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SELECTED EXAMPLES OF PRACTICAL APPROACHES FOR THE ASSESSMENT OF MODEL RELIABILITY - PARAMETER UNCERTAINTY ANALYSIS

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ABSTRACT

The uncertainty analysis of model predictions has to discriminate between two fundamentally different types of uncertainty. The presence of stochastic variability (Type 1 uncertainty) necessitates the use of a probabilistic model instead of the much simpler deterministic one. Lack of knowledge (Type 2 uncertainty), however, applies to deterministic as well as to probabilistic model predictions and often dominates over uncertainties of Type 1. The term "probability" is interpreted differently in the probabilistic analysis of either type of uncertainty.

After these discriminations have been explained the discussion centers on the propagation of parameter uncertainties through the model, the derivation of quantitative uncertainty statements for model predictions and the presentation and interpretation of the results of a Type 2 uncertainty analysis. Various alternative approaches are compared for a very simple deterministic model.

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1. INTRODUCTION

This paper is adapted from a more extensive working draft manuscript [1] in preparation for the International Atomic Energy Agency (IAEA) which reviews a series of practical concepts and procedures for the assessment of the reliability of predictions produced by environmental transfer models. The factors affecting the reliability of model predictions have been identified as belonging to five distinct categories: (1) uncertainty due to improper definition and conceptualization of the assessment problem or scenario, (2) uncertainty due to improper formulation of the conceptual model (3) uncertainty involved in the formulation of the computational model (4) uncertainty inherent within the estimation of parameter values and (5) calculational and documentation errors in the production of results.

Because of the difficulties in recommending general rules or prescriptions how to assure that the formulated assessment problem and the formulation of the mathematical models are correct, the IAEA manuscript [1] emphasizes procedures for reliability assessment when the predominant sources of uncertainty in model predictions are due to uncertainties in the estimation of parameter values. Despite this emphasis, the manuscript recommends that the best procedure for assessing the extent of potential misprediction due to both improper model structure and parameter estimation is the testing of model predictions against independent (and appropriately derived) sets of data. This procedure is referred to as "model validation". Many of the examples pertaining to model validation in the IAEA manuscript have been taken from [2].

When model validation is impossible or impractical, a "parameter uncertainty analysis" is recommended. Model intercomparisons and the quality assurance of computation and documentation of model results can be seen as complementary methods.

Considering the objectives of the present workshop, this paper is predominantly concerned with the discussions devoted within the IAEA manuscript to parameter uncertainty analysis.

2. NECESSARY DISCRIMINATIONS

Prior to conducting an uncertainty analysis it is essential to distinguish between two fundamentally different types of uncertainty [3] subsequently referred to as Type 1 and Type 2.

Type 1 uncertainty is due to stochastic[1] variability. That is, the quantity of interest exhibits stochastic variability within the system to be modelled and therefore it is uncertain which value to use in the model.

Type 2 uncertainty is due to a lack of knowledge about deterministic components of the system. That is, the quantity of interest is de-

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(1) The adjective "stochastic" implies the presence of a random variable.

termined within the system to be modelled but since it is only vaguely or imprecisely known it is uncertain which value to use in the model.

In order to classify properly uncertainties with respect to the distinction of Type 1 and Type 2 it is necessary to have a thorough understanding of the assessment question. The following simple example should serve the illustration.

Example:

The assessment question:	What is the value of the sum $S = K_1 + K_2$ if K_1 is the number shown by a die that will be thrown many times during the prediction period and K_2 is the unknown number shown by another given die that was thrown in the past and will be left untouched and unseen?
The system to be modelled:	The die (no. 1) being repeatedly thrown and the die (no. 2) lying untouched and unseen.
Type 1 uncertainty:	The uncertainty of K_1 is of Type 1 since K_1 varies stochastically within the system to be modelled.
Type 2 uncertainty:	The uncertainty of K_2 is of Type 2 since K_2 is determined within the system to be modelled but is unknown.

The need for this discrimination becomes most obvious in the case of risk assessments. Type 1 uncertainty is a constituent of the risk to be assessed while Type 2 uncertainty inheres to the assessment process. The discrimination is therefore essential for decision making.

"Probability" can be thought of as the mathematical language of uncertainty [4]. The practical interpretation of the term "probability" is, however, different for Type 1 as compared to Type 2 uncertainty. For Type 1 "probability" is interpreted as the relative frequency(2)of values from a specified interval among a sample of randomly selected values (i.e. the frequentistic interpretation). For Type 2 "probability" is interpreted as the degree of belief that a determined but vaguely or imprecisely known value is within a specified interval (i.e. the subjectivistic interpretation).

If "probability" is used in the classical (frequentistic) interpretation it is simply called "probability". If it is used in the subjectivistic (or

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(2) More precisely, the limiting value the relative frequency approaches as the sample size increases.

Bayesian) interpretation it is called "subjective probability"(3).

A model that uses probabilities to represent Type 1 uncertainty is called "probabilistic". A probabilistic model produces predictions in distributional form. In the context of probabilistic environmental radiological assessments it is of interest to ask "What is the probability for Y (concentration, dose, consequence etc.) to exceed a level of concern y?". For this reason it is preferred to present the distributions provided by probabilistic models in complementary form. While the customary cumulative distribution function (cdf) F_Y provides the probability $P(Y \leqq y)$ the complementary cumulative distribution function (ccdf) provides the probability $P(Y>y)$ which is $1-F_Y$ (see Fig. 1c).

Often, however, quantities that exhibit stochastic variability are depicted in the model as a single value (i.e., the quantities are modelled as being deterministic). This procedure is frequently adopted for the sake of simplicity. In the extreme case, the entire model neglects the stochastic variability in the system and thus produces only a single prediction value for a given assessment question. In this case the model is deterministic. This extreme simplification is the one most commonly adopted for environmental radiological assessment modelling.

Modelling Type 2 uncertainty via subjective probabilities has been practiced only in the past few decades. The reason for this is the increasing need to predict the behaviour of complex systems under conditions in which a substantial lack of knowledge prevails. In this situation stochastic variability of a quantity about its mean value is quite often negligible when compared to the lack of knowledge about the mean value itself. The uncertainty in this quantity is then dominantly of Type 2. In the context of Type 2 uncertainty analyses, it is of interest to ask "What is the degree of belief (subjective probability) for $Y \leqq y$?" if the model is deterministic, and "What is the degree of belief for $P(Y>y) \leqq p$?" if the model is probabilistic. Consequently, it is preferred to present the subjective probability distributions provided by Type 2 uncertainty analyses in the form of a cumulative distribution function.

Figure 1 illustrates the typical prediction format for the four analysis situations:

1 a deterministic model is adequate (Fig. 1a)

2 a deterministic model is adequate but Type 2 uncertainty is to be modelled by subjective probabilities (Fig. 1b)

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(3) In practice we shall additionally differentiate with respect to the basis for quantification of Type 1 and Type 2 uncertainty. If, in the frequentistic case, the probability value is largely based on relevant observations (sample evidence) we simply call it an "estimate". If, however, it is largely based on expert judgment (rather than sample evidence) we call it a "subjective estimate". If, in the subjectivistic case, the probability value is largely based on relevant observations (sample evidence) we simply call it a "confidence level". If, however, it is largely based on expert judgment (rather than sample evidence) we call it a "subjective confidence level".

3 a probabilistic model is required to account for Type 1 uncertainty while Type 2 uncertainty is negligible (Fig. 1c)

4 a probabilistic model is required to account for Type 1 uncertainty and Type 2 uncertainty is to be modelled by subjective probabilities (Fig. 1d)

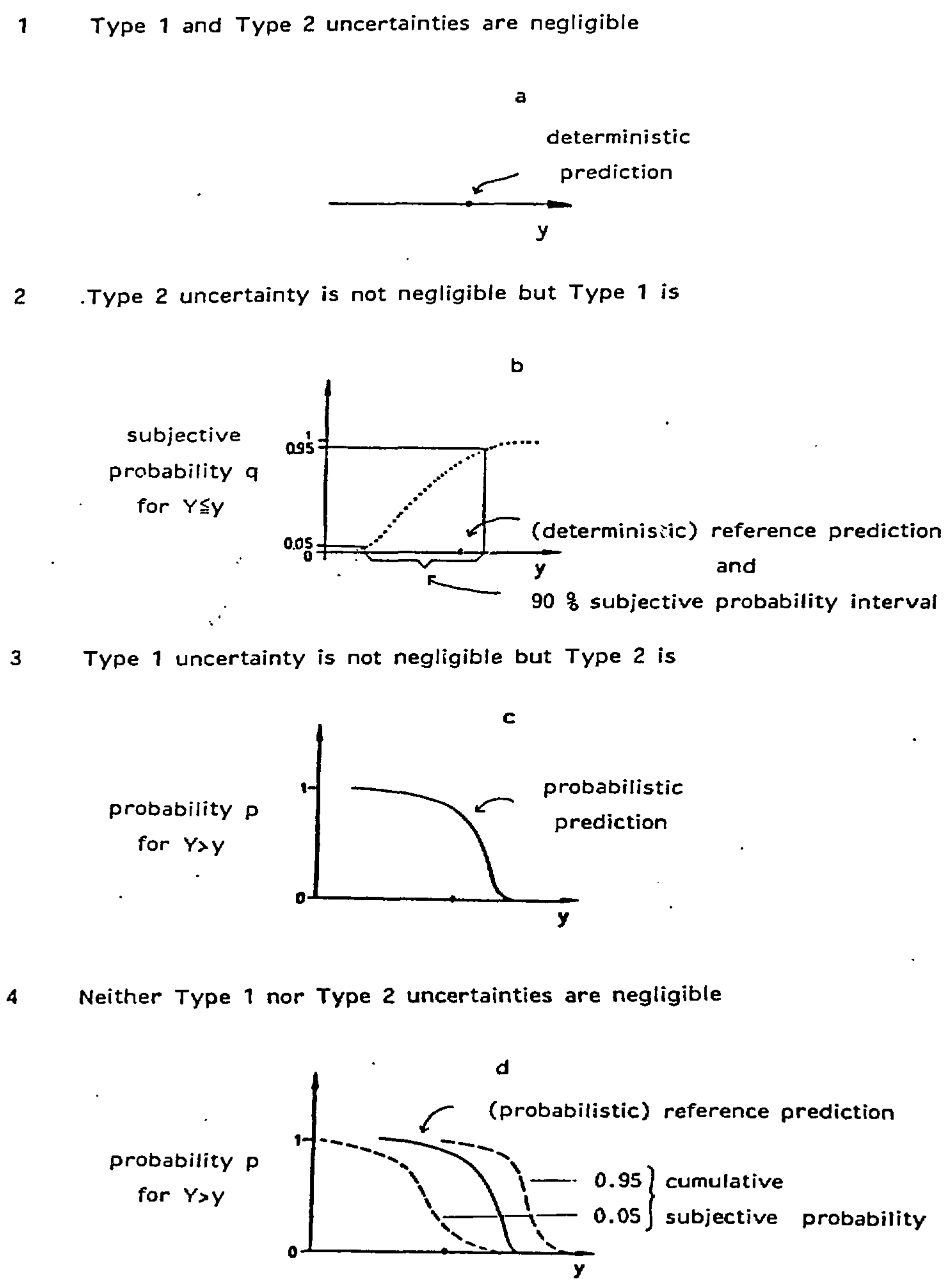

Figure 1: Prediction formats for different analysis situations

In this paper it is assumed that either a deterministic model is adequate and employed or that the model is probabilistic, expressing all relevant Type 1 uncertainties by probability distributions. The model prediction will then be either a single value or a ccdf (see Figs. 1a and 1c). Consequently, all that is left for an uncertainty analysis is to assess Type 2 uncertainty. Although the sources of Type 2 uncertainty are those associated with (1) scenario specification, (2) formulation of the conceptual model, (3) formulation of the computational model, (4) value selection for model constants (subsequently referred to as model parameters), and (5) computation errors, presently, only those uncertainties associated with parameter value selection (point (4)), and certain aspects of model formulation (points (2) and (3)), are amenable to a rigorous quantitative uncertainty analysis. This is why the subtitle "parameter uncertainty analysis" was chosen for this paper.

3. MAIN STEPS OF A PARAMETER UNCERTAINTY ANALYSIS

The main steps involved with conducting a parameter uncertainty analysis are:

1. List all of the parameters that potentially contribute to uncertainty in the final model prediction.

2. For each parameter listed, specify the maximum conceivable range of possibly applicable alternative values.

3. Specify the degree of belief (in percentage) that the appropriate parameter value is not larger than specific values selected from the range established in Step 2 above and select a probability distribution that best fits the quoted degrees of belief.

4. Account for correlations among model parameters by introducing suitable restrictions, by quoting appropriate conditional degrees of belief, or by estimating correlation coefficients, respectively.

5. At this stage a subjective probability density function (pdf) is set up for the combined range of parameter values. This will subsequently be referred to as a joint pdf. Propagate this joint pdf through the model to generate a subjective probability distribution of predicted values.

6. Derive quantitative statements about the effect of parameter uncertainty on the model prediction.

7. Rank the parameters with respect to their contribution to the uncertainty in the model prediction.

8. Present and interpret the results of the analysis.

These steps are discussed in detail in the IAEA draft manuscript [1]. See also [5] for a comparison of uncertainty and sensitivity analysis techniques for computer models and [6] for a relevant practical example of sensitivity analysis.

For the purpose of this paper the discussion will be restricted to Steps 5, 6 and 8. Particularly, it will be of interest to compare the following alternative approaches

- Variance Propagation
- Moment Matching
- Distribution-free Fractile Estimates from a Simple Random Sample (SRS)
- Distribution-free Statistical (u%,v%) Tolerance Limits from an SRS
- Fractile Estimates from an SRS (under the assumption of a normal or lognormal distribution)
- Statistical (u%,v%) Tolerance Limits from an SRS (under the assumption of a normal or lognormal distribution)
- Distribution-free Fractile Estimates from a Latin Hypercube Sample (LHS)
- Fractile Estimates from an LHS (under the assumption of a normal or lognormal distribution)

with respect to the kind and quality of information they provide about the effect of parameter uncertainties on the model prediction. These approaches are described in the IAEA manuscript [1] where their application is illustrated for the very simple fictitious model Y = ABC/D. In this model Type 1 uncertainty is negligible (so that a deterministic model is adequate) and parameters A, B, C and D are subject to Type 2 uncertainty. Their joint subjective pdf (resulting from Steps 1 to 4 of a parameter uncertainty analysis) is presented in Table I.

Table I:

JOINT SUBJECTIVE PDF AND "BEST ESTIMATE" VALUES OF THE UNCERTAIN PARAMETERS OF THE DETERMINISTIC MODEL Y = ABC/D.

A correlation was seen between parameters B and C only. It was expressed via the ordinary product moment correlation coefficient $\rho(B,C) = 0.7$.

Parameter	Distr. Type	Min. Value	5% Fractile	"best estimate"	95% Fractile	Max. Value
A	log triangular	10^2		10^3 ($\ln 10^3$ = Mode of lnA)		10^4
B	log. normal (truncated)	5.10^{-4}	8.10^{-4}	4.10^{-3}	4.10^{-2}	10^{-1}
C	log. normal (truncated)	0	10^{-6}	10^{-5}	2.10^{-4}	10^{-3}
D	log.uniform	5.10^{-6}		10^{-5}		5.10^{-5}

With the "best estimate" values of the uncertain parameters the model predicts Y = 4. The total resulting uncertainty range of Y is $[0 \leq Y \leq 20{,}000]$. A more informative quantitative expression of the uncertainty in the model prediction is the resulting subjective probability distribution of Y. The fractiles of this unknown cdf immediately provide subjective confidence limits or end points of subjective confidence intervals. The percentage of the confidence levels is given by the fractile percentages or their differences, respectively. This permits statements like:

> "At a subjective confidence level of 95 %, the value Y to be predicted is below y_{95}."(4)

with y_{95} being the 95 % fractile of the subjective probability distribution of Y. Therefore, it must be the aim of an uncertainty analysis to obtain the desired fractile values or sufficiently safe estimates thereof. How do the aforementioned practical approaches compare in this respect?

Variance Propagation only provides mean value and variance of Y and thus only permits the construction of " ± z standard deviation" intervals about the mean value. The fractile percentages corresponding to the end points of these intervals are unknown as Variance Propagation does not provide information on the type of distribution of Y. The assumption of a normal or lognormal distribution is typically employed, but such an assumption requires sufficiently rigorous justification. This necessitates information in addition to mean and variance. Variance Propagation applied to lnY of the simple model provided above produces $E\{\ln Y\} = 1.62$ (mean value of lnY) and $D\{\ln Y\} = 2.95$ (standard deviation of lnY). The "± 2 standard deviation" interval [-4.3, 7.5] of lnY about $E\{\ln Y\}$ corresponds to the interval [0.014, 1808] for Y(5). For simplicity the truncations of parameters B and C were neglected.

Moment Matching [7,8] does provide an approximation to the cdf of Y and thus approximate fractile values are obtained. Applied to lnY it indicates that a normal distribution (with mean value $E\{\ln Y\}$ and standard deviation $D\{\ln Y\}$ as above) is a reasonable approximation to the subjective probability distribution of lnY. For simplicity the truncations of parameters B and C are again neglected. The 95 % fractile of the approximate lognormal cdf of Y is $\exp(1.62 + 1.645 \cdot 2.95) = 647$. It may serve as an estimate of the 95 % fractile of the subjective probability distribution of Y.

Both, Variance Propagation and Moment Matching, are basically analytical methods requiring that the relationship between the model prediction and the set of uncertain parameters be expressed as an algebraic equation. However, the so-called numerical methods for Step 5, involve the selection of m-tuples of parameter values (where m is the number of uncertain parameters) and computation of the corresponding model prediction. Generally the m-tuples are selected at random according to the joint subjective pdf of the uncertain parameters. There are also non-random techniques like Fractional Factorials or the Discrete Variable Approach but these are not discussed in the IAEA manuscript [1]. The number n of m-tuples of parameter values selected via simple random sampling (SRS) or Latin Hypercube Sampling (LHS) [9, 10] is called the sample size. Computation of the model prediction for each m-tuple in the sample provides a

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(4) provided, all uncertainties not quantified may be neglected

(5) Note, that the interval for Y is not a "± 2 standard deviation interval" of Y.

sample of equal size from the subjective probability distribution of the model prediction Y and thus an empirical cdf of Y. The k-th ordered (ascending order of magnitude) value of the model prediction is the $k/n \cdot 100$ % sample fractile. The sample fractiles may serve as estimates of the corresponding desired fractiles of the subjective probability distribution of the model prediction.

Since Type 2 uncertainty analyses generally deal with very wide distributions and since interest usually centers on rather extreme fractile percentages like 95 or 97.5 etc., these estimates will, however, only be satisfactory if n is large (say, on the order of a thousand or more). When model computations are inexpensive n can be arbitrarily large. Frequently, however, models are complex and the computation of the model prediction to only one m-tuple of parameter values is expensive, thereby severely limiting the feasible sample size (say, to less than 100). In this case sample fractiles will often not be good estimates of, for instance, the 95 % fractile. Consequently the sampling(6) error will be a matter of concern. How close an estimate (from a sample of size n) is to the true fractile value is entirely a matter of chance and is unknown in practice. The chance, however, depends on the estimation function, the cdf of the model prediction and on the sampling technique. If underestimation of the uncertainty in the model prediction is considerably more undesirable than overestimation, an estimation function that has a sufficiently low chance to underestimate the desired fractile values must be selected. For instance, there is only a chance of less than 5 % that a statistical tolerance limit at a confidence level of 95 % will in fact be an underestimate of the desired fractile value.

A simple random sample (SRS) of size n=59 for the model $Y = ABC/D$ provided the 95 % sample fractile $\hat{y}_{95} = 212$ which is a <u>Distribution-free Estimate</u> of the 95 % fractile y_{95} of the subjective probability distribution of the model prediction Y. However, there is only insufficient (less than 50 %) confidence that this is not an underestimate.

The <u>Distribution-free Statistical (u=95%, v=95%) Tolerance Limit [11]</u>, obtained from the same SRS of size n = 59, is the 59th ordered prediction value which is 2080. If this limit is used as the fractile estimate $\hat{y}_{95}$ one can be v% confident that it is not an underestimate of the true u% fractile of Y. The number n of model runs, that is necessary for the n-th ordered prediction value to be the (u%,v%) limit, depends on u and v only and not on the number m of uncertain parameters.

The empirical distribution of the corresponding 59 values of lnY, plotted on probability paper, turns out to be reasonably well approximated by a straight line. Therefore the subjective probability distribution of Y may be assumed to be lognormal. With $\overline{\ln Y}$ being an estimate of the mean value $E\{\ln Y\}$ and $S_{\ln Y}$ an estimate of the standard deviation $D\{\ln Y\}$ one obtains the <u>Fractile Estimate (assuming a normal distribution of lnY)</u>

$\hat{y}_{95} = \exp(1.94 + 1.65 \cdot 2.59) = 499$ where $\overline{\ln Y} = 1.94$ and $S_{\ln Y} = 2.59$.

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(6) That part of the difference between the true fractile value and the estimate which is due to the fact that only a sample of values is observed.

However, there is only insufficient confidence that this is not an underestimate.

Assuming a normal distribution of lnY, the Statistical (u=95%,v=95%) Tolerance Limit is provided by 1.94 + 2.026 · 2.59 = 7.19 [12]. If this limit is used as an estimate of the u% fractile of the assumed normal cdf of lnY one can be v% confident that it is not an underestimate. Consequently, $\hat{y}_{95}$ = exp 7.19 = 1320 serves as a sufficiently safe estimate of the desired 95 % fractile of Y.

A Distribution-free Fractile Estimate from an LHS is the corresponding sample fractile. An LHS of size n = 59 provided the 95 % sample fractile $\hat{y}_{95}$ = 285. We cannot quantify our confidence that this is not an underestimate as we do not know, how to obtain a (u%,v%) statistical tolerance limit from an LHS. However, for model predictions that are monotonic functions in each of the uncertain parameters it has been shown [9] that LHS compares favourably with SRS with respect to the variance of estimates of mean value and cumulative probabilities.

If a lognormal distribution is assumed for the model prediction Y, with mean value and standard deviation of the normal distribution of lnY estimated from the LHS mentioned above, $\hat{y}_{95}$ = exp(1.64 + 1.65 · 2.80) = 523 is the Fractile Estimate from the LHS (assuming a normal distribution of lnY). Again, we cannot quantify our confidence that this is not an underestimate as we do not know how to obtain (u %,v %) statistical tolerance limits for fractiles of normal distributions with mean value and variance estimated from an LHS. However, in this particular case and for the particular sample the estimate happens to be close to the true 95 % fractile of Y.

Table II:

SUMMARY OF QUANTITATIVE UNCERTAINTY STATEMENTS OBTAINED FOR THE DETERMINISTIC MODEL Y=ABC/D

(cf. Table I for the joint subjective pdf of the uncertain parameters A, B, C and D)

Approach	Quantitative Uncertainty Statement for Y
Analytical Methods	
- Variance Propagation: ± 2 standard deviation interval for lnY: [-4.3, 7.5] => truncations of B and C neglected	[0.014, 1808]
- Moment Matching: (1.62, 2.95) normal distribution as an approximation to the subjective probability distribution of lnY => truncations of B and C neglected	$\hat{y}_{95}$ = 647

table continued

Numerical Methods

- **SRS of size n=59**
  - Distribution-free fractile estimate $\hat{y}_{95} = 212$
  - Distribution-free statistical (95%,95%) tolerance limit $\hat{y}_{95} = 2080$
  - Fractile estimate, if the subjective probability distribution of lnY is assumed to be normal $\hat{y}_{95} = 499$
  - Statistical (95%,95%) tolerance limit, if the subjective probability distribution of lnY is assumed to be normal $\hat{y}_{95} = 1320$

- **LHS of size n=59**
  - Distribution-free fractile estimate $\hat{y}_{95} = 285$
  - Fractile estimate, if the subjective probability distribution of lnY is assumed to be normal $\hat{y}_{95} = 523$

Distribution-free fractile estimate from an SRS of size n = 50.000 $\hat{y}_{95} = 564$

Application of the aforementioned practical approaches for Steps 5 and 6 to the simple model Y = ABC/D provided estimates of the 95 % fractile of the subjective probability distribution of Y that range from 200 to 2000. The true 95 % fractile is about 564 (if the truncations of B and C are neglected the 95 % fractile, obtained from an SRS of size 50.000, is about 648). What are the conclusions to be drawn from this comparison for the more realistic situation of many uncertain parameters of a complex computational model? Since in most cases the model will not be provided as an algebraic expression Variance Propagation and Moment Matching will not be applicable in their analytical form. If the model requires long CPU-times per run the permissable sample size n of numerical methods will be severely limited. Consequently, the sampling error will be a matter of concern. Usually little is known in advance (before the sample is drawn) about the type of distribution of the model prediction. After the sample has been drawn a lognormal or normal distribution may quite often seem to be a reasonable assumption. In a considerable number of instances the sample may, however, not justify this assumption. In any case, we recommend the use of (u%,v%) statistical tolerance limits (possibly assuming a lognormal or normal distribution, if justified) at a sufficiently high level (v/100) as estimates of the desired u % fractiles of the subjective probability distribution of the model prediction. If the affordable sample size is small one can be sufficiently confident (v%) that this estimate is not an underestimate of the desired u% fractile. The larger the sample size the smaller the difference to be expected between the statistical tolerance limit and the true fractile value.

4. PRESENTATION AND INTERPRETATION OF ANALYSIS RESULTS

The results produced by a Type 2 parameter uncertainty analysis are:

- quantitative statements about the effect of parameter uncertainty on the model prediction
- ranking of the uncertain parameters with respect to their contribution to the uncertainty in the model prediction.

In the IAEA manuscript [1] a detailed discussion is given on formats for presenting the results of a Type 2 uncertainty analysis and on their interpretation. This discussion differentiates the two analysis situations (see Figs. 1b and 1d):

(1) Type 1 uncertainty is negligible (a deterministic model is adequate)

(2) Type 1 uncertainty is not negligible (the prediction model must be probabilistic).

For the purpose of this paper it suffices to comment on the situation where Type 1 uncertainty is negligible. In this case a deterministic prediction is adequate. The quantitative uncertainty statements of the Type 2 uncertainty analysis are generally in the form of subjective confidence limits and intervals at a high (≧ 95 %) subjective confidence level. The confidence limits and/or the end points of the confidence intervals are given by corresponding fractiles of the subjective probability distribution of the prediction value (Fig. 2). For instance, the 5 % and 95 % fractiles are the end points of a 90 % subjective confidence interval while the 95 % fractile is an upper 95 % subjective confidence limit.

Some of the uncertainty propagation methods (Step 5) provide a random sample from the subjective probability distribution of the prediction value. There the fractiles must be estimated from the sample. For large sample sizes (in the order of thousand and more) the sample fractiles may serve as fractile estimates. They are easily read from the empirical distribution function. In the case of small sample sizes, however, statistical tolerance limits are to be used as fractile estimates to properly provide against underestimation of the uncertainty in the model prediction.

The subjective confidence limits and intervals permit statements(7) like:

"At a subjective confidence level of 95 %, the value to be predicted is below y_{95}"

"At a subjective confidence level of 90 % the value to be predicted is between y_5 and y_{95}".

where y_{95}, and y_5 are the 95 % and 5 % fractiles of the subjective probability distribution of the prediction value Y.

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(7) provided all uncertainties not quantified may be neglected

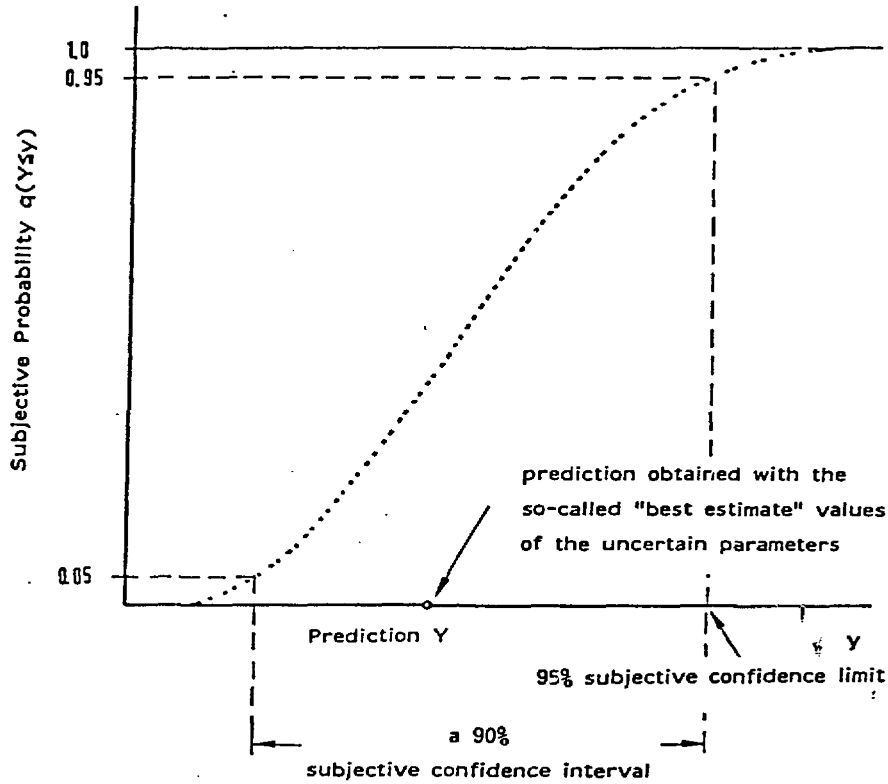

Figure 2: Illustration of quantitative uncertainty statements from a Type 2 uncertainty analysis of a deterministic model prediction.

The results of a Type 2 parameter uncertainty analysis determine a subjective level of confidence (degree of belief) that the value to be predicted is within a specified range or, that it is in compliance with specified limiting values. This level of confidence is of course highly dependent upon the uncertainty quantifications for the parameters and the subjective probability distributions given by the experts. It also assumes, that all uncertainties not quantified may be neglected. These caveats always need to accompany the analysis results in order to permit a proper interpretation of the results.

Once subjective confidence limits have been established the results should lead to one of three basic conclusions:

(1) At a high subjective level of confidence the value to be predicted is in compliance with the limiting value (e.g. dose limit)

or

(2) At a high subjective level of confidence the value to be predicted is not in compliance with the limiting value.

or

(3) The subjective levels of confidence for violation of and for compliance with the limiting value are of the same order of magnitude. Additional studies are necessary to improve the knowledge base for the most important parameters in the model prior to making decisions about compliance with the limiting value.

The ranking of the uncertain parameters provides direction for further research efforts in case (3) to improve the knowledge base and thus to efficiently reduce uncertainty in the deterministic model prediction. A practical example is provided in [1].

The situation where Type 1 uncertainty is not negligible is discussed in detail in the IAEA manuscript [1]. This document presents some of the numerous results that were obtained from a Type 2 uncertainty analysis of the probabilistic atmospheric dispersion module of an accident consequence code [13]. A Type 2 parameter uncertainty analysis of a probabilistic foodchain model prediction has also been presented in [13]. Results from an uncertainty analysis of a probabilistic consequence assessment may be found in [14].

In the interpretation of the quantitative statements of uncertainty obtained in Step 6 and the rankings of uncertain parameters in Step 7, it is important to note that these results will be affected by each modification of the question asked of the model and account for only those uncertainties that have been quantified in the analysis. Different quantitative uncertainty statements and parameter rankings should therefore be expected for model applications to critical group dose assessment than for applications to questions involving exposures of large populations. Different quantitative uncertainty statements and parameter rankings should also be expected for generic assessment questions than for assessment questions that are site-specific.

The results of a parameter uncertainty analysis will also be highly dependent on the subjective information obtained in Steps 1 to 4. For example, the subjective distributions quantifying parameter uncertainties may undergo changes over time even if the question asked of the model and the members of the group of experts remain unchanged. This is because the knowledge base will change over time.

When subjective confidence limits approach a decision-making criterion, such as a dose limit, this should be incentive to improve the knowledge base available on the uncertain parameters. Again, this activity should be guided by the ranking of uncertain parameters provided in Step 7.

5. CONCLUSIONS

Two fundamentally different types of uncertainty are to be discriminated in an uncertainty analysis of a model prediction. Type 1 uncertainty is due to stochastic variability within the system to be modelled while Type 2 uncertainty is due to a lack of knowledge.

Type 2 uncertainty analyses provide quantitative statements about the effect of parameter uncertainty on the model prediction in form of fractiles of its subjective probability distribution. Additionally a ranking of the uncertain parameters with respect to their contribution to the uncertainty in the model prediction is obtained. So-called numerical methods, based on random samples, are preferred for Type 2 uncertainty analyses of complex models. The affordable sample size will be limited by the CPU-time required per model run. Consequently, the sampling error in estimates of fractiles of the subjective probability distribution will generally be a matter of concern. If a statistical (u%,v%) tolerance limit is used as an estimate of the desired u% fractile, one can be v% confident that it is not an underestimate. For the customary values of v these limits are therefore to be preferred over the sample fractiles in the frequently encountered situation where the affordable sample size is small. The minimum sample size required to obtain a distribution-free statistical (u%,v%) tolerance limit [11] is independent of the number of uncertain parameters and is determined by u and v only. For u=v=95 59 model runs are sufficient.

The results from a Type 2 uncertainty analysis permit a comparison of model predictions with limiting values (and/or limit lines [15]) in the situation where lack of knowledge prevails. Where the subjective levels of confidence for compliance with and for violation of the limiting value are of the same order of magnitude the ranking of the uncertain parameters provides direction for further research efforts to improve the knowledge base and thus to reduce the Type 2 uncertainty in the model prediction.

6. REFERENCES

[1] A working document on Procedures for Assessing the Reliability of Radionuclide Environmental Transfer Model Predictions, IAEA, Division of Nuclear Fuel Cycle, 1987

[2] HOFFMAN, F.O., GARDNER, R.H., "Evaluation of Uncertainties in Environmental Radiological Assessment Models", In: Radiological Assessment: A Textbook on Environmental Dose Assessment, J.E. Till and H.R. Meyer (Eds), U.S. Nuclear Regulatory Commission, Washington, D.C., NUREG/CR-3332, ORNL-5968, pp. 11-1 - 11-55 (1983).

[3] VESELY, W.E., RASMUSON, D.M., "Uncertainties in Nuclear Probabilistic Risk Analyses", Risk Analysis, Vol. 4, No. 4 (1984)

[4] WINKLER, R.L., HAYS, W.L., Statistics, Holt, Rinehart and Winston, New York (1975)

[5] IMAN, R.L., HELTON, J.C., "A Comparison of Uncertainty and Sensitivity Analysis Techniques for Computer Models", NUREG/CR-3904, Sandia National Laboratories (March 1985)

[6] HELTON, J.C., IMAN, R.L., BROWN, J.B., "Sensitivity Analysis of the Asymptotic Behaviour of a Model for the Environmental Movement of Radionuclides", Ecological Modelling, 28; 243-278 (1985)

[7] HAHN, G.J., SHAPIRO, S.S., Statistical Models in Engineering, John Wiley and Sons, Inc., New York (1967)

[8] JOHNSON, N.L., NIXON, E., AMOS, D.E., PEARSON, E.S., "Table of Percentage Points of Pearson Curves, for Given $\sqrt{\beta_1}$ and β_2, Expressed in Standard Measure", Biometrika, Vol. 50, Nos. 3 and 4, p. 459 (1963)

[9] McKAY, M.D., BECKMAN, R.J., CONOVER, W.J., "A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code", Technometrics, Vol. 21, No. 2, 239-245 (1979)

[10] IMAN, R.L., CONOVER, W.J., "Small Sample Sensitivity Analysis Techniques for Computer Models, with an Application to Risk Assessment", Communications in Statistics, A9 (17), 1749-1842 (1980), ("Comments", 1843-1861, "Rejoinder to Comments", 1863-1874)

[11] CONOVER, W.J., Practical Non-parametric Statistics, John Wiley & Sons, Inc., New York, (1980)

[12] GUTTMAN, I., Statistical Tolerance Regions, Classical and Bayesian, Griffin, London (1970)

[13] HOFER, E. et al. "Uncertainty and Sensitivity Analysis of Accident Consequence Sub-Models", Proceedings of the International ANS/ENS Topical Meeting on Probabilistic Safety Methods and Applications, San Francisco, 24 February - 1 March 1985, EPRI (1985)

[14] ALPERT, D.J., IMAN, R.L., HELTON, J.C., JOHNSON, J.D., "A Demonstration Uncertainty/Sensitivity Analysis Using the Health and Economic Consequence Model CRAC2", NUREG/CR-4199, Sandia National Laboratories, May 1985

[15] COX, D.C., BAYBUTT, P., "Limit Lines for Risk", Nuclear Technology 57 (1982)