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SUMMARY

The fuel elements of a High-Temperature Gas-Cooled Reactor (HTGR) consist of monolithic graphite moderator blocks that are drilled with a pattern of holes having various diameters. The stress analysis of this structure is demanding from a computational standpoint. The geometry, even with a two-dimensional approximation, is sufficiently complicated that an accurate representation requires several thousand finite elements. A two-dimensional stress analysis could require a stiffness matrix containing 50,000 degrees of freedom. This was beyond the capability of most finite element programs. As a result, stress analysts sought various modeling approximation techniques including a method of local mesh refinement and homogenizing the holes with an equivalent solid material. These methods were employed largely without verification because a complete fine mesh analysis was not economically feasible.

Two steps have been taken to quantify and improve the accuracy in the analysis. First, the limitations of various approximation techniques have been studied with the aid of smaller benchmark problems containing fewer holes. Second, a new family of computer programs has been developed for handling such large problems.

This paper describes the accuracy studies and the benchmark problems. A review is given of some proposed modeling techniques including local mesh refinement, homogenization, a special-purpose finite element, and substructuring. Some limitations of these approaches are discussed. The new finite element programs and the features that contribute to their efficiency are discussed. These include a standard architecture for out-of-core data processing and an equation solver that operates on a peripheral array processor.

The central conclusions of the paper are:

1. Modeling approximation methods such as local mesh refinement and homogenization tend to be unreliable, and they should be justified by a fine mesh benchmark analysis.
2. Finite element codes are now available that can achieve accurate solutions at a reasonable cost, and there is no longer a need to employ modeling approximations in the two-dimensional analysis of HTGR fuel elements.

1. Introduction

The fuel elements of a High-Temperature Gas-Cooled Reactor (HTGR) consist of hexagonal prismatic blocks of graphite that are drilled with a triangular pattern of holes having different diameters. In the standard fuel element, shown at the bottom of Fig. 1, about two-thirds of the holes contain fuel rods, and about one-third are coolant channels. In the control element, top of Fig. 1, the typical pattern is interrupted by three or four larger channels for control rods and reserve shutdown material. The purpose of this paper is to describe some recent benchmark studies pertaining to numerical accuracy (defined below) in the stress analysis of HTGR fuel blocks. The current studies are limited to a two-dimensional, linear, static analysis, although the final application is more complex.

The loads on the block include spatially varying thermal and irradiation-induced strains plus seismic loads. The temperature fields, the stress fields, and the geometry of the block have both local and global characteristics. In addition, the graphite material is nonlinear and viscoelastic, and its properties are functionals of the temperature and irradiation history.

These circumstances result in a large numerical analysis problem. Even with a two-dimensional approximation, a significant number of finite elements is required to model the geometry and to resolve the local and global aspects of the stress field. An example of a fuel block mesh is shown in Fig. 2. This mesh contains 3774 eight-node isoparametric elements with 13976 nodes total. The stiffness matrix has 27955 degrees of freedom.

Until recently, the solution of such a problem, particularly in a nonlinear time-dependent case, was not economically feasible with the available finite element codes. As a result, analysts sought various modeling approximation techniques, including a method of local mesh refinement [1] and homogenizing the holes with an equivalent solid material [2]. These methods were employed largely without verification because a complete fine mesh analysis would have been too costly.

In the stress analysis of a safety-related component there is a need to quantify errors from all sources. Therefore, two steps have been taken to resolve the question of numerical accuracy. First, limitations of various approximation techniques have been studied with the aid of smaller benchmark problems containing fewer holes. Second, a new family of computer programs has been developed that is capable of efficiently solving very large problems.

This paper reports the results of these activities. Section 2 contains a brief review of modeling techniques that have been considered. The limitations of these techniques are discussed along with reasons why they have not been adopted. Section 3 describes the features of our latest finite element codes. Finally, Section 4 summarizes the results of the small problem benchmark studies, which were used to compare the efficiencies of various element types and to estimate the numerical accuracy achievable with full-sized problems. Two sources of numerical error are considered. The first is discretization error, which is the theoretical difference between a finite element solution and an exact solution. The second is roundoff error, caused by the finite precision of the computer.

2. Review of Modeling Techniques

2.1 Local Mesh Refinement

A popular method to solve complicated structural problems is to first employ a coarse model to determine displacements or forces in a global sense. The coarse model results are then used as boundary conditions for a fine mesh analysis of a local region. Such a method was proposed in Ref. [1] for the analysis of HTGR fuel blocks. However, an evaluation of this method by Sullivan [3] indicates that it must be used with extreme caution.

Local mesh refinement is accurate only to the extent that the global response is well-approximated by the coarse model analysis. A coarse mesh of finite elements may be too stiff for modeling a multiple-holed structure, and, as a consequence, the displacements and stresses may be underestimated. It is possible to gain some efficiency by reducing the error in the stress to the same magnitude as the error in the global displacements, but the gain is problem-dependent. Sullivan [3] reported one case in which the coarse mesh was responsible for errors of 39% in the displacement and 65% in the peak stress, yet local mesh refinement had essentially no effect on the calculated peak stress.

It was concluded that local mesh refinement must be separately justified for each case to which it is applied, and for this purpose a globally fine mesh is desirable.

2.2 Equivalent Homogenous Material

The "equivalent solid plate" method is recommended by the ASME for the stress analysis of tube sheets [4]. In this method the holes are replaced by a homogenous material having an equivalent stiffness. Smeared stresses are readjusted using stress concentration factors.

As explained in Ref. [4], the equivalent plate method is intended for the elastic analysis of an extensive repeating hole pattern subject to in-plane mechanical loads. A similar method was used by Gwaltney and McAfee [2] for the thermal-stress analysis of an HTGR control block subject to a cure-in-place furnace cycle. However, to perform an accurate stress analysis under reactor conditions some additional complicating factors must be considered:

1. The material is nonlinear and viscoelastic,
2. The stress field includes local and global, thermal and mechanical contributions in varying proportion,
3. No portion of a control block hole pattern is far from an edge or discontinuity.

In our judgment the effort that would have been required to account for these factors in the equivalent plate method would have been excessive. A fine mesh analysis would be required as a benchmark in any event.

2.3 Hybrid Super-Element

Tzung [5] proposed the use of a hybrid super-element to achieve an improved accuracy with fewer degrees of freedom. The element had the shape of a hexagon with one central circular hole. A triangular pattern of holes could be spanned by a combination of these super-elements with standard solid elements around the rim.

Tzung's super-element was derived using a complementary energy formulation, using stress functions to satisfy internal equilibrium along with a condition of zero traction on the surface of the hole. Displacement mismatch with adjacent elements was minimized using an auxilliary variational principle.

This element was developed through the trial phase, but the effort was discontinued. Owing to algebraic and numerical complexities, this would have been a long-term effort, whereas a short-term solution was desired. Continued development may be warranted.

2.4 Substructuring

An alternative way to reduce the number of degrees of freedom is to use substructuring to construct a super-element from an assembly of simpler elements. The interior degrees of freedom are eliminated prior to the assembly of the super-element stiffness into the global stiffness matrix.

Substructuring is beneficial only to the extent that it avoids repetitive calculations. It can be shown that, for nonlinear analysis or problems with spatially varying properties, the number of arithmetic operations in the equation solving is not reduced by substructuring. Accordingly, substructuring was also abandoned.

3. Efficient Finite Element Codes

From the above considerations we concluded that the first priority was to develop the capability for a fine mesh benchmark analysis of the complete problem. Accordingly, a new family of two-dimensional finite element codes was developed with the objective of efficiently solving large problems. It is recognized that some good general purpose programs such as MARC and ADINA are available. However, better performance can be achieved using streamlined programs that are optimized for a particular class of problems and that take advantage of specific computer hardware.

The new programs, TWOD and HEAT2 [6], are patterned after similar codes developed at General Atomic Company for three-dimensional analysis. Some of the features that contribute to their efficiency are summarized below:

1. Separate driver programs are used for each class of problem such as 2-D, 3-D, heat transfer, static analysis, or dynamic analysis. These draw on a common library of utility routines such as the matrix solver or assembler.
2. Linear triangular elements have been discarded in favor of isoparametric quadrilateral elements.
3. An efficient algorithm for solving nonlinear problems, based on Newton's method, is employed. The details are given in [7].
4. A Univac 1100/82, with two million words of high-speed memory, is used as a host computer. This provides powerful file management and data storage capability.
5. A Floating Point Systems array processor, model AP 190L, is used for vectorized "number crunching" operations such as equation solving. The AP uses a combination of parallel processing and pipeline processing to achieve a rapid throughput in repetitive computations. With its low capital cost, the AP is extremely cost effective in this application [8].

6. A special purpose matrix solver has been written for the AP. Computing and I/O are performed simultaneously, and the data storage is optimized so that computing is nearly continual. The method is an extension of that in [7].

7. Matrix operations are performed in single precision, and the nonlinear solution algorithm is used to correct for roundoff errors. Rapid convergence to seven-place accuracy is achieved.

At this writing, the AP routines are in the final stages of development, so no timing statistics are available. Without the AP an elastic analysis of the 27955 D.O.F. mesh shown in Fig. 2 can be achieved in about 1.5 Univac hours at a cost of about $1400. The AP should reduce the cost of solving such large problems by about a factor of ten.

4. Small Problem Benchmark Studies

Two benchmark studies have been performed to assess the accuracy of finite elements for modeling multiple-holed geometries. In the first, Sullivan [3] evaluated the accuracy of linear triangular elements using the SAFIRE code [9]. The test problem was a one-half sector of a seven-hole block, because this was the most complex geometry for which adequate convergence could be assured. Sullivan concluded that linear triangles were unacceptable for multiple-hole applications and that local mesh refinement did not offer much hope for improvement. The convergence characteristics of linear triangles, in comparison to isoparametric quadrilaterals, are shown in Fig. 7.

A second benchmark study was recently completed to assess the accuracy of isoparametric quadrilaterals using the TWOD code. Owing to the improved efficiency of TWOD it was possible to evaluate a sequence of test problems with increasing numbers of holes (7, 19, and 37). Two examples, the smallest and the largest, are shown in Figs. 3 and 4. As expected, it was concluded that the number of holes influences the size of the problem (hence, the roundoff error) but the discretization error is governed only by the nature of the discretization in a typical ligament.

The convergence characteristics of three element types are shown in Fig. 7. Here, the "number of nodes" refers to a quarter-section of a seven-hole block, as shown in Fig. 3, and the "error" is estimated by taking the finest mesh to be exact. It can be seen that the higher-order, eight-node quads are considerably more efficient than the four-node quads in this application. For the same level of accuracy, eight-node quads require a factor of 4-10 fewer degrees of freedom, and the cost of using them is a factor of 10-100 lower, than four-node quads. Thus, four-node quads were ruled out, and the remainder of the study was completed using eight-node quads alone.

Contour plots of the stress fields are shown in Figs. 5 and 6. The mesh of Fig. 5 has two subdivisions of 8-node quads across a mesh, and the mesh of Fig. 6 has eight subdivisions. The discontinuities in the two-division case portray the actual finite element discretization. The discontinuities are greatest in regions of small or slowly varying stress. The peaks are reasonably accurate.

An alternative plot of the stress field shape is shown in Fig. 8. This plot is derived from the integration point stresses. It can be seen that the stress field shapes in all cases are nearly coincident. For the two-division case the 10% error (Δ) in the peak stress can be reduced to about 1% either by extrapolation to the hole surface or by a local mesh refinement.

Finally, Figs. 9 and 10 permit one to distinguish between roundoff error and spatial discretization error. The curves labeled "1ST GS" are the result of the single-precision matrix solution, including roundoff error. The curves labeled "CONVER" are obtained after the roundoff error has been effectively removed using the nonlinear solution algorithm. Figure 9 is taken from a thick cylinder test problem. Comparison with an exact solution confirmed that seven-place accuracy was achieved for the finest mesh. Figure 10 was taken from the seven-hole, eight-node quad series. Convergence behavior was similar to the cylinder, indicating that the multiple-holed configuration is also well-behaved.

5. Conclusions

Modeling approximation techniques tend to be unreliable, and they should be justified by a fine mesh benchmark analysis. The TWOD code is capable, without modeling approximations, of calculating peak stresses with about 10% error and displacements with about 1% error in an HTGR fuel element.

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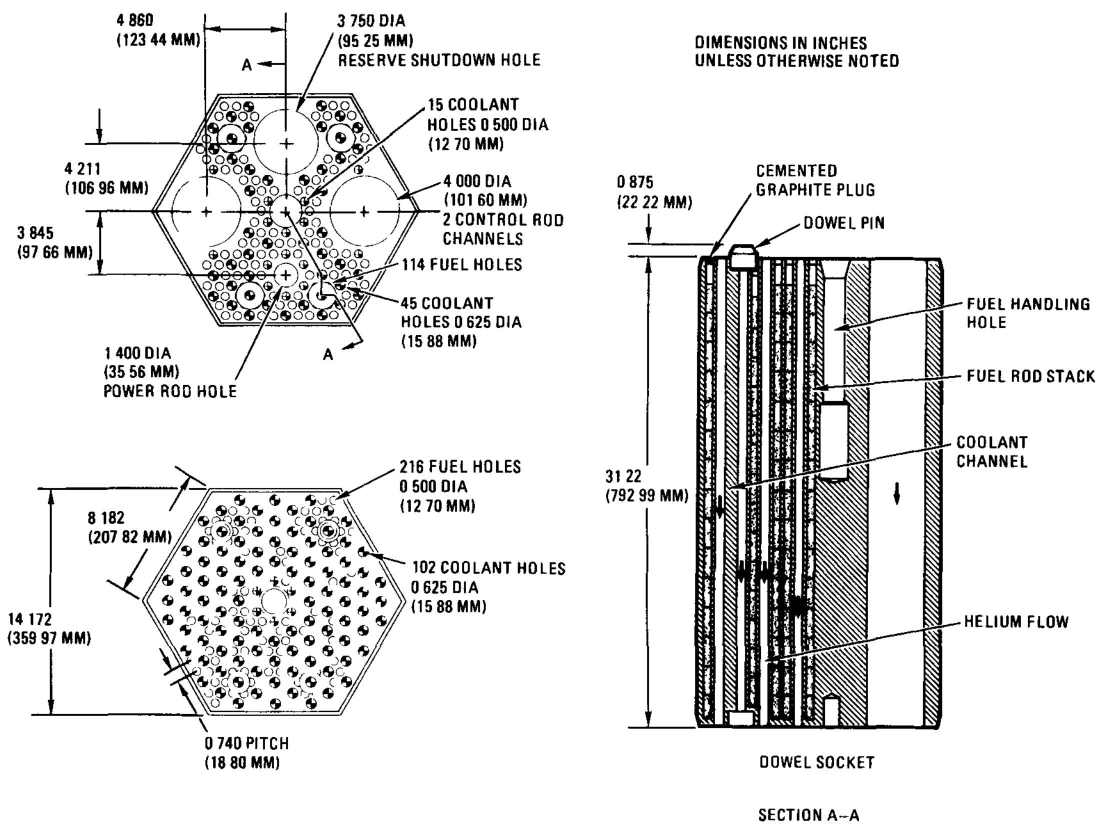

Fig. 1   HTGR Fuel Elements

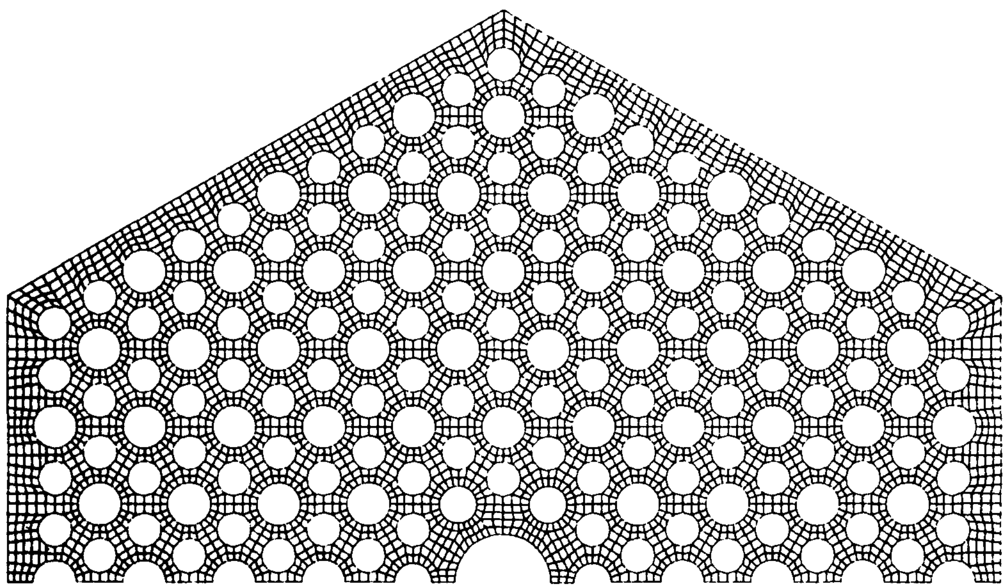

Fig. 2   Mesh for Standard Fuel Element

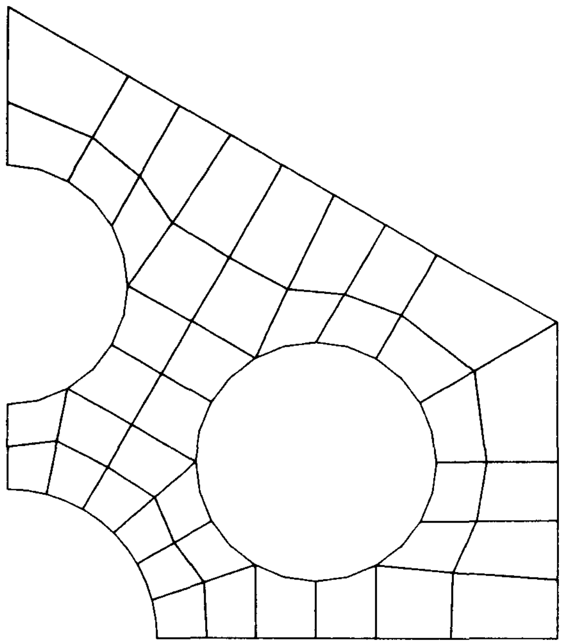

Fig. 3 One Quarter of a Seven-Hole Block
Two Divisions of Eight-Node Quads
Across Minimum Web

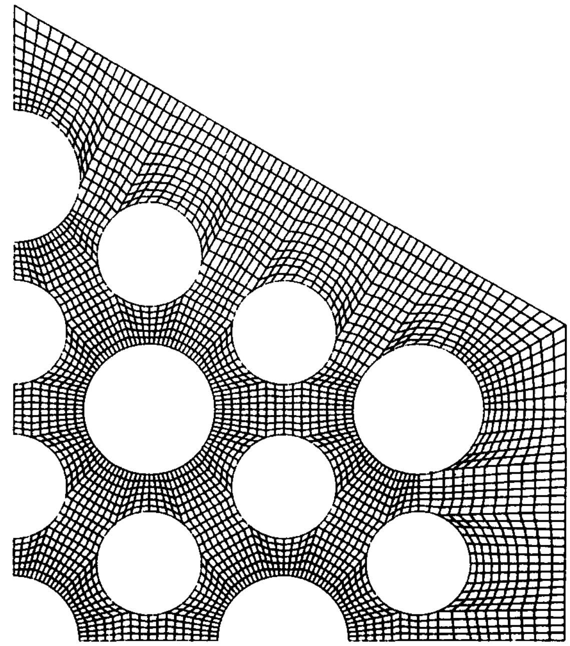

Fig. 4 One Quarter of a Thirty-Seven Hole Block
Six Divisions of Eight-Node Quads
Across Minimum Web

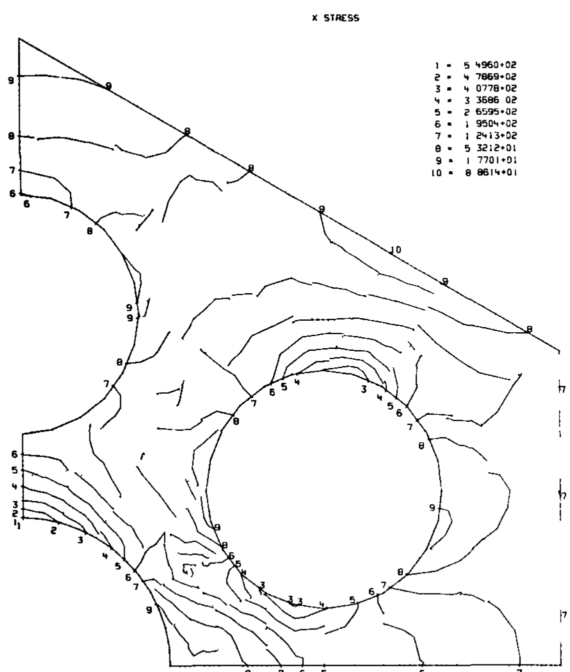

Fig. 5 Contour Plot of σ_{xx}
Two Divisions of Eight-Node Quads

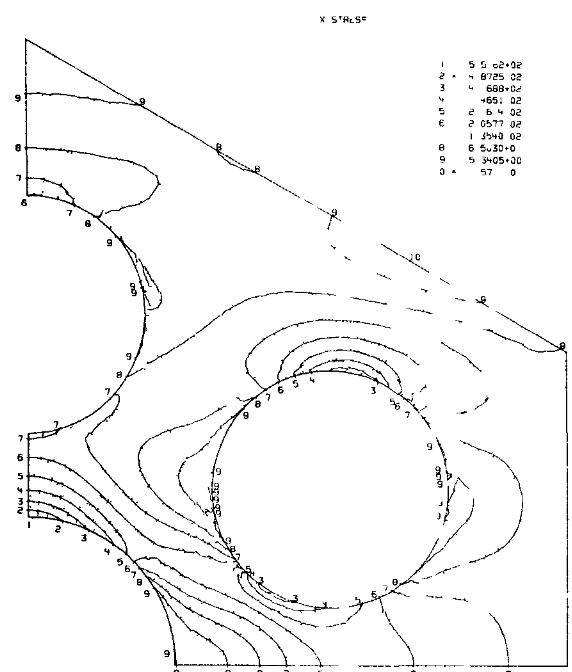

Fig. 6 Contour Plot of σ_{xx}
Eight Divisions of Eight-Node Quads

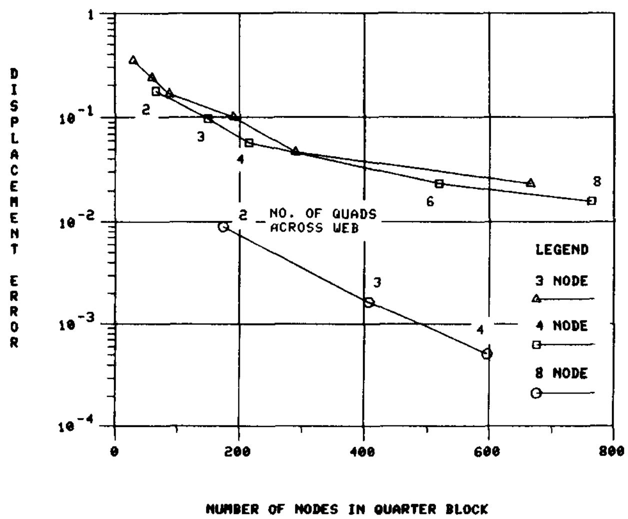

Fig. 7 Convergence Characteristics of Three Element Types Triangles, 4-Node, and 8-Node Seven-Hole Benchmark Problem

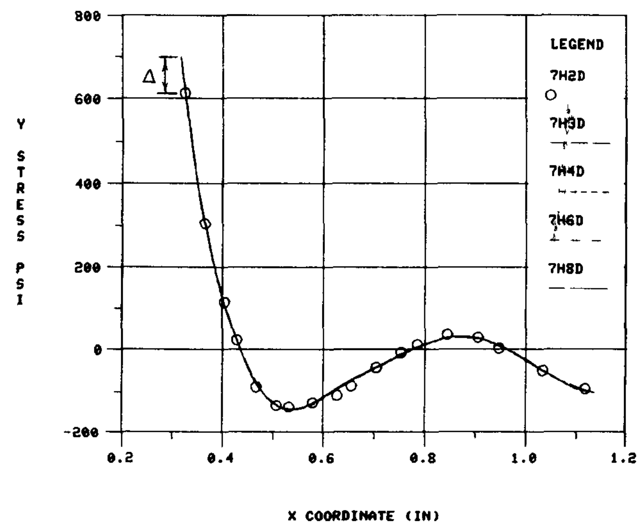

Fig. 8 Integration Point Stress, σ_{yy}, Along X Axis Seven-Hole Benchmark Problem Eight-Node Quadrilaterals

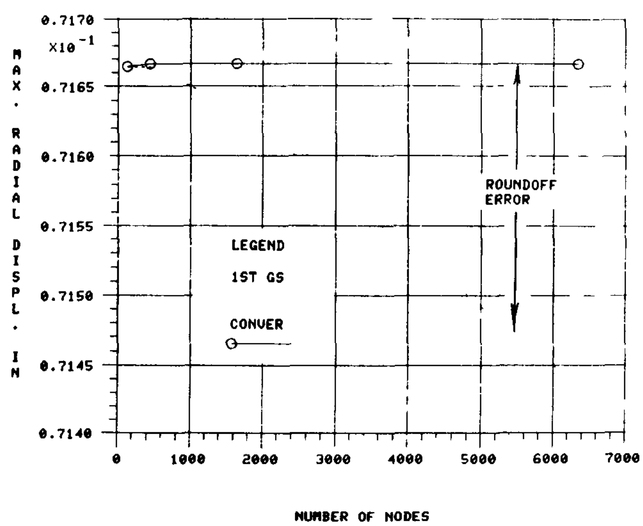

Fig. 9 Roundoff vs Discretization Errors Thick Cylinder Under Internal Pressure Eight-Node Quadrilaterals

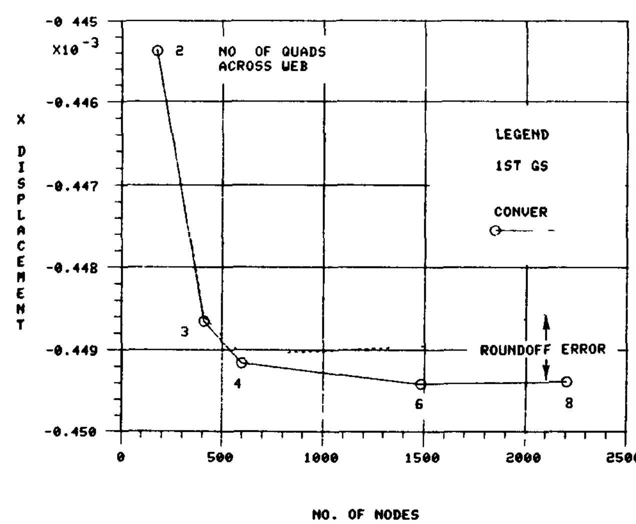

Fig. 10 Roundoff vs Discretization Errors Seven-Hole Benchmark Problem Eight-Node Quadrilaterals