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Scattering Amplitudes to All Orders in Meson Exchange

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ABSTRACT

As the number of colors in QCD, N_C, becomes large, it is possible to sum up *all* meson-exchange contributions, however arbitrarily complicated, to meson-baryon and baryon-baryon scattering. A semi-classical structure for the two-flavor theory emerges, in close correspondence to vector-meson-augmented Skyrme models. In this limit, baryons act as extended static sources for the classical meson fields. This leads to non-linear differential equations for the classical meson fields which can be solved numerically for static radial (hedgehog-like) solutions. The non-linear terms in the equations of motion for the quantized meson fields can then be simplified, to leading order in $1/N_C$, by replacing all factors of the meson field but one by the previously-found classical field. This results in linear, Schrödinger-like equations, which are easily solved. For the meson-baryon case the solution can be subsequently analyzed to obtain the phase shifts for the scattering and, from these, the baryon resonance spectrum of the model. As a warm-up, we have carried out this calculation for the simple case of σ mesons only, finding sensible results.

1. Introduction

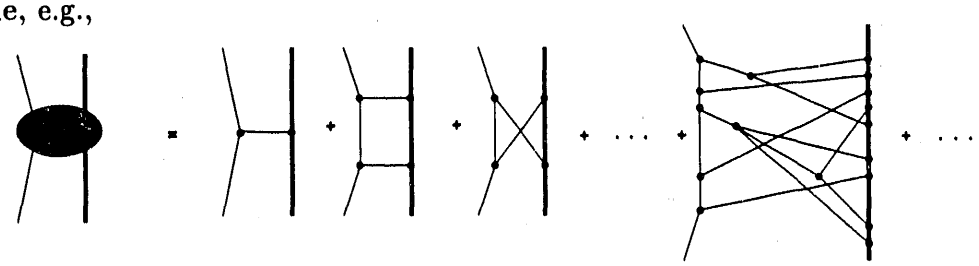
In the large-N_C limit of QCD a number of interesting things happen.[1] The quark and gluon degrees of freedom "freeze out" of the theory, leaving behind an effective chiral field theory of mesons. The nucleon (and other baryons) become heavy objects, with renormalized masses of $O(N_C)$, while mesons remain "light" (masses of $O(1)$). In fact, for two-flavor QCD, the nucleon is but one member of a degenerate tower of baryons with $I = J = \frac{1}{2}, \frac{3}{2}, \cdots$ with masses of $O(N_C)$. In quark-model language these are the states formed from N_C quarks in relative S-waves, antisymmetrized in color, and symmetrized in spin $\otimes$ isospin; in Skyrme-model language these states are the quantized excitations of the rotational collective coordinates of the hedgehog.

Two lesser-known facts regarding meson-baryon vertices in the large-N_C limit have been pointed out by Mattis and collaborators,[2] motivated by work of Donohue.[3] The first of these is the so-called "$I_t = J_t$ Rule", which says that those partial wave amplitudes for meson-baryon scattering which have the isospin in the t-channel *un*equal to the total (spin plus orbital) t-channel angular momentum go to zero like $O(1/N_C)$. [For example, this rule therefore says that the ρNN coupling is dominantly tensor-like ($\sigma_{\mu\nu}$), while the ωNN coupling is vector-like (γ_μ).] The second fact is that meson-baryon coupling constants are related by a "Proportionalty Rule",

$$g_{xNN'} \sim [(2i_N + 1)(2i_{N'} + 1)]^{1/2} \ , \tag{1}$$

for meson x interacting with a baryon N with isospin i_N to make a final baryon N' with isospin $i_{N'}$. For example, $g_{\pi NN} : g_{\pi N\Delta} : g_{\pi\Delta\Delta} = 1 : \sqrt{2} : 2$. [Since we don't know much about the $\pi\Delta\Delta$ coupling constant, the latter ratio is better considered as a prediction.]

Using these two large-N_C rules, Arnold and Mattis[4] recently showed how to sum up *all* the meson-exchange contributions in, say, meson-baryon scattering amplitude, e.g.,



Graphs which are *not* included in the above t-matrix summation are the "Compton-like" graphs, in which the external meson lines attach directly to the baryon-line.

At first glance, this claim of summability seems *a priori* outrageous. Consider, for example, the set of diagrams for exchange of n boson exchanges. For one thing, each of the $n!$ tanglings (i.e., orderings of how the n bosons attach to the baryon line) involves a different integrand for the multiple integral over the loop momenta. Also, for meson-baryon couplings which depend on spin and isospin, each of the $n!$ tanglings involves a different overall group-theoretic factor. (This merely reflects the fact that the Pauli matrices, for spin or for isospin, do not commute.) Nonetheless, as $1/N_C \to 0$, these complexities disappear.[4] We do not go into the proofs here, but they depend on the validity of the $I_t = J_t$ and Proportionality Rules mentioned above, as well as the assumption that the momenta of interest are all of the order of a typical meson mass, i.e., $O(N_C^0)$.

In the following we will sketch how the summation over all meson-exchange graphs goes for the simple case of σN scattering. We have carried out this case numerically as a warm-up problem before tackling a more realistic model which also includes π, ρ, and ω mesons. After presentation of numerical results, we will wrap up, indicating how we would extend the meson-baryon scattering work and what some of the additional complications are in applying this technique to baryon-baryon scattering.

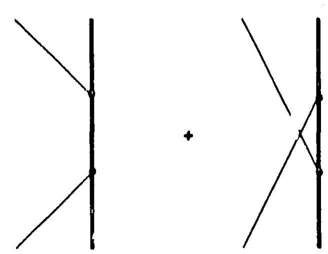
2. Qualitative Remarks on Vertices and Amplitudes for Large N_C

First, a diversion to provide some insight as to the sizes of things. As the number of colors in QCD grows large, one finds[1] that the theory requires the three-meson vertex coupling constant to be $O(1/\sqrt{N_C})$, the four-meson vertex to be $O(1/N_C)$, and the meson-baryon coupling to be $O(\sqrt{N_C})$. [The latter is reasonable, *vis-a-vis* the g_{MMM} coupling, because of the N_C quarks necessary to make up the baryon, compared with the $\bar{q}q$ pair in the meson.] This means that the meson-baryon scattering amplitude (and thus the scattering phase shifts) will be of $O(1)$, as can be seen by considering powers of N_C for each of the various terms in the diagram in the last section.

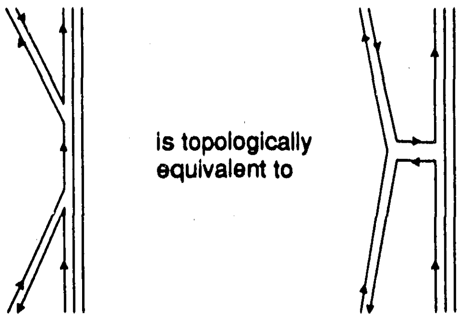
Note in particular that, with the coupling constants going as stated above, meson-loop contributions to the scattering amplitude are suppressed by a power of $1/N_C$ or more. This implies that, in the large-N_C limit, the relevant diagrams (as far as the mesons are concerned) are tree diagrams. That is, the meson-meson interactions do not involve loop integrations and hence can be well described by classical fields for the mesons.

By the same arguments, a meson-meson scattering amplitude is of $O(1/N_C)$, i.e., is small. On the other hand, a baryon-baryon scattering amplitude iterates meson exchange, and each exchange brings in a factor going like $iO(N_C)$. Thus the (unitary) S-matrix goes like $\exp(iO(N_C))$ and its phase shifts δ_l are of $O(N_C)$.

Note also that, by the same sort of power counting, one might expect that the contribution to the meson-baryon scattering amplitude from the Compton-like graphs,



would be $O(N_C)$, not $O(1)$. However, there is a cancellation of the leading orders that leaves these graphs contributing only terms of $O(1)$ (which are, in the calculation below, nevertheless neglected). This cancellation could have been anticipated by consideration of the topological equivalence (duality) between the quark diagrams representing these Compton-like graphs and the usual graphs representing meson exchange:



Recall that the meson-exchange graphs are manifestly of order unity.

The neglect of the Compton-like $O(1)$ contributions to the meson-baryon scattering amplitude may not be so bad an approximation as one might first imagine. In the large-N_C limit, the Δ is just as elementary a baryon as the N; both are members of the same (degenerate) tower of $I = J$ (non-strange) baryons. We do not need the crossed-π-exchange graph as a driving term to build up the Δ as a Chew-Low type of πN resonance.

3. Sketch of a σ-Nucleon Scattering Amplitude Calculation

Reference 4 really presents only an outline of an extended numerical program. We have begun work on this program by considering first the warm-up problem of σ-nucleon scattering (through arbitrarily complex σ exchange). The σ meson has $I = S = 0$ and the σNN coupling is S-wave (i.e., satisfies the $I_t = J_t$ rule). As a result of $I = 0$ there can be no transitions to any of the higher-isospin baryons (such as Δ's). Also, because the only mesons present in this simple model are σ's, one doesn't have to worry about transitions to other meson states.

The Lagrangian we want to solve, then, has the form

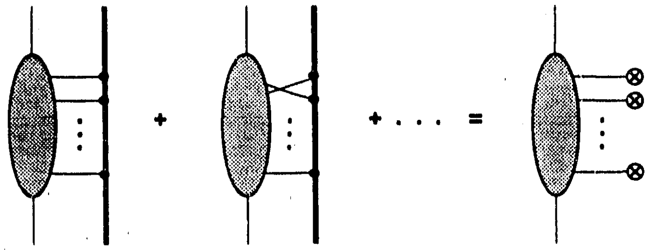
$$\mathcal{L}_{\sigma N} = \frac{1}{2}(\partial_\mu \sigma)^2 - V(\sigma) + \overline{N}(i\gamma \cdot \partial - m_N)N - g\sigma\overline{N}N \ , \tag{2}$$

where the σ self-interactions are described by the potential

$$V(\sigma) = \frac{1}{2}\mu^2\sigma^2 + \frac{1}{6}\kappa\sigma^3 + \frac{1}{24}\lambda\sigma^4 + \dots \ . \tag{3}$$

where μ is the σ meson mass.

We first note the simplification that, because of the large mass m_N compared with the momenta in the nucleon propagators, together with use of a remarkable identity,[4] the sum of all n-boson tanglings simplifies to



Here the symbol $\otimes$ indicates a static source $j(\mathbf{x})$ for the σ's due to the (heavy) N's. (We have chosen to work in the rest frame of the nucleon.) That is, the nucleon field "freezes out", meaning that the $\mathcal{L}_{\sigma N}$ Lagrangian can be replaced by

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu \sigma)^2 - V(\sigma) - g\sigma j(\mathbf{x}) \ , \tag{4}$$

to leading order in $1/N_C$.

The equation of motion for the σ field for $\mathcal{L}_{\text{eff}}$ is

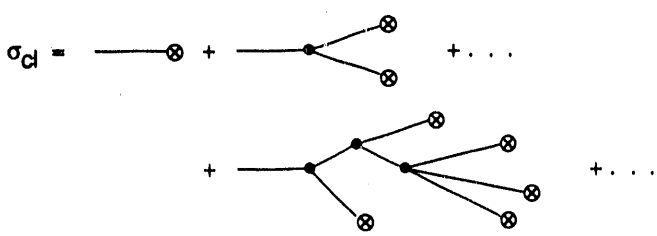
$$\partial_\mu \partial^\mu \sigma(x) + V'(\sigma) + gj(\mathbf{x}) = 0 \ . \tag{5}$$

Here $j(\mathbf{x})$ will be taken as an *extended* source reflecting the size of the nucleon, e.g.,

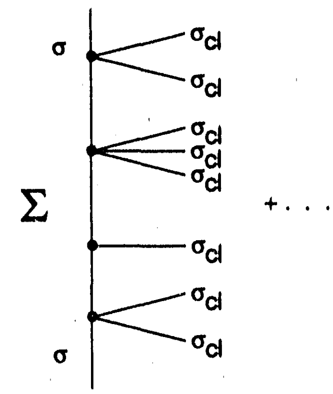
$$j(\mathbf{x}) \propto e^{-r^2/a^2} \ , \tag{6}$$

where, say, $a \approx 0.5$fm.

In the large-N_C limit, the solution of Eq.(5) involves summing up all the tree graphs. This is equivalent to solving Eq.(5) for the *semi-classical* solution,



i.e., σ_{cl} is shorthand for the complete sum of tree-level one-point graphs. With this in hand, it is a reasonable approximation to treat the σN scattering problem as the dressing of a (quantized) σ-field by all possible attachments of σ_{cl}:



Together, these two pictures motivate a two-step numerical program, which we now describe.

We first solve for the classical, static $\sigma_{\rm cl}(\mathbf{x})$ from the equation

$$\Delta\sigma_{\rm cl} - V'(\sigma_{\rm cl}) = gj(\mathbf{x}) \ , \tag{7}$$

which is the classical equation whose solution sums up the tree-level one-point graphs shown above. If one then assumes a radial solution, $\sigma = \sigma(r)$, this equation becomes a one-dimensional non-linear ordinary differential equation, which can be solved by standard techniques.[5] In particular, for the potential V up to quartic terms, this ODE is

$$u'' - \mu^2 u - \kappa\frac{u^2}{r} - \lambda\frac{u^3}{r^2} = \frac{g}{(a\sqrt{\pi})^3} r e^{-r^2/a^2} , \tag{8}$$

where we have made the usual replacement of $\sigma_{\rm cl}(r) = u(r)/r$.

Given $\sigma_{\rm cl}$, we then solve for the *quantized* σ field from the induced quadratic Lagrangian

$$\mathcal{L}_{\rm quad} = \frac{1}{2}(\partial_\mu)^2 - \frac{1}{2}\sigma^2 V''(\sigma_{\rm cl}) \ , \tag{9}$$

or its equivalent linearized equation of motion

$$[\partial_\mu\partial^\mu + V''(\sigma_{\rm cl}(\mathbf{x}))]\ \sigma(x) = 0 \ . \tag{10}$$

(This equation is equivalent to solving for the renormalized propagator depicted above.) In the radial, static limit the latter equation becomes

$$\left[\frac{d^2}{dr^2} + k^2 - \kappa\sigma_{\rm cl} - \lambda\sigma_{\rm cl}^2 - \frac{l(l+1)}{r^2}\right] u_l(r) = 0 \ , \tag{11}$$

where we have assumed

$$\sigma(\mathbf{x},t) = e^{i\omega t}\sigma(\mathbf{x}) = e^{i\omega t}\,\frac{u_l(r)}{r}\,Y_{lm}(\hat{\mathbf{r}}) \tag{12}$$

and set $k^2 = \omega^2 - \mu^2$. This is a Schrödinger-like linear differential equation that can also be solved in standard ways[5,6] to get the partial-wave scattering amplitudes or phase shifts for σN scattering.

4. Fixing Parameters, Method of Solution, and Numerical Results

The differential equation for $\sigma_{\rm cl}$, i.e., the $u(r)$ of Eq.(8), involves five parameters, which is too many for a simple model with only σ mesons. We have appealed to the linear σ-model[7] to reduce this number to three, namely,

- a, the nucleon size parameter (nominally 0.5 fm),
- g, the σN coupling constant (here the same as the πN coupling, 13.6), and
- μ, the σ-meson mass (typically 600 MeV/c^2).

In terms of these parameters, the meson self-coupling constants are

$$\lambda = 3g^2(\mu^2 - m_\pi^2)/m_N^2 \ , \qquad (13a)$$

$$\kappa = \lambda < \sigma > /4 \ , \qquad (13b)$$

where $< \sigma >= m_N/g$ is the vacuum expectation value that breaks the chiral symmetry.

We recognize Eq.(8) as a non-linear differential equation which is of the "two-boundary-value" type.[5] The boundary conditions on $u(r)$ are that $u(0) = 0$ (the solution $\sigma_{\rm cl}$ is regular at the origin) and that, as $r \to \infty$, $u(r) \to Be^{-\mu r}$. The latter condition reflects the confinement of the σ field and can be understood as resulting from the shorter-ranged nature of the non-linear terms on the LHS as compared with the $\mu^2 u$ term. [In fact, dropping the non-linear terms (i.e., setting $\kappa = \lambda = 0$) and replacing $j(\mathbf{x})$ by a point source ($a = 0$), the solution for the field $\sigma_{\rm cl}$ would be a Yukawa-function with the range controlled by the mass μ and with normalization controlled by the coupling g.]

The numerical method used[5] involves a Runge-Kutta integration, shooting out from the origin and in from the asymptotic region to a matching radius at $r = a$, given initial guesses for $u'(0) = A$ and the asymptotic normalization B. Requiring u and u' to be smooth at $r = a$ fixes the next iteration with refined values of A and B. One then repeats the Runge-Kutta process iteratively until the solution converges. Typically, the computer finds a solution in less than five iterations. The solution for $\sigma_{\rm cl}$ for the nominal parameter values mentioned above is shown in Fig. 1. The quadratic behavior at the origin comes from the finite size of the nucleon source. The effects of the non-linear terms are hard to see but come in between 0.3 and 1.0 fm.

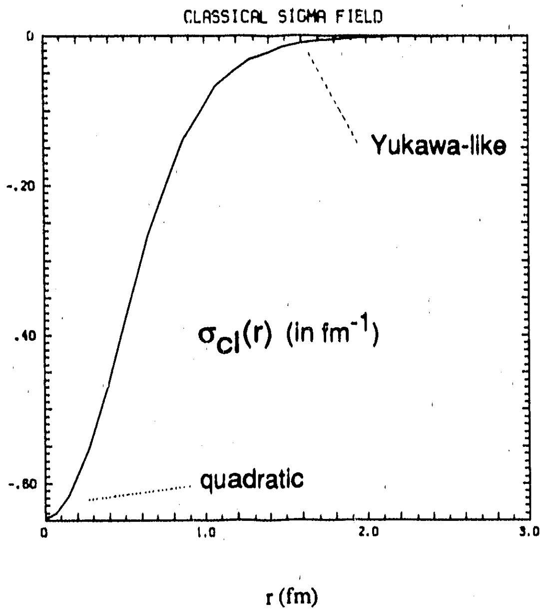


Fig. 1. Solution of Eq.(8) for $\mu = 600$ MeV, $g = 13.6$, and $a = 0.5$ fm.

The linearized scattering equation for the quantized σ field, Eq.(11), can be solved directly for $u_l(r)$, from which the phase shift δ_l can be extracted by comparing the numerical solution with the asymptotic form. However, it is not actually necessary to solve this equation *per se* if one only wants phase shifts. These can be found by the variable-phase method,[6] solving a first-order non-linear differential equation for the phase shift itself.

The "potential" in the Schrödinger equation, Eq.(11), consists of a short-range repulsive piece (coming from the quartic term in the Lagrangian) and a longer-ranged attractive piece (from the cubic term, reflecting $\sigma_{\rm cl}(r) < 0$). This leads to a combination which changes sign, as in Fig. 2, showing moderate attraction at 1 fm.

For such a potential one finds sensible phase shifts as shown, for the standard parameter set, in Fig. 3. For example, the S-wave phase shift at low energies is positive because of the medium-range attraction, but it soon turns over and looks like the phase shift for a hard-core repulsive potential. (At higher energies, not shown, the phase shift returns to 0, since the short-range repulsive core is not infinite.) For the higher partial waves, $l = 1, \ldots, 4$ there is a similar behavior, but delayed because of the angular momentum barrier.

One can get somewhat more interesting phase shifts by adjusting the parameters in this model to get more attraction in the "potential". For example, making g smaller (i.e., the σ less strongly interacting) gives an s-wave phase shift which peaks slightly higher than shown in Fig. 3 ($g = 7$ gives δ_0 peaking at 16°). Making a larger (spreading out the nucleon) also gives a bigger phase shift; δ_0 peaks at 57°

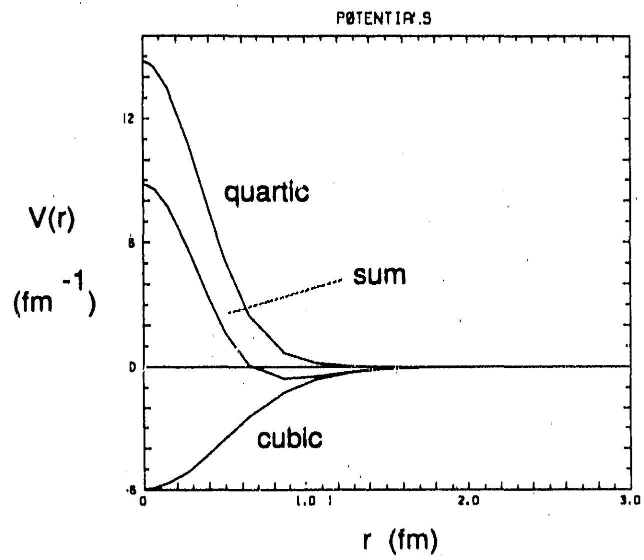


Fig. 2. The "potential" in Eq.(11), for $\mu = 600$ MeV, $g = 13.6$, and $a = 0.5$ fm.

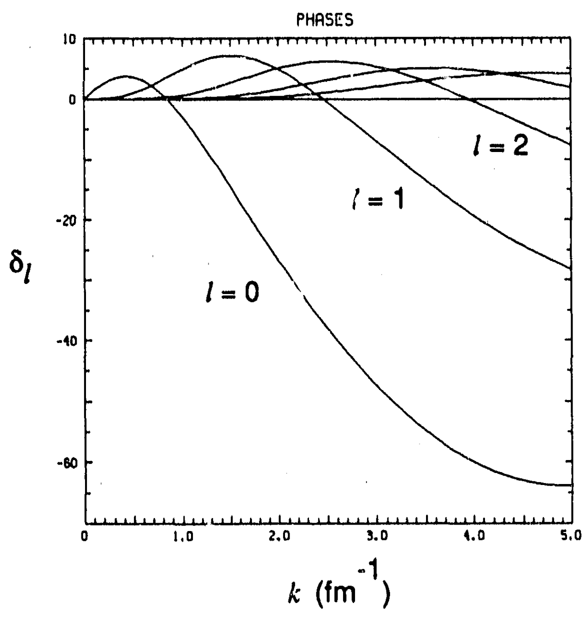


Fig. 3. Phase shifts of Eq.(11) versus meson momentum, for standard parameters of Figs. 1 and 2.

for $a = 1$ fm. Making the σ-meson more massive is the most interesting variation. By $\mu = 7\text{fm}^{-1} = 1400$ MeV the "potential" is mostly attractive. In fact, it appears that there is an $l = 0$ bound state at this value of μ, with δ_0 dropping from 180° to

$0°$ in going from $k = 0$ to $k = \infty$, in accordance with Levinson's theorem.

5. Outlook and Summary

For the warm-up problem of σN scattering, there was a significant simplification with respect to spin and isospin dependences. We do not propose to push the σN model any further. It is better at this point to extend the computations for meson-baryon scattering (and reactions) to the more realistic case which includes π's, ρ's, and ω's as well as σ's. There will then be need to deal with the spin-isospin complications, but the $I_t = J_t$ and Proportionality Rules will lead to simplifications. Instead of a single, non-linear differential equation, as in Eqs.(8) or (11), we will now have coupled-channel equations. (E.g., reactions such as $\pi N \to \rho\Delta$ will now be calculable.) These equations will again be solved using a static approximation with a hedgehog Ansatz. An interesting question is whether this extended model will reproduce the known properties of the higher N^* and Δ^* resonances (or make predictions about yet unknown properties).

For extension of these techniques to baryon-baryon scattering there will be some complications. Unlike the meson-baryon case, there is now no natural restframe; at least one of the baryons will be in motion. This means that the "remarkable identity" must be modified so that what was previously a static source, $j(\mathbf{x})$, becomes time dependent. There will be rewards for the extra work involved. Unlike the Skyrme model for the two-nucleon interaction (in the Heitler-London approximation), one does expect a full σ, π, ρ, ω model to provide the medium-range attraction (in the central potential) that is necessary to understand the existence of atomic nuclei. What one does not know, but would be interested in seeing, is whether the higher-meson-exchange contributions result in significant differences from, say, the Bonn potential (which has only been fully developed to the two-meson-exchange level[8]).

To summarize, we have seen how, to leading order in $1/N_C$, one can sum arbitrarily complicated meson-exchange contributions to meson-baryon scattering. The program can also be applied to baryon-baryon scattering, but with some complications. We have carried out, as a warm-up calculation, the numerical program for solving the non-linear and linear equations for the simple model of σN elastic scattering. It is clear how to proceed now with more realistic models for meson-baryon and baryon-baryon systems, and this program is underway.

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