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EFFECT OF INITIAL FLUID-SYSTEM PRESSURES ON THE BEHAVIOR OF A RUPTURE-DISC PRESSURE-RELIEF DEVICE

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by

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Summary

Rupture disc assemblies are used in piping network systems as a pressure-relief device to protect the system from being exposed to excess pressures. Among the various disc assemblies, the reverse-buckling type is chosen for application in the Clinch River Breeder Reactor. This rupture-disc assembly consists of a portion of a thin spherical shell with its convex side subjected to the fluid system. A cutting-knife setup is placed immediately near the concave side of the disc. When the pressure on the disc is of certain magnitude and frequency composition, the disc may develop large displacement, and is consequently torn open by the knife setup.

The reverse-buckling type rupture disc assemblies have been used successfully in environments where the fluid is gas, i.e. highly compressible, and their performances have been judged as adequate in the liquid environment. To analyze the piping system, an analysis method is needed taking into consideration of the fluid/disc interaction, the nonlinear dynamic buckling phenomenon of the disc, and the possible cavitation of the fluid.

A computer code SWAAM-I had been written at the Components Technology Division, Argonne National Laboratory. Among its many functions, one is to compute the response of 1-dimensional pressure pulse propagation including the effects of many different types of boundary conditions and possible pipe plasticity. Rupture disc assembly is treated as one of the boundary conditions where rather than either the pressure or the velocity is specified, the pressure and velocity is required to satisfy the equation of motion of the disc. The SWAAM-I code in most cases has been observed to run well and yield good agreement between computer's prediction and that of the test results.

However, in certain situations, when the initial pipe system pressure is not low, say compared to the buckling pressure of the disc, the performance of the code becomes doubtful, since the dynamic analysis of the disc uses the assumption that the disc was stress free initially or when the disturbance within the pipe arrives at the disc. This paper reports our finding from studying the effect of initial stress in an efficient way.

1. Introduction

The part of SWAAM-I computer code used in computing response in a fluid system is based on the method of characteristics [1, 2]. In this method, the solution, i.e., pressure and velocity, of the fluid is computed by simultaneously solving two equations which represent the conditions the fluid must satisfy along certain specific "paths." At a regular boundary, only one or the other of these equations applies depending on whether the boundary node is located at "left" or at "right"; the other condition necessary to uniquely determine the solution is provided by the so-called boundary condition specifying that either the pressure or the velocity is to be equal to a prespecified value. A natural extension of this is to specify the boundary pressure or velocity as a function of time or even the solution of certain (differeential) equations. Another extension that may appear more complicated in form, but is not in essense, is the rupture-disc boundary condition. Instead of specifying either the pressure or velocity to satisfy certain prescribed condition, a rupture-disc boundary requires both the pressure and the velocity to satisfy certain condition, namely the equation of motion of the rupture disc.

The equation of motion of the rupture disc is numerically represented by a convective or corotational finite-element model [3, 4]. This approach assumes that the disc has an initial natural state and upon this state loadings are applied. Because of its formulation, the dynamic disc model does not reduce to a static one when zero time-derivatives are introduced into the system. Therefore, to obtain static solution using this model, a slow-enough loading must be applied to the disc and the "equilibrium" solution of the disc then be taken as the static solution. This method may require certain number of trials to define the slowness of the loading and may consume computer resources; however, this is also the most natural way of treating the "static" loading, considering the definition of a static load is more a concept than a reality.

The limitation on the rupture-disc modeling, i.e., inability to treat the static loading as it is conventionally treated, also reflects the same limitation on the SWAAM-I which contains the rupture disc as a module. Therefore, SWAAM-I is expected to perform reasonably well when the initial pipe system pressure is small such that the corresponding initial stress in the disc is negligible. However, in reality, there exists situations where the initial pipe system pressure is large, say about 70% of the buckling pressure of the rupture disc used. The existence of high initial pipe system pressure prompts the needs to improve the rupture-disc module to incorporate the ability to treat the effect of initial condition, namely the equation of motion of the rupture disc.

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2. Approach

A natural approach is to develop a computer code using the same methodology (i.e., corotational- or convective-coordinates formulation), but computing directly the static solution of the rupture disc. This approach is straightforward for a methodology of which the final equation of motion is linear and of which the global stiffness matrix is explicitly available. Unfortunately, the corotational-coordinates method does not require, hence does not have, an explicit stiffness matrix. Therefore, the efforts necessary to develop a new static code or to reduce a static code from the existing dynamic one is not a trivial task and may require considerable work.

However, this approach is applicable to thin shells of any geometry and boundary condition subjected to arbitrarily distributed static loads. This feature is more than needed for the rupture discs, since they are assumed to be thin spherical shells with fixed edge subjected to uniformly distributed loadings. This simplification provides the possible existence of analytical solution for statically loaded rupture discs. Analytical static solutions for clamped spherical shells subjected to uniform pressure are, therefore, sought for to provide the initial condition for the dynamic response of the rupture disc.

In short, a compatible static solution for a spherical shell is needed. And the compatibility of the solution with the dynamic analysis is explained next.

3. Compatibility between static solution and dynamic analysis

The dynamic response of a rupture disc is computed by the following sequence:

1. Obtain the (initial) displacement d(i), velocity v(i), and acceleration a(i) for time step i.
2. Compute the displacement d(i+1) for the next time step using

$$d(i + 1) = d(i) + v(i)h + a(i)h^2/2$$

where h is the time-step size.
3. Compute internal nodal forces $f^{int}(i+1)$ from the nodal displacement d(i + 1). This is accomplished through the use of the corotational-coordinates formulation.
4. Compute external nodal forces $f^{ext}(i+1)$ from the applied loading at time step i+1.
5. Compute the acceleration by $a(i+1) = M^{-1}[f^{ext}(i+1) - f^{int}(i+1)]$ where M^{-1} is the inverse of the mass matrix M.
6. Compute the velocity by $v(i+1) = v(i) + h[a(i) + a(i+1)]/2$
7. Use d(i+1), v(i+1), and a(i+1) as the new initial conditions and repeat steps 1 to 7 until the final time is reached.

The external nodal equivalent force f^{ext} due to the "static" force applied to the disc and the internal nodal equivalent force f^{int} due to the displacement of the disc under the static pressure can be computed automatically by the dynamic analysis code as a part of the initial condition computation.

As can be shown by a direct substitution that nonzero velocity and acceleration would be generated for the next time step if a condition corresponding to $f^{int} \neq f^{ext}$ is used. Therefore, the compatibility of a static solution and the dynamic analysis is defined when $f^{int} = f^{ext}$, i.e., no fictitious motion results from a nonchange of loading.

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4. Analytical static solution

Many analytical solutions of a clamped spherical shell of constant thickness subjected to a uniformly distributed pressure are computed numerically through finite-element or finite-difference methods. The solutions obtained may be different from each other depending upon the particular shell theory or equations used. That is, everyone of the solutions thus obtained is only an approximation and its applicability should be determined upon its performance.

Three approximate static solutions of an elastic shell are used in this study and they are given below

(a) Asymptotic integration solution

One of the approximate methods is the method of asymptotic integration [5,6]. This solution is accurate when applied to regions except the vicinity of the apex. The solution for the in-plane membrane force and bending moments and rotation of the shell generator is

$$N_\phi = -\cot(\alpha - \psi) C e^{-\lambda\psi} \sin(\lambda\psi + \gamma)$$

$$N_\theta = -\lambda\sqrt{2}\, C e^{-\lambda\psi} \sin(\lambda\psi + \gamma - \frac{\pi}{4})$$

$$M_\phi = \frac{a}{\lambda\sqrt{2}} C e^{-\lambda\psi} \sin(\lambda\psi + \gamma + \frac{\pi}{4})$$

$$M_\theta = \nu M_\phi$$

$$V = -\frac{2\lambda^2}{Eh} C e^{-\lambda\psi} \cos(\lambda\psi + \gamma)$$

where the subscripts ϕ and θ represent the meridional and circumferential directions, respectively, and the other symbols are

- α – half opening angle
- ψ – positional angle measured from edge to apex
- λ – defined by $[3(1 - \nu^2)(a/h)^2]^{1/4}$, representing thinness of the shell
- ν – Possion's ratio; E – Young's modulus
- a – radius of the disc; h – thickness of the disc
- C, γ – integration constants to be determined by the edge and load conditions

For the fixed-edge spherical shell subjected to uniform pressure, the values for C and γ are

$$C = Pa(1-\nu)/(2\lambda); \quad \gamma = \pi/2$$

where P is the magnitude of the applied uniform load.

The axial and normal displacements u and w of the shell can be computed by

$$u = \sin\phi\left[\int_\alpha^\phi \frac{f(\phi)}{\sin\phi}\, d\phi\right]$$

$$w = u\cot\phi - a\varepsilon_\theta$$

where

$$f(\phi) = \frac{(1+\nu)a}{Eh}(N_\phi - N_\theta)$$

$$\varepsilon_\theta = \frac{1}{Eh}(N_\theta - \nu N_\phi)$$

Therefore, the needed displacement distributions are obtained from the results for u, subjected to uniform pressure, the values for C and γ are

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$$f(\phi) = \frac{(1+\nu)a}{Eh}(N_{\phi} - N_{\theta})$$

$$\varepsilon_{\theta} = \frac{1}{Eh}(N_{\theta} - \nu N_{\phi})$$

Therefore, the needed displacement distributions are obtained from the results for u, w, and v, which can be computed either explicitly or by numerical integration.

Note that the asymptotic-integration solution is not valid near the apex of the shell. However, for thin shells, i.e., those with high radius to thickness ratio (a/h), the solution should be governed by membrane stress only. The region where bending stresses become important can be identified by studying the results given by the asymptotic-integration solution and the membrane solution. For displacement solution a region that spans five degrees from the apex is considered the membrane region for a disc with a/h = 225.

(b) Membrane solution

The solution for the shell is taken as the membrane part of the above solution, i.e., no bending stress is present. This means that the boundary condition may be violated in this approximation.

(c) Trivial membrane solution

The solution for the shell is taken as a special case of the membrane solution in which the equilibrium of forces is also violated. Specifically we take the solution to be zero; i.e., the static load does not generate any response from the disc.

5. Numerical Examples and Discussions

Numerical examples are studied using a straight pipe with one of its ends connected to a rupture disc and the other end considered as a pressure source. The pipe is filled with liquid under a certain pressure. The displacement due to the initial pipe pressure is computed and used as the initial condition for the rupture disc.

The numerical results for the pressure at the disc obtained for various initial pipe pressures show that the disc pressure first drops and then oscillates about the initial pipe pressure when the initial pressure is small compared to the characteristic, say buckling, pressure of the disc. The disc develops into a large displacement and opens itself when the knife assembly is reached if the initial pipe pressure is not small. In all test runs using solution-A, the drop of the disc pressure at an early time is about 10-30%, and the time required for the disc to reach an "equilibrium" pressure of the pipe is reasonable.

The results for the membrane solution, solution-B, indicate that the disc buckles for pressures that do not cause disc failure for the other two, solutions-A and -C. This, of course, means that the effect of the edge on the static displacement, and subsequently on the dynamic response of the disc, is large for the disc used in this study (even though the membrane solution is a good approximation for a good central portion of the disc). The most interesting and striking phenomenon, however, is reflected through the comparison of results from the asymptotic-integration displacement, solution-A, and the zero displacement, solution-C, the latter being trivial as noted before. The disc pressure is expected to drop when the computation starts, with the drop for solution-C larger than that for solution-A. Nonetheless, the numerical results indicate the reverse is true. For example, the drop of disc pressure from the initial pipe pressure of 1.47 MPa (213 psi) is about 26% for solution-A, and is about 10% for solution-C. The disc failed for the solution-B displacement.

Since none of the three approximate solutions is compatible with the dynamic analysis in the sense defined previously. This observation suggests that the incompatibility between the internal nodal forces computed from the approximate displacement and the external nodal forces from lumping the uniform pressure is very important particularly for imperfectionmembrane solution is a good approximation for a good central portion of the disc). The most interesting and striking phenomenon, however, is reflected through the comparison of results from the asymptotic-integration displacement, solution-A, and the zero displacement, solution-C, the latter being trivial as noted before. The disc pressure is expected to drop when the computation starts, with the drop for solution-C larger than that for solution-A. Nonetheless, the numerical results indicate the reverse is true. For example, the drop of disc pressure from the initial pipe pressure of 1.47 MPa (213 psi) is about 26% for solution-A, and is about 10% for solution-C. The disc failed for the solution-B displacement.

Since none of the three approximate solutions is compatible with the dynamic analysis in the sense defined previously. This observation suggests that the incompatibility between the internal nodal forces computed from the approximate displacement and the external nodal forces from lumping the uniform pressure is very important particularly for imperfection-sensitive structures. This difficulty can be circumvented only if compatible formulas for internal and external nodal forces are used. Due to the complexity of the factors involved, this is not an easy task.

The numerical results also indicate that the equilibrium of the disc pressure can be reached within a reasonable time. In other words, for all cases except those in which the disc failed, the pressure at the disc converges quickly to small oscillations about the initial pipe pressure. This convergency is identified by observing the moving average of the disc pressure.

In the hope to reduce the magnitude of the pressure drop at the disc when the computation starts and to reduce the amplitude in the residual pressure oscillation, damping is introduced into the disc material. However, numerical results indicate the opposite effect is introduced, i.e., the more material damping introduced into the disc material, the larger the magnitude of pressure drop. In fact there appears to exist a certain level of damping value above which the disc becomes unstable in this disc-fluid interaction problem (though it is stable when no damping is used).

This "contradiction" can be explained through the following reasons. First, the concept or presumption that damping always reduces the "response amplitude" is incorrect. To understand what damping will do to a response, the response must first be defined. For a one-degree-of-freedom system, if one examines the displacement response at the mass due to a sinusodial force applied at the same mass, one then finds that the magnitude of the displacement is a monotonically decreasing function of the damping for forces at any frequency. However, different situations occur, if one looks at the transmissibility between the mass and its foundation (connected to the mass by a massless spring and a massless dashpot). Here the transmissibility is defined as the ratio of the force transmitted to the foundation to the imposed sinusoidal force at the mass (or the ratio of the corresponding displacements). An examination of the transmissibility of such a system [7] reveals that the magnitude of the transmissibility is not a monotonic function of the damping. In other words the transmissibility reduces as the damping value increases for imposed forces with a frequency less than $\sqrt{2}$ times the natural frequency of the system; however the transmissibility increases as the damping value increases when the frequency of the force is greater than the aforementioned value. Therefore, the transmissibility due to an arbitrary force can not be predicted unless the frequency content of that force and the natural frequency of the system are known. Since the transmissibility between the foundation of the spring mass system and the mass can also be identified with that between the fluid and the disc, it is clear that the "response," i.e., the magnitude of pressure drop, may not necessarily be reduced since the effective equivalent force imposed on the disc is a step force when computation starts.

The next "reason" to be discussed is that common sense derived from experience of structures vibrating in air or vacuum can not be directly extended to vibrations in nearly incompressible fluid such as water or liquid sodium. For example, the initial peak displacement at the apex of an elastic rupture disc connected to a water-filled pipe is greater than that corresponding to an elastoplastic disc in a similar situation [1]; also there is a pressure reduction at the disc when the disc starts to fail in a liquid-filled system, but not in a gas-filled system [8]. This suggests that the interaction between the fluid and disc is very important particularly when the fluid is near incompressible such as an arbitrary force can not be predicted unless the frequency content of that force and the natural frequency of the system are known. Since the transmissibility between the foundation of the spring mass system and the mass can also be identified with that between the fluid and the disc, it is clear that the "response," i.e., the magnitude of pressure drop, may not necessarily be reduced since the effective equivalent force imposed on the disc is a step force when computation starts.

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6. Conclusion

The methods of incorporating the effect of initial pipe system pressure are studied and many numerical examples are examined. We found the "high accuracy" of certain methods in providing certain solution components may be less desirable than the need for compatibility between the method chosen and the dynamic integration algorithm of the SWAAM-I code. Also noted is that great caution must be exercised when extrapolating experience obtained from one condition to another.

In addition to the SWAAM-I code's original natural capability of handling the initial system pressure (i.e., treating the initial system pressure as a slowly applied dynamic load and use SWAAM-I code to run this problem as was), an additional control is introduced allowing the rupture disc to "relax" to its equilibrium position with the initial system pressure. The equilibrium is considered reached when a moving average of the pressure at the disc stays almost a constant. This approach is considered as sufficient if the initial system pressure is not very near the buckling pressure of the disc. If the pressure is that high initially, a more compatible method must be developed.

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