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INVESTIGATIONS OF THE STRUCTURE AND ELECTROMAGNETIC
INTERACTIONS OF FEW-BODY SYSTEMS

Progress Report

1 September 1989 to 31 August 1990

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Principal Investigator

The George Washington University

Washington, D.C. 20052

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INTRODUCTION

This progress report describes the work of the George Washington University (GWU) nuclear theory group during the period 1 September 1989 to 31 August 1990 under DOE Grant No. DE-FG05-86-ER40270. The GWU nuclear theory group during this period was composed of six faculty members: D.R. Lehman (P.I. of DOE grant), H. Haberzettl, E. Harper, L. Maximon, W.C. Parke, and F. Prats. One graduate student, John Woloschek, is working under the direction of Professor Lehman. Outside collaborators with the group during this year were B.F. Gibson of Los Alamos National Laboratory, A. Fonseca of the U. of Lisbon, Y. Koike of Hosei University, and W. Sandhas of the U. of Bonn.

The format of the Progress Report is as follows:

1. Papers published, papers submitted for publication, and papers in preparation;
2. Invited talks at Conferences;
3. Invited talks at Universities and Laboratories;
4. Posters contributed;
5. Visitors to the group; and
6. Research progress by topic.

PAPERS PUBLISHED

H. Haberzettl, "Separable expansion for the two-body T matrix based on a nonsingular scattering equation", Phys. Rev. C40,1147(1989).

D.P. Heddle and L.C. Maximon, "Differences in straggling for positrons and electrons", Phys. Rev. C40,1632(1989), C41,1320(E)(1990)

Chr. von Ferber, H. Haberzettl, and W. Sandhas, "Calculating deuteron photodisintegration amplitudes using nonsingular scattering equations", Phys. Rev. C40,1923(1989).

D.R. Lehman and W.C. Parke, "Angular Reduction in Multiparticle Matrix Elements", J. Math. Phys. 30,2797 (1989).

E.A. Bartnik, H. Haberzettl, and W. Sandhas, "Comment on 'Unified treatment of bound-state and scattering problems'", Phys. Rev. C41,398(1990).

PAPER IN PRESS

L.C. Maximon, "On the evaluation of the integral

$$I_{\ell,\ell'}(k,k') = \int j_{\ell}(kr) j_{\ell'}(k'r) r^2 dr \quad ,$$

J. Math Phys. 31, in press (1990).

PAPERS SUBMITTED

H. Haberzettl and U. Kerwath, "Comment on 'Triton model calculation test of the Born W-matrix rank-one approximation'", submitted to Phys. Rev. C.

H. Haberzettl, "Calculating three-body binding energies using the W matrix: Proof of a variational principle", submitted to Phys. Rev. C.

PAPERS IN PREPARATION

D.R. Lehman and B.F. Gibson, "Formalism underlying the A=4 Λ hypernuclei 1^+ equations", to be submitted to Few-Body Systems.

W.C. Parke, Y. Koike, D.R. Lehman, and L.C. Maximon, "Quality of the ^3H wave function as obtained for the Paris potential with the EST expansion", to be submitted to Phys. Rev. C.

INVITED TALKS AT CONFERENCES

D.R. Lehman, "Asymptotic normalization constants in few-body systems: D-wave", Invited Talk, Western Regional Nuclear and Particle Conference, Lake Louise, Alberta, Canada, 16-18 February 1990.

D.R. Lehman, "Evidence for and explication of the D-state in few-nucleon systems", Invited Talk, PARIS 90, Seventh International Conference on Polarization Phenomena in Nuclear Physics, 9-13 July 1990; to be published in the proceedings.

INVITED TALKS AT UNIVERSITIES AND LABORATORIES

D.R. Lehman, "Asymptotic normalization constants in few-body systems: S-wave", Invited Seminar, University of Saskatchewan Accelerator Laboratory, 15 February 1990.

D.R. Lehman, "Coincidence reactions and the three-body structure of ${}^6\text{Li}$ ", Invited Talk, Saskatoon Accelerator Laboratory Users Meeting, Lake Louise, Alberta, Canada, 18 February.

CONTRIBUTED PAPERS OR POSTERS

L.C. Maximon, "Theoretical aspects of Tagged Photon Facilities", Proceedings of the 7th Seminar on Electromagnetic Interactions of Nuclei at Low and Medium Energies, Moscow, USSR, 1990.

C.R. Chen, D.R. Lehman, and W.C. Parke, " ${}^4\text{He}(d,\gamma){}^6\text{Li}$ Polarization Observables", presented at the Gordon Research Conference on Photonuclear Reactions, 5-10 August 1990.

A.C. Fonseca and D.R. Lehman, " ${}^1\text{H}(d,\gamma){}^3\text{He}$ Polarization Observables from a Faddeev Continuum Calculation", presented at the Gordon Research Conference on Photonuclear Reactions, 5-10 August 1990.

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RESEARCH PROGRESS BY TOPIC

TOPIC: Three-Nucleon Rearrangement, Breakup, and
Photodisintegration Reactions.

INVESTIGATORS: H. Haberzettl, T.N. Frank (U. of Bonn), Th.
Januschke (U. of Bonn), C.B. Kuhl (U. of
Bonn), and W. Sandhas (U. of Bonn).

OBJECTIVE: To calculate three-nucleon scattering data with
realistic potentials using the W-matrix
prescription for the two-body input.

SIGNIFICANCE: The exact calculation of three-nucleon
scattering processes requires advanced
computational facilities and large amounts of
computing time, in particular, if one uses
realistic nucleon-nucleon forces. In many
circumstances, however, one would like to have
a method which allows one to get reasonably
accurate results relatively fast and
inexpensively, e.g., if one wants the
theoretical predictions to serve as a
guideline for choosing experimental
configurations. Using the semi-realistic
Malfliet-Tjon potentials, the separable
approximation of the two-body T matrix
following from its W-matrix representation was
shown to provide such a simplified approach,
with the corresponding three-body results
agreeing extremely well with the exact
calculations¹⁻³. The extension and
corroboration of these findings for more
realistic interactions is of great practical
importance.

DESCRIPTION OF PROGRESS: Much progress has been achieved
regarding the feasibility of three-body bound state
calculations within the W-matrix approach. Following the
findings of Ref. 2, the adopted procedure for calculating
these binding energies has been to search for a minimum of
the binding energy as a function of the negative-energy W-
matrix parameter k . It was conjectured in Ref. 2 that the
feasibility of this method was due to an underlying
variational principle and that this procedure guaranteed an
optimal value for the binding energy bounded from below by
the corresponding exact result. All our test calculations
(using the Malfliet-Tjon, Paris, and Bonn potentials) had
borne out our conjecture. Moreover, a recent counter-
example⁴ by Gibson, Pearce, and Payne indicating a violation
of this variational principle for the Reid-soft-core
interaction was subsequently shown by us⁵ to be in error.

In addition, most recently, on the theoretical side a break-
through was achieved with a mathematical proof showing that,
indeed, there exists a variational principle which explains

our findings, thus putting our adopted minimum procedure for calculating three-nucleon binding energies on a solid theoretical foundation. A manuscript⁶ detailing the proof was submitted for publication. The proof consists in an investigation of bounds on the eigenvalues of the energy-dependent Sturmian functions as employed within a Hilbert-Schmidt expansion; it makes essential use of the expansion of the remainder term of the W-matrix representation of the two-body T matrix as put forward in Ref. 7. The proof is of fairly general nature, and one outcome of it is that the variational principle is applicable not just to the W-matrix approach, but to all well-defined separable expansions of the two-body T matrix which have a block-diagonal structure and which are controlled by a free parameter. Furthermore, it also explains the empirical finding why binding energy values obtained with the Hilbert-Schmidt expansion converge monotonically towards the exact result.

As far as the variational parameter k is concerned, it is found that the minimum procedure for the binding energy calculations determines k completely only if one restricts oneself to constant values of k , as had been done in the investigations of Refs. 2-5 for reasons of simplicity. This restriction, however, is by no means necessary. It is obvious that at negative two-body energies one may allow for an energy dependence of k , $k=k(E)$. While this additional freedom does not seem to matter in binding energy calculations, i.e., choosing k as a (reasonable) function of the energy rather than a constant does not significantly alter the optimal values achieved, our investigations suggest that for certain scattering observables it may actually be of considerable importance for improving the results. Of course, in order to be able to exploit this additional freedom fully and make it viable independently of whether there exist benchmark solutions or not, one needs a criterion to distinguish various functional forms from one another. Clearly, the optimal binding energy values alone, having been found insensitive to different functions $k=k(E)$, are no longer adequate in this respect. We have found in our investigations that a certain variant of the Schmidt norm of the kernel of the three-body scattering equations seems to provide such an independent criterion. With its help, we are able to choose $k(E)$ such that the separable part of the W-matrix representation is optimized much beyond what is achievable with a constant choice for k , thus drastically minimizing possible effects of the neglected remainder. Moreover, it turns out that the optimal functional form $k(E)$ following from this procedure is fairly independent of the actual two-body interaction used, suggesting that it is determined almost exclusively by the kinematical structure of the three-body kernel. Our results in this respect are not quite finalized yet, but if these findings withstand further scrutiny, they would be of significant practical

importance. A manuscript explaining this part of our work will be completed within the next few weeks⁸.

As to the detailed numerical results for rearrangement scattering reactions obtained so far, we have recently concluded a complete set of five-channel calculations for all independent three-body observables. Employing the Paris interactions and exploiting the aforementioned freedom in choosing $k=k(E)$, it was found that within the respective numerical errors, our W-matrix results are in complete agreement with published values⁹ from the Bochum group using a Pade solution of the two-dimensional Faddeev equations. A manuscript on our findings is in the planning stage¹⁰. Also, the numerical investigations up to a total subsystem angular momentum $J \leq 2$ are progressing well. The code has been set up and is currently undergoing a thorough testing stage. We expect to have definitive numerical results within the next few months. Currently, we are also discussing the feasibility of extending our calculations up to $J \leq 3$. The limitations in this respect stem from the availability of computing resources, not from our code. The code is capable of handling any value for J ; in principle, therefore, we could even go to still higher angular momenta. Concurrently with these investigations for the Paris interactions, we have begun calculations with the Bonn boson-exchange potentials. For the latter interactions, we expect to produce definitive numerical results at about the same time the testing phase with the Paris potential will be concluded.

For breakup calculations, we have extended the existing code for s-wave two-body input used to produce the results of Ref. 3 to accommodate higher partial waves. At the moment, the code is being tested using the five-channel half-on-shell rearrangement T matrices obtained with the Paris potentials. By the time the rearrangement tests are finalized (see above) we also expect the breakup code to be in working order, capable of treating also higher subsystem angular momenta. Production runs will then follow within a very short period of time.

In view of all the other details of this extensive project, we have not pushed ahead with the conversion of the existing s-wave code for three-nucleon photodisintegration processes to higher partial waves. This will be undertaken once the testing phase for the rearrangement and breakup codes is completed.

This project presumably will be completed within the next year.

¹ E.A. Bartnik, H. Haberzettl, and W. Sandhas, "Two-body bound state problem and nonsingular scattering equations", Phys. Rev. C34, 1520(1986).

- ² E.A. Bartnik, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, "Neutron-deuteron scattering calculations with W-matrix representation of the two-body input", Phys. Rev. C36,1678(1987).
- ³ T.N. Frank, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, "Neutron-deuteron breakup calculations with W-matrix representation of the two-body input", Phys. Rev. C38,1112(1988).
- ⁴ B.F. Gibson, B.C. Pearce, and G.L. Payne, "Triton model calculation test of the Bonn W-matrix rank-one approximation", Phys. Rev. C40,2877(1989).
- ⁵ H. Haberzettl and U. Kerwath, "Comment on 'Triton model calculation test of the Bonn W-matrix rank-one approximation'", submitted to Phys. Rev. C.
- ⁶ H. Haberzettl, "Calculating three-body binding energies using the W matrix: Proof of a variational principle", submitted to Phys. Rev. C.
- ⁷ H. Haberzettl, "Separable expansion for the two-body T matrix based on a nonsingular scattering equation", Phys. Rev. C40,1147(1989).
- ⁸ T.N. Frank, H. Haberzettl, and Th. Januschke, "Calculating three-body observables using the W matrix: Choosing an optimal variational parameter", in preparation.
- ⁹ H. Witala, W. Gloeckle, and Th. Cornelius, Few-Body Systems 3, 123(1988); Nucl. Phys. A508,118c(1990).
- ¹⁰ T.N. Frank, H. Haberzettl, Th. Januschke, and W. Sandhas, "Results of five-channel three-nucleon scattering calculations with W-matrix representation of the Paris potential", in preparation.

TOPIC: Four-Nucleon Rearrangement and Photodisintegration Reactions.

INVESTIGATORS: H. Haberzettl, U. Kerwath (U. of Bonn), W. Sandhas (U. of Bonn), and S. Sofianos (U. of South Africa).

OBJECTIVE: To calculate four-nucleon scattering data from an exact four-body theory using the W-matrix prescription for the two- and three-body input.

SIGNIFICANCE: Owing to the numerical complexity of the problem, four-body scattering calculations so far have only been undertaken for very simple Yamaguchi-type separable interactions. Moreover, except for some calculations at very low energies (i.e., below the lowest breakup threshold), the dynamics of the required three-body subsystem amplitudes was never taken into account fully: for processes above the lowest breakup threshold existing investigations always resorted to a K-matrix approximation for these amplitudes, and therefore entirely neglected all contributions from channels containing asymptotically unbound clusters. The present project will subject both the two- and three-body subamplitudes to a W-matrix¹⁻³ treatment. Thus, not only will all channels be taken into account, but, by construction, it will be done in a way that preserves the full off-shell unitarity of the respective subsystem dynamics. The interactions to be used will be the semi-realistic Malfliet-Tjon I and III potentials. This approach, therefore, goes a significant step beyond previous investigations.

DESCRIPTION OF PROGRESS: Not much progress has been achieved beyond what was reported in last year's progress report. The reason for it is simply a lack of manpower. Most of the numerical work on this project had been done by U. Kerwath at the University of Bonn. He was offered a lucrative position by IBM and has decided to leave physics. For the time being, therefore, this project is on hold until we find a replacement.

This project will continue into the next year.

¹ E.A. Bartnik, H. Haberzettl, and W. Sandhas, "Two-body bound state problem and nonsingular scattering equations", Phys. Rev. C34, 1520(1986).

- ² E.A. Bartnik, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, "Neutron-deuteron scattering calculations with W-matrix representation of the two-body input", Phys. Rev. C38,1678(1987).
- ³ T.N. Frank, H. Haberzettl, Th. Januschke, U. Kerwath, and W. Sandhas, "Neutron-deuteron breakup calculations with W-matrix representation of the two-body input", Phys. Rev. C38,1112(1988).

TOPIC: High accuracy comparison of electron and positron scattering from nuclei

INVESTIGATORS: L.C. Maximon, in collaboration with experimentalists at the Nuclear Physics Laboratory and Dept. of Physics, University of Illinois at Urbana-Champaign, Urbana, IL; DPhN/HE, CEN Saclay, France; CERI-CNRS, Orleans, France; Dept. of Physics, University of Basel, Switzerland

OBJECTIVE: To provide the theoretical cross sections which are needed for a highly accurate analysis of the elastic scattering of electrons and positrons from nuclei at 450 MeV. These include Coulomb corrections to Landau straggling, thick target bremsstrahlung, and the Schwinger radiative correction, as well as multiple soft photon emission in the case of the Schwinger correction.

SIGNIFICANCE: The present experiment, in which electron and positron beams having the same energy and emittance are scattered from both light and heavy nuclei, permits us to understand the limits of validity of the standard description of the electron-nucleus interaction. This description approximates the scattering process by the exchange of a single hard photon with a static nuclear charge and current density and uses partial wave analysis for elastic scattering and DWBA for inelastic scattering. This experiment also provides information on the importance of dispersive effects, which are not taken into account by the standard description; they require the exchange of at least two virtual photons.

DESCRIPTION OF PROGRESS: A short article describing the experiment and its analysis is currently being prepared. Our goal is to finish this manuscript and submit it for publication in October 1990.

TOPIC: Integrals over products of Bessel functions

INVESTIGATOR: L.C. Maximon

OBJECTIVE: To evaluate integrals over the product of two spherical Bessel functions in a form suitable for application to nuclear physics problems, i.e., in terms of distributions such as the delta function and step function, and familiar functions of mathematical physics, such as Legendre functions.

SIGNIFICANCE: In both atomic and nuclear physics, the partial wave decomposition of wave functions and operators in matrix elements results in integrals over the product of spherical Bessel functions. The integral had direct application to the calculation of multipole moments of few-body systems in momentum space.

DESCRIPTION OF PROGRESS: A technique for separating the delta functions from the integral was developed; the remaining part could then be evaluated by a limiting procedure from known results in terms of hypergeometric functions. It was then shown that these could all be written in terms of Legendre functions of the first and second kind. An internal report was prepared, and a paper submitted for publication to the Journal of Mathematical Physics has been accepted.

TOPIC: D-state components in ^4He .

INVESTIGATORS: D.R. Lehman, W.C. Parke, A. Raskin, and E. Tomusiak

OBJECTIVE: Solve the four-nucleon bound state equations including the NN tensor force.

SIGNIFICANCE: Due to the intrinsic difficulty in an exact treatment of the four-body problem, as yet, only one calculation (Fonseca) goes beyond including only the S-wave NN interactions in the calculation of the ^4He ground-state binding energy and wave function, but this work is limited to simple rank-1 separable interactions.¹ On the other hand, separable expansion techniques or 'realistic' local interactions have been shown to be a viable alternative to direct local potential calculations, but with the advantage of giving a more tractable calculational formalism. Thus, the tools now exist to go beyond the work of Fonseca and carry out an exact four-body bound-state calculation that involves 'realistic' two-nucleon interactions. Since we already know that the tensor component of the NN interaction plays a significant role in the three-body bound state configuration, it definitely should be included in any realistic four-body calculation of ^4He . Once this has been accomplished with the best phenomenological two-nucleon interactions available, it should then be possible to understand better the D-state structure of the alpha-particle by calculation of the D to S asymptotic-normalization-constant ratio for $^4\text{He} \rightarrow d + d$. To date, the related DWBA parameter, D_2 , has been extracted from experiment.² Though the sign of D_2 is determined to be negative, its magnitude remains uncertain.

DESCRIPTION OF PROGRESS: We began the project this summer at the Gordon Conference on Photonuclear Reactions by setting up the framework for the calculation and by starting the derivation of the coupled integral equations. We expect to do much of the derivation and calculation in two parallel groups. In this way, we can make cross-checks at each stage. We intend to make the results extendible to arbitrary rank potentials such as the PEST representations,³ or the Koike expansions for local potentials.⁴

- ¹ B.F. Gibson and D.R. Lehman, Phys. Rev. C 18, 1042(1978); ibid. C 14, 685(1976); ibid. C 15, 2257(1977); A.C. Fonseca, Phys. Rev. C 40, 1390(1989); Nucl. Phys A508, 141c(1990).
- ² B.C. Karp, E.J. Ludwig, W.J. Thompson, and F.D. Santos, Phys. Rev. Lett. 53, 1619(1984); J.A. Tostevin, Phys. Rev. C 28, 961(1983); F.D. Santos, Prog. Theor. Phys. 70, 1679(1983).
- ³ J. Haidenbauer and W. Plessas, Phys. Rev. C 30, 1822(1984).
- ⁴ Y. Koike, private communication.

TOPIC: $\alpha + d \rightarrow {}^6\text{Li} + \gamma$ Scattering Cross Section and Polarization Observables at Low Energies.

INVESTIGATORS: C.R. Chen, D.R. Lehman, and W.C. Parke.

OBJECTIVE: To calculate the cross section and polarization observables for the reaction $\alpha + d \rightarrow {}^6\text{Li} + \gamma$ using consistent three-body dynamics in both the initial and final states.

SIGNIFICANCE: The understanding of photocapture of alpha particles on deuterons has importance in both astrophysics and nuclear physics.

DESCRIPTION OF PROGRESS: The capture reaction ${}^2\text{H}(\alpha, \gamma){}^6\text{Li}$ at low incident α energies has been a focus of interest to both astrophysicists and nuclear physicists.¹ The value of the cross section below center-of-mass energies of about 2 MeV is important in determining the abundance of ${}^6\text{Li}$ from the standard big-bang model and from cosmic ray generation. The reaction is also clearly important in a controlled-fusion plasma. The analysis of the gammas could be used to determine the plasma temperature.²

Measurements at center-of-mass energies from 1 to 8 MeV have been made, showing a cross section between 4 and 100 nanobarns which is apparently dominated by an E2 direct capture mechanism.³ Another series of measurements closer to the reaction threshold has been proposed.²

Besides the cross-section measurement, there has been recent interest in using radiative capture of polarized deuterons on the alpha particle⁴ to get a handle on the D-state component in ${}^6\text{Li}$, especially the sign of the D to S asymptotic-normalization-constant ratio for ${}^6\text{Li} \rightarrow \alpha + d$. Hopefully, such information will shed light on the underpinnings of the difficult-to-predict electric quadrupole moment of ${}^6\text{Li}$. Experiments at Duke studying the reaction ${}^4\text{He}(d, \gamma){}^6\text{Li}$ are imminent.⁵

As yet, there is no fundamental and self-consistent calculation of the photocapture reaction, although there have been several effective two-body calculations published.⁴ A three-body calculation for both the incoming and outgoing nuclear wave functions, using the same two-body interactions in the continuum and bound state would satisfy self-consistency. This approach has already been successful in the case of the calculation for the two-body and three-body photodisintegration of ${}^3\text{H}$ and ${}^3\text{He}$.⁶ Moreover, use of a three-body continuum wave function in the initial state would mean that the ${}^6\text{Li}(3^+)$ resonance is automatically included in the αd continuum. This state enhances the

reaction cross section by almost two orders of magnitude near threshold (at about 700 keV in the center-of-mass).

We are currently engaged in a full three-body derivation with the bound-state and incoming scattering states both determined by the same fundamental interactions. As a first step, we have projected the exact E2 three-body Born term for the reaction amplitude into an effective two-body model Born term, including contributions from the finite size of the deuteron, e.g. the deuteron quadrupole moment. We will then be prepared to compare effective two-body models that have consistent initial and final states with the full three-body calculation to see the effects of three-body dynamics. Our results already show that the contribution to the reaction cross section can be sensitive to the quadrupole moment of the deuteron, particularly at center of mass energies which probe the radial node in the α -d wave function (this node coming from the underlying Pauli exclusion principle for the nucleons in the two clusters).⁷ We have also calculated the tensor analyzing power T_{20} . As expected, this polarization observable is controlled by the D-state components in the α -d system, with the deuteron quadrupole term swamping the measured value near 90°, even at center-of-mass energies of 4 MeV.

- ¹ D.N. Schramm and R.V. Wagener, *Annu. Rev. Nucl. Sci.* 27,37(1977); R.G.H. Robertson *et. al.*, *Phys. Rev. Lett.* 47,1867(1981).
- ² E. Cecil, private communication.
- ³ Q.K.K. Liu, H. Kanada, and Y.C. Tang, *Phys. Rev. C* 33,1561(1986).
- ⁴ K. Langanke, *Nucl. Phys. A* 457,351(1986); K. Langanke and C. Rolfs, *Z. Phys. A* 325,193(1986); R. Crespo, A.M. Eiro, and F.D. Santos, *Phys. Rev.* 39,306(1989); R. Crespo, A.M. Eiro, and J.A. Tostevin, preprint.
- ⁵ H. Weller, private communication.
- ⁶ B.F. Gibson and D.R. Lehman, *Phys. Rev. C* 11,29(1975); *ibid.* 13,477(1976).
- ⁷ C.R. Chen, D.R. Lehman, and W.C. Parke, "⁴He(d, γ)⁶Li Observables", Gordon Research Conference on Photonuclear Reactions, Tilton, New Hampshire, 1990.

TOPIC: A Study of Pion Elastic and Charge-Exchange Scattering from the Bound Trinucleon.

INVESTIGATOR: E. Harper

OBJECTIVE: To study the possibility of extracting information relating to the three-nucleon bound state and/or the pion-nucleon interaction in the presence of other nucleons.

SIGNIFICANCE: Pion-nucleus scattering calculations are usually carried out by deriving an optical pion-nucleus potential calculated with the use of nuclear shell-model wave functions¹. In the case of two- and three-nucleon systems, it is feasible to make use of more realistic descriptions of the nucleus, i.e., wave functions obtained from solutions of the Faddeev equations in the three-body case. I am attempting to implement this procedure by using wave functions derived previously from the Bonn OBEP². I intend to compare the results obtained thereby with those resulting from a calculation I propose to carry out based on an effective two-body formulation of the N-body problem due to Haberzettl and Sandhas³.

DESCRIPTION OF PROGRESS: I have obtained algebraic expressions for the non-flip and spin-flip contributions to the positive and negative pion-trinucleon optical potential. The potentials obtained are non-local and energy-dependent. The usual treatments of the energy-dependence⁴ are not satisfactory and I hope to make some improvements. In addition, the expression obtained for the optical potential is quite complicated, involving, in addition to sums over the trinucleon and pion-nucleon partial waves, also sums over internal angular momenta. It is of importance for comparison with other representations of the problem that these sums be carried out accurately.

It is usual to use the optical potential so obtained in a Lippmann-Schwinger equation in order to get a T-matrix from which scattering and polarization observables may be obtained⁵. I am considering using the W-matrix representation of the pion-nucleus T-matrix using the methods of Haberzettl *et al.*⁶ as an efficient means of arriving at a solution.

¹ A.W. Thomas and R.H. Landau, Phys. Reports 58,121(1980).

- ² K. Erkelenz, Phys. Reports 130,194 (1974); R. Machleidt, K. Holinde and Ch. Elster, Phys. Reports 149,1(1987).
- ³ H. Haberzettl and W. Sandhas, Phys.Rev.C24,359(1981).
- ⁴ R.H. Landau, Ann.Phys.92,205(1975); F. Geffen, B. Bakker, H. Boersma, and R. van Wageningen, Nucl. Phys. A468,683 (1987).
- ⁵ A.K. Kerman, H. McManus and R. Thaler, Ann. Phys. 8,551(1959).
- ⁶ E. Bartnik, H. Haberzettl and W. Sandhas, Phys. Rev. C34,1520(1986).

TOPIC: Deuteron Photodisintegration in the energy region
 $300 < E_\gamma < 700$ MeV.

INVESTIGATOR: E. Harper

OBJECTIVE: To obtain a unified and systematic description of the differential cross-section, photon asymmetry and nucleon polarization in the photodisintegration of the deuteron at intermediate and high photon energies starting from a meson-exchange model of the N-N interaction.

SIGNIFICANCE: This has been described in our last report [DOE/ER/40270--17] p. 22.

DESCRIPTION OF PROGRESS: The formalism outlined in the above-mentioned report has been used to calculate deuteron photodisintegration. Though we obtain fair agreement with the data for the differential cross-section, we get partial agreement with the experimental data on the proton polarization at 90° only by using a value for $f_{\pi N\Delta}$ well outside the limits usually obtained for this coupling constant. In addition, our results for the photon asymmetry are in complete disagreement with experiment. This latter disagreement indicates that our treatment of meson-exchange currents is in error. If time permits, we hope to devote some additional study to these problems.

TOPIC: Long-range behavior of the effective nucleon-deuteron interaction for the three-nucleon system

INVESTIGATORS: D.R. Lehman, L.C. Maximon, Walid Younes (senior undergraduate physics major at GWU), and B.F. Gibson (Los Alamos National Laboratory)

OBJECTIVE: The aim of this work is to derive rigorously the asymptotic behavior of the effective interaction between a nucleon and deuteron within the bound three-nucleon system.

SIGNIFICANCE: One aim in attempts to understand the low-energy properties of the three-nucleon system is to determine whether these properties can be understood within the framework of two-body models with effective interactions. Until recently, such two-body models have met with no success. However, Tomio, Delfino, and Adhikari,^{1,2} recently published an analysis of the low-energy properties of the three-nucleon system based on an ad-hoc effective two-body interaction that has an $-\exp(-\mu r)/r^2$ long-range tail which is essential for the success of the description. In particular, the three-parameter potential is found to predict correctly the value of the doublet p-d scattering length obtained in exact three-body calculations and the low-energy scattering length - binding energy correlations for both the n-d and p-d cases, after having been fit to the ³H and ³He binding energies and the n-d doublet scattering length. In an attempt to justify the long-range behavior of the n-d effective interaction used by Tomio *et al.*^{1,2}, Delfino, Frederico, and Tomio³ have derived the asymptotic behavior starting from a zero-range representation of the three-nucleon bound-state wave function. The essence of their approach is to project the bound-state wave function with a nucleon-deuteron state (freely moving relative to each other) and then substitute this resulting two-body wave function into the two-body Schrodinger equation to derive the effective potential that would lead to that effective n-d wave function. They find by this approach that the effective n-d potential at large separation distances behaves as $-\exp(-\lambda r)/r^{3/2}$. Though their result differs from that of Tomio *et al.*^{1,2}, they argue that it is similar in that both contain the qualitative feature of depending on a range associated with the size of the deuteron, i.e., through μ and λ . They also argue that their result must be valid for any short-range interaction since it is based on minimal assumptions about the three-nucleon wave function, i.e., a zero-range form. However, it is particularly these assumptions about the form of the three-nucleon wave function which cast doubt on the reliability of their conclusions. It is well known that in zero-range approximation, the three-nucleon system has infinite binding

energy.* Historically, it was this fact that lead the early workers on the two-nucleon interaction to the conclusion that the two-nucleon interaction had a short, but finite, range.* Therefore, it is important to attempt another approach towards deriving asymptotic behavior of the effective n-d potential in the three-nucleon system.

DESCRIPTION OF PROGRESS: The approach taken in the current work is to begin from the three-body Schrodinger equation for the bound-state wave function without making specific assumptions about the two-nucleon interactions (possible three-nucleon interactions are neglected). The equation is projected with a bra state composed of a nucleon plus deuteron moving freely relative to each other. The aim is to obtain an equation for this effective n-d wave function, i.e., for the overlap amplitude of the n-d state and the three-nucleon bound state. Except for the two terms containing the potentials in the coordinates that involve the 'external' nucleon and one of the nucleons in the deuteron, this falls out directly. In the latter 'permuted-potential' terms, a complete set of nucleon-correlated pair states is inserted to affect the sought after equation. (The correlated pair terms are the deuteron and an np scattering state.) It is then immediately apparent that we must deal with a coupled set of effective two-body equations, one corresponding to the effective nucleon-deuteron wave function and the other to the effective nucleon-np-continuum wave function. In the effective nucleon-deuteron wave function equation, the interaction potential has a local piece (if the underlying two-nucleon interactions are local) that couples the effective nucleon-deuteron wave function to itself, plus a piece that couples through a nonlocal (in excitation energy) to the breakup channel, i.e., to the effective nucleon-np-continuum wave function. Prior to this year, we had completed a thorough study of the local part of the effective potential.

Our aim in studying the local part of the n-d effective potential was to determine its long-range behavior. We first started with specific potentials that were analytically tractable: 1. Zero-range; 2. Square-well; and 3. Hulthen. Except for the zero-range potential, we immediately found that the other two potentials lead to an asymptotic behavior of $-(\text{constant}) \exp(-4\gamma r)/r^2$, where the binding energy of the deuteron defines γ , i.e., $E_d = \gamma^2/M$. Moreover, the 'constant' multiplying the asymptotic form is known. Based on these analyses, it was then decided to assume an arbitrary short range two-nucleon interaction, e.g., with Yukawa behavior at large separation distances, to see if the result can be made potential independent. The answer is that it can. The final expression has the above form with the constant being determined by a particular integral over the potential. The general result has been

checked with the specific cases listed above and they have been reproduced.

What have we learned from this investigation so far? Firstly, and most importantly, we found that for the zero-range potential the local-part of the effective interaction is identically zero. Since it is expected, from our cursory study so far, that the nonlocal contribution to the effective potential falls off differently and possibly more rapidly asymptotically than the local piece, and since the authors of Ref. 3 neglected this piece, this demonstrates that the Ansatz used by these authors is invalid. Secondly, if we forget about the nonlocal piece for the time being, our result substantiates the adhoc assumption of Tomio et al.^{1,2} as to the asymptotic behavior of the effective nucleon-deuteron potential. Moreover, the precise parameter to be used in the exponential is obtained. It differs in numerical value significantly (approximately a factor of 4 larger) from that found by the fits in Refs. 1 and 2. The "magnitude" of the potential is explicitly given by the multiplicative constant. Thirdly, our asymptotic result and the exact expression for the potential can be used in combination to find at what separation distance the asymptotic behavior begins to dominate. This has been done numerically. Fourthly, in the appropriate Efimov limit⁶, i.e., when the deuteron binding energy (equivalently ϵ) approaches zero and r is large but smaller than $1/\epsilon$, we would expect to find the sought after $1/r^2$ tail that appears to be essential for the explanation of the Efimov effect⁶. So far, after much work this year, we have not yet been able to recover the $1/r^2$ behavior. Our statement in last year's progress report that we had succeeded in this derivation turned out to be in error. As a consequence, we spent considerable time this past year looking at the next correction term to the effective potential with no definite conclusions to date. This work will continue with total focus on the recovery of the Efimov $1/r^2$ interaction from our current formulation.

¹ L. Tomio, A. Delfino, and S.K. Adhikari, Phys. Rev. C38,441(1987).

² S.K. Adhikari, A.Delfino, and L. Tomio, in Proceedings of the International Workshop on Few-Body Approaches to Nuclear Reactions in Tandem and Cyclotron Energy Regions, edited by S.Oryu and T. Sawada (World-Scientific, Singapore, (1987), p.82.

³ A. Delfino, T. Frederico, and L. Tomio, Phys. Rev. C38,11(1988).

⁴ L.H. Thomas, Phys. Rev. 47,903(1955).

⁵ H.A. Bethe and R.F. Bacher, Rev. Mod. Phys. 8,82(1936).

• V. Efimov, Nucl. Phys. A362, 48(1981).

TOPIC: Quadrupole Moment of ${}^6\text{Li}$.

INVESTIGATORS: John Woloschek (Ph.D. student), and D.R. Lehman.

OBJECTIVE: To explain the quadrupole moment of ${}^6\text{Li}$ within the framework of three-body models.

SIGNIFICANCE: The quadrupole moment of ${}^6\text{Li}$ is one of its most difficult properties to predict. Its small negative value seems to imply, within the context of effective two-body (α -d) models, a subtle interplay between the S- and D-wave components of the ${}^6\text{Li}$ wave function. Successful prediction of this observable should lead to a deeper understanding of the D-wave component of the ${}^6\text{Li}$ wave function.

DESCRIPTION OF PROGRESS: One aspect of our previous work¹ on the elastic electromagnetic form factors of ${}^6\text{Li}$ was to extract the quadrupole moment from the slope of the quadrupole form factor as the electron momentum transfer goes to zero. Unfortunately, this turned out to be a numerically difficult problem due to the fact that the expressions for the form factors are five-fold integrals. Nevertheless, it appears that the three-body models predict a positive quadrupole moment of $\approx 0.5 \text{ e-fm}^2$ in contrast to the experimental value of $-0.0644 \pm 0.0007 \text{ e-fm}^2$. To get a better grasp of this result, the alpha-deuteron-component contribution to the quadrupole moment was extracted from the three-body model. When only the alpha-deuteron contribution is present, the ${}^6\text{Li}$ quadrupole moment essentially originates from two terms: 1.) The intrinsic quadrupole moment of the deuteron reduced by the fraction of alpha-deuteron component in the three-body wave function; and 2.) An interference contribution that originates from the possibility of either s-wave or d-wave relative motion between the alpha-particle and deuteron. In the three-body models, both these terms are positive! However, it has been argued recently,^{2,3} on the basis of alpha-deuteron cluster models, that the interference contribution must be negative. For this alpha-deuteron-projected term to be negative in the three-body models, it implies that the s-wave and d-wave effective alpha-deuteron wave functions must have opposite signs, especially at large distances. This is not the case; in particular, the s-wave and d-wave alpha-deuteron asymptotic normalization constants are of the same sign. As a consequence, it emphasizes the importance of having a reliable experimental determination of the d-wave alpha-deuteron asymptotic norm, relative to the s-wave, to check this prediction and to indicate the plausibility of the alpha-deuteron cluster-model explanations of the ${}^6\text{Li}$

quadrupole moment.⁵ Nevertheless, one must keep in mind that the three-body wave functions also have the contribution coming from the alpha-(correlated np pair in the continuum) projection that makes up 35-40% of the norm.

As a consequence of the last point in the previous paragraph, we first carried out a standard calculation of the ${}^6\text{Li}$ quadrupole moment, i.e., directly from the quadrupole operator expectation value with the original three-body ground-state wave functions. The aim was to confirm the results extracted from the quadrupole form factor extrapolations. We found that the previous results are confirmed, i.e., the quadrupole moment does turn out to be positive. We then improved the representation of the ${}^3\text{S}_1$ - ${}^3\text{D}_1$ np interaction used in generating the three-body ground-state wave function. The np interaction underlying the original three-body ${}^6\text{Li}$ wave functions is that of Yamaguchi-Yamaguchi⁴ which is known to give a poor representation of the mixing parameter ϵ_1 and the wrong sign for the barred D-wave phase shift. Our first improvement was to allow for an arbitrary-rank separable interaction so that an interaction that gives a reasonable representation of ϵ_1 at low energies can be used. Interestingly, even with an excellent representation of the np interaction, the EST expansion of the Paris potential,⁶ the quadrupole-moment value is predicted to be positive with a magnitude of approximately 0.5 e-fm^2 . In addition, the s-wave and d-wave alpha-deuteron asymptotic normalization constants retain the same sign though their values change somewhat compared to the original models. Thus, the inadequacy of existing three-body models in understanding the quadrupole moment of ${}^6\text{Li}$ cannot be attributed to the simplicity of the original np interaction. On further reflection and backed up by our new calculations, this should not be surprising. Our new calculations with the Paris interaction indicate that the bulk of the quadrupole-moment value is already given by the alpha-deuteron-projected piece, i.e., approximately 80%. Thus, since the main component of the effective interaction between the alpha and deuteron comes from the alpha-nucleon interaction folded with the deuteron wave function, it immediately becomes clear that the np interaction enters at a secondary level.

With the above understanding at hand, we have returned to the work of Merchant and Rowley² in an attempt to understand more fully their conclusions. They imply that one only requires the folded alpha-nucleon interaction to recover the proper sign and rough magnitude of the ${}^6\text{Li}$ quadrupole moment. Our results, obtained within the context of exact three-body dynamics seem to cast doubt on this conclusion. In addition, we are now looking into the question of inelasticity effects in the alpha-nucleon interaction. We plan to repeat our calculations with alpha-nucleon interactions that fit the phase-shifts and inelasticities

up to approximately 50 MeV. This will be accomplished by considering the alpha-nucleon interaction to be of a coupled-channel form. In one sense, this allows for the distortion of the alpha particle, or in another sense, allows for three-body forces within the three-body framework.

- ¹ A. Eskandarian, D.R. Lehman, and W.C. Parke, Phys. Rev. C38,2341(1988).
- ² A.C. Merchant and N. Rowley, Phys. Lett. 150B,35(1985).
- ³ T. Mertelmeir and H.M. Hofmann, Nucl. Phys. A459,387(1986).
- ⁴ Y. Yamaguchi and Y. Yamaguchi, Phys. Rev 95,1635(1954).
- ⁵ R. Crespo, A.M. Eiro, and F.D. Santos, Phys. Rev. C39,305(1989).
- ⁶ J. Haidenbauer and W. Plessas, Phys. Rev. C30,1822(1984);
ibid, C32,1424(1985); J. Haidenbauer, Y. Koike, and W.
Plessas, Phys. Rev. C33,439(1986).

TOPIC: Low-Energy ($0 \leq E_\gamma \leq 50$ MeV) Two-Body
Photodisintegration of ${}^3\text{He}$ and ${}^3\text{H}$

INVESTIGATORS: D.R. Lehman, W.C. Parke, L.C. Maximon, A.C. Fonseca (U. of Lisbon), and Y. Koike (Hosei University, Tokyo),

OBJECTIVE: The scope of this program is to generate theoretical predictions for the key observables in the low-energy photodisintegration of ${}^3\text{He}$ from 'realistic' nucleon-nucleon interactions like the Paris potential. Specifically, exact Faddeev calculations (bound-state and continuum) will be performed including the $E1$ and $E2$ operators and all components of the ground-state wave function connected through these operators to the allowable continuum states.

SIGNIFICANCE: Attainment of the above objective will permit us to uncover the mechanism of the $E2$ rescattering effect and to examine D-state effects in the disintegration process through the data available on the polarization observables.

DESCRIPTION OF PROGRESS: During the period of this progress report three major accomplishments can be reported:

1. A paper is almost completed that describes our work on generating the ${}^3\text{H}$ wave function with the EST expansion¹⁻³ of the Paris potential. We reported on some of these results last year. We have added to the results of last year's calculation of the S- and D-wave asymptotic normalization constants. They are in excellent agreement with the results of the Los Alamos group for the Paris potential. This gives us complete confidence in our ${}^3\text{H}$ wave function so obtained.
2. The above calculation of the ${}^3\text{H}$ wave function is based on the ${}^3\text{S}_1$ - ${}^3\text{D}_1$ and ${}^1\text{S}_0$ components of the two-nucleon interaction. As explained in our original proposal, it will be necessary to include the P-wave components of the two-nucleon interaction to check the role played by the D-wave component in the ground-state wave function that is made up of a pair of nucleons in a P-wave with the spectator nucleon moving in a P-wave relative to the pair. As a consequence, we have now written a code that allows us to include not only the above stated partial waves of the two-nucleon interaction, but the ${}^1\text{P}_1$, ${}^3\text{P}_0$, ${}^3\text{P}_1$, and ${}^3\text{P}_2$ components as well. We are currently testing this code by making comparisons with a code that handles rank-1 interactions that was developed by Fonseca. Our code, as before, handles arbitrary-rank interactions.

3. We now have a working code that calculates all the capture (photodisintegration) observables assuming the underlying two-nucleon interaction is rank one and includes only the 1S_0 and 3S_1 - 3D_1 partial waves, while the electromagnetic operator is pure E1. Of course, this is a fully consistent Faddeev calculation for both the bound and continuum states. We have calculated cross sections plus vector and tensor analyzing powers for polarized deuterons. These results were presented as a poster at the Gordon Conference on Photonuclear Reactions this summer. Where appropriate, all results are compared with existing data. These comparisons make it clear that at least the E2 operator must be included to understand the angular distributions of the cross sections and vector analyzing powers. Interestingly, the tensor analyzing power T_{20} at low energies is well described by the theory. As one moves upward in energy, this agreement deteriorates, either indicating a role for the E2 contributions and/or the P-wave contributions to the D-state of the triton. Currently, we are proceeding to test and check the current code in preparation for including the P-wave components of the two-nucleon interaction. Following successful inclusion of these pieces of the amplitudes, we shall begin to generalize the code to allow for higher-rank interactions in order to carry out our original objective.

¹ D.J. Ernst, C.M. Shakin, and R.M. Thaler, Phys. Rev. C8, 46(1973); ibid. C9, 1780(1974).

² J. Haidenbauer and W. Plessas, Phys. Rev. C30, 1822(1984); ibid. C32, 1424(1985).

³ Y. Koike and J. Haidenbauer, Nucl. Phys. A463, 365c(1987); J. Haidenbauer and Y. Koike, Phys. Rev. C34, 1187(1987).

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