas the notion of “degree or level of lifeplan-satisfaction” – a numerical entity – would seem, if operationally meaningful intrapersonally (premise (2)), to be operationally meaningful interpersonally (premise (3)) as well.

⁵² There are extended Rawlsian passages that provide independent ground for ascribing some such (complex) intuition to Rawls. It strikes me as particularly noteworthy that Rawls fuses epistemic and moral-political considerations in discussing primary goods. See, e.g., note 14; and “The Priority of Right and Ideas of the Good,” *Philosophy & Public Affairs* 17 (1988): 251-76, reprinted in Rawls, *Collected Papers*, note 57, pp. 449-72, at 455: “What is crucial is that in introducing [additional primary goods] we recognize the limits of the political and the practicable: First, we must stay within the limits of justice as fairness as a political conception of justice that can serve as the focus of an overlapping consensus; and second, we must respect the constraints of simplicity and availability of information to which any practicable political conception (as opposed to comprehensive moral doctrine) is subject.” See also the later reference, on the same page, to “the *practical* nature of primary goods” (emphasis supplied). And page 456: “Nor does [justice as fairness] try to *estimate* the extent to which individuals *succeed* in advancing their way of life – their overall scheme of final ends” (emphasis again supplied). And Rawls, note 1, p. 94: “[Justice as fairness] does not look behind the use which persons make of [primary goods] in order to *measure*, much less to maximize, the satisfactions they achieve” (emphasis again supplied). And see Risse, note 2, pp. 10-13, where it is observed that if a “magic machine” able to “look into peoples’ minds” were to appear, a “Pandora’s Box” would be opened and the morally relevant informational “circumstances of politics” radically altered in such wise as to force us to reconsider, even if not to change, our politico-theoretic commitments.

⁵³ Thus by alternatively characterizing the group of transformations that can be applied to individual satisfaction-indices without affecting a composite social ordering, we can represent the analytical distinctions of intra- and interpersonal measures of satisfaction. E.g., the group of transformations represented by $\forall \alpha, \beta \in \mathbb{R}^+ \wedge \beta > 0, \forall u, v \in \mathbb{N}: uO^*v \equiv (\alpha_1 + \beta_1 u_1, \alpha_2 + \beta_2 u_2, \dots, \alpha_n + \beta_n u_n) O^* (\alpha_1 + \beta_1 v_1, \alpha_2 + \beta_2 v_2, \dots, \alpha_n + \beta_n v_n)$, where \mathbb{N} is a Euclidian space indexed by the names of n persons, u and v are individual satisfaction-functions likewise indexed, and O^* is a social ordering over \mathbb{N} – i.e., the group of all positive affine transformations of the ordering – preserves intrapersonally cardinal but interpersonally incomparable (“CN,” or “CNC”) satisfaction-information across individuals 1 through n . And the group represented by $\forall \Phi \in \{\text{strictly increasing numeric functions}\}, \forall u, v \in \mathbb{N}: uO^*v \equiv (\Phi(u_1), \Phi(u_2), \dots, \Phi(u_n)) O^* (\Phi(v_1), \Phi(v_2), \dots, \Phi(v_n))$ – i.e., all strictly increasing transformations applied one at a time to all satisfaction-indices – preserves intrapersonal ordinal but interpersonally full-comparable (“OC,” “OFC,” or “co-ordinal”) satisfaction-information across individuals 1 through n . Thorough summaries of means by

which mathematically to characterize various combinations of various forms of intrapersonal and interpersonal measurability are Claude d'Aspremont, "Axioms for Social Welfare Ordering," in Leonid Hurwicz et al., eds., *Social Goals and Social Organization: Essays in Memory of Elisha Pazner* (Cambridge: University Press, 1985), pp. 19-76, particularly 42f.; and Roemer, note 1, pp. 16f. The method appears to have originated with Professor Sen. E.g., Sen, note 28; A. K. Sen, *Collective Choice and Social Welfare* (Amsterdam: North-Holland, 1970); and A. K. Sen, "On Weights and Measures: Informational Constraints in Social Welfare Analysis," *Econometrica*, 45 (1979): 1539-72. A very useful collection of essays treating of the technical and philosophic concerns surrounding intrapersonal and interpersonal measurement is Jon Elster & John E. Roemer, eds., *Interpersonal Comparisons of Well-Being* (Cambridge: University Press, 1991). For skepticism about the fundamentality of putative differences between separate persons and separate temporal "phases" of individual persons, see D. Parfit, *Reasons and Persons* (Oxford: University Press, 1984); and John Broome, *Weighing Goods* (Cambridge: University Press, 1991). I do not wish to be taken to endorse the latter skepticism. The skepticism that I express is only as to the political, operational or pragmatic significance of the formal distinction between measurements of intrapersonal, and of interpersonal, "subjective" phenomena, not the metaphysical distinction between persons and "person-phases." An early work assimilating persons and time-dated person-phases simply for theoretic-cum-expository purposes is Maurice Allais, *Economie et Intérêt* (Paris: Gallimard, 1947). The latter in effect extends to persons Hicks's method of temporally individuating commodities in *Value and Capital* (Oxford: Clarendon, 1931), and of course anticipates Arrow's and Debreu's method of individuating state-claims, respectively, in "Le Rôle de Valeurs Boursières par la Répartition la Meilleure des Risques," *Econometrie, Colloques Internationaux du Centre National de la Recherche Scientifique* 40 (1953): 41-80; and *Theory of Value* (New Haven: Cowles, 1954), Chapter 7. (Allais also, incidentally, presented a paper along the same lines as Arrow's at the same colloquium; it follows Arrow's in the cited journal of proceedings.)

⁵⁴ Please see note 59 for the group-theoretic meanings of these expressions.

⁵⁵ Risse, note 2, argues that Rawls cannot renounce (1) because primary goods would then "lose their ability to guide 'interpersonal comparisons required for workable political principles,'" p. 16, quoting Rawls, *Restatement*, note 14, p. 60. Neither can Rawls renounce (3), holds Risse, because "insisting on utility comparability (which is what the rejection of that condition amounts to) would unacceptably undermine the subjective element in any plausible conception of happiness," p. 16. Risse concludes that Rawls must renounce (2), even though this "comes at the expense of severing primary goods from happiness," p. 17. Roemer's derived contradiction, Risse observes, thus "mak[es] clear how far-reaching [Rawls's contractarian] commitment is," p. 22. The possible employment of non-classical logics, I think, gives the lie to Risse's parenthetical (that "insisting on utility comparability" is what rejection of premise (3) "amounts to"). It also casts doubt upon his equating "meaningful[ness]" with "being true or false, rather than pointless," at the top of p. 15. And employing such a logic in revising both (2) and (3) enables us both to retain all of Rawls's premises – albeit in "softened" form – and to avoid the curious tension that seems to me to afflict not only Rawls's premises (2) and (3) considered jointly, but also Risse's desire to retain "the subjective element in any plausible conception of happiness" while at the same time "severing" it completely (rather than only "softly," as I do here) from the primary goods which Rawlsians – reasonably, it seems to me – take to conduce, in some moral-theoretic and pragmatic sense not susceptible of precise and reliable measurement, to plausible happiness. (Note again, in this connection, e.g., the Rawlsian passage quoted by Roemer, note 1, at p. 166, and in note 7 supra.) The final three paragraphs of this paper, I believe, convincingly indicate how the formalizations proposed in the remaining pages not only spare Rawls the Roemerian inconsistency, but also lend themselves to a more plausible, more intuitively complete account of primary goods and their relation to lifeplan-satisfaction than either Rawls or Risse has quite mustered. Each of them, we might say, is ambivalent about subjectivity, while Risse (at least) is unnecessarily bivalent about logic; solution lies in univalence about subjectivity and trivalence (or its functional equivalent) about logic.

⁵⁶ There might, of course, be commensurability problems between non-identical bundles – that is, there might be an indexing problem where one primary goods vector includes more of component x_1 and less of component x_2 than does another primary goods vector – but this (considerable) problem need not

concern us here. I treat it at length in my forthcoming “The Grammar of Distribution: A Metatheory of Justice,” and “Market-Able Justice: Endowment Equality, Value-Grounded Departures from Equality, and Social Exchange.” For more on the matter, see Richard Arneson, “Primary Goods Reconsidered,” *NOUS* 24 (1990): 429-57; Richard Arneson, “Rawls versus Utilitarianism in the Light of *Political Liberalism*,” in V. Davian & C. Wolf, eds., *The Idea of Political Liberalism: Essays on Rawls* (New York: Rowman & Littlefield, 2000). Also Roemer, note 1, and Risse, note 2. More generally, Broome, note 28, and Chang, note 45. All of these contributions can be taken, in effect, for close critiques of premise (1), while the present paper is primarily a critique of premises (2) and (3).

⁵⁷ Please see note 10.

⁵⁸ Rawls, *Restatement*, note 14, pp. 60-61.

⁵⁹ Professor Roemer has suggested, in conversation, that reformalizing the Rawlsian premises pursuant to a non-classical logic might constitute too radical or far-reaching a maneuver, leaving “everything up for grabs.” I don’t think that we need harbor such worries – worries which, incidentally, seem often to bedevil gifted and scrupulous mathematicians when confronted by non-classical logics. (Cf. Ramsey on intuitionism’s “Bolshevik menace,” note 48.) As I hope to have indicated in the discussion throughout Parts III and IV, the non-classical logics considered in the present paper depart from classical logic only in their formal recognitions of meaningful propositions of somehow (metaphysically, epistemically, politically-pragmatically, etc.) indeterminate truth-value, their consequent assignments of mildly divergent truth-functional meanings to the classical sentential connectives, and/or their sacrifice of certain classical tautologies which, in view of the possibility of the aforementioned meaningful propositions of indeterminate truth-value, probably ought not to be maintained in any event. Those logics do not seem to pose any threat of unpleasant surprise derivable from Rawlsian premises interpreted in light of the countenanced indeterminacies, nor do they seem to threaten any other surprise likely to be found unpleasant by Rawlsians. (As I noted earlier, a full-blown metalogical proof might yield a bit more formal confidence on this point – at least as regards possible contradictions – but the intuitive arguments ought to suffice.) All that they do is to offer a means of giving formal expression to what looks to be Rawls’s actual three-fold understanding of primary goods and their relation to lifeplan-satisfaction among persons, and to do so in a manner that both a) preserves such relations of logical consequence as we should wish to retain and b) avoids such paradox as that to which the aforementioned Rawlsian understanding, if mis-formalized bivalently, would conduce. Professor Roemer also has suggested, again in conversation, that a more critical inconsistency of which Rawls is guilty comes with his attempt to integrate considerations of responsibility into his theory of justice via his accounts both of primary goods and of the “veil of ignorance.” With this claim I am in considerable sympathy, and see no way out for Rawls via any plausible formalization or reformalization. See generally cites to Arneson’s, Roemer’s and my own work at note 62; also John Roemer, “Three Egalitarian Views and American Law,” *Law and Philosophy*, xx (2001): 433-60; T. M. Scanlon, *What We Owe to Each Other* (Cambridge: Harvard, 1998); and Risse, note 2, as well as Mathias Risse, “Rawls and Responsibility” (draft, 2 March 2003, on file with the author), who is at pains to defend Rawls against this inconsistency charge. While Professor Risse’s emphasis on the political nature of Rawlsian justice constitutes an important corrective to much criticism of Rawls, and provides what I believe to be an additional rationale for reformalizing the Rawlsian premises as I have done in this paper (rather than for simply dropping premise (2), as Professor Risse proposes), I do not believe that it spares Rawls the difficulties over responsibility from which Risse hopes to extricate him. But that matter is reserved for Hockett, note 62.