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OPTIMAL CONTROL STUDIES OF A SOLAR HEATING SYSTEM

by

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ABSTRACT

This paper presents the results of an optimal control study of a solar heating system. The model used was for the National Security and Resources Study Center at Los Alamos Scientific Laboratory. The study shows that the use of optimal control can significantly reduce the requirements for auxiliary energy as compared with conventional control systems.

1. INTRODUCTION

About 12% of the total energy usage in the United States is directed towards the heating and cooling of commercial buildings. A design for heating, ventilating and air conditioning (HVAC) systems that yields a significant saving of energy would therefore make possible a considerable reduction in the total national energy usage. Previous studies using adaptive optimal control techniques have indicated that such an energy-conservative HVAC system design is indeed possible [1] - [3]. The logical next step is, therefore, to establish performance bounds in order to determine if further refinement of control techniques is appropriate. Such bounds may be obtained by carrying out an optimal control study.

This paper describes an optimal control study of the HVAC system in a solar heating building. Perfect knowledge of environmental conditions such as ambient temperature, wind velocity and insolation has been assumed in order to determine whether prior knowledge of such information can be effectively employed to reduce the amount of auxiliary energy used. The study has been based on a model of the 66000 sq. ft. National Security and Resources Study Center at the Los Alamos Scientific Laboratory, a solar heated and cooled building.

2. OPTIMAL CONTROL TECHNIQUE

Many systems are referred to as "optimal" and the sense in which optimality is understood depends on how the performance specification is stated. In the context of this paper, optimal control is understood to mean controlling the interior temperature within some region of comfort while minimizing the amount of auxiliary energy used, and the performance index has been stated in the familiar integral cost functional form.

Let the system be expressed by the usual nonlinear state equation as

$$\dot{\underline{x}}(t) = \underline{F}[\underline{x}(t),\underline{u}(t)] \tag{1}$$

and the performance index by

$$J(\underline{u}) = \int_0^T G[\underline{x}(t),\ \underline{u}(t)]\ dt \tag{2}$$

Then the optimal control is that control $\underline{u}^*(t)$, $0 \leq t \leq T$ that minimizes $J(\underline{u})$. This optimal control may be found by employing Pontryagin's Maximum Principle [4] - [5] which states that corresponding to the optimal control $\underline{u}^*$ and the resultant state trajectory $\underline{x}^*$, there exists a costate trajectory $\underline{p}^*$ such that

$$\text{(a)}\quad \dot{\underline{x}}^*(t) = \frac{d}{dt}\,\underline{x}^*(t) = \frac{\partial}{\partial \underline{p}^*}H[\underline{x}^*(t),\ \underline{p}^*(t),\underline{u}^*(t)]$$

$$\dot{\underline{p}}^*(t) = \frac{d}{dt}\,\underline{p}^*(t) = -\frac{\partial}{\partial \underline{x}^*}H[\underline{x}^*(t),\underline{p}^*(t),\underline{u}^*(t)] \tag{3}$$

$$\text{(b)}\quad \underline{x}^*(0) = \underline{x}_0$$

$\underline{p}^*(T)$ is normal to $\underline{x}^*(T)$

(or if $\underline{x}^*(T)$ is unspecified, $\underline{p}^*(T) = 0$ (4)

$$\text{(c)}\quad H[\underline{x}^*(t),\ \underline{p}^*(t),\ \underline{u}^*(t)] \leq H[\underline{x}^*(t),\underline{p}^*(t),u(t)]$$

for any $\underline{u}(t) \neq \underline{u}^*(t)$. (5)

Here H is the Hamiltonian function:

$$H[\underline{x}(t),\underline{p}(t),\underline{u}(t)] = G[\underline{x}(t),\underline{u}(t)] + \underline{p}^T(t)F[\underline{x}(t),\underline{u}(t)] \tag{6}$$

There are a variety of iterative methods available that utilize these three conditions to generate the optimal control $\underline{u}^*$. Broadly these methods may be classified into three approaches.

In the first, $\underline{u}^*$ is expressed in terms of $\underline{x}^*$ and $\underline{p}^*$, so that $\dot{\underline{x}}^*$ and $\dot{\underline{p}}^*$ may be expressed in terms of $\underline{x}^*$ and $\underline{p}^*$ alone. Starting with an initial guess of $\underline{p}^*_1(0)$, this method attempts to find progressively better guesses $\underline{p}^*_2(0)$, $\underline{p}^*_3(0)$ .... so that the resulting $\underline{p}^*_n(T)$ and $\underline{x}^*_n(T)$ satisfy Eq. (4).

In the second approach, again $\underline{u}^*$ is expressed in terms of $\underline{x}^*$ and $\underline{p}^*$. But here the entire trajectories $\underline{x}^*$ and $\underline{p}^*$ are guessed, attempting to match the trajectories with the specified boundary conditions.

In the third, the control $\underline{u}^*_0$ is guessed and the system integrated using the boundary conditions in (4). Here the aim is to guess progressively better controls $\underline{u}^*_1$, $\underline{u}^*_2$ ... so that Eq. (5) is satisfied.

The system under study is sufficiently complicated to make the explicit expression of $\underline{u}^*$ in terms of $\underline{x}^*$ and $\underline{p}^*$ impractical if not impossible. For this reason, the first two approaches are unsuitable in this application and hence the third approach has been employed here.

Within the third approach there are a number of related algorithms. The algorithm selected here is the Time-Weighted Steepest Descent (TWSD) method [6]. The method consists of these broad steps:

1. Guess $\underline{u}(t)$ and integrate the state equation (1) forward in time to obtain the trajectory $\underline{x}_1(t)$.

2. Using $\underline{p}(T) = 0$ from (4), and the state trajectory $\underline{x}_1(t)$ integrate the costate equation (3) backward in time to obtain the costate trajectory $\underline{p}(t)$.

3. Evaluate

$$\left(\frac{\partial H}{\partial \underline{u}}\right)_1(t) = \frac{\partial H}{\partial \underline{u}}\left[\underline{x}_1(t)\ \underline{p}_1(t), \underline{u}_1(t)\right] \text{using}$$

$\underline{x}_1$, $\underline{p}_1$ and $\underline{u}_1$.

4. Determine a new guess of the control from

$$\underline{u}_2(t) = \underline{u}_1(t) - (K_1^1 + K_1^2\ t)\left(\frac{\partial H}{\partial \underline{u}}\right)_1(t)$$

where K_1^1 and K_1^2 are diagonal matrices whose diagonal entries are found by a parameter optimization technique such as the Rosenbrock Hill Climber [7]. The objective in this step is to obtain the best second guess given $\underline{u}_1$ and $\left(\frac{\partial H}{\partial \underline{u}}\right)_1$. Since "best" is defined as minimum value of the performance index, the system will have to be integrated several times with trial values of the second guess (corresponding to trial values of K_1^1 and K_1^2) before the optimum second guess can be made.

5. Repeat Steps 1 through 4 using $\underline{u}_2$ instead of $\underline{u}_1$.

This process is continued ideally until $\frac{\partial H}{\partial \underline{u}}$ becomes zero. In practice, the iteration is continued until progress has slowed considerably.

Most constraints may be expressed as a cost term so that the method described above may be used without any modification. In the case of magnitude constraints on the controls, it is worthwhile employing a simpler technique called the Clipping-off Technique [8], which is readily incorporated in a gradient method such as the TWSD. The essence of the method is to use the gradient technique as long as the control so obtained is within the allowable magnitude range. If the control violates the magnitude constraint, then the control is allowed to take on only the extreme values of the range, as below:

$$u = \begin{cases} u_{min}, & u < u_{min} \\ u, & u_{min} < u < u_{max} \\ u_{max}, & u > u_{max} \end{cases}$$

3. HEATING SYSTEM AND BUILDING MODEL

A simplified model of a solar heating system for the building is used. The system consists of an unglazed selective surface solar collector, collector coolant loop, a one-node storage tank, an air heating coil, and a heated and ventilated enclosure with air recirculation. It is assumed that the same fluid is used in the collector, storage tank, and heating coil. To meet minimum fresh air requirements, the air flow contains a quantity of outside air which is at least 10% of the maximum air flow rate. The rest may be recirculated. Auxiliary energy is added through heated water from an auxiliary heat exchanger. Mixing from the auxiliary heat exchanger and the storage tank is allowed. A schematic of the system is shown in Fig. 1.

Four differential equations describe the system:

$$M_c C_{p_c} \dot{T}_c = A_c[\alpha Q_c - \sigma\varepsilon_c((T_c + 460)^4 - (T_a + 460)^4) - C_w(1+0.3V)(T_c - T_a) - 2U_c(T_c - T_{cc})] \quad (7)$$

$$M_{cc}C_{p_{cc}}\dot{T}_{cc} = C_{p_{cc}} W_c (T_s - T_{cc}) + 2U_c A_c (T_c - T_{cc}) \quad (8)$$

$$M_s C_{p_s} \dot{T}_s = C_{p_{cc}} W_c (T_{cc} - T_s) + C_{p_s} W_{hws} (T_{hwr} - T_s) \quad (9)$$

$$M_r C_{p_r} \dot{T}_r = C_{p_r} W_s (T_{rs} - T_r) - (UA)_b (T_r - T_a) + Q_{ri} \quad (10)$$

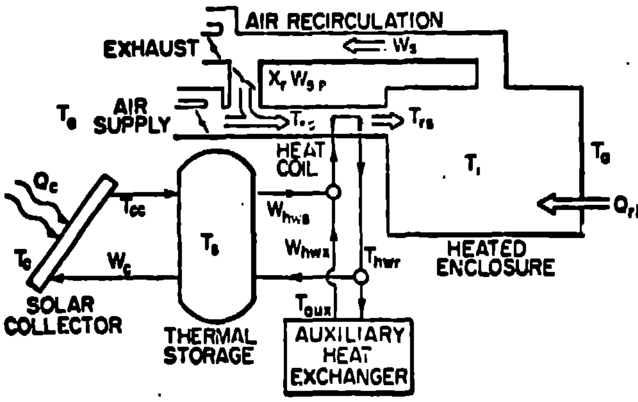

Fig. 1. Solar heating system.

The algebraic equations which describe the performance of the heating coil are

$$T_{rs} = \frac{z_w}{(1+z_w)(1+z_a)-1} T_{wsp} + \left[1 - \frac{z_w}{(1+z_w)(1+z_a)-1}\right] T_{ss} \quad (11)$$

$$T_{hwr} = \left[1 - \frac{z_a}{(1+z_w)(1+z_a)-1}\right] T_{wsp} + \frac{z_a}{(1+z_w)(1+z_a)-1} T_{cs} \quad (12)$$

where

$$z_w = \frac{C_{p_s} (W_{hws} + W_{hwx})}{(UA)_h} \quad (13)$$

$$z_a = \frac{C_{p_r} W_s}{(UA)_h} \quad (14)$$

$T_{wsp} = T_s$, T_{aux}, or a mass flow weighted sum of T_s and T_{aux}, as appropriate (15)

and the mixed supply in temperature is

$$T_{ss} = \frac{[13200+(1-X_r)(W_s - 13200)]T_a + X_r(W_s - 13200)T_r}{W_s} \quad (16)$$

The algebraic relation for computing X_r, the air recirculation fraction is

$$X_r = \frac{8 - P_o + C_{ss}(T_{ss} - 60) + C_r(T_r - 70)}{5} \quad (17)$$

Equations (16) and (17) have to be simultaneously solved for T_{ss} and X_r.

The variables are

- T_c = Collector surface temperature
- T_{cc} = Collector coolant outlet temperature
- T_s = Mixed storage temperature
- T_{aux} = Auxiliary hot water temperature
- T_r = Room air temperature
- T_{rs} = Room supply air (heating coil outlet) temperature
- T_{wsp} = Heating coil water supply temperature
- T_{hwr} = Hot water return temperature
- T_{ss} = Mixed air supply temperature
- T_a = Ambient temperature
- Q_c = Collector solar flux
- Q_{ri} = Interior heat load
- W_c = Collector coolant mass flow rate
- W_{hws} = Heating coil water mass flow rate from storage
- W_{hwx} = Heating coil water mass flow rate from auxiliary
- W_s = Ventilating air mass flow rate
- X_r = Air recirculation fraction
- V = Outside wind velocity

The parameters and some nominal values are given below. All values have been expressed in BTU, feet, hours, lbs. and degrees Farenheit.

A_c	= Collector surface area	
M_c	= Collector mass	
C_{p_c}	= Collector specific heat	
U_c	= Collector heat transfer coefficient	= 37.0
$2U_cA_c$		= 5.93×10^5
$A_c/(M_cC_{p_c})$		= 3.69
α	= Collector surface absorptance	= 0.9
σ	= Stefan-Boltzman radiation constant	
ϵ_c	= Collector surface emittance	
$\sigma\epsilon_c$	=	= 3.12×10^{-10}
C_w	= Wind convection coefficient	= 0.5
M_{cc}	= Collector coolant mass	= 5780.0
$C_{p_{cc}}$	= Collector coolant specific heat	= 1.0
M_s	= Storage mass	= 83333.0
C_{p_s}	= Storage specific heat	= 1.0
M_r	= Room air mass	= 42190.0
C_{p_r}	= Room air specific heat	= 0.24
$(UA)_h$	= Reheat coil total heat transfer coefficient	= 90086.0
$(UA)_b$	= Building overall heat loss coefficient	= 4143.0
P_o		= 2
C_{ss}		= 1
C_r		= 16

The weather and solar insolation data are taken from actual measurements on a day which had temperature extremes of 8°F and 23°F, wind velocities between 2 and 12 miles per hour and maximum solar insolation of 284.8 BTU/hr - ft^2. The auxiliary storage temperature is fixed at 140°F based on previous studies of this system.

The variable controls are W_c, W_{hws}, W_{hwx} and W_s. The magnitude constraints on these controls are:

$$0 \leq W_c \leq 160\,000$$

$$0 \leq W_{hws} + W_{hwx} \leq 100,000$$

$$66,000 \leq W_s \leq 132,000$$

Since one constraint has been placed on $W_{hws} + W_{hwx}$, it was decided to compute the four controls as:

$$u_1 = W_c$$

$$u_2 = W_{hws} + W_{hwx}$$

$$u_3 = W_{hwx}$$

$$u_4 = W_s$$

with constraints:

$$0 \leq u_1 \leq 160,000$$

$$0 \leq u_2 \leq 100,000$$

$$0 \leq u_3 \leq u_2$$

$$66,000 \leq u_4 \leq 132,000$$

The performance index was computed as the weighted sum of the total auxiliary energy used and a term defining closeness of the room temperature to 70°:

$$J = \int_0^{24} \{W_{hwx}(T_{aux} - T_{wr})C_{ps}X10^{-3} + (T_r - 70.)^4\}dt$$

4. RESULTS

The results of the optimal control study are indicated in Table I. It is seen that auxiliary energy savings of about 30% as compared with conventional control [1] is possible.

TABLE I

	CONVENTIONAL	OPTIMAL
ROOM TEMP		
AVERAGE	70.0	69.6
MINIMUM	69.8	68.5
MAXIMUM	70.3	70.2
ENERGY USED		
AUXILIARY	3.74	2.59
SOLAR	1.04	2.12
INTERNAL HEAT	6.12	6.12
TOTAL	10.91	10.83
PERCENTAGE		
AUXILIARY	34.3	23.9
SOLAR	9.6	19.6
INTERNAL	56.1	56.5
SAVINGS OF AUX. ENERGY		31%

The most important factor for this considerable savings in energy is the more effective utilization of solar energy. Optimal control keeps the storage tank at a much lower temperature [see Fig. 2] so that there is a larger temperature differential between the collector coolant and the storage tank and, consequently, more solar energy is collected. This improvement in collection efficiency is indicated in Fig. 3.

In this study, perfect knowledge of the weather parameters was assumed in order to investigate whether such advance knowledge can help control temperature in a more energy-conservative manner. Comparison of these results with those of adaptive control studies [2] that did not assume such information imply that prior knowledge of weather conditions did not significantly aid in reducing auxiliary energy usage.

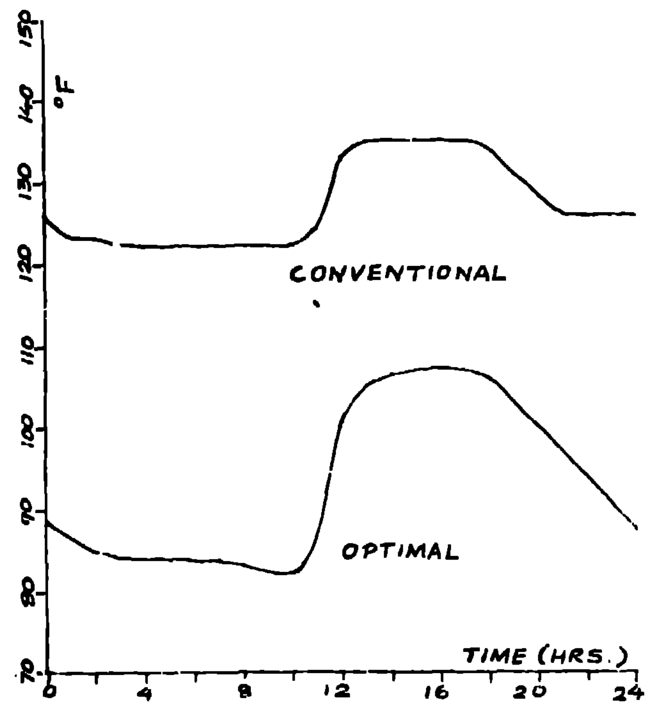

Fig. 2 Storage Tank Temperature

5. ACKNOWLEDGEMENT

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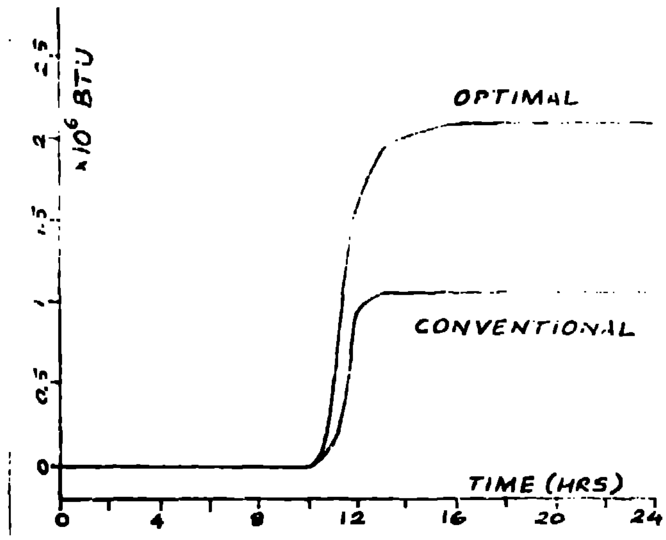

Fig. 3 Solar Energy Collected

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