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AN ANALYTICAL FORMULA FOR FUEL PARTICLE TO
LIQUID SODIUM HEAT TRANSFER*

by

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An Analytical Formula for Fuel Particle to Liquid Sodium Heat Transfer

by

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Introduction

In the analysis of unprotected loss-of-flow and overpower transients of liquid metal fast breeder reactors, the modeling of heat transfer from molten fuel, ejected into the coolant channel after cladding rupture, to liquid sodium is an important part of fuel-coolant interaction (FCI). Some of the ejected molten fuel fragments into small particles and gets dispersed in the coolant. In the PLUTO2 code model¹ and other modeling efforts² of FCI it is assumed that the fuel particle to liquid sodium heat transfer is limited only by the thermal conduction resistance of the fuel because the thermal conductivity of liquid sodium is about 30 times higher than that of mixed oxide. The fuel particle (assumed to be a sphere) surface temperature, under this assumption, equals the coolant temperature. This is a reasonable approximation as long as the fuel particle is in direct contact with liquid sodium and is not blanketed by a film of sodium vapor or fission gas. Furthermore, the PLUTO2 code model uses constant fuel thermal properties and evaluates the fuel-coolant heat transfer coefficient h using the following equation:

$$h = C_1 \frac{K}{a} V_\ell^{C_2}, \quad (1)$$

where K is fuel thermal conductivity, a is fuel particle radius, V_ℓ is liquid sodium volume fraction in the coolant channel, and C_1 , C_2 are constant parameters.

Purpose:

The purpose of the present analysis is to obtain the value of the parameter C_1 (for $V_\ell = 1$) by solving the linear transient heat conduction equation

$$Hv(r,t) = \frac{A}{K}, \quad (2)$$

for constant heat generation rate A per unit time per unit volume in the fuel particle, uniform initial temperature v_0 of the particle (taking the coolant temperature as the zero reference) and zero particle surface temperature. Here the surface temperature equals the coolant temperature which is assumed to remain constant and equal to the zero reference, $v(r,t)$ is the fuel temperature at time t after contact with the liquid sodium and at radius r of the spherical particle, and the linear differential operator H is given by

$$H \equiv \frac{1}{\alpha} \frac{\partial}{\partial t} - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right), \quad (3)$$

where α is the fuel thermal diffusivity $K/\rho c$, ρ is fuel density and c fuel specific heat.

Analytical Derivation:

The solution of the above linear problem can be obtained by adding the solutions of the two simpler transient conduction problems stated by Eqs. (4) and (5).

$$Hv_1(r,t) = 0, \quad v_1(r,0) = v_0, \quad v_1(a,t) = 0, \quad (4)$$

$$Hv_2(r,t) = \frac{A}{K}, \quad v_2(r,0) = 0, \quad v_2(a,t) = 0, \quad (5)$$

and

$$v = v_1 + v_2. \quad (6)$$

The following solutions of the problems stated by Eqs. (4) and (5) are available in the literature.³

$$v_1 = \frac{2 a v_0}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n \pi r}{a} \exp \left(- \frac{\alpha n^2 \pi^2 t}{a^2} \right), \quad (7)$$

$$v_2 = \frac{A}{6K} (a^2 - r^2) - \frac{2 A a^3}{K \pi^3 r} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} \sin \frac{n \pi r}{a} \exp \left(- \frac{\alpha n^2 \pi^2 t}{a^2} \right). \quad (8)$$

The rate of increase of the total energy E of a fuel particle equals the heat generation rate in the particle minus the rate of heat transfer from the particle to the coolant.

$$\frac{dE}{dt} = \frac{4 \pi a^3 A}{3} - 4 \pi a^2 h \bar{v}, \quad (9)$$

where h is the desired conduction-limited heat transfer coefficient, \bar{v} is the spatially averaged temperature of the fuel particle and represents the temperature difference between the particle and the coolant. Averaging Eqs. (6) to (8) one obtains $\bar{v} = \bar{v}_1 + \bar{v}_2$,

where

$$\bar{v}_1 = \frac{6v_0}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left(- \frac{\alpha n^2 \pi^2 t}{a^2} \right), \quad (10)$$

and

$$\bar{v}_2 = \frac{Aa^2}{15K} - \frac{2Aa^2}{\pi^4 K} \sum_{n=1}^{\infty} \frac{1}{n^4} \exp \left(- \frac{\alpha n^2 \pi^2 t}{a^2} \right) \quad (11)$$

The total energy E of a fuel particle can be written as the product of its volume, density, specific heat and average temperature.

$$E = \frac{4 \pi a^3 \rho c \bar{v}}{3} \quad (12)$$

On substituting the value of E from Eq. (12) into Eq. (9) and solving for h one obtains

$$h = \frac{a}{3\bar{v}} \left(A - \rho c \frac{d\bar{v}}{dt} \right) \quad (13)$$

Result and Discussion:

On substituting the value of \bar{v} from Eqs. (10) and (11) into Eq. (13), the following expression for the heat transfer coefficient is obtained:

$$h = \frac{\frac{2 v_0 K}{a} \sum x_n + \frac{Aa}{3} \left(1 - \frac{2}{\pi^2} \sum x_n/n^2 \right)}{\frac{6v_0}{\pi^2} \sum x_n/n^2 + \frac{Aa^2}{15K} \left(1 - \frac{30}{\pi^4} \sum x_n/n^4 \right)} \quad (14)$$

where x_n is simply a notation for $\exp(-\alpha n^2 \pi^2 t/a^2)$ and all summations run from $n=1$ to infinity. For transient time $t \geq 10$ msec and typical values of mixed oxide fuel thermal properties and particle radius ($K = 0.04$ w/cm 2 -K, $\rho = 10$ gm/cm 3 , $c = 0.5$ J/gm-K, $a = 250$ microns), one obtains $\alpha \pi^2 t/a^2 = 1.26$, and each of the three infinite series in Eq. (14) becomes nearly equal to the first term x_1 . The second terms of the series $\sum x_n$, $\sum x_n/n^2$ and $\sum x_n/n^4$ are less than 2.3%, 0.57% and 0.14% of x_1 respectively. So, for $t > 10$ msec for typical fuel particles Eq. (14) approximates to $(2/\pi^2 = 0.2026, 30/\pi^4 = 0.3080)$ the equation

$$h = \frac{\frac{K}{a} \left[2x_1 + \frac{Aa^2}{3v_0K} (1 - 0.2026 x_1) \right]}{\frac{6x_1}{\pi^2} + \frac{Aa^2}{15v_0K} (1 - 0.3080 x_1)} \quad (15)$$

One more consideration for simplification of Eq. (15) comes from the relative magnitude of the heat source compared to the magnitude of the initial temperature. If the dimensionless ratio $Aa^2/(v_0K) \gg 1$, then the second term is much greater than the first term in the numerator and also in the denominator of Eq. (15). If the dimensionless ratio $Aa^2/(v_0K) \ll 1$, then the first term is dominant in the numerator and the denominator of Eq. (15). For these two extreme cases Eq. (15) reduces to

$$h = \begin{cases} \frac{5K}{a} & \text{if } \frac{Aa^2}{v_0K} \gg 1, \\ \frac{\pi^2}{3} \frac{K}{a} & \text{if } \frac{Aa^2}{v_0K} \ll 1. \end{cases} \quad (16)$$

In the first case in Eq. (16), the ratio $(1 - 0.2026 x_1)/(1 - 0.3080 x_1)$ equals 1.033 at $t=10$ msec for typical fuel particles and approaches unity later into the transient, and hence it has been set equal to 1.0.

Eq. (16) gives the desired fuel particle to liquid sodium heat transfer coefficient for the two extreme cases of (i) fuel heat source dominant, and (ii) initial fuel temperature dominant. For intermediate situations, the factor C_1 (see Eq. (1)) appearing before K/a in the heat transfer coefficient should lie between $\pi^2/3$ and 5.

References

1. H. U. Wider, "PLUTO2: A Computer Code for the Analysis of Overpower Accidents in LMFBRs," Trans. Am. Nucl. Soc., 27, 533 (November, 1977).
2. D. H. Cho, R. O. Ivins, and R. W. Wright, "A Rate-Limited Model of Molten Fuel/Coolant Interactions: Model Development and Preliminary Calculations," ANL-7919, Argonne National Laboratory, Argonne, Illinois (March 1972).
3. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd edition, pp. 235 and 243, Oxford University Press, London (1959).