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TITLE **STUDIES OF SPECTRAL ALTERATION IN ULTRASONIC WAVES RESULTING FROM NONLINEAR ELASTIC RESPONSE IN ROCK**

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Experimental and Theoretical Studies of Spectral Alteration in Ultrasonic Waves Resulting From Nonlinear Elastic Response in Rock

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Experiments in rock show a large nonlinear elastic wave response, far greater than that of gases, liquids and most other solids. The large response is attributed to structural defects in rock including microcracks and grain boundaries. In the earth, a large nonlinear response may be responsible for significant spectral alteration at amplitudes and distances currently considered to be well within the linear elastic regime.

Recordings of seismic waves at various distances from the source are used to estimate the magnitude of the source, characterize high frequency roll-off and model source parameters. It is generally assumed that beyond a few source radii, seismic waves propagating outward from the source reside in an elastically linear material, i.e., in a material that has a linear relationship between stress and strain. A nonlinear relationship between stress and strain (nonlinear elasticity) implies wave multiplication leading to the creation of sum and difference frequency waves, i.e., the failure of wave superposition. Thus, if seismic waves propagate at sufficient strains in a suitably large volume for nonlinear elastic effects to be important, then important features in a seismogram will be modified by elastic nonlinearity. Estimates of magnitude, assessment of roll off, and models of source parameters should be consistent with nonlinear elastic behavior.

Several laboratory studies of nonlinear elastic behavior in solids already exist[1]. These studies show that spectrum alteration in the form of harmonics increases with wave propagation distance. However, study of nonlinear elastic behavior in rock is relatively new[2-9]. In contrast to most solids, rock is strikingly heterogeneous on scales from millimeters to tens of meters due in part to structural defects in the form of microcracks, grain boundaries and fractures. These structural defects cause a large compliance and therefore a large nonlinear elastic response in rock. As a result, the cubic anharmonicity (the first order nonlinear elastic contribution) in rock is typically many orders of magnitude greater than in other solids[2,7,10].

Our goal in this work is to examine the extent to which source frequency content is altered during nonlinear seismic wave propagation. To this end, we have conducted ultrasonic experiments to study the spectral changes that take place along the wave propagation path. For a plane wave propagating in an elastic material with cubic anharmonicity, the amplitude of the 2ω harmonic is proportional to xk^2U^2, where x is the propagation distance, $k=\omega/c$ is the wave vector, and U is the displacement amplitude of the source at frequency ω. Our initial experiments focused on confirming this result. We confirmed that the amplitude of the 2ω harmonic scaled with x. At fixed x, the amplitude of the 2ω harmonic scaled as frequency squared (k^2) and as source amplitude squared (U^2). Thus the fundamental prediction for the behavior of the 2ω harmonic in rock was confirmed. The compressional nonlinear modulus β was measured to be 7000 +/ 25% ($|\beta|$ is less than 10 for most solids[1,10]). We also observed the strong growth of odd harmonics 3ω and 5ω, suggesting that a higher order term in the stress strain relationship (quartic anharmonicity) may be necessary to give a complete theoretical description of nonlinear elastic behavior in rock. We argue, based on our observations at ultrasonic frequencies, that the effect of nonlinear elasticity on seismic wave propagation may be large, and should be considered in modeling.

Theory

The equation of motion for a homogeneous elastic solid, to second order in the displacement (cubic anharmonicity in the elastic moduli), is derived in several texts[11,12,13]. The inclusion of linear attenuation leads to a straightforward modification of this equation[14,15,16]. For a longitudinal plane wave propagating in the x direction, the equation of motion in the absence of attenuation is[16]

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2} = -\beta \frac{\partial}{\partial x}\left(\frac{\partial u(x,t)}{\partial x}\right)^2 . \qquad (1)$$

where β is the nonlinear coefficient defined as

$$\beta = \frac{3(\lambda+2\mu)+2(l+2m)}{2(\lambda+2\mu)} .$$

$u(x,t)$ is the displacement, c is the compressional velocity, λ and μ are second order elastic moduli (Lame coefficients), and l and m are third order elastic moduli (Murnaghan coefficients).

The term on the right hand side of Eq. (1), the (nonlinear) interaction of the displacement with itself, causes the creation of sum and difference frequency waves. Equation (1) can be solved analytically, for example, by an iterative Green function technique[15]. Solution to this equation for a source at the origin of frequency ω and amplitude U is, to first order in the nonlinearity,

$$u(x,t) = u_0(x,t) + u_1(x,t) = Ue^{i(kx-\omega t)} + \frac{\beta U^2 k^2 x}{2} e^{i(2kx-2\omega t)} , \qquad (2)$$

where $u_0(x,t)$ is the displacement solution to the linearized equation of motion and $u_1(x,t)$ is the first order correction to $u_0(x,t)$ due to the nonlinear interaction. Note that a source at the origin of frequency ω and initial displacement amplitude U generates a plane wave at frequency ω with amplitude U and a second plane wave at frequency 2ω whose amplitude grows linearly with the distance of propagation x, the square of the fundamental wavevector k and the square of the fundamental amplitude U. In our experiment, we test for these signatures of a 2ω harmonic due to cubic anharmonicity.

Experimental Apparatus

The apparatus used in the experiments is shown in Fig. 1. A 2-m long by 6-cm diameter rod of Berea sandstone was used for the experiments. One end of the sample was tapered in order to minimize reflections. To accommodate the detectors (pin transducers), 11 small holes were drilled into the rod at 5 cm intervals, up to a distance of 58 centimeters from the source transducer.

A self-monitoring drive transducer composed of a piezoelectric crystal with a backload and capable of direct displacement measurement was constructed for use as the source[9]. A small diameter hole was drilled through the center of the backload and transducer and a fiber optic probe was positioned in the hole to directly measure the rock displacement at the source. This probe is sensitive to 10^{-9} meters over a frequency range of 0 - 200 kHz.

The source transducer was amplified and driven with a single frequency, amplitude modulated N cycle wave train; N ranged from 8 to 24. Frequencies of 8 to 24 kHz were used and care was taken to assure that the measured signals were not contaminated by reflections from the opposite end of the sample. Detected signals were output to a 16 bit waveform analyzer. Source displacements ranged from 10^{-9} - 10^{-6} m. Strain levels at the source were measured to be 6-60 x 10^{-6}, and, at the limit of the measurement range (58 cm), strain levels were 0.6-9 x 10^{-6}.

Experimental Observations

The most fundamental experimental observation providing evidence for nonlinear elastic behavior in the sample is shown in Figs. 2 and 3. Fig. 2 shows the frequency spectrum measured at the source for a drive frequency $\omega/2\pi$ of 13.75 kHz. The five different curves correspond to five amplitudes of the source transducer varying over a factor of approximately 50. The source displacement spectrum is relatively monochromatic containing only a small fraction of 2ω at large drive levels (from electronic distortion). Fig. 3 shows the displacement frequency spectrum at 58 cm, also at increasing drive level. For detected displacements at the fundamental frequency as small as 10^{-8} m, the composition of the displacement frequency spectrum at 58 cm is extremely rich in harmonics not present at the source, out to at least 6ω. Further, these higher harmonic displacement fields have amplitudes that are a sensitive function of the drive amplitude. Similar results were obtained for 30 frequencies in the range 8 - 24 kHz.

In order to emphasize how little wave distortion is required to produce the harmonics shown in Fig. 3, Fig. 4 shows a portion of the time signal at large input drive level collected at 58 cm (large amplitude solid line) in relation to a pure sine wave of equal amplitude (dashed line). The difference between the two signals is the low amplitude solid line.

The amplitude of the displacement field at 2ω was found to increase linearly with source - receiver distance x. A representative result is shown in Fig. 5, where the relative amplitude R of the 2ω harmonic is plotted as a function of distance from the source transducer for a 13.75 kHz drive. R is the ratio of the amplitude u_1 at 2ω (source frequency of ω) to the amplitude u_0 at 2ω (source frequency of 2ω). This ratio was taken in order to correct for detector site effects and intrinsic attenuation. According to Eq. (2), this ratio is proportional to the propagation distance:

$$R = \frac{|u_1(x,2\omega)|}{|u_0(x,2\omega)|} \sim x \tag{3}$$

The results in Fig. 5 confirm this prediction. Measurements throughout the frequency range 8 - 24 kHz showed similar results. [The fluctuations about the fit line in Fig.5 are attributed to positional and frequency dependent elastic scattering from the periodic array of detectors. Periodic scatterers cause rapid spatial fluctuations in wave amplitude along the length of the rod and produce an effective increase in absorption in the rod[9]].

The two other predictions for 2ω harmonic behavior in rock with cubic anharmonicity, described at the end of the theory section, were also confirmed: at fixed x, u_1 scaled as frequency squared (k^2) and as source amplitude squared (U^2)[9]. As a consequence of the agreement between the behavior of the observed 2ω harmonic amplitude and Eq. (2), we are confident that a significant portion of the observed response is due to cubic anharmonicity in the elastic response of the rock.

The compressional nonlinear parameter β was calculated from Eq. (2),

$$\beta = -\frac{2c^2|u_1(x,2\omega)|}{\omega^2 x |u_0(x,\omega)|^2} \tag{4}$$

where u_1 is the amplitude at 2ω and u_0 is the amplitude at ω when the source frequency is ω, c = 2600 +/- 50 m/s, ω/2π = 13.75 +/- 0.002 kHz, and x = 3.0 +/- 0.1 cm. We find β = -7000 +/- 25% for our sandstone sample.

We have used the simplest form of the theory, Eq. (2), to discuss and analyze the data, neglecting the effects of attenuation and higher order terms in the solution to Eq. (1). We have available the results of a complete treatment of the theoretical problem including attenuation[16] and a full set of experimental studies of the linear response of our sample. Numbers derived from experimental results are used in the theory to provide a guide to the conduct of the experiment. Values of β obtained by others are consistent with the above value [2,7]. The value of β is also similar to that obtained for other extremely nonlinear media such as a liquid containing gas bubbles[17,18].

The results in Figs. 2 and 3 show spectral growth at harmonics higher than 2ω. In particular, we find that the 3ω harmonic amplitude grows proportionally with U^3, a result which is in agreement with a second order correction to Eq. (2)[16]. We also observe strong growth in amplitude of other odd harmonics and, when the source is excited at two frequencies ω_1 and ω_2, intermodulation terms such as $2\omega_2 - \omega_1$ are observed to grow strongly in amplitude. This suggests that higher order terms (i.e. quartic anharmonicity) in the stress strain relationship may be necessary to give a complete description of nonlinear elastic behavior in rock.

Discussion

What do our observations imply for seismic wave propagation in the earth? In order to answer this question, we examine some important considerations affecting nonlinear elastic behavior in seismic waves.

How does the nonlinear response of rock vary as a function of depth in the earth? It is known from laboratory experiments[19] that the dependence of velocity on applied stress in rock is very large due to structural defects such as grain boundaries and cracks which act to make the material compliant. At pressures of order 0.5 - 1.0 kBar, depending on the degree or type of structural defect, velocity becomes relatively independent of applied pressure, as is the case for materials such as metals or single crystals. Therefore, if the nonlinear response is similar at seismic frequencies we expect that the strongest nonlinear response due to structural defects in the earth will occur to depths of several kilometers. At deep crustal and upper mantle depths, the question of the presence of defects in the form of open cracks and fractures, which could cause a large nonlinear response, is under discussion[20,21,22,23]. Evidence suggesting the existence of open cracks at deep crustal and upper mantle depths is based on larger than expected (from laboratory experiments) electrical conductivity measurements[20,21,22,23], deep borehole observations of open cracks and fractures[24,25], inferences from metamorphic processes where chemically bound water is forced into defects [26], direct observation of cracks in upper mantle xenoliths[27], seismic source inferences requiring large fluid pressures[28], and anisotropy in wave propagation attributed to aligned cracks and fractures[29]. Based on the above evidence, it is possible that a large nonlinear response is present as deep as the base of the crust and upper mantle due to the existence of structural defects that act to make the material more compliant.

Are strains large enough to produce a significant nonlinear response at large distances from a seismic source, and will the ω^2 dependence of the nonlinear response, Eq. (2), make nonlinear effects at seismic frequencies negligible? We have numerically modelled the propagation of a plane wave produced by a broadband source (Blackman - Harris window) at seismic frequencies including nonlinear elasticity and attenuation[16]. We chose $\beta = 10^3$, and specific dissipation Q (equivalent to energy loss per cycle) for an active tectonic region of order 10^2. The initial source displacement amplitude was chosen to be consistent with a typical displacement from a magnitude 5.5 source[30,31]. In Fig. 6, we show the evolution of the displacement frequency spectrum at 1, 10, and 100 km including the nonlinear response to first order. Note especially the change in corner frequency with propagation distance. Clearly, the effect of nonlinear elasticity on the displacement frequency spectrum with distance is significant, especially at low frequency and in the high frequency roll-off portion of the spectrum.

In conclusion, our results indicate that the progressive effect of nonlinear response in rock on the frequency spectra of seismic waves may be significant for waves propagating in the earth's crust and upper mantle. Remote observations of seismic frequency spectra may be substantially different from the originating source frequency spectrum. Further studies of nonlinear elastic response in rock as a function of pressure, fluid saturation, structural defect, and dimension are forthcoming.

Stress - Strain Hysteresis

We have also studied the relationship between applied stress and the resulting strain as a function of time for the case of a continuous sine wave source. The following has application to modeling of hysteresis observed in static stress - strain data in addition to illustrating another manner of viewing the nonlinear response for transient or continuous waves. For this example the external stress applied at the origin is

$$\sigma = 2(\lambda + 2\mu)U \left[\chi(\omega_0)A(\omega_0)e^{-i\omega_0 t} - \chi(-\omega_0)A(-\omega_0)e^{-i\omega_0 t}\right]. \tag{5}$$

Here,

$$\chi(\omega) = 1 - \frac{\Delta}{1 + (\pm)\, i|\omega\tau|},$$

Δ can be thought of as the fractional amount of the displacement derivative which is retarded in time, τ is the characteristic damping time, and the $\pm$ always refers to the sign of the angular frequency[15]. By making the assumption that τ and Δ are the same for all displacement components, we are assuming that Q is the same for both compressional and shear waves, and that both types of waves experience the same velocity shift from low to high frequencies. In the absence of attenuation Eq. (5) becomes,

$$\sigma = 2(\lambda + 2\mu)Uk_0 \cos(\omega_0 t). \qquad (6)$$

We have taken measured strain to be the linear strain defined by $\varepsilon_L = \frac{\partial u}{\partial x}$. In Fig. (7), we show the time trace of strain as a function of applied stress. The stress is applied at x = 0 and the strain is calculated one wavelength away (λ = 200 m). Fig. (7a) is a plot of strain as a function of stress in the absence of attenuation. Fig. (7b) is a plot of strain as a function of stress when attenuation is present. The input parameters for the calculation were $\omega_0/2\pi$ = 30 Hz, U=10^{-3} m, c = 6000 m/s, β = -10^3, δ = -10^6 (second order nonlinear coefficient)[32], τ = 0.01 s, Λ = 0.1, x=200 m, and $\lambda + 2\mu$ = 10^5 MPa. Note that in Fig. (7a), the first order nonlinear contribution to the strain consists of symmetric lobes about the origin, while the second order nonlinear contribution is asymmetric, causing the total strain to roll over toward an asymptote. The direction of the roll over of the second order term depends on the sign of the nonlinear coefficient δ which is always negative for rock in our experience. In Fig (7b) we see that attenuation adds hysteresis to the linear strain. At these levels of attenuation, the first order nonlinear contribution is changed very little from its contribution in the absence of attenuation.

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Figure Captions

FIG. 1. Experimental configuration.

FIG. 2. Source displacement spectrum at increasing applied voltages as shown by the different line types. Drive at 13.75 kHz.

FIG 3. Displacement spectrum 58 cm from source at increasing applied voltages corresponding to FIG. 2. Note the richness of the spectrum created by nonlinear elasticity in the rock during wave propagation. Drive is at 13.75 kHz.

FIG. 4. Time series showing distorted wave at 58 cm from source (large amplitude solid line) versus pure sine wave (dashed line). The difference between the two signals is the low amplitude solid line.

FIG. 5. Dependence of harmonic amplitude on propagation distance ($R = u_1/u_0 \propto x$) for drive at 13.75 kHz. The dashed line is a least squares linear fit to the data. The harmonic amplitude is normalized to unity at the position nearest the source.

FIG. 6. Model result showing evolution of the displacement spectrum with distance for a broad band source at seismic frequencies. The source function was a Blackmann-Harris window. The pulse propagates to x = 1km, 10 km and 100 km, respectively, producing sum and difference frequencies through a first order nonlinear interaction. (a) in the absence of attenuation. (b) including attenuation. The initial displacement was chosen to correspond to a displacement from a magnitude 5.5 seismic source. The sharp drop in amplitude near 6, 8 and 9 Hz may be due to wave interference.

FIG. 7. Stress - strain response over one cycle from a continuous sine wave source. The linear, first and second order nonlinear, and total strain response are shown. (a) In the absence of attenuation. (b) Including attenuation.

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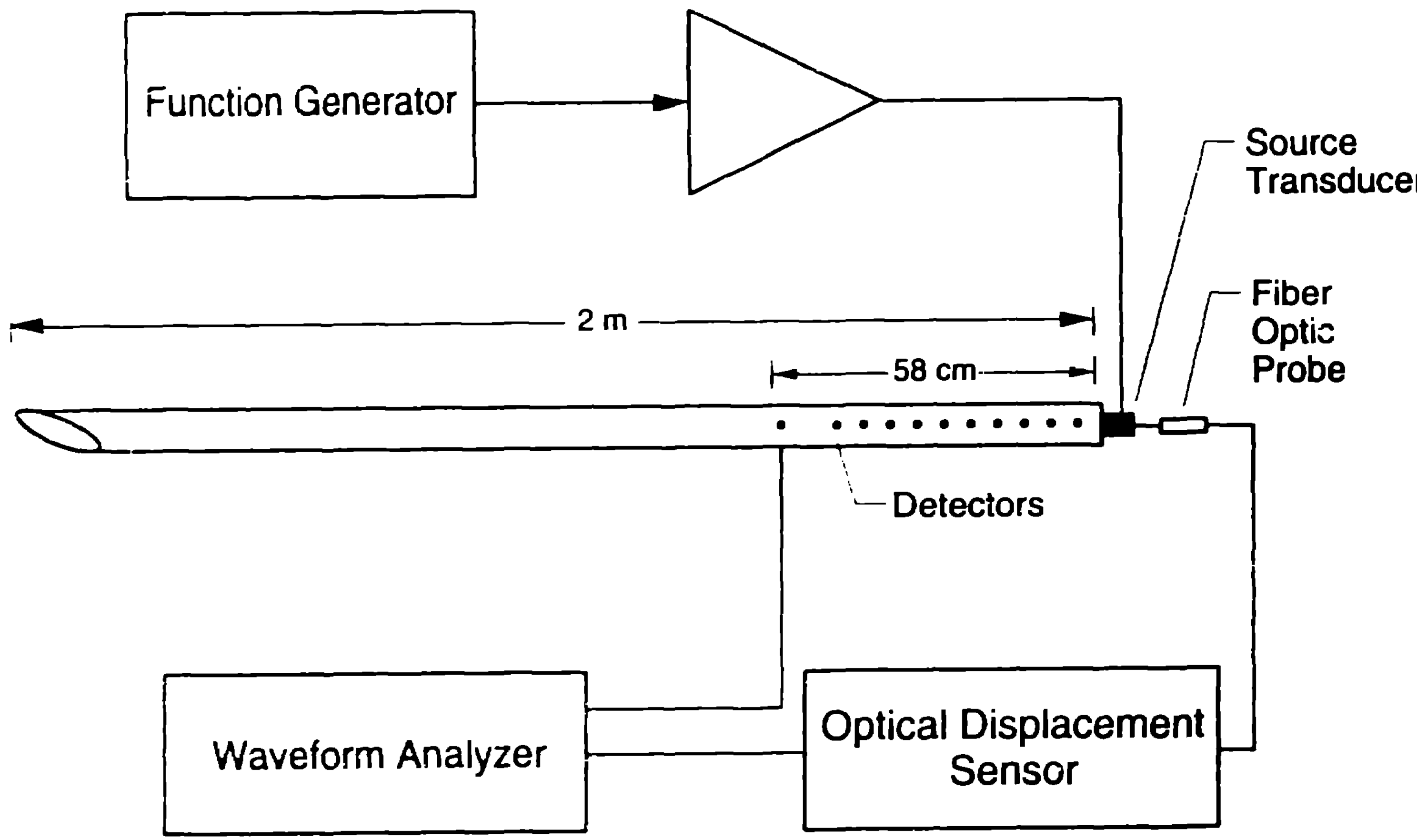
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Source Transducer
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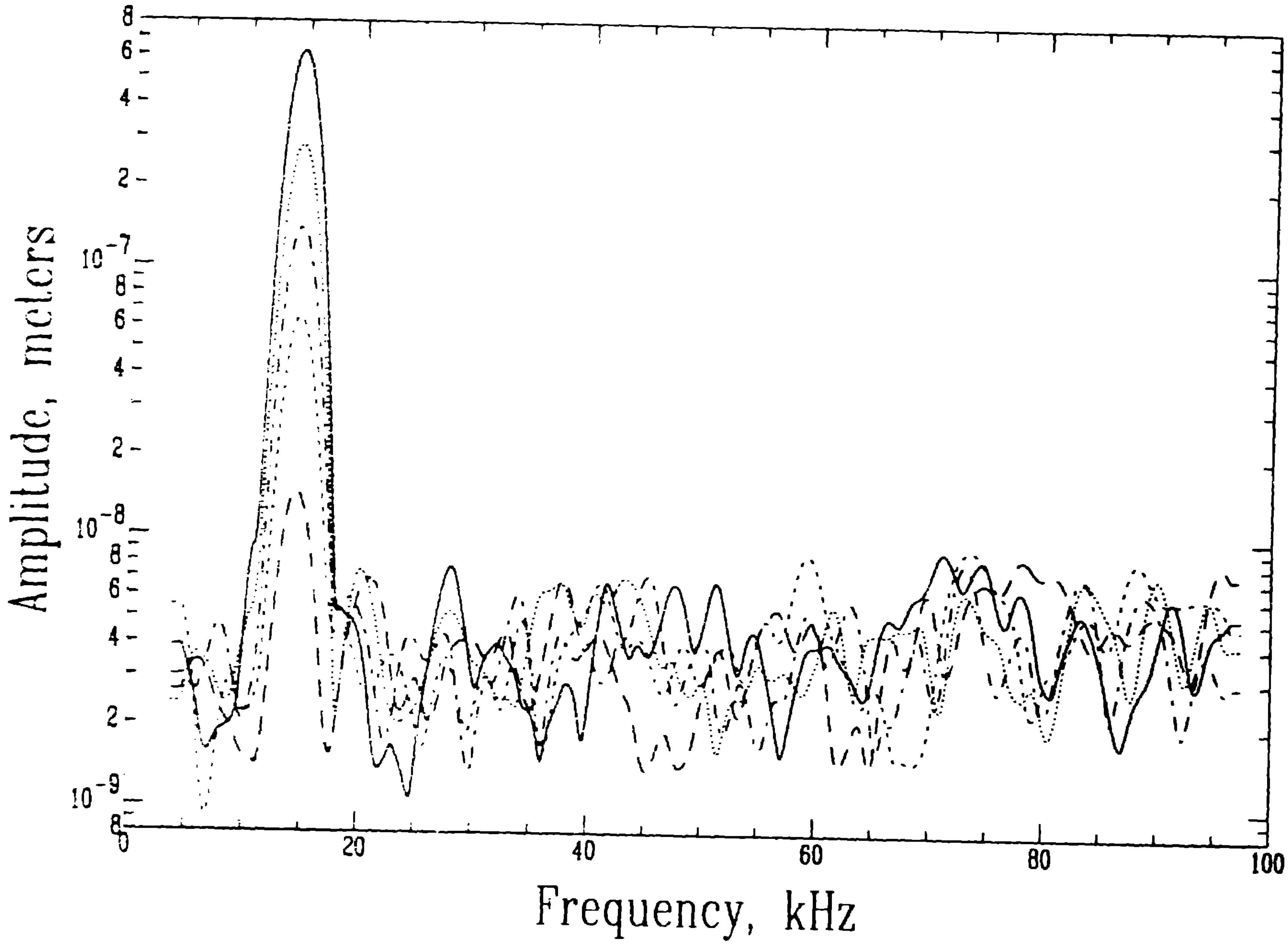
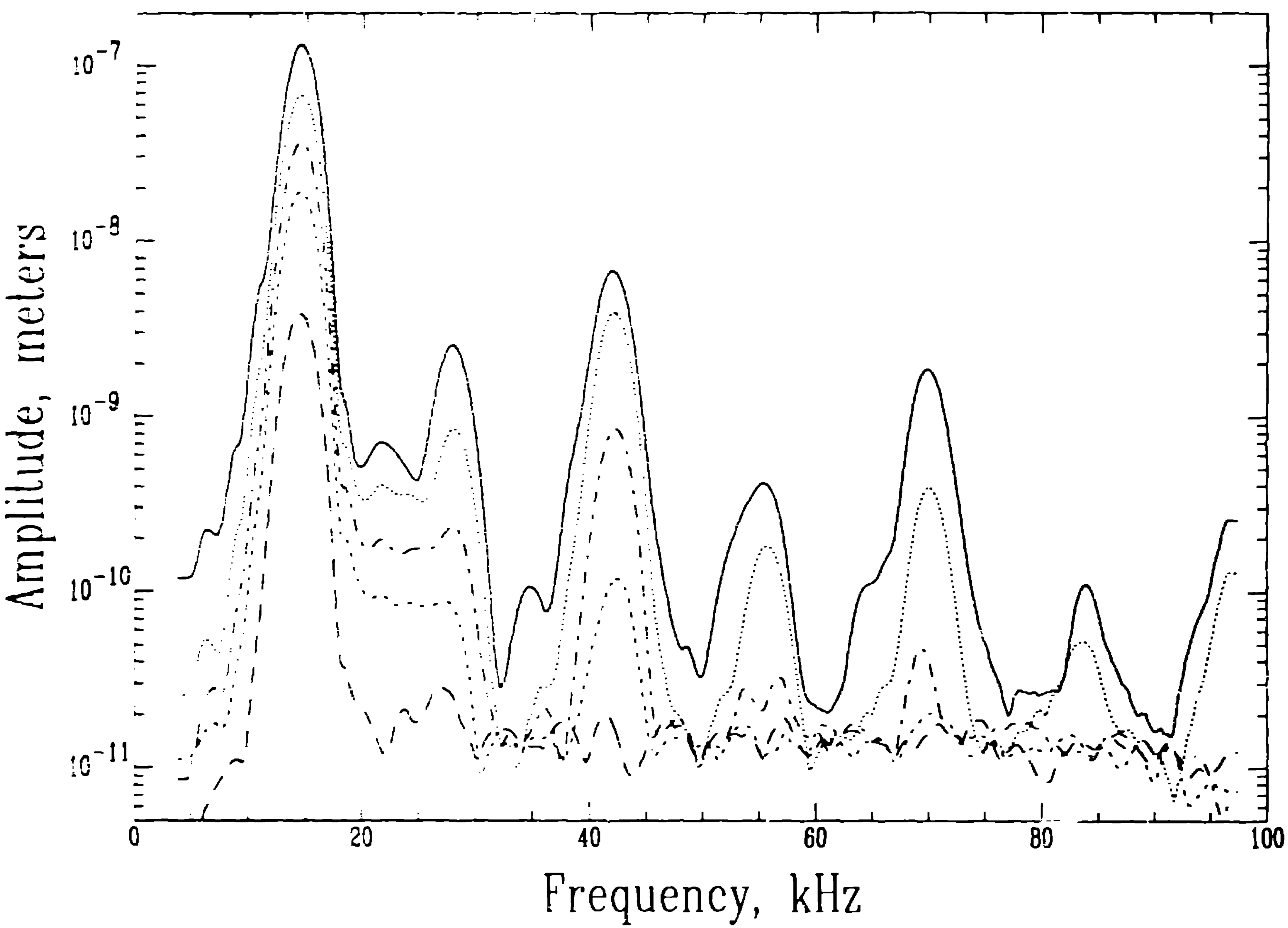
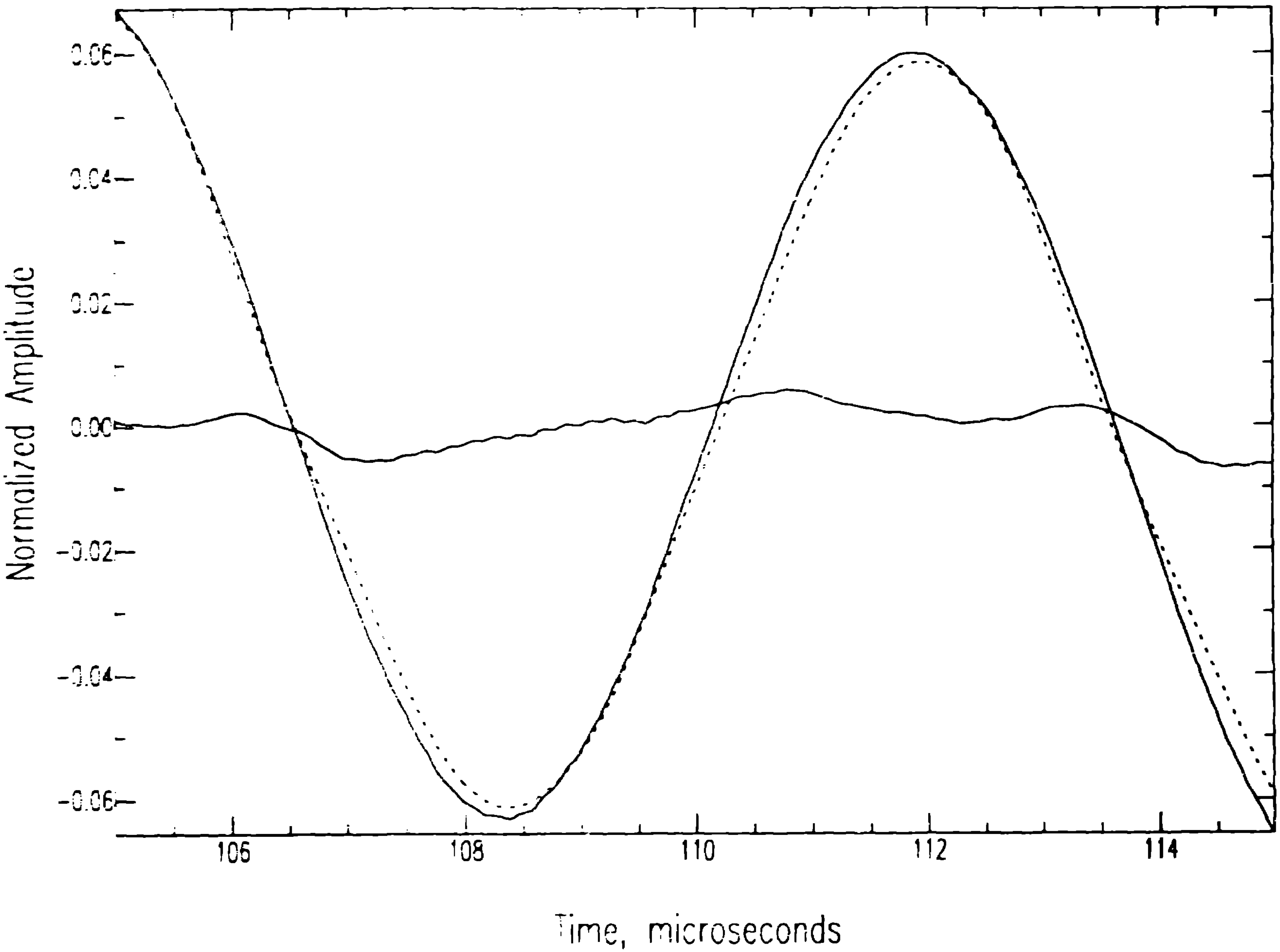
58 cm
Fiber Optic Probe
Detectors
Waveform Analyzer
Optical Displacement Sensor



Amplitude, meters
Frequency, kHz



10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
Amplitude, meters
0
20
40
60
80
100
Frequency, kHz



Normalized Amplitude
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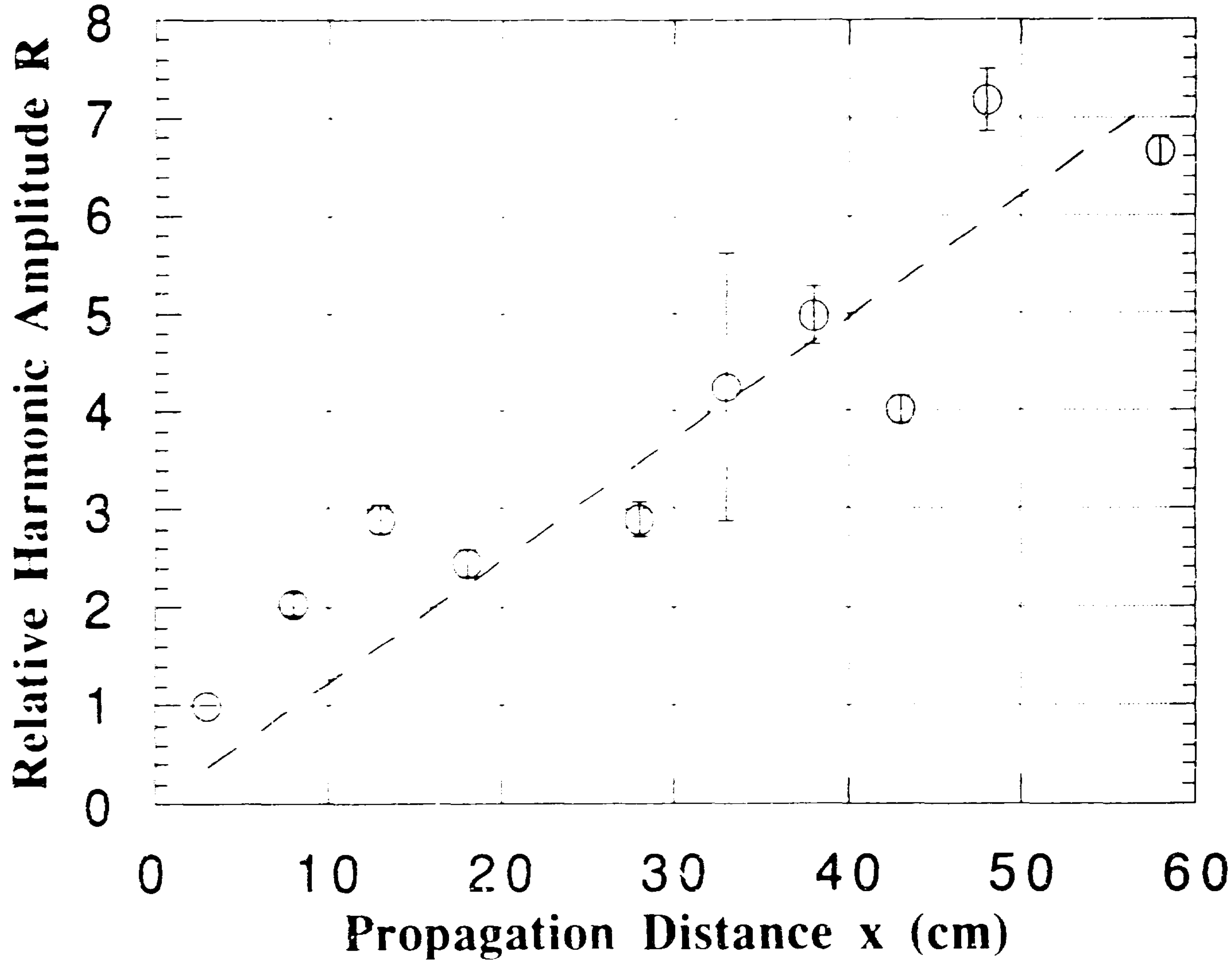
0.02
0.00
-0.02
-0.04
-0.06
106
108
110
112
114
Time, microseconds



Relative Harmonic Amplitude R
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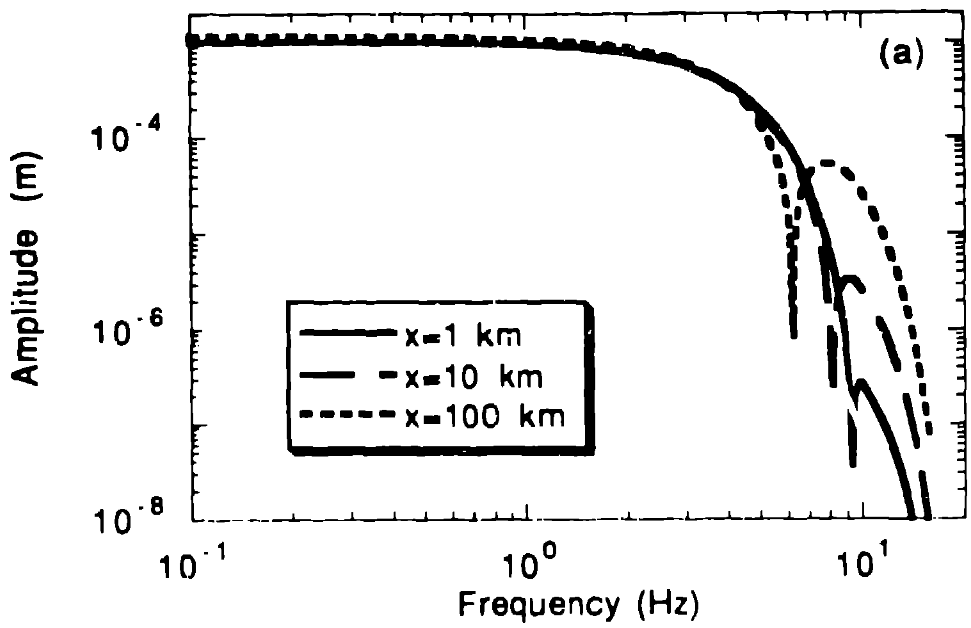
2
3
4
5
6
7
8
0
10
20
30
40
50
60
Propagation Distance x (cm)


(a)
Amplitude (m)
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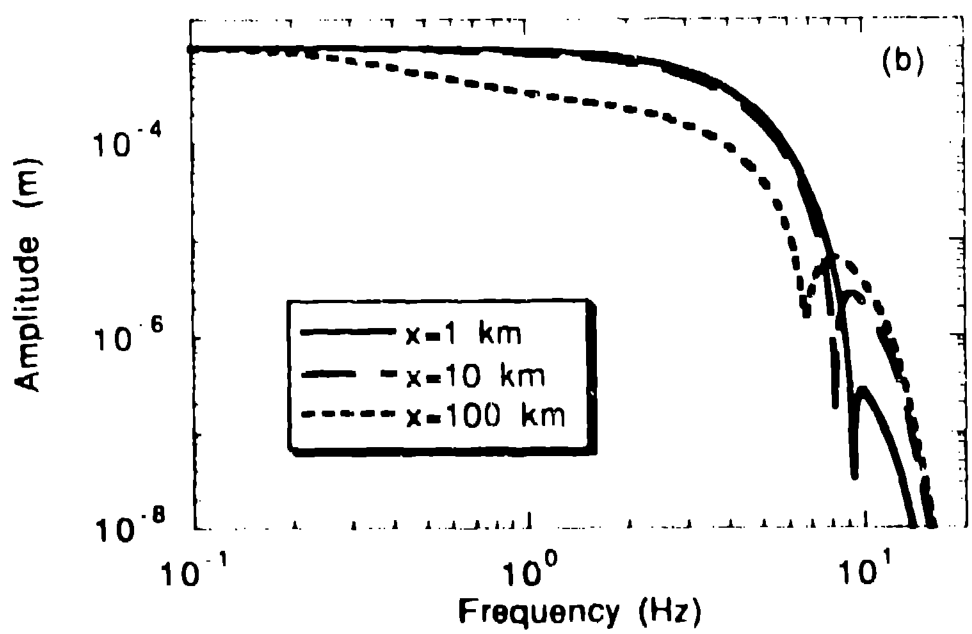
10^-6
10^-8
10^-1
10^0
10^1
Frequency (Hz)
x=1 km
x=10 km
x=100 km


(b)
Amplitude (m)
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10^-6
10^-8
10^-1
10^0
10^1
Frequency (Hz)
x=1 km
x=10 km
x=100 km