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A DISCRETE NODAL INTEGRAL TRANSPORT-THEORY METHOD FOR MULTIDIMENSIONAL REACTOR PHYSICS AND SHIELDING CALCULATIONS

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ABSTRACT

A coarse-mesh discrete nodal integral transport theory method has been developed for the efficient numerical solution of multidimensional transport problems of interest in reactor physics and shielding applications. The method, which is the discrete transport theory analogue and logical extension of the nodal Green's function method previously developed for multidimensional neutron diffusion problems, utilizes the same transverse integration procedure to reduce the multidimensional equations to coupled one-dimensional equations. This is followed by the conversion of the differential equations to local, one-dimensional, in-node integral equations by integrating back along neutron flight paths. One-dimensional and two-dimensional transport theory test problems have been systematically studied to verify the superior computational efficiency of the new method. It typically required a computational effort that was significantly reduced from that of the widely-used discrete S_N method for the one-dimensional test problems studied, and an even greater reduction was achieved for the two-dimensional transport test problems.

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INTRODUCTION

Prompted by the high accuracy obtained on very coarse meshes and concomitant high computational efficiencies achieved in multidimensional thermal and fast reactor neutron diffusion problems by the nodal Green's function method,[1-6] two nodal integral transport methods, the discrete nodal transport method (DNTM) and the piecewise polynomial nodal transport method (PNTM) have been developed recently and applied to one-dimensional transport problems.[5,6] This paper extends and applies one of those methods, the DNTM, to two-dimensional problems. This method is developed for multidimensional geometries by first integrating the transport equation over all but one of the independent Cartesian spatial variables[4-7] within a computational node to convert the multidimensional transport equations to coupled one-dimensional transport equations, and then discretizing the angular flux in a manner analogous to the discretization in the original Wick-Chandrasekhar discrete-ordinates method. The resulting coupled local ordinary differential equations are then integrated back along the discretized neutron flight directions within the node to obtain local within-node one-dimensional integral equations for the discretized transverse-integrated directional fluxes that are coupled to nearest-neighboring node fluxes through the continuity of the transverse-integrated discretized surface directional fluxes.

As a result of the transverse integration procedure,[4-7] and the formal mathematical details which follow, the required local coupling matrices are more easily calculated and the resulting global solution more easily obtained than when the full multidimensional transport equation is treated as in a direct multidimensional coarse-mesh method.[9-11] The present method is related to the Nodal Discrete-Ordinates Method;[7] however, the assumptions, on the azimuthal symmetry of the transverse-integrated flux, on the isotropy of the incoming node-surface angular currents and on the double-P_1 representation of the incoming transverse-integrated node-surface angular fluxes, made in the development of that method[7] are not made here. Hence, numerical solutions for integral quantities obtained via the present DNTM converge with decreasing node size to the exact solution of the Wick-Chandrasekhar discrete-ordinates equations.[8]

FORMALISM

Although the formalism for the multidimensional discrete nodal transport method previously was presented briefly in Appendix A of Reference 6, it is repeated here for completeness. The development is for two-dimensional Cartesian geometry; however, the extension to three-dimensional Cartesian geometry is straightforward. The formalism begins by partitioning the system into K homogeneous nodes and writing the two-dimensional multigroup transport equation for the k-th node

$$\frac{2\mu}{\Delta x}\frac{\partial}{\partial x}\psi_g^k(x,y,\mu,\eta) + \frac{2\eta}{\Delta y}\frac{\partial}{\partial y}\psi_g^k(x,y,\mu,\eta) + \Sigma_g^{t,k}\psi_g^k(x,y,\mu,\eta) =$$

$$S_g^k(x,y,\mu,\eta),\ -1\le x\le +1,\quad -1\le y\le +1,\quad k=1,\ \ldots,\ K,\ g=1,\ \ldots,\ G, \tag{1}$$

where the dimensionless (local) variables x and y are written in terms of the node half-widths $\frac{\Delta x}{2}$ and $\frac{\Delta y}{2}$. The source term $S_g^k(x,y,\mu,\eta)$ includes contributions from inscattering, fission, and external sources. Analogous to the diffusion-theory NGFM, the multidimensional equation is reduced to two coupled one-dimensional equations by integrating Eq. (1) seperately over each of the two co-ordinate directions. For example, integrating Eq. (1) over y from -1 to +1 yields the x-dependent equation

$$\frac{2\mu}{\Delta x}\frac{\partial}{\partial x}\psi_{gx}^k(x,\mu,\eta) + \Sigma_g^{t,k}\psi_{gx}^k(x,\mu,\eta) = S_{gx}^k(x,\mu,\eta) - L_{gx}^k(x,\mu,\eta), \tag{2}$$

where ψ_{gx}^k and S_{gx}^k are partially-integrated angular fluxes and sources, e.g.

$$\psi_{gx}^k(x,\mu,\eta) = \frac{1}{2}\int_{-1}^{1} dy\ \psi_g^k(x,y,\mu,\eta), \tag{3}$$

and the leakage in the transverse direction is

$$L_{gx}^k(x,\mu,\eta) = \frac{\eta}{\Delta y}\left[\psi_g^k(x,1,\mu,\eta) - \psi_g^k(x,-1,\mu,\eta)\right]. \tag{4}$$

Equation (2) is then solved separately for $\mu<o$ and $\mu>o$. For $\mu>o$, the solution is

$$\psi_{gx}^k(x,\mu,\eta) = \frac{\Delta x}{2}\int_{-1}^{1} dx_o\ \frac{1}{\mu}\ e^{-\Sigma(x-x_o)/\mu}\left\{S_x^k(x_o,\mu,\eta) - L_{gx}^k(x_o,\mu,\eta)\right\}$$

$$+\ \psi_{gx}^k(-1,\mu,\eta)\ e^{-\Sigma(x+1)/\mu}, \qquad \mu>o, \tag{5}$$

where $\Sigma \equiv \frac{\Delta x}{2}\Sigma_g^{t,k}$. Equation (5) gives the node-interior transverse-integrated group angular flux in terms of the transverse-integrated sources within the node and the group angular fluxes on the three surfaces of the

node. An equation for the outgoing transverse-integrated flux on the right surface is obtained by evaluating Eq. (5) at x=1:

$$\psi^k_{gx}(1,\mu,\eta) = \frac{\Delta x}{2}\int_{-1}^{1} dx_o \frac{1}{\mu} e^{-\Sigma(1-x_o)/\mu}\left\{ S^k_{gx}(x_o,\mu,\eta) - L^k_{gx}(x_o,\mu,\eta)\right\}$$

$$+ \psi^k_{gx}(-1,\mu,\eta)\ e^{-2\Sigma/\mu}, \qquad \mu>o. \tag{6}$$

Two analogous equations are obtained by solving Eq. (2) for $\mu>o$. Finally, four additional equations are derived for the y-dependent partially-integrated fluxes by integrating Eq. (1) over x from -1 to +1. Thus, a total of eight coupled equations must be solved for the two-dimensional case. The nodes are coupled by requiring that the angular fluxes calculated using Eq. (6) be continuous across the node boundaries.

APPROXIMATION TECHNIQUES

The angular dependence in Eqs. (5) and (6) is approximated by projecting onto discrete ordinates, and the one-dimensional spatial dependence of the node-interior fluxes and sources, and the spatial dependence of the transverse leakages are expanded in local Legendre polynomials:

$$\psi^k_{gx}(x,\mu_i,\eta_i) = \sum_{n=o}^{N} \frac{2n+1}{2}\ \psi^k_{gxn}(\mu_i,\eta_i)\ P_n(x) \tag{7a}$$

$$S^k_{gx}(x,\mu_i,\eta_i) = \sum_{n=o}^{N} \frac{2n+1}{2}\ S^k_{gxn}(\mu_i,\eta_i)\ P_n(x) \tag{7b}$$

$$L^k_{gx}(x,\mu_i,\eta_i) = \sum_{n=o}^{NS} \frac{2n+1}{2}\ L^k_{gxn}(\mu_i,\eta_i)\ P_n(x). \tag{7c}$$

An equation for the expansion coefficients of the angular fluxes is obtained by substituting Eqs. (7) into Eq. (5), weighting with $P_n(x)$, n=0,...,N, and integrating over x from -1 to +1:

$$\underline{\psi}^k_{gx}(\mu_i,\eta_i) = [G^{xx}_g(\mu_i)]\left\{ \underline{S}^k_{gx}(\mu_i,\eta_i) - \underline{L}^k_{gx}(\mu_i,\eta_i)\right\}$$

$$+[G^{x+}_g(\mu_i)]\psi^k_{gx}(-1,\mu_i,\eta_i), \qquad \mu_i>o. \tag{8}$$

where $\underline{\psi}^k_{gx}$, $\underline{S}^k_{gx}$, and $\underline{L}^k_{gx}$ are vectors of expansion coefficients.

Substituting Eqs. (7b) and (7c) into Eq. (6) yields the discretized equation for the outgoing node-surface angular flux:

$$\psi^k_{gx}(1,\mu_i,\eta_i) = [G^{+x}_g(\mu_i)]^T\left\{ \underline{S}^k_{gx}(\mu_i,\eta_i) - \underline{L}^k_{gx}(\mu_i,\eta_i)\right\}$$

$$+ \psi^k_{gx}(-1,\mu_i,\eta_i)\ e^{-2\Sigma/\mu_i}, \qquad \mu_i>o. \tag{9}$$

The entries of the matrices in Eqs. (8) and (9) are given by

$$\left[G_g^{xx}(\mu_i)\right]_{mn} = \frac{\Delta x}{2}\,\frac{2n+1}{2}\int_1^1 dx\; P_m(x)\int_{-1}^{1} dx_o\;\frac{1}{\mu_i}\; e^{-\Sigma(x-x_o)/\mu_i}\; P_n(x_o) \qquad (10a)$$

$$\left[G_g^{x+}(\mu_i)\right]_m = \int_{-1}^{1} P_m(x)\; e^{-\Sigma(x+1)/\mu_i} \qquad (10b)$$

$$\left[G_g^{+x}(\mu_i)\right]_n = \frac{\Delta x}{2}\,\frac{2n+1}{2}\int_{-1}^{1} dx\;\frac{1}{\mu_i}\; e^{-\Sigma(1-x)/\mu_i}\; P_n(x). \qquad (10c)$$

The expansion coefficients for the space-dependent transverse leakage must be determined using the information available from the solution of Eq. (9), i.e. the average values of the node-surface angular fluxes. From Eqs. (4) and (7c), it is clear that the P_o spatial moment of the transverse leakage is

$$L_{gxo}^{k}(\mu_i,\eta_i) = \frac{2\eta_i}{\Delta y}\left[\psi_{gy}^{k}(1,\mu_i,\eta_i) - \psi_{gy}^{k}(-1,\mu_i,\eta_i)\right]. \qquad (11)$$

Hence a simple "flat" transverse leakage approximation can be constructed using Eq. (11) with NS≡0 in Eq. (7c). In the NGFM developed for diffusion theory,[6] a quadratic approximation for the transverse partial currents across a node surface in two-dimensional problems was constructed by fitting partial currents on the node surface and on the two colinear adjacent-node surfaces. However, the use of exactly analogous fits to construct higher-order transverse leakage approximations in the two-dimensional DNTM is not appropriate since the transverse leakage for a node depends upon the spatial shape of the angular flux on the surface intersected back along the ordinate and not upon the angular fluxes on the two colinear adjacent-node surfaces. Only the flat transverse leakage approximation given by Eq. (11) is used here. However, higher-order approximations utilizing information obtained by tracing back along the ordinate would most likely yield more accurate results.

Equations (8) and (9) are solved using directed sweeps through the spatial mesh similar to those used to solve the discrete S_N equations.[13] The outgoing angular fluxes along an ordinate are calculated by solving Eq. (9) and its y-counterpart simultaneously for these angular fluxes on the two "outgoing" surfaces (the two surfaces which neutrons cross when leaving the node in the direction of the ordinate). These surface angular fluxes are then used to update the leakage terms needed in Eq. (8) and its y-counterpart for the calculation of the expansion coefficients of the node-interior angular fluxes. Both inner (in-group scatter) and outer (fission source) iterations are accelerated using coarse-mesh rebalancing[13] and asymptotic source extrapolation.

The equations for one-dimensional applications are obtained from Eqs. (8) and (9) by eliminating the transverse leakage terms and suppressing the

η-dependence of the angular fluxes and sources. An error analysis for these one-dimensional equations is summarized in Appendix A. As shown there the global truncation error of the one-dimensional DNTM equations is $O(\Delta x^{2N+2})$, where N is the order of the flux and source expansions introduced in Eqs. (7a) and (7b).

A simple device that leads to a slightly modified formulation of the DNTM can be used to decrease the actual error in calculations for scattering-dominated systems. As discussed in Appendix B, this simple intuitive device leads to a more accurate representation of the actual removal along neutron flight paths by approximately accounting for inscattering and fission as well as outscattering and absorption along the paths.

In many applications the discretization of the angular dependence of the flux leads to poor results. This difficulty can be overcome by expanding the angular flux in piecewise continuous functions. This has been done in the development of the one-dimensional piecewise polynomial nodal transport method (PNTM)[6] which in all cases studied[6] was superior to the DNTM. The extension of that method to two-dimensional applications would utilize piecewise continuous spherical harmonics expansions of the node-surface and node-interior angular fluxes on subdomains (or patches) on the unit $\hat{\Omega}$-sphere and therefore would be free of the ray effects that result from discretization of the angular variables.

NUMERICAL RESULTS

Numerical studies[6] based on applications on the DNTM and PNTM to a series of one-dimensional test problems have shown that both methods represent significant improvements over the conventional diamond-difference discrete-S_N method. For a one-dimensional eight-group representation[5] of a 1000-Mwe heterogeneous-design fast reactor, the DNTM and PNTM required seven and fourteen times less computer time than ILLSN (ILLinois S_N), a special-purpose in-house discrete-S_N code which uses the same convergence-acceleration schemes as those incorporated in the DNTM and PNTM. For this same problem, the DNTM and PNTM were one hundred and two hundred times faster than the widely-used discrete-S_N code ANISN[14] for comparable accuracy requirements.

Table I summarizes DNTM D_8 and TWOTRAN[13] S_8 results[15] for the two-dimensional ray-effect benchmark problem.[15] (In order to avoid confusion with the discrete-S_N method, we use D_N to denote a DNTM calculation using an S_N quadrature set.) The DNTM results were obtained using a quadratic expansion of the node-interior fluxes and the flat transverse leakage approximation. Although the errors in both the 16-node DNTM calculation and the 6804-cell TWOTRAN calculation are 1.5×10^{-2}, the DNTM calculation required 150 times less computer time than the TWOTRAN calculation. These results show that the DNTM, even with the lowest-order (flat) approximation for the spatial dependence of the angular fluxes on the transverse surfaces, is capable of very high computational efficiency (high accuracy in small computing times) relative to standard discrete-S_N methods for multidimensional problems.

TABLE I

SUMMARY OF DNTM D_8 AND TWOTRAN[a] S_8 RESULTS FOR THE TWO-DIMENSIONAL RAY-EFFECT BENCHMARK PROBLEM

Method	No. of Mesh Cells	Total Leakage (Group 1)	Relative Error[a]	Execution Time[b] (s)
DNTM	16	5.918×10^{-4}	1.5×10^{-2}	2.3
DNTM	64	5.829×10^{-4}	3.4×10^{-4}	6.1
DNTM	256	5.827×10^{-4}	$<2.0\times10^{-4}$	25.1
DNTM	1024	5.827×10^{-4}	Ref. Sol.	120.2
TWOTRAN[a]	756	5.10×10^{-4}	1.3×10^{-1}	≅39
TWOTRAN	3024	5.64×10^{-4}	3.3×10^{-2}	≅154
TWOTRAN	6804	5.74×10^{-4}	1.5×10^{-2}	≅356
TWOTRAN	(∞)[c]	5.82×10^{-4}	$<2.0\times10^{-3}$	-

[a] TWOTRAN results taken from Ref. 13 in which total leakages were reported to only three digits.

[b] CYBER 175. The TWOTRAN times for the CYBER 175 were obtained by dividing the CDC 6600 execution times given in Ref. 13 by 2.5. Pointwise convergence criterion of 10^{-5} was used in DNTM calculations.

[c] Obtained by h^2-extrapolation of the three-digit TWOTRAN results.

Table II summarizes results for a light water reactor cell problem[17] consisting of a fuel region with 4x4 homogenized pin cells surrounded by a light water moderator. The DNTM results show that the calculations using a 6x6 mesh layout are accurate to 10^{-4}. The DNTM D_8 calculation is in excellent agreement with the results obtained using QP1[10] and COXY[17], both of which use expansions in angle to approximate the node-surface angular fluxes. The DNTM (D_8) execution time is slightly greater than the times for QP1 and COXY; however, the spatial errors in the DNTM calculation appear to be smaller. This is probably because the DNTM uses a quadratic source approximation whereas the QP1 and COXY use a linear source approximation.

TABLE II

SUMMARY OF RESULTS FOR A LWR CELL PROBLEM

Method	No. of Mesh Cells	k-eff	Execution Time[a] (s)
DNTM (D_2)	6x 6	1.2186	0.9
DNTM (D_2)	12x12	1.2186	4.1
DNTM (D_4)	6x 6	1.2138	3.0
DNTM (D_4)	12x12	1.2139	10.8
DNTM (D_8)	6x 6	1.2127	9.4
DNTM (D_8)	12x12	1.2127	37.5
QP1	6x 6	1.2128	≅2.7
QP1	10x10	1.2124	≅6.5
COXY	6x 6	1.2129	≅2.8
COXY	10x10	1.2140	≅7.7

[a]CYBER 175. The QP1 (COXY) times for the CYBER 175 were obtained by dividing the CDC 6500 (CDC 6600) execution times given in Ref. 10 (17) by 5.0 (2.5). A pointwise convergence criterion of 10^{-3} was used in the DNTM and QP1 calculations.

SUMMARY

A new discrete nodal transport method for the numerical solution of the two-dimensional neutron transport equation in Cartesian geometry has been developed and numerically tested. A transverse integration procedure was used in the formalism to reduce the multidimensional transport equation within a node to coupled one-dimensional transport equations defined in ficticious node-engulfing slabs. These slab transport equations were then formally solved to yield one-dimensional integral equations. The integral equations were solved approximately by projecting the angular dependence onto discrete ordinates and expanding the one-dimensional spatial dependence in Legendre polynomials. Numerical results have demonstrated that the method is capable of very high accuracy on coarse spatial meshes and significantly increased computational

efficiency relative to standard discrete-S_N methods for systems that have large homogeneous subregions. Hence the DNTM is potentially most useful for shielding calculations and for global reactor calculations where transport theory effects are important such as heterogeneous-core fast reactors.

APPENDIX A: ERROR ANALYSIS

For isotropic scattering and one-dimension geometry, Eq. (9) becomes

$$\psi^k_{gx}(1,\mu_i) = \sum_{n=o}^{N} \left[G_g^{+x}(\mu_i)\right]_n S_n + \psi^k_{gx}(-1,\mu_i)e^{-2\Sigma/\mu_i} \tag{A-1}$$

where the source moments are given by

$$S_n = \int_{-1}^{1} dx\ P_n(x)S(x) \tag{A-2}$$

and $\left[G_g^{+x}(\mu_i)\right]_n$ is defined by Eq. (10a). The local truncation error in Eq. (1) is composed of contributions from (1) truncating the source expansion after N terms, and (2) errors in the calculation of all source moments S_n for n=0,...,N. Hence the local error is

$$E_{Loc} = \left[G_g^{+x}(\mu_i)\right]_{N+1} S_{N+1} + \sum_{n=o}^{N} \left[G_g^{+x}(\mu_i)\right]_n E_n \tag{A-3}$$

where E_n is the error in approximating S_n. An expression for this error can be obtained from the one-dimensional form of Eq. (8); the result is

$$E_n = \sum_{i=1}^{I} w_i \left\{ c\left[G_g^{xx}(\mu_i)\right]_{n,N+1} S_{N+1} + c\sum_{n'=o}^{N} \left[G_g^{xx}(\mu_i)\right]_{nn'} E_{n'} \right\} \tag{A-4}$$

where w_i are the discrete ordinate weights and c is a constant relating flux moments to source moments. Since

$$\begin{aligned} S_{N+1} &= \int_{-1}^{1} dx\ P_{N+1}(x)\ S(x) \\ &= \int_{-1}^{1} dx\ P_{N+1}(x) \left[S(o) + x\ S'(o) + \tfrac{1}{2}x^2 S''(o) + \ldots\right] \\ &= O(\Delta x^{N+1}), \end{aligned} \tag{A-5}$$

and $$\left[G_g^{+x}(\mu_i)\right]_n = O(\Delta x^{n+1}), \tag{A-6}$$

$$\left[G_g^{xx}(\mu_i)\right]_{mn} = O(\Delta x^{1+|m-n|}), \tag{A-7}$$

Eq. (A-3) yields

$$E_{Loc} = O(\Delta x^{2N+3}) + \sum_{n=o}^{N} E_n\, O(\Delta x^{n+1}). \tag{A-8}$$

Using Eqs. (A-5)-(A-7) in Eq. (A-4), it follows that the second term in Eq. (A-8) is $O(\Delta x^{2N+4})$; thus the local error E_{Loc} of the one-dimensional method is $O(\Delta x^{2N+3})$, and the global error is $O(\Delta x^{2N+2})$. Hence, for N=2 corresponding to the quadratic source approximation used here, the global error in the one-dimensional DNTM is $O(\Delta x^6)$. Numerical results for a simple test problem have confirmed this behavior. This $O(\Delta x^6)$ convergence contrasts the $O(\Delta x^4)$ convergence, established by an error analysis[12] and confirmed for the same simple test problem,[12] for the quadratic method reported in Ref. 16. As discussed by Larsen and Miller,[12] the "flaw" in the quadratic method is the $O(\Delta x^3)$ error in the difference formula used to calculate the first spatial moment at the source; in the DNTM, the first moment is calculated to $O(\Delta x^6)$ by using Eq. (8) rather than the lower-order difference relationship.

APPENDIX B: ALTERNATE FORMULATION FOR SCATTERING-DOMINATED PROBLEMS

Although the error analysis done in Appendix A shows that the DNTM has an $O(\Delta x^6)$ spatial error irrespective of whether the problem is absorption-dominated or scattering-dominated, intuition indicates that the actual error in the latter would be greater than in the former since the exact inversion of the left hand side of the transport equation using the total cross section more closely represents the actual removal along a neutron flight path in strongly absorbing regions. In scattering-dominated regions the outscatter removal along a flight path is partially compensated by inscatter contributions. A similar statement applies for the fission part of the absorption cross section and the fission source term. Hence, intuition suggests subtracting $\Sigma_s + \nu\Sigma_f/k$ from the total cross section on the left hand side of the equation, to represent the actual physical removal of neutrons along a flight path, and compensating exactly for this in the source term. Hence we write the one-dimensional transport equation (with isotropic scattering) in the form

$$\mu \frac{\partial}{\partial x}\psi(x,\mu) + \left[\Sigma^a - \frac{1}{\lambda_o}\nu\Sigma^f\right]\psi(x,\mu)$$

$$= -\left[\Sigma^s + \frac{1}{\lambda_o}\nu\Sigma^f\right]\psi(x,\mu) + \frac{1}{2}\left[\Sigma^s + \frac{1}{\lambda}\nu\Sigma^f\right]\int_{-1}^{1} d\mu'\psi(x,\mu'), \tag{B-1}$$

where λ_o is a reference value of the eigenvalue. Expanding the angular flux as

$$\psi(x,\mu) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} \psi_\ell(x) P_\ell(\mu) ,$$

and then substituting into the right side of Eq. (B-1) yields after simplication

$$\mu \frac{\partial}{\partial x} \psi(x,\mu) + \left[\Sigma^a + \frac{1}{\lambda_o} \nu\Sigma^f\right] \psi(x,\mu)$$

$$= \Sigma^s \sum_{\ell=0} \frac{2\ell+1}{2} \left[\delta_{\ell o} - P_\ell(\mu)\right] \psi_\ell(x)$$

$$+ \nu\Sigma^f \sum_{\ell=0} \frac{2\ell+1}{2} \left[\frac{1}{\lambda_o} \delta_{\ell o} - \frac{1}{\lambda} P_\ell(\mu)\right] \psi_\ell(x) . \qquad \text{(B-2)}$$

It is clear from Eq. (B-2) that the scattering source term does not include a contribution from the isotropic component of the angular flux which is generally large for scattering-dominated systems. Hence this term does not contribute to the truncation error of the method, and the leading coefficient of the error will be smaller since it involves only the higher angular moments at the flux. However, for problems with strong absorption, in which the flux is highly anisotropic these can be large, and the truncation error can actually be increased due to the inclusion of the error component resulting from the approximation of the angular flux term on the right hand side of the equation in the modified formulation. This behavior has been verified numerically for a simple fixed-source test problem[12] which was solved over a range of values of c, the mean number of secondaries per collision. As expected the results showed for this problem, that while the modified formulation based on Eq. (B-1) actually increased the error for $c \lesssim 0.2$, a significant reduction in the error was achieved for $c \gtrsim 0.5$. Hence for problems in which scattering is important the slightly modified formulation should be used.

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