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ABSTRACT

The interplay between resonant solar neutrino oscillations and neutrino oscillations in laboratory experiments is investigated in a 3 generation model. Due to the assumed hierarchy of neutrino masses, together with our choice of a convenient parameterization of the 3 generation mixing matrix, we can derive a simple analytic formula which reduces the solar neutrino problem to an effective 2 generation problem. The reduction makes it apparent that the allowed range of mixing and mass parameters crucially depend on whether the survival probability of solar neutrinos S satisfies $S \gtrsim 1/3$ or not. The formulae for probabilities of laboratory neutrino oscillations are also greatly simplified. We argue that a combination of the observed solar neutrino depletion and data obtained from reactor experiments seems to rule out some range of neutrino masses. If a sizeable $\nu_\mu \to \nu_e$ oscillation is observed at accelerators, as suggested at this Workshop, it severely restricts the range of 2 mixing angles.

INTRODUCTION

The main purpose of this talk is to clarify in the 3 generation model of leptons the interplay between the probabilities of neutrino "matter oscillations" within the solar interior and neutrino "vacuum oscillations" on the earth, i.e., in accelerator or nuclear reactor experiments. We assume throughout this talk that the resonant enhancement of neutrino oscillations in the presence of matter, advocated by Mikheyev, Smirnov[1] and Wolfenstein[2] is the source of the depletion of solar ν_e flux.[3] Define a "survival probability" S by

$$S \equiv \text{(detected solar neutrino flux) / (prediction of solar model).} \qquad (1)$$

In contrast to the case of a 2 generation model,[1,2,3,5,6,7] in a realistic model with 3 generations, as we will see, the survival probability S around 1/3 will not necessarily lead to very small neutrino mass-squared differences (like $\Delta m^2 \lesssim 10^{-4}\mathrm{eV}^2$),[5] thereby encouraging laboratory neutrino oscillation experiments. In the 3 generation model, we at least need to diagonalize a 3×3 matrix to analyze the solar neutrino problem. Furthermore, the formulas for probabilities of $\nu_\alpha \to \nu_\beta$ $(\alpha, \beta = e, \mu, \tau)$ oscillations in laboratory experiments, denoted by $P_{\alpha\to\beta}$, are in general complicated functions of neutrino masses and generation mixing angles (including even a CP violating phase), even though the experimental bounds on such parameters are usually given as if there are only 2 generations.[8]

In our analysis, the following very reasonable hierarchical structure of mass scales will be assumed:[9]

$$A,\ \Delta m_{21}{}^2 \ll \Delta m_{31}{}^2 \simeq \Delta m_{32}{}^2, \tag{2}$$

where $\Delta m_{ij}{}^2 = m_i^2 - m_j^2$ $(i, j = 1, 2, 3)$ with m_i being the mass of ν_i, a mass eigenstate in the vacuum, and $A = 2\sqrt{2} G_F\, N_e\, k$ (N_e: electron density, k: neutrino momentum) is the famous "matter effect."[1,2,5] The hierarchical structure in Eq. (2) will not only simplify the expressions for $P_{\alpha\to\beta}$, but also will enable us to derive a very simple analytic formula, which reduces the solar neutrino problem in 3 generation model to the one in "effective" 2 generation system for arbitrary mixing angles. In this way, existing results in the 2 generation model obtained by sophisticated procedures[6,7] can be utilized to analyze the solar neutrino problem in the 3 generation model.

We also propose to take advantage of using a parametrization of generation mixing matrix; weak eigenstates ν_α $(\alpha = e, \mu, \tau)$ are related to ν_i by $\nu_\alpha = V_{\alpha i}\nu_i$, where[10]

$$V = \begin{pmatrix} c_1c_3 & s_1c_3 & s_3e^{-i\delta} \\ -s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\ s_1s_2 - c_1c_2s_3e^{i\delta} & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3 \end{pmatrix} \tag{3}$$

and $c_i = \cos\theta_i$, $s_i = \sin\theta_i$ $(i = 1, 2, 3)$. The reasons why this parametrization is so well suited for our discussion are two-fold; (i) among matrix elements of V, only V_{ei} $(i = 1, 2, 3)$ are responsible for the parameter S.[11,12] This is because only $\nu_e \to \nu_e$ is relevant for S and the matter effects proportional to A are invariant under any $\nu_\mu \leftrightarrow \nu_\tau$ U(2) rotations.[13] (ii) Eq. (3) is most convenient to describe $P_{\alpha\to\beta}$. In particular, when $\Delta m_{21}{}^2 \ll 1\ \mathrm{eV}^2$ (by 1 eV^2 we just mean a typical Δm^2 scale, which laboratory experiments are sensitive to) as is inspired by the solar neutrino puzzle, these probabilities are approximated by functions of $V_{\alpha 3}$ $(\alpha = e,\ \mu,\ \tau)$ and $\Delta m_{31}{}^2$ only, since these functions should be invariant under $\nu_1 \leftrightarrow \nu_2$ U(2) rotations. The parametrization will also make it possible to clarify what generation mixings are really responsible for some given oscillation.

This talk is based upon the work with W.J. Marciano. The details including the argument on the possible generalization to higher generation models, will be reported elsewhere.[14] The solar neutrino problem in the 3 generation model has also been discussed by C.W. Kim in this Workshop (see also Refs. 11 and 12), relying mainly on numerical computations.

SOLAR NEUTRINO PROBLEM

First, let us focus our attention on the solar neutrino problem. The key idea here is to consider the (time) evolution of vacuum mass eigenstates ν_i. Note that the relevant "Hamiltonian" matrix, which governs the evolution, is dominated by the (3,3) matrix element $\Delta m_{31}{}^2/2k$. Let us emphasize that this statement is true for arbitrary mixing angles, as long as (2) is maintained, since we are working in the base of ν_i, instead of ν_α. Thus, the ν_3 state turns out to be decoupled from the remaining two states, linear combinations of ν_1 and ν_2. Namely, when $P_{e\to e}$ is averaged over the detection points and integrated over the neutrino energy spectrum, it yields a relation[14]

$$S = \cos^4\theta_3 \cdot S_{\text{eff}} + \sin^4\theta_3, \tag{4}$$

where the second term on the r.h.s. is nothing but the contribution of a "decoupled" ν_3, and the factor $\cos^4\theta_3$ reflects the incompleteness of unitarity in the subspace of ν_1 and ν_2, ie., $\sum_{i=1,2}|V_{ei}|^2 = \cos^2\theta_3$. S_{eff} is defined as a survival probability in the effective 2 generation system, whose (time) evolution is governed by $H^{(2)}$,

$$H^{(2)} = \frac{1}{4k}\begin{pmatrix} A_{\text{eff}} - \Delta m_{21}{}^2\cos 2\theta_1 & \Delta m_{21}{}^2\sin 2\theta_1 \\ \Delta m_{21}{}^2\sin 2\theta_1 & -A_{\text{eff}} + \Delta m_{21}{}^2\cos 2\theta_1 \end{pmatrix}, \tag{5}$$

where $A_{\text{eff}} = \cos^2\theta_3 \cdot A$. Thus, the possible contours in $\left(\Delta m_{21}{}^2,\ \theta_1\right)$ space can be translated from known 2 generation results. The net effects of the 3rd generation's presence are re-scalings of parameters, $S \to S_{\text{eff}}$ and $A \to A_{\text{eff}}$. For adiabatic solutions, the change $A \to A_{\text{eff}}$ is equivalent to $\Delta m_{21}{}^2 \to \Delta m_{\text{eff}}{}^2 = \Delta m_{21}{}^2/\cos^2\theta_3$, and the contour in a 2 generation model,[6,7] corresponding to S_{eff} (not S itself), can be immediately re-interpreted as the contour in the plane of $\left(\Delta m_{\text{eff}}{}^2,\ \theta_1\right)$. As for the non-adiabatic case, such a re-interpretation is not so easy, in general. However, if we assume $N_e \propto \exp\left(-x/R_s\right)$, R_s: scale height, as in Ref. 6, the re-scaling of mass-squared is not needed for the non-adiabatic case, since $-d\left(\ln N_e\right)/dx = 1/R_s$ is position independent.

To get a rough idea, in Figs. 1 and 2 we have shown the iso-S (survival probability) contours, in 2 generation[6] and 3 generation models, respectively. For illustrative purposes, S has been calculated for fixed neutrino momentum k and we have taken $\sin^2\theta_3 = 1/3$ in Fig. 2. We learn from these figures that a plausible value of S around 1/3 has quite different consequences in two kinds of models. Namely, in the 3 generation model the order of magnitude of $\Delta m_{21}{}^2$ is very sensitive to S; if $S < 1/3$, $\Delta m_{21}{}^2 \lesssim 2/3 \cdot 2\sqrt{2}kG_F N_C$, where the factor 2/3 is due to the re-scaling of adiabatic solution and N_C is the electron density at the center of the sun, while $S \gtrsim 1/3$ allows a solution with large $\Delta m_{21}{}^2$. This should be compared to the result in the 2 generation model, where $S \approx 1/3$ always requires very small Δm^2.[5,6,7] Such a qualitative difference of results in the two kinds of models just reflects the fact that a "large" mass-squared difference means the irrelevance of matter effect and is possible only if vacuum oscillation is by itself capable of explaining

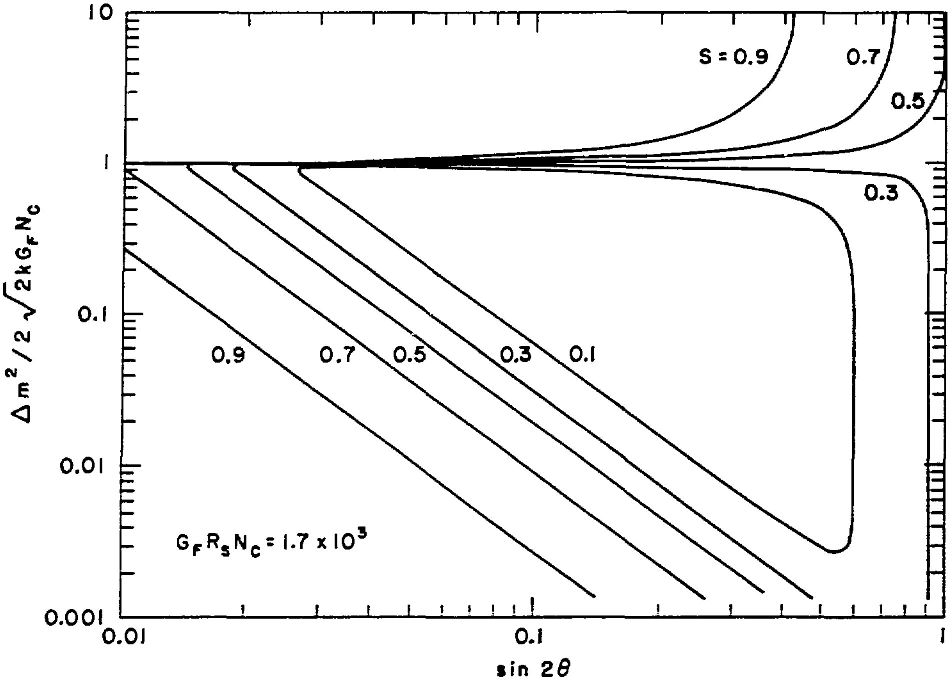

Fig. 1: The iso-S (survival probability) contour plot in the $(\Delta m^2/2\sqrt{2}kG_FN_C, \sin 2\theta)$ plane derived in Ref. 6 (we have added a contour for $S = 0.5$), in the 2 generation model. N_C is the electron number density at the center of sun, and we have assumed an exponential density distribution, $N_e \propto \exp(-x/R_s)$, with $G_FR_sN_C = 1.7 \times 10^3$.

the depression of solar neutrino flux. Let us remember that vacuum oscillation implies $S \geq 1/N_g$ (N_g: the number of generations).

To be more quantitative, we take 2.1 SNU[15] as the detected solar neutrino flux for ^{37}Cl experiment, and take two typical predictions of solar models: (i)[16] $S = 2.1/5.9 > 1/3$ and (ii)[17] $S = 2.1/7.5 < 1/3$. We also rely on the sophisticated procedure by Parke and Walker[7] to get possible contours. In accordance with the above argument, we find that in case (ii) we have an upper bound: $\Delta m_{21}{}^2 \lesssim 1 \cdot 10^{-4} \cos^2\theta_3$ eV^2 ($S_{\text{eff}} \lesssim 0.39$ from Eq. (4)), while in case (i), S_{eff} can exceed 1/2 ($S_{\text{eff}} \lesssim 0.55$), and $\Delta m_{21}{}^2$ can be arbitrarily large provided θ_1 and θ_3 satisfy $S = 2.1/5.9 = \cos^4\theta_3 \left(\cos^4\theta_1 + \sin^4\theta_1\right) + \sin^4\theta_3$, which is nothing but the relation describing vacuum oscillations and gives $0.89 \lesssim \sin^2\theta_1$, $0.21 \lesssim \sin^2 2\theta_3 \lesssim 0.46$.

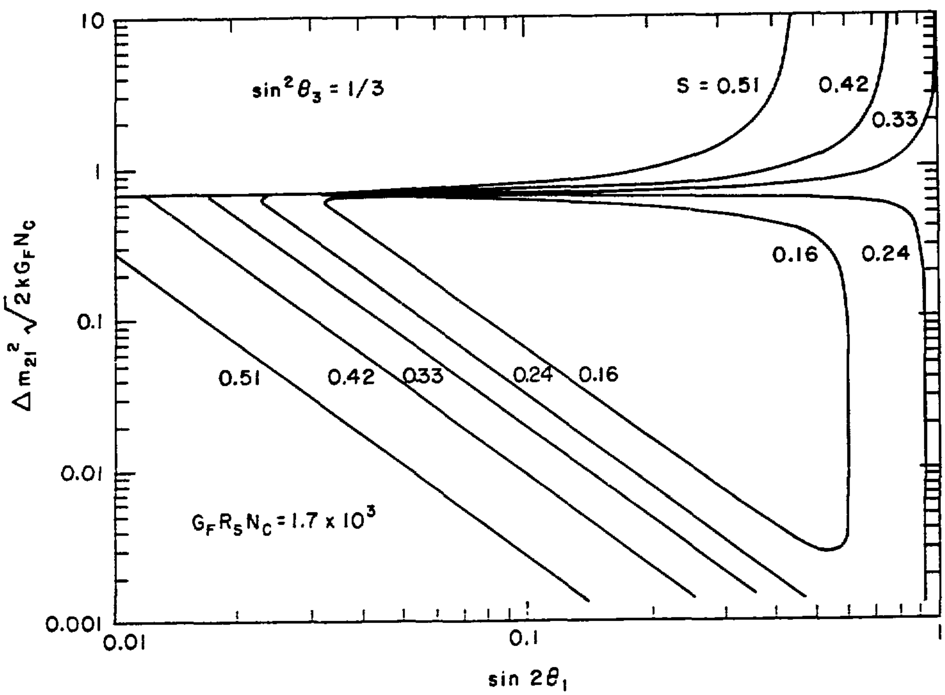

Fig. 2: The iso-S contour plot in the $(\Delta m_{21}^2/2\sqrt{2}kG_FN_C,\ \sin 2\theta_1)$ plane in the 3 generation model. We have taken the same value for $G_FR_sN_C$ as in Fig. 1, and for simplicity $\sin^2\theta_3$ has been fixed to be $1/3$.

LABORATORY NEUTRINO OSCILLATION EXPERIMENTS (A=0)

Now let us turn to a discussion of neutrino oscillations in accelerator or reactor experiments. From our study of the solar neutrino problem we have learned that the magnitude of Δm_{21}^2 crucially depends on the values of S (and θ_3). So we will consider two possible scenarios: (a) $\Delta m_{21}^2 \ll 1$ eV^2 and (b) $\Delta m_{21}^2 \gtrsim 1$eV^2, where we have chosen 1eV^2 to mean a typical Δm^2 studied in laboratory experiments and $\Delta m_{21}^2 \ll \Delta m_{31}^2$ is assumed.

We will first discuss case (a), which is required in the scenario (ii) solution to the solar neutrino problem. Since ν_1 and ν_2 can be treated as almost degenerate states, in this case, expressions for oscillation probabilities depend only on $V_{\alpha 3}$ and Δm_{31}^2:

$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4|V_{\alpha 3}|^2\left(\delta_{\alpha\beta} - |V_{\beta 3}|^2\right)\sin^2\left(\frac{\Delta m_{31}^2}{4k}x\right) \tag{6}$$

and our parametrization simplifies the expressions, e.g.,

$$P_{\mu\to e} = \sin^2\theta_2 \sin^2 2\theta_3 \sin^2\left(\frac{\Delta m_{31}^2}{4k}x\right), \tag{7a}$$

$$P_{\mu\to\tau} = \sin^2 2\theta_2 \cos^4\theta_3 \sin^2\left(\frac{\Delta m_{31}{}^2}{4k}x\right), \tag{7b}$$

$$P_{e\to e} = 1 - \sin^2 2\theta_3 \sin^2\left(\frac{\Delta m_{31}{}^2}{4k}x\right), \tag{7c}$$

where x is the distance from the neutrino source to the detector. Since only $\Delta m_{31}{}^2$ is responsible for the oscillations, ν_α can oscillate only through their couplings to ν_3. Therefore, these probabilities behave very differently from corresponding 2 generation results, although Eq. (7) mimics the form of 2 generation results. In fact, in the "2 generation limit", which is realized when θ_2 and θ_3 are very small, we find $P_{\mu\to e}$ is greatly suppressed;

$$P_{\mu\to e} \simeq \sin^2\theta_3 \cdot P_{\mu\to\tau}. \tag{8}$$

For illustrative purposes, we take values suggested by quark generation mixings, $\sin\theta_2 \simeq 0.05$, $\sin\theta_3 \simeq 0.01$, which yield for averaged probabilities $\bar{P}_{\mu\to e} \simeq 5\cdot 10^{-7}$ and $\bar{P}_{\mu\to\tau} \simeq 5\cdot 10^{-3}$. The result implies the relative importance of $\nu_\mu \to \nu_\tau$ experiment.

Although the oscillation probabilities in Eq. (7) depend only on θ_2 and θ_3, scenario (a) has no immediate contradiction with all existing experimental upper bounds.[8] In fact, if $\Delta m_{31}{}^2$ is sufficiently small (say $\lesssim 0.1\mathrm{eV}^2$) the bounds on θ_2 and θ_3 are not very restrictive. We should also note that bounds on $(\Delta m_{31}{}^2,\ \theta_2,\ \theta_3)$ derived from laboratory experiments will be consistent with the observed solar neutrino depletion, as long as $\sin^4\theta_3 \leq S$ is satisfied. Once θ_3 is fixed, e.g., by reactor $\nu_e \to \nu_e$ experiments (see (7.c)), an observed S will be easily translated to $S_{\rm eff}$ through (4), which in turn determines the contour in $(\Delta m_{21}{}^2,\ \theta_1)$ space. But, whether $S_{\rm eff} > 1/2$ or not, there always exists a solution with small $(\ll 1\ \mathrm{eV}^2)$ $\Delta m_{21}{}^2$, which is needed to be consistent with our definition of case (a).

Next, we will study another scenario (b), which can happen only if $S \gtrsim 1/3$ (as in the case (i)) for some range of θ_3. In this case the same phenomenon as in solar neutrino oscillation occurs; after the oscillatory terms associated with $\Delta m_{31}{}^2$ are averaged $(\Delta m_{31}{}^2 \gg 1\ \mathrm{eV}^2)$, the contribution of ν_3 decouples from those of the remaining two neutrinos, i.e.,

$$P_{\alpha\to\beta} = \left|\sum_{j=1,2} V_{\alpha j}V_{\beta j}^* \cdot \exp\left(-i\frac{m_j^2}{2k}x\right)\right|^2 + |V_{\alpha 3}V_{\beta 3}^*|^2. \tag{9}$$

In particular, the probability of $\nu_e \to \nu_e$ has exactly the same expression as Eq. (4),

$$P_{e\to e} = \cos^4\theta_3 P_{\rm eff} + \sin^4\theta_3, \tag{10}$$

where $P_{\rm eff}$ is the probability in an effective 2 generation system,

$$P_{\rm eff} = 1 - \sin^2 2\theta_1 \sin^2\left(\frac{\Delta m_{21}{}^2}{4k}x\right). \tag{11}$$

The other oscillation probabilities are rather complicated functions of all angles, including even δ, and $\Delta m_{21}{}^2$, e.g.,

$$
\begin{aligned}
P_{\mu\to e} = &(\sin^2 2\theta_1[\cos^2\theta_2\cos^2\theta_3 - \frac{1}{4}\sin^2\theta_2\sin^2 2\theta_3] \\
&+ \frac{1}{4}\sin 4\theta_1 \sin 2\theta_2 \cos\theta_3 \sin 2\theta_3 \cos\delta)\sin^2(\frac{\Delta m_{21}{}^2}{4k}x) \\
&+ \frac{1}{4}\sin 2\theta_1 \sin 2\theta_2 \cos\theta_3 \sin 2\theta_3 \sin\delta \sin(\frac{\Delta m_{21}{}^2}{2k}x) \\
&+ \frac{1}{2}\sin^2\theta_2 \sin^2 2\theta_3.
\end{aligned}
\tag{12}
$$

It is easily understood that in the "2 generation limit", $\nu_\mu \to \nu_e$ just reproduces the 2 generation result and is the dominant oscillation, in contrast to case (a);

$$
\bar{P}_{\mu\to\tau} \ll \bar{P}_{\mu\to e} \simeq (1/2)\cdot\sin^2 2\theta_1. \tag{13}
$$

The reason is simply because in this limit ν_e, ν_μ sector and ν_τ are almost disconnected.

However, this scenario seems to be ruled out, or at least very nearly so, once information from both the solar neutrino depletion and data from reactor $\bar{\nu}_e \to \bar{\nu}_e$ experiments are taken into account. In scenario (b), since $A \ll \Delta m_{21}{}^2 \ll \Delta m_{31}{}^2$, the survival probability S of solar neutrino should obey the formula in vacuum oscillation. Combining with the most recent prediction of the solar model, 8.2 ± 2.5 SNU, by Bachall,[18] we get constraints; $S = \cos^4\theta_3\left(\cos^4\theta_1 + \sin^4\theta_1\right) + \sin^4\theta_3 < 0.37$ (again "detected solar neutrino flux" is set to be 2.1 SNU) gives $0.83 \le \sin^2 2\theta_1$ and $0.18 \le \sin^2\theta_3 \le 0.49$. The allowed range of θ_3 will lead, through (10) and (11), to the inequality

$$
P_{e\to e} \le \cos^4\theta_3 + \sin^4\theta_3 \le 0.70, \tag{14}
$$

which contradicts data from reactor experiments. For example, the Gõsgen experiment[19] gives more or less $1 - P_{e\to e} < 0.1$ for the $\Delta m_{21}{}^2$ mass range considered here.

Finally, let us ask what are the implications of the reported possible excess of ν_e events in the $\nu_\mu \to \nu_e$ experiments BNL E816[20] and E776.[21] We will take the suggested values[21] $\Delta m^2 \simeq 0.5\text{eV}^2$ and $\sin^2 2\theta \simeq 0.05$, given in the 2 generation assumption. From our formula (7a) we find $\Delta m_{31}{}^2 \simeq 0.5\text{eV}^2$ and

$$
\sin^2\theta_2 \sin^2 2\theta_3 \simeq 0.05. \tag{15}
$$

Since $\Delta m_{31}{}^2$ has been fixed, upper bounds on other processes impose additional constraints through (7b) and (7c);

$$
\begin{aligned}
&\nu_\mu \to \nu_\tau : \sin^2 2\theta_2 \cos^4\theta_3 < 0.3 \qquad \text{Ref. (8)}, \\
&\nu_e \to \nu_e : \sin^2 2\theta_3 < 0.1 \qquad \text{Ref. (19)},
\end{aligned}
\tag{16}
$$

which have a consequence, consistent with $\sin^4\theta_3 \le S < 0.37$,

$$
0.92 \lesssim \sin^2\theta_2,\ 0.013 \lesssim \sin^2\theta_3 \lesssim 0.014. \tag{17}
$$

We realize that the allowed range is not so wide and is very sensitive to $P_{\mu\to\tau}$, therefore a dedicated $\nu_\mu \to \nu_\tau$ experiment will be very desirable to settle the situation.

SUMMARY

We learned that assumed neutrino mass hierarchy Eq. (2), and our choice of parametrization of the mixing matrix Eq. (3) greatly help our qualitative understanding concerning both the solar neutrino problem and neutrino oscillations in laboratory experiments in the framework of the 3 generation model. Thanks to these key ingredients, the following points were revealed.

(i) The solar neutrino problem in the 3 generation model actually reduces to the problem in an "effective" 2 generation system. As a result, possible contours in a $(\Delta m_{21}{}^2,\ \theta_1)$ plane is quite easily derived from the existing results in the 2 generation model.

(ii) The allowed range of parameters, especially $\Delta m_{21}{}^2$, consistent with the solar neutrino depletion, crucially depends on whether $S \gtrsim 1/3$ or not (and also on θ_3), as is seen in Fig. 2. Thus the settlement of the solar model predictions looks like a very urgent issue.

(iii) As for neutrino oscillations at laboratories, $P_{\alpha\to\beta}$ behave very differently in the two possible scenarios examined: (a) $\Delta m_{21}{}^2 \ll 1\text{eV}^2$, required for $S < 1/3$, and (b) $\Delta m_{21}{}^2 \gtrsim 1\text{eV}^2$, possible only if $S \gtrsim 1/3$. For example, in the "2 generation limit", where $|\theta_2|,\ |\theta_3| \ll 1$, scenario (b) reproduces the 2 generation result; $\bar{P}_{\mu\to\tau} \ll \bar{P}_{\mu\to e} \simeq \sin^2 2\theta_1/2$, while in (a) $\nu_\mu \to \nu_e$ is highly suppressed; $\bar{P}_{\mu\to e} \ll \bar{P}_{\mu\to\tau} \ll 1$.

(iv) As a matter of fact, scenario (b) appears to be ruled out, once information from the solar neutrino depletion and reactor experiments are combined.

(v) The possible excess of ν_e events, reported in this Workshop, is compatible with all upper bounds from other types of oscillations (so far). However, it was argued that the allowed range is very restricted and is very sensitive to the bound on $P_{\mu\to\tau}$. Thus, a dedicated $\nu_\mu \to \nu_\tau$ experiment looks very warranted and is strongly motivated. Finally, our arguments based on two key ingredients Eqs. (2)and (3) can be easily generalized to higher generation models.[14]

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