3.3 CONSTITUTIVE EQUATIONS FOR MEETING ELEVATED-TEMPERATURE-DESIGN NEEDS

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ABSTRACT

Constitutive equations for representing the inelastic behavior of structural all-ys at temperatures in the creep regime are discussed from the viewpoint of advances made over the past decade. An emphasis is placed on the progress that has been made in meeting the needs of the liquid-metal fast-breeder reactor (LMFBR) program whose design process is based in part on a design-by-inelastic-analysis approach. In particular, the constitutive equations that have been put into place for current use in design analyses are discussed along with some material behavior background information. Equations representing short-term plastic and long-term creep behaviors are considered. Trends towards establishing improved equations for use in the future are also described. Progress relating to fundamentals of continuum mechanics, physical modeling, phenomenological modeling, and implementation is addressed.

3.3.1. Introduction

The introduction to this chapter (Section 3.1) notes that individual papers will frequently draw heavily from the progress that has been made in connection with liquid-metal fast-breeder reactor (LMFBR) technologies. That is the case with this paper. This is consistent with the concept of a decade of progress since continuous and coordinated efforts have been underway since the late 1960's to provide reference constitutive equations for use in LMFBR design projects. The emphases have been on equations capable of representing the high-temperature inelastic response of specific

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alloys and on practicability for design project usage. The equations developed have been used in the performance of inelastic analyses (see Dhalla and Gallagher [1]*) as well as in the development of other design tools such as simplified methods (see Yahr and Nickell [2]).

The LMFBR program recognized early [from the beginning of the design phase of the Fast-Flux Test Facility (FFTF)] that inelastic aspects of high-temperature structural responses had to be accommodated in the design process. The most relevant ASME Code Case at that time for elevated temperature design was the ASME Code Case 1331-4. As described by Snow and Jakub [3], that code case was very limited and subsequent FFTF activities contributed heavily to criteria developments that carried the document through several revisions. It was later issued as Code Case 1592, and several additional revisions have evolved in more recent years while the Clinch River Breeder Reactor Project (CRBRP) has been underway. It is presently issued as ASME Code Case N-47. Throughout this evolution, those documents have included rules and limits that may require inelastic (elastic-plastic-creep) analyses for their satisfaction. However, the ASME Code Cases have not included guidance on how such analyses should be carried out. The Department of Energy (DOE) [earlier as the Atomic Energy Commission (AEC) and the Energy Research and Development Administration (ERDA)] has maintained a continuous effort to provide the needed inelastic analysis methods including constitutive equations. The resulting design guidelines have been placed in use in design and technology development activities.

The likelihood of inelastic deformations occurring in LMFBR components is enhanced by three factors. The first is that temperatures may often be above that where creep becomes significant. The second factor is

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*Numbers in brackets refer to references at the end of this paper.

that the temperature difference that exists across the core of an LMFBR is sufficient to cause large changes in temperatures (thermal transients) to occur in both primary and secondary coolant circuits upon shut downs of the reactor. The third factor is that the good heat transfer properties of the sodium coolant combine with the large thermal transients to produce large thermal gradients through components walls. These thermal gradients can cause plastic yielding which creates, in addition to plastic strains, residual stresses that greatly influence subsequent creep after the system returns to operating conditions. Therefore, time-independent plasticity, time-dependent creep, and interactions between plasticity and creep have been recognized as important considerations. This paper discusses developments that have taken place in providing constitutive equations for use in carrying out analyses of LMFBR components undergoing such inelastic loadings. Whereas these developments are based mostly on classical theories of continuum mechanics that separate elastic, plastic, and creep strains, the paper also discusses progress that has been made in developing potentially improved constitutive euqations on the basis of unified measures of inelastic strains and state variables that characterize history dependence.

Three materials have been at the center of LMFBR applications and ASME elevated-temperature code case developments over this decade. These are types 304 and 316 stainless steel and 2 1/4 Cr—1 Mo steel. Much of the discussion given below relates to these materials, and examples of inelastic material response data are given for them.

3.3.2. Approach to Developing Constitutive Equations for Project Usage

The approach employed in developing constitutive equations for LMFBR applications has combined analytical and experimental efforts. The Oak

Ridge National Laboratory (ORNL) has had the lead in this endeavor, and the efforts have built upon the progress made by various laboratories and investigators. The status of understanding material behavior and development of constitutive equations were addressed early in the decade in surveys such as Refs. [4] and [5]. Subsequently, experimental information such as that discussed by Swindeman and Brinkman [6] has been combined with assessment of models written in terms of continuum constitutive variables to identify candidate equations. The assessments have been initially made for uniaxial situations, and then the candidate equations are examined in terms of their ability to predict results from tests involving homogeneous multiaxial stress states and tests of structural configurations. Some multiaxial tests that have been carried out are discussed in Refs. [7-10] and related structural testing is addressed in Refs. [11,12]. The structural testing is aimed at validating the entire inelastic analysis process that includes constitutive equations, computation process/programs, and materials data correlations.

At the point in the process when a substantial number of comparisons have been made between experimental data and predictions, candidate constitutive equation formulations have been judged to fall in one of three categories: (a) capable of representing the basic features of the observed material and structural responses to a degree that warrants recommendation for use in design; (b) a promising method for future recommendation but currently possesses uncertainties that must be resolved on the basis of further exploratory and structural test data; or (c) unacceptable for recommendation because of its inability to represent important features of the experimental observations, because of unacceptable computational times, or because of excessive properties data requirements.

A part of recommending a constitutive formulation for use in design analyses has been identifying the companion properties data (and correlations) needed for implementation. If data and correlations are available for appropriate ranges of variables, a constitutive equation formulation is implemented upon acceptance by a special DOE appointed task force (membership includes designers, DOE personnel and researchers). However, if the types of properties data required to implement a formulation are not available, then application is delayed until a sufficient data set is generated. The process of updating recommendations has been continuous, and feedback has been obtained from component designers who have been implementing the recommendations.

The first recommendations were for FFTF applications and were for two stainless steels; these are discussed in Ref. [13]. An updated discussion of those equations is given in Ref. [14]. With the steam generators of the CRBRP to be made of 2 1/4 Cr—1 Mo steel, initial equation recommendations were made for this alloy, Ref. [14]. The initial equations for each of these alloys have been updated to incorporate improved understanding of behavioral features; some of these are discussed in Refs. [12, 15-16].

3.3.3. Equations for Elastic-Plastic Behavior

Although they differ in details, elastic-plastic models of the same basic form were initially recommended for the three alloys cited above. The form was based on capturing the overall observed dependencies (of plastic yield and flow) on prior deformation, stress and temperature histories. Briefly stated that model includes the von Mises yield condition, a modified kinematic hardening law, and an associated flow rule.

In particular, the initial yield surface and subsequent loading surfaces are given by

$$f = \frac{1}{2}(\tau_{ij} - \alpha_{ij})(\tau_{ij} - \alpha_{ij}) = \kappa, \qquad (1)$$

where $\underset{\sim}{\tau}$ is the deviatoric stress tensor, $\underset{\sim}{\alpha}$ is a kinematic-hardening parameter (tensor) that locates the center of the yield surface in stress-space as a function of the temperature and deformation histories, and κ specifies the size of the yield surface. In keeping with experimental evidence, such as that shown in Fig. 3.3.1, the size of the yield surface (the range of elastic response as defined by bilinear idealizations of successive stress-strain cycles) generally increases with initial accumulation of inelastic strain, but the *rate* of increase diminishes rapidly with continued strain accumulation and eventually approaches zero. The hardening law, therefore, is based on kinematic hardening after limited isotropic changes in the size of the yield surface occur. That is to say, the yield parameter κ increases isotropically until it reaches a limiting value, κ_1, after which κ remains unchanged.

Mathematically, the hardening rules are expressed by

$$\kappa = \kappa \left(\varepsilon^{P}_{ij}, T \right) \qquad \text{for } \kappa \leq \kappa_1 \qquad (2)$$

and

$$d\alpha_{ij} = C d\varepsilon^{P}_{ij} - \frac{1}{2\kappa} \frac{\partial \kappa}{\partial \varepsilon^{P}_{k\ell}} d\varepsilon^{P}_{k\ell} (\tau_{ij} - \alpha_{ij}) \qquad (3)$$

Equation (2) represents the isotropic-hardening contribution, while Eq. (3) represents the kinematic-hardening portion of the law. The material constant C (dependent on the temperature and strain range) is related to the plastic slope of a bilinear representation of the initial tensile stress-strain curve. The flow rule is based on the assumption that the plastic strain increment is normal to the loading surface:

$$d\varepsilon_{ij}^{P} \propto \frac{\partial f}{\partial \sigma_{ij}} ,$$

or explicitly,

$$d\varepsilon_{ij}^{P} = \frac{[(\tau_{k\ell} - \alpha_{k\ell})\, d\sigma_{k\ell} - \frac{\partial \kappa}{\partial T}\, dT]\ (\tau_{ij} - \alpha_{ij})}{2C\kappa} \qquad (4)$$

where f is given by Eq. (1) and σ_{ij} are the components of the total stress tensor.

The material properties needed to apply those equations in elastic-plastic analyses are the elastic constants (e.g., E and ν), the bilinear coefficeint C, and the yield surface measure κ [Eq. (2)]. These properties are determined from uniaxial stress-strain tests conducted at discrete temperatures. E is the initial elastic slope and C is the plastic slope of the bilinear deviatoric stress-plastic strain representation. The yield parameter κ is also determined from uniaxial test results with the initial value $\kappa = \kappa_0$ determined from the bilinear representation of the initial tensile stress-strain curve. The equations initially recommended captured the occurrence of limited isotropic hardening by imposing a one-time stepwise change in κ from κ_0 to κ_1. Options were provided later for changing κ during initial inelastic deformations. Those options include making changes on the basis of either the accumulated inelastic strain or inelastic work. The extent of the κ change is limited in each option to $\kappa = \kappa_1$, the value corresponding to a specified number of cycles of fixed strain-range ($\Delta\varepsilon$) cycling. The κ_1 value is taken to be representative of a partially-hardened material and to provide a balance between the tendency for further hardening and possible softening effects due to high-temperature exposure.

Figure 3.3.2 illustrates an option that uses $\kappa(\int d\varepsilon)$ and that was developed as an accumulative method of incorporating the limited isotropic hardening. By using successive loops of several tests, $\Delta\kappa$ is plotted as the change in κ that has occurred between initial loading and the current point in the cyclic loading history; $\Delta\kappa_s$ is the total change that would occur if cycling continued until saturation is reached ($\kappa = \kappa_s$). The ratio $\Delta\kappa/\Delta\kappa_s$ is plotted in Fig. 3.3.2 against the accumulated plastic strain for different tests, and the points form a trend that is independent of the strain range. Thus, the properties have less dependence on the strain range than do those in Ref. [15]. Furthermore, tests to failure are not required to establish κ_s; Fig. 3.3.3 gives an indication of the number of cycles required to approach saturation at 593°C (1100°F) for type 304 stainless steel. An option that bases the isotropic hardening on accumulated inelastic work, has been discussed in Ref. [12].

The reasonableness of applying kinematic hardening models has been examined through multiaxial testing at ORNL, especially for the stainless steels (e.g., see Refs. [7-9, 14-16]) and to a lesser extent for 2 1/4 Cr-1 Mo steel. The experiments have provided yield surfaces for various histories of combined tensile and torsional loadings while using a small-offset definition of yield (usually 10 $\mu\varepsilon$). Some typical ORNL experimental results are shown in Fig. 3.3.4 for a type 304 stainless steel specimen subjected to a nonradial loading at room temperature. The results have shown that the initial-yield surface is represented well by a von Mises surface [in keeping with Eq. (1)] and that subsequent plastic loadings do not alter the size and shape of the surface in a major and permanent manner. This latter observation supports the kinematic-hardening rule when coupled with the observation that the surfaces generally move in the direction of plastic straining.

In summary, multiaxial constitutive equations have been provided to LMFBR designers for consistent project usage in elastic-plastic analyses of structures. Analysts are now given options with respect to the dependence of κ on the plastic strain history; the options emphasize different parts of the history dependence and require different correlations of data. References [12-13, 17-18] discuss these options: the dependence on the accumulated *inelastic* work in Ref. [12], on the accumulated *inelastic* strain in Ref. [17], and on a critical inelastic strain history (stepwise κ variation) in Ref. [18].

3.3.4. Equations for Time-Dependent (Creep) Behavior

A common basic form of constitutive equation has also been established for representing the time-dependent behavior of these three LMFBR alloys. The approach is an equation-of-state approach that uses stress, temperature, and an accumulated strain-hardening measure as state variables. The equations have been discussed in Refs. [13-15, 19], and they have been incorporated into design guidance along with the elastic-plastic equations. The creep rate tensor $\dot{\varepsilon}^{C}_{ij}$ is observed [10] to be essentially colinear with the deviatoric stress tensor τ_{ij},

$$\dot{\varepsilon}^{C}_{ij} = \lambda \tau_{ij} . \quad (5)$$

For loading situations that do not involve stress reversals, the proportionality function λ has been taken to depend upon the current values of effective stress $\bar{\sigma}$, effective creep strain $\bar{\varepsilon}^{C}$, and temperature T,

$$\lambda = \frac{3}{2} \frac{\dot{\bar{\varepsilon}}^{C} (\bar{\sigma}, \bar{\varepsilon}^{C}, T)}{\bar{\sigma}} . \quad (6)$$

The analytical form for $\dot{\bar{\varepsilon}}^{C}$ is the same as that for the time rate of change of axial strains in constant uniaxial-load creep tests. The initial recommendation for FFTF use (Ref. [13]) and subsequent updates augmented this classical strain-hardening model with auxiliary rules for use when stress reversals occur. Under reversed loadings λ becomes dependent on $\bar{\varepsilon}^{H}$ rather than $\bar{\varepsilon}^{C}$, where $\bar{\varepsilon}^{H}$ is an effective strain measured from either of two reference strain origins (see Ref. [19]).

Correlation of creep data $\varepsilon^{C}(\sigma, t, T)$ from constant uniaxial-load tests under isothermal conditions is sometimes referred to as the *creep equation* and represents required *properties* information. Both primary and secondary creep strains are included, and in principle, any mathematical form has been considered acceptable for use in LMFBR design analyses as long as it represents the available creep data for the given material. Different forms have been used [20,21] for different materials and data bases. While some forms of the creep equation are used more easily in computational schemes than others, this has not been judged to be a major economic or time factor in performing structural analyses.

3.3.5. Interactions Between Classical Creep and Plastic Deformations

It has been recognized for a long time that the various modes of inelastic deformations influence one another for structural alloys at elevated temperatures. It has, however, been much more challenging to understand the nature of these influences to the point where they can be incorporated into constitutive equations intended for general design use. The challenges have included identifying potentially important interactions between deformation modes, understanding their magnitude and longevity, representing them with mathematical models, and understanding the consequences of the interaction models for loading conditions other than the ones from which they were initially developed.

The equations first recommended for LMFBR use, coupled the creep and plastic strains only through the observation that prior creep strains increase the range of elastic responses for type 304 stainless steel. Typical uniaxial data that support that observation are shown in Fig. 3.3.9. The equations in Ref. [13] allow the size of the yield surface to change to the maximum allowed value after a critical amount of creep strain has been accumulated. Further studies led to this interaction model being modified to permit κ to change (limited isotropic hardening) continuously with accumulated inelastic (including creep) work or strain (see Refs. [12] and [15]).

Further investigations of 2 1/4 Cr—1 Mo steel led to the conclusion that *reversed plastic strains* reduced the resistance to subsequent creep strains in much the same way as if the reversed strains were *due to creep*. Therefore, appropriate modifications were made to the equations for this material. In particular, the auxiliary rules for creep under load reversals were modified so that $\bar{\varepsilon}^{H}$ became dependent upon reversed plastic strains as well as reversed creep strains. Some examples of the improved response predictions provided by this mofication are given in Ref. [12].

The interaction between creep and plastic strains that was next incorporated into LMFBR design analysis guidelines has as its origin the proper representation of cyclic uniaxial stress-strain responses for fixed strain range cycling when hold times are imposed at extreme strain values. During such hold periods relaxation occurs and, without interactions between creep and plastic strains, the predicted stress point moves below the yield stress (interior to the yield surface in multiaxial stress space) as the relaxation takes place. Predictions for numerous such cycles lead to stress-strain loops that sequentially shift along the uniaxial-stress axis in the direction away from the end of the cycle where the hold period occurs. This behavioral feature is not in keeping with experimental data which show relatively stable stress-strain loops. The modification made specifies that the yield surface will shift (up to a limit) with creep strains. The shift is made as though plastic strains were occurring and Eq. (3) becomes

$$d\alpha_{ij} = C d\varepsilon^{P}_{ij} + H d\varepsilon^{C}_{ij} - \frac{1}{2\kappa}\frac{\partial\kappa}{\partial\varepsilon^{P}_{k\ell}}\, d\varepsilon^{P}_{k\ell}\,(\sigma_{ij} - \alpha_{ij}), \tag{8}$$

where $H = C$ if $\bar{\alpha} \leq$ limit and $H = 0$ if $\bar{\alpha} >$ limit. This condition, of course, leads to associated terms in the expanded form of the plastic-flow rule [Eq. (4)]; in particular the flow rule encompasses a dependence upon the creep strain via α_{ij}. The ability of the updated coupled equations to predict essential characteristics of material response is illustrated by comparing the predictions shown in Fig. 3.3.10 with the data shown in Fig. 3.3.11 for relaxation-hold cycling about a mean strain. Whereas these comparisons are very good, Fig. 3.3.12 illustrates that predictions based on the uncoupled theories of plasticity and creep are not adequate for this type of loading.

More comprehensive studies of treating creep and plastic strains as unified inelastic strains have been underway during the last decade. Considerable progress has been made, and the next section summarizes a great deal of that work.

3.3.6. Bases for Improved Constitutive Equations for Design Usage

The constitutive equations discussed in the previous paragraphs have been identified over the past decade and used to carry out analyses for the design of fast breeder reactor components. The equations have evolved over that period with modifications and improvements being made periodically as more was learned from material behavior studies and design applications. The intent of this section is to outline trends that now exist toward further development and refinement of constitutive equations.

The progress that has been made during the past decade has come from many investigators (within industrial, research, and academic communities) on a number of fronts. The fronts that have received notable attention include: (1) implementation of the principles of continuum mechanics and thermodynamics, (2) identification of concepts within the classical framework of elasticity, plasticity, and creep to represent history-dependent material responses, (3) analytical representation of behavior due to specific physical mechanisms, (4) identification of variables appropriate to defining the *state* of a material undergoing general inelastic deformation, and (5) identification of appropriate *growth laws* for describing the evolution of state variables during general thermomechanical loadings.

3.3.6.1 Fundamental Considerations

Although impacts began to be felt in applied works only in the past decade, a fundamental emphasis has been in place, especially within the academic community, for more than two decades to promote the understanding and use of complete and proper statements of the principles of continuum mechanics and thermodynamics in all fields of solid mechanics. This emphasis has been characterized by such writings as Truesdell and Toupin [22], Green and Rivlin [23], Eringen [24], and Green and Naghdi [25]. The application of the principles of thermodynamics to the behavior of solid materials has been addressed by Biot [26,27] and by Schapery [28]. It has been recognized that since many of the variables are tensorial, it is important in formulating constitutive equations to adhere to the principles of tensor and matrix analyses. This aspect of constitutive equation formulation was given especially heavy attention during the 1960s and is addressed in writings such as Refs. [22-28,29-30]. Studies such as these provide fundamentals upon which *multiaxial* constitutive equations must be formulated. Often, especially in mechanistic approaches to constitutive equation development, multiaxial aspects are treated in a much less fundmental way. For example, uniaxial observations have often been generalized to multiaxial situations by mere replacement of uniaxial variables with *effective* values; although in some instances this is admissible, it by no means covers the full set of possibilities or in some events even the most appropriate one. In the case of anisotropic materials, for example, much more is involved than replacement of a single term by another scalar quantity.

In keeping with the continuum mechanics approach followed in the works referenced above, the development of improved constitutive equations can be thought of as being composed of three steps. The first is to derive guidance, mostly from experimental observations, on selecting variables capable of characterizing material behavior and the fashion in which the history dependence is manifested. The second step is to select forms of these variables (state variables) that possess the characteristics of being *objective* in the sense of continuum mechanics. The third step is to relate these variables in functional ways that properly capture the history dependence as well as adhere to the rules of tensor analysis.

3.3.6.2 Extensions of Classical Theories

Much of the work that has been done in the last decade can be categorized as extending or refining concepts within the framework of elasticity, plasticity, and creep analysis. Whereas those works have adhered to the above principles to varying degrees, each have made their contribution to improved understanding and modeling of inelastic material behavior. Only a few are cited here to illustrate areas that have received significant attention.

In the case of plasticity, the LMFBR equations discussed above used representations which described essential characteristics of the material behavior as portrayed by information available. At the same time cognizance was given to the data base required for design implementation.

Those equations were formulated to be consistent with the principles described in references such as [25,31-33], but the experimental information was not sufficient to identify a nonlinear hardening theory for general applications; therefore the assumptions of linear kinematic hardening were recommended for situations involving nonradial loadings. Extensions of those earlier works have continued. A major challenge is to develop sufficient information on which to establish nonlinear representations appropriate for cyclic as well as nonradial loadings.

Multiaxial yield and flow characteristics in austenitic stainless steels have been extensively studied by Liu and Ellis (Refs. [7] through [9]). Although their studies support the reasonableness of the idealizations employed in the LMFBR plasticity model, they also have revealed some of the characteristics of the nonlinear behavior of stainless steels. Representation of the detailed nonlinear yield characteristics has not yet been satisfactorily formulated for incorporation into general design practice. However, the LMFBR equations do allow for the use of nonlinear kinematic hardening models for radial loading situations, and particular models have been addressed for type 304 stainless steel. Other considerations of nonlinear models (such as Refs. [34,35]) have been reported during the last decade. These have also been restricted by the lack of sufficient experimental information for general loading situations.

The creep constitutive equations used in LMFBR design analyses fall within the broad category of equation-of-state approaches, many of which are discussed by Rabotnov [36]. In addition to these types of equation-of-state approaches, considerable attention has been given in the last decade to methods that are based on hereditary integral formulations. Among these are the works of Rogers and Pipkin [37], Lockett [38], and Krempl [39]. Whereas, many of these works draw upon or extend the work that had been developed for polymer materials, they involve multiple integral formulations in some cases, and nonlinear single integral formulations in others. Although the latter is perhaps on less rigorous mathematical grounds, its practical application is more feasible. It is noteworthy that there are important mechanistic and phenomenological differences between the behavior of metals at elevated temperature and the behavior of high polymers and these must be accommodated in the functional representations.

Rashid [40] has incorporated a nonlinear integral representation of creep into finite element computer techniques. Whereas Rashid shows the ability to reproduce creep responses under a number of variable load situations by this type of formulation, general multiaxial relations applicable to cyclic and nonradial loadings have not been established. Therefore, this is not at present a leading candidate for design utilization.

There have been attempts to devise improved models for plasticity and creep on the basis of sublayer modeling over the past decade. Some of the investigators working in this area are Besseling [41], Mroz [42], and Meijer [43]. These types of models have been reported in terms of their use within finite element techniques by Meijer [43] and Janssen [44]. While these works show promise, sublayer models are not yet employed within the European breeder reactor programs which have sponsored most of the cited works.

3.3.6.3 Physical Modeling

Over the past decade considerable effort has been dedicated to the understanding and modeling of the physical mechanisms which govern inelastic deformation. Indeed, mobile microstructural imperfections such as dislocations and vacancies almost exclusively give rise to inelastic deformations in metals. Much of this work has been addressed by Swindeman and Brinkman [6] earlier in this chapter. Some of the specific works included in the category of physically based models are given by Refs. [45-50].

Constitutive equations which form the basis of structural analysis in support of design must be macroscopic and phenomenological in nature. If one starts from a strictly microstructural standpoint there seems to be no way to presently sort the details on that level to arrive at quantitative predictions of gross behavior as required by the design analyses. The best approach is to use mechanistic-based (or -inspired) phenomenology to lead to physically sound constitutive relations that can be tested for interpolation or extrapolation. Although microstructural information may sometimes be employed in a suggestive rather than rigorous way, it, nevertheless, should always be looked upon to provide guidance toward physical soundness and toward defining realistic ranges of applicability.

Careful definition of the range of applicability of constitutive relations and the extent to which they allow extrapolation to neighboring ranges of variables is an important aspect in long-term, high-temperature design situations. Much of the data on which design correlations are based are generated, for example in stress-temperature regions where a set of specific physical mechanisms may be operative; however, the design conditions may be in regions where others are also active or even dominant.

Thus constitutive equations should ideally embrace the ability to transcend from one region to another. The work of Swindeman [51] is one example where such continuity is being explored in terms of contour maps. In that work Swindeman suggests different mechanisms that dominate the creep deformation process in different regions in a stress-temperature space for type 304 stainless steel. On the basis of such an approach, even if extrapolation capabilities cannot be verified, the hope is that guidance can be obtained in terms of stress and temperature ranges where specific constitutive equations may be appropriate.

3.3.6.4 Development Trends for LMFBR Application

The activity in constitutive equation development in the LMFBR program is basically two pronged. One part of the effort involves making refinements of the current constitutive relations in response to continued materials testing and feedback from design analysts. The second part is more fundamental and is concerned with the development of alternate constitutive models aimed at providing more complete representations of high-temperature material behavior.

The constitutive equations outlined in Section 3.3.3 through 3.3.5 above are essentially *classical* and rest on the assumption that the inelastic strain can be decomposed into two additive contributions, one time-independent (plasticity) and one time-dependent (creep). Experimental evidence does not fully support this assumption, however, and suggests representations in which creep and plasticity are characterized as occurring simultaneously and interactively. Section 3.3.5 discusses creep/plasticity interactions and outlines how they have been dealt with in the short run in terms of modifications of the design equations.

The more fundamental approach to creep/plasticity interactions being pursued under LMFBR programs and elsewhere disregards the assumption concerning the distinction and additivity of creep and plastic strains and writes the flow equations in terms of the *total* inelastic strain rate. Such constitutive equations have been termed *unified* equations in as much as they consider creep and plasticity at elevated temperatures to arise from the same physical mechanisms.

Unified equations are being developed from the standpoint of identifying appropriate variables capable of describing the *state* of a material during inelastic deformation processes, and of establishing appropriate *evolutionary* or *growth laws* that describe the way these variables evolve during those deformation processes. The framework in which such a state variable theory should be formulated has been discussed in detail by Onat in Refs. [4], [52], and [53]. The method that is presently being pursued most vigorously is in the category of constitutive equations that embody the concept of an *internal (or back)* stress being one of the state variables. This approach is conceptually capable of including models of physical mechanisms in comprehensive equations. For example, the work of Robinson [14,15] shows that the dislocation-based models of Lagneborg [45] and of Störackers [50] can be incorporated in a unified multiaxial constitutive model. Additionally, Robinson's work recognizes the flow potential formulation of Rice [54] and the extensions to that work by Ponter and Leckie [55]. While the conventional stress and a generalized internal stress are taken as state variables, the growth law is expressed in multiaxial terms as representing competing hardening and softening terms. This concept is in keeping with the early Bailey-Orowan theories [56,57]. Robinson is developing specific forms of state variables and growth laws based on data from DOE experiments as well as other experimental results such as those due to Mitra and McLean [58], Williams and Wilshire [59], and Poirier [60]. The overall status of this developmental work was discussed in 1978 in Ref. [15] and the work is continuing.

As noted earlier, the fundamental model being developed falls within a broad category of models being examined by various investigators. Krieg [61] surveyed a number of related investigations and cataloged them according to the manner in which they treat the state variables chosen.

One of these investigations, Hart [62], does not rest on the concept of back stress but includes a tensorial state variable representing a stored strain that is partly recoverable upon unloading. Hart's principal variable of state is a scalar, termed hardness, that accounts for isotropic strain hardening. Provision is also made in Hart's model for static thermal recovery of the scalar hardness. Material parameters are determined mainly from relaxation testing which allows information over several decades of strain rate to be collected in a relatively short time. Hart and Solomon [63] have also found that a trajectory can be identified in a log stress-log strain-rate plot, along which curves from tests on specimens with different amounts of pre-hardening can be translated into coincidence. This greatly facilitates the quantification of functional relationships and parameters in the Hart model. Yamada and Li [64] have conducted experiments on several bcc metals in efforts to verify Hart's theoretical model.

Another notable material model is that under development at Stanford University by Miller and co-workers [65-67]. Miller's model is physically based and limited to uniaxial deformation. It includes two (scalar) state variables, one representing a "rest" stress and the other a "friction" stress due to deformation; static thermal recovery of both state variables is accounted for. Emphasis in the development of the Miller model has been on monotonic deformation, creep and dynamic strain aging effects and their mutual interactions.

The desire of LMFBR programs is to bring together the best features of such investigations as described above in order to identify equations that can be employed under general long-time design conditions after they have been adequately verified through varied experimental investigations; these include uniaxial, multiaxial, and structural test conditions.

3.3.7. Summary

A description has been presented of some of the progress that has been made in the evolution of constitutive equations for representing the behavior of structural alloys undergoing inelastic deformations at high temperatures. It has been emphasized that a need for practicable inelastic analyses in the design process has developed during the past decade especially for LMFBR systems. Fulfillment of that need has provided force to the development of equations for general application under long-term elevated-temperature conditions and to the generation of associated material behavior information.

The constitutive equations that have been established for LMFBR design use have been discussed. Those equations are interim and improved as more information becomes available. Present trends toward replacement equations have been described in terms of unified constitutive equations based on total inelastic strains rather than on the summation of plastic and creep strains. A goal is the incorporation of characteristic features of physical mechanisms known to be active while retaining a phenomenological model capable of being applied to general multiaxial design situations. The future equations must, of course, consider all the elements required for design implementation. This includes the ability to represent general material behavior characteristics, ability to be employed in practicable analytical techniques (e.g., finite element methods) and require amounts and types of properties data that are obtainable from materials programs.

The theoretical, analytical, and experimental bases are much better today than they were a decade ago to make progress in this area. Therefore, it is anticipated that the next decade will be a productive period. Projects such as the U.S. LMFBR program will, undoubtedly, be looking to utilize the combined progress made by investigators working in the various disciplines that relate to establishment of improved inelastic design analysis capabilities.

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List of Figure Captions for ASME Decade of Progress Paper

CONSTITUTIVE EQUATIONS FOR MEETING ELEVATED TEMPERATURE DESIGN NEEDS

C. E. Pugh
D. N. Robinson

3.3.1. Stress-strain loops for fixed strain-range cycling of type 304 stainless steel illustrate characteristic hardening with initial accumulation of inelastic strains.

3.3.2. The dependence of the yield parameter κ on accumulated plastic strain is (in one option) simplified by allowing strain range dependence to enter only through the κ_s values for saturated cycles. A master curve gives the isotropic hardening ($\Delta\kappa = \kappa - \kappa_0$) in terms of a fraction of the hardening reached at saturation ($\Delta\kappa = \kappa_s - \kappa_0$) as a function of accumulated plastic strain. Data shown are from three continuous fixed strain-range cycling tests.

3.3.3. The total stress range for fixed strain-range cycling of type 304 stainless steel saturates with continued cycling; data are shown for five separate tests at 593° (1100°F). Although the stress range and κ are not identical measures, saturation of the stress range approximates saturation of κ.

3.3.4. Yield surfaces have been determined from specimens subjected to combined tensile and torsional loads to examine multiaxial aspects of elastic-plastic constitutive equations. Surfaces corresponding to initial yield and yield subsequent to three plastic loadings are shown for type 304 stainless steel. A normalized axial stress is shown to allow the von Mises surface to appear as a circle.

3.3.5. Strain-hardening models have been shown to give reasonable predictions of long-time variable-load creep test results especially when the stress levels are low to moderate. The sample uniaxial test shown is for type 316 stainless steel (heat 8092297) at 593°C (1100°F).

3.3.6. Strain-hardening models have been evaluated by comparing predictions with results from creep tests in which the stress and temperature are varied. The comparisons shown are for 2 1/4 Cr—1 Mo steel (heat 20017).

3.3.7. Auxiliary rules have been provided in constitutive equations for predicting creep responses under loadings that involve stress reversals.

3.3.8. Multiaxial aspects of the constitutive equations for creep have been compared against data from tests that involve combined tensile and torsional loadings and combined tensile and internal pressure loadings. The tests shown are for type 304 stainless steel at 593°C (1100°F) and illustrate that creep rates after 120 h of constant stress state loadings are essentially normal to the corresponding von Mises ellipses.

3.3.9. The elastic stress range for type 304 stainless steel is increased by creep deformations. Creep and plastic loadings of the three specimens took place at 593°C (1100°F) in this example.

3.3.10. Stress-strain loops predicted by constitutive equations which couple the kinematic motion of the yield surface to creep deformations have desirable stability characteristics. The conditions shown correspond to fixed strain range cycling (0.4%) with a mean strain and a hold period of 24 hr at the peak tensile strain for 2 1/4 Cr–1 Mo steel at 538°C (1000°F).

3.3.11. Measured stress-strain loops for 2 1/4 Cr–1 Mo steel (heat 20017) for the loading conditions considered in Fig. 3.3.10.

3.3.12. Stress-strain loops predicted by uncoupled constitutive equations for creep and plasticity drift away from the end of the cycle where the relaxation hold period occurs for the loading conditions considered in Figs. 3.3.10 and 3.3.11.

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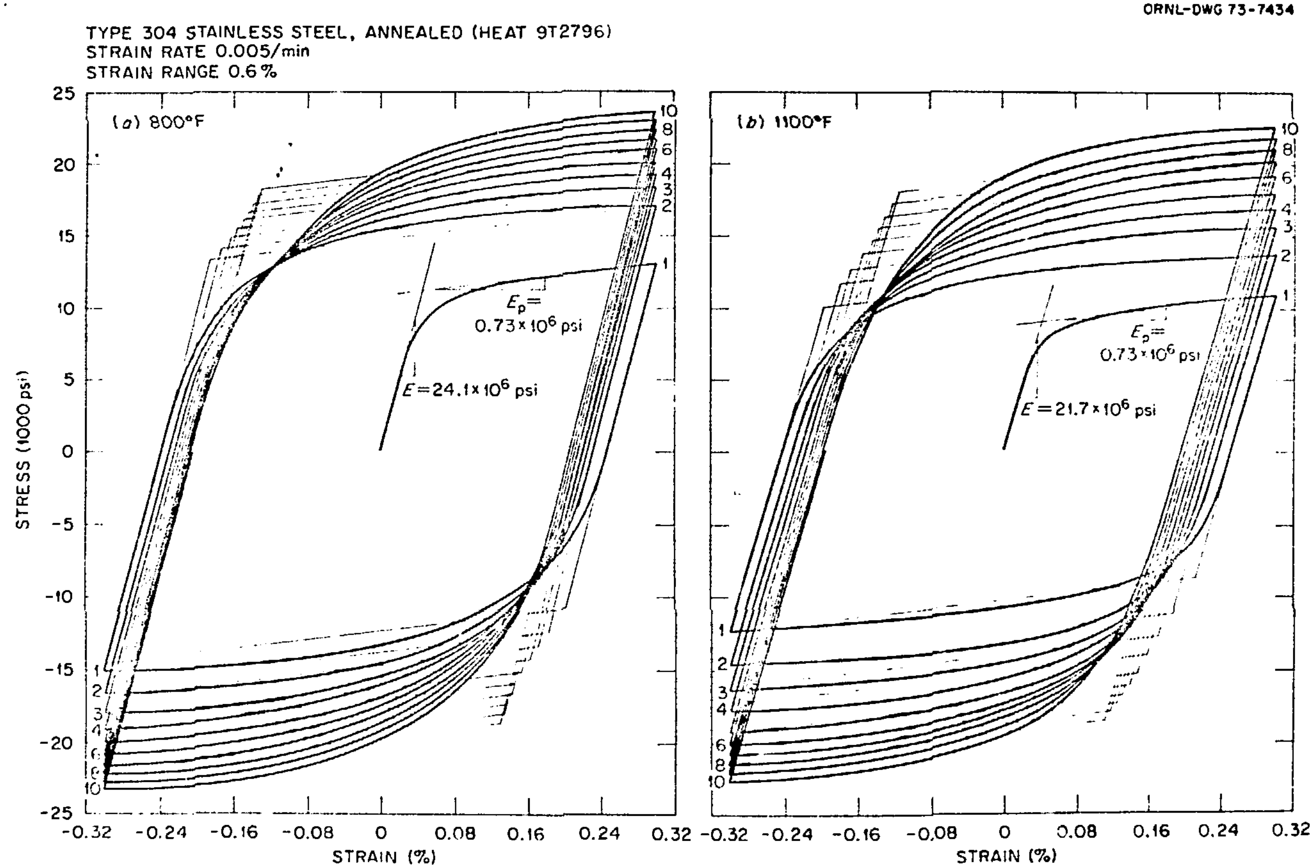

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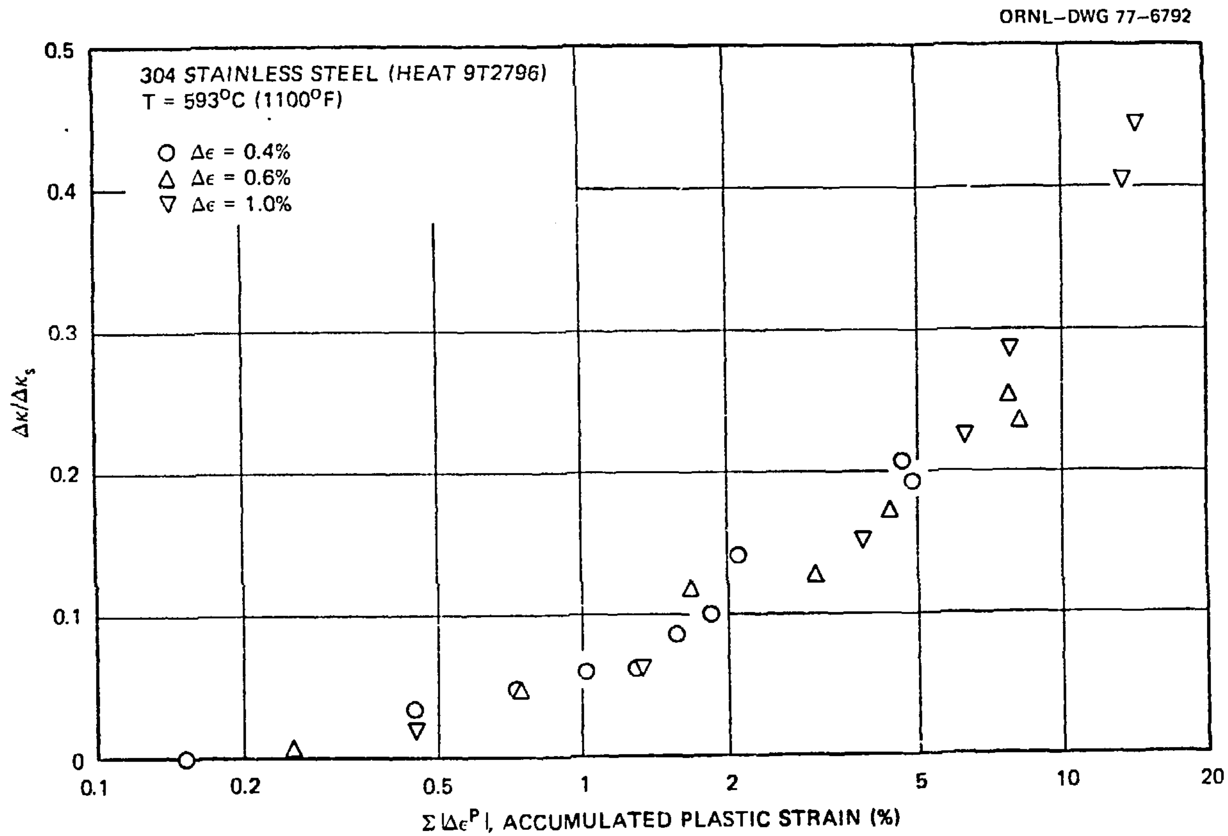

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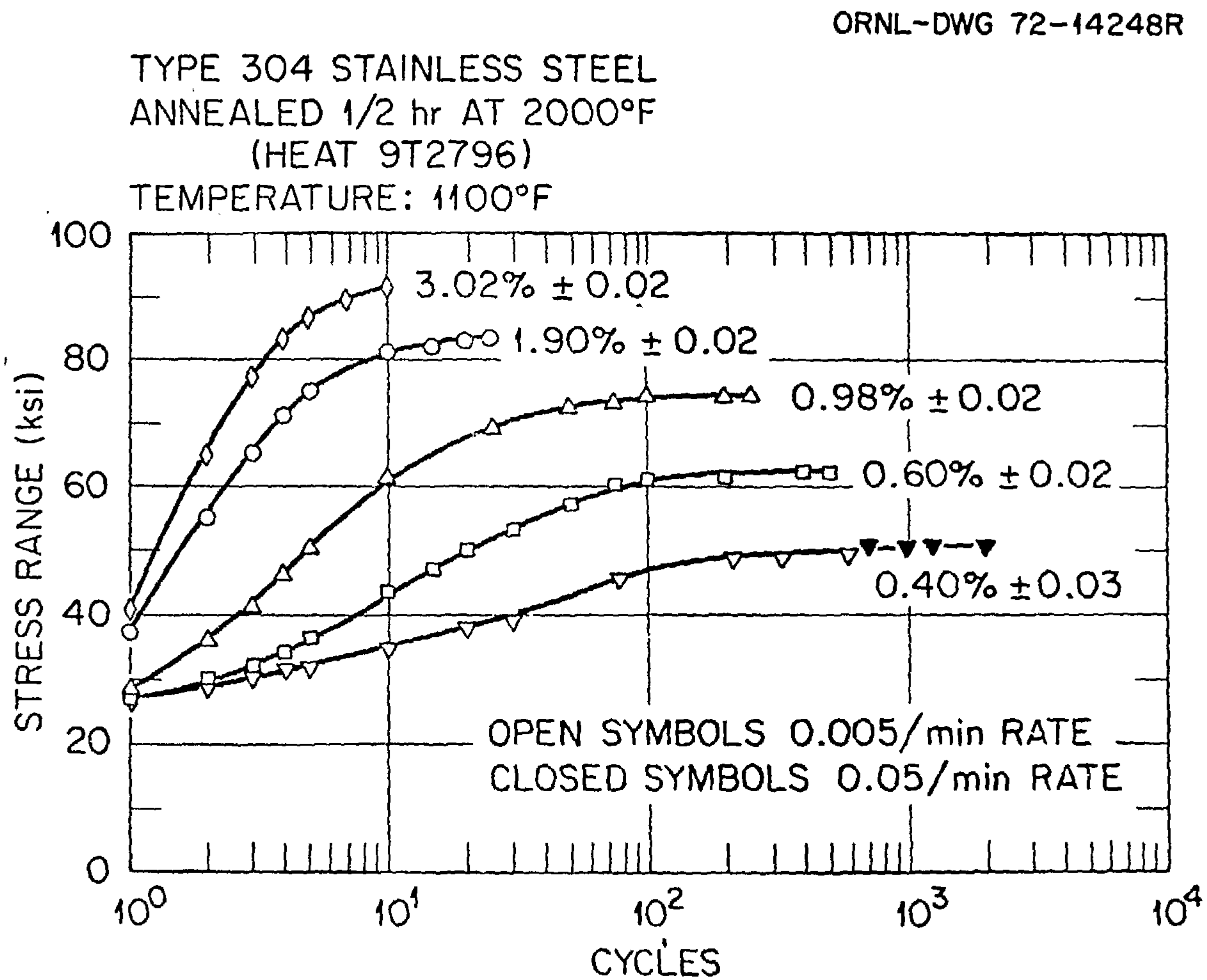

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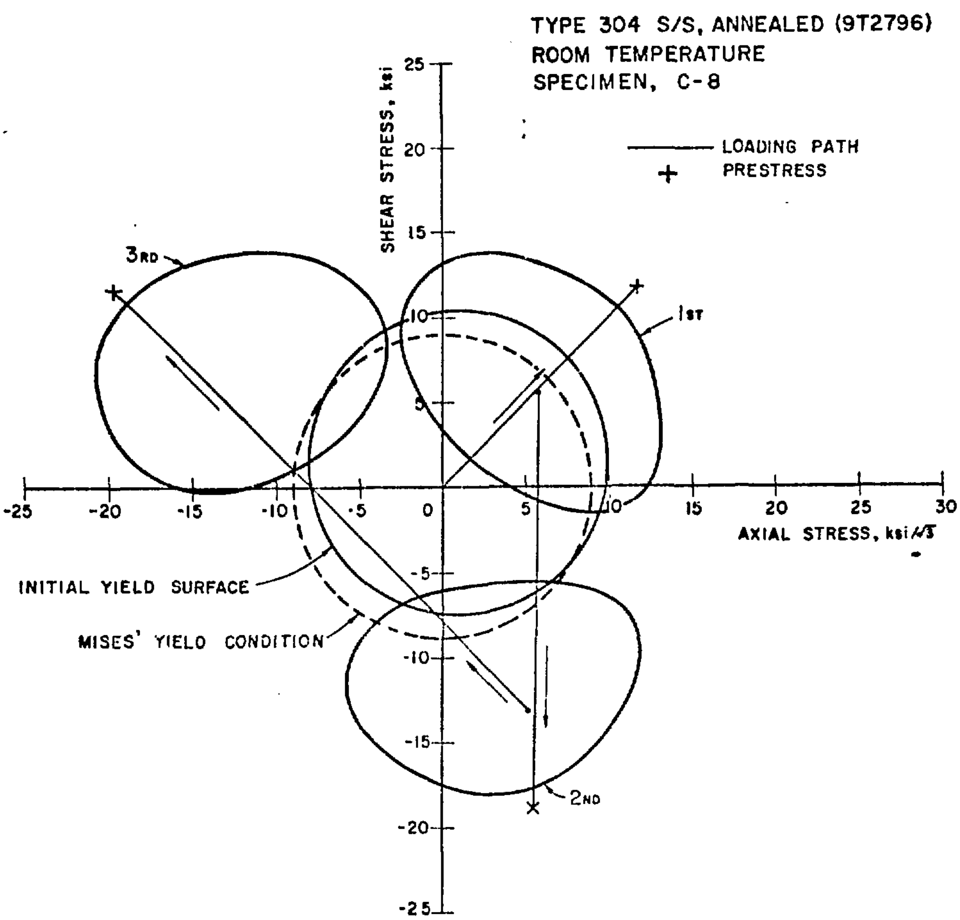

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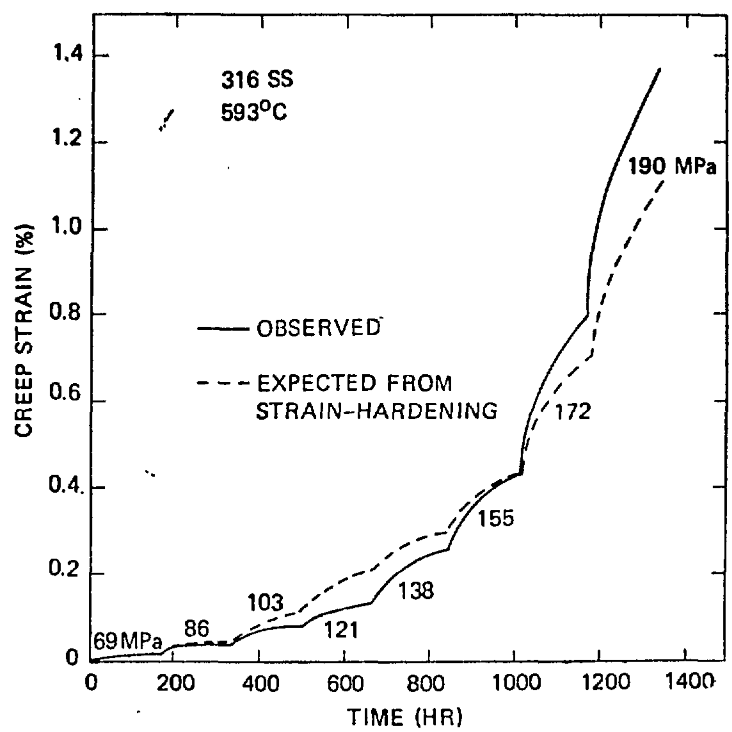

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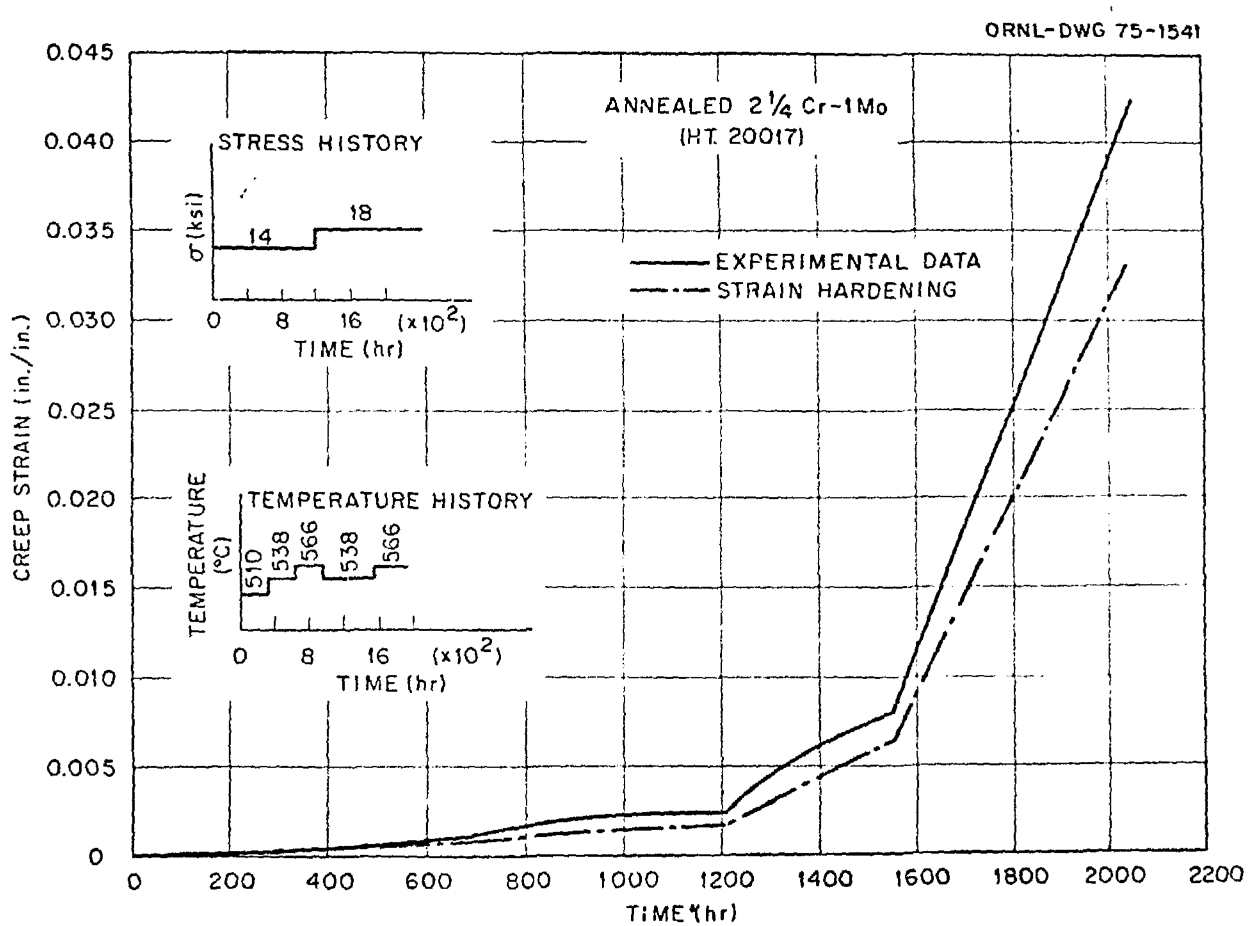

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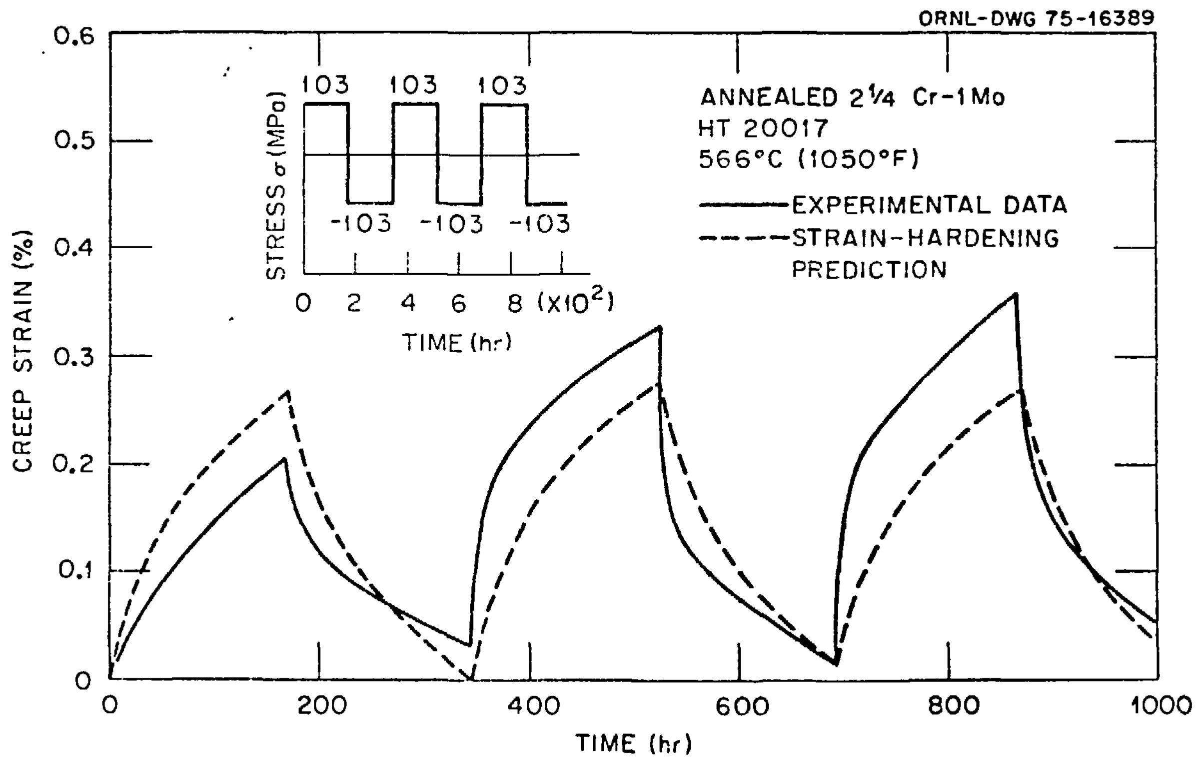

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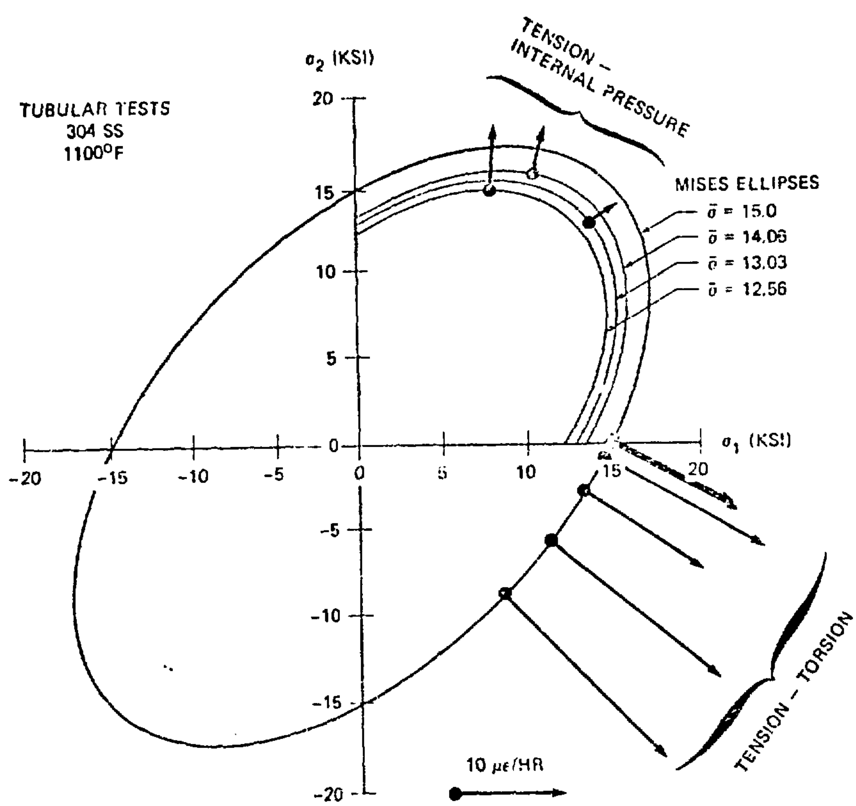

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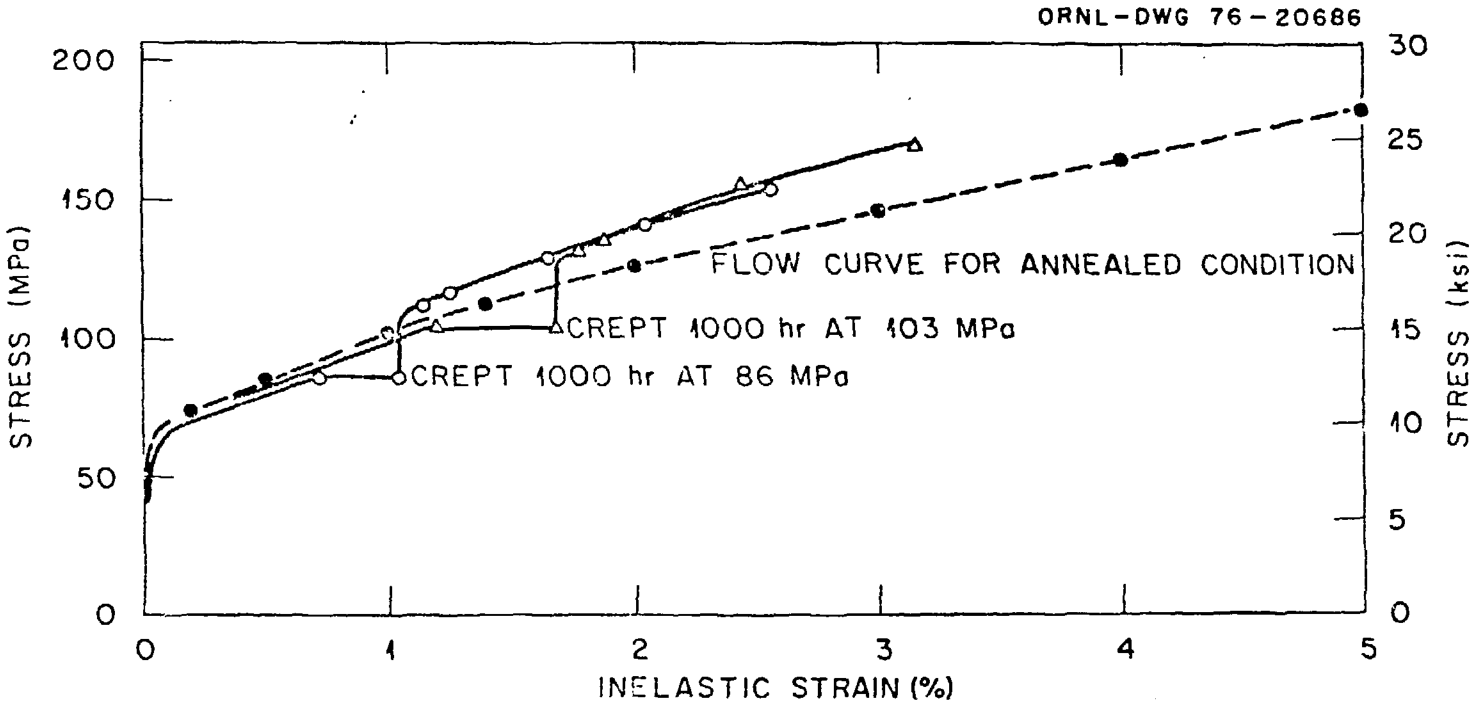

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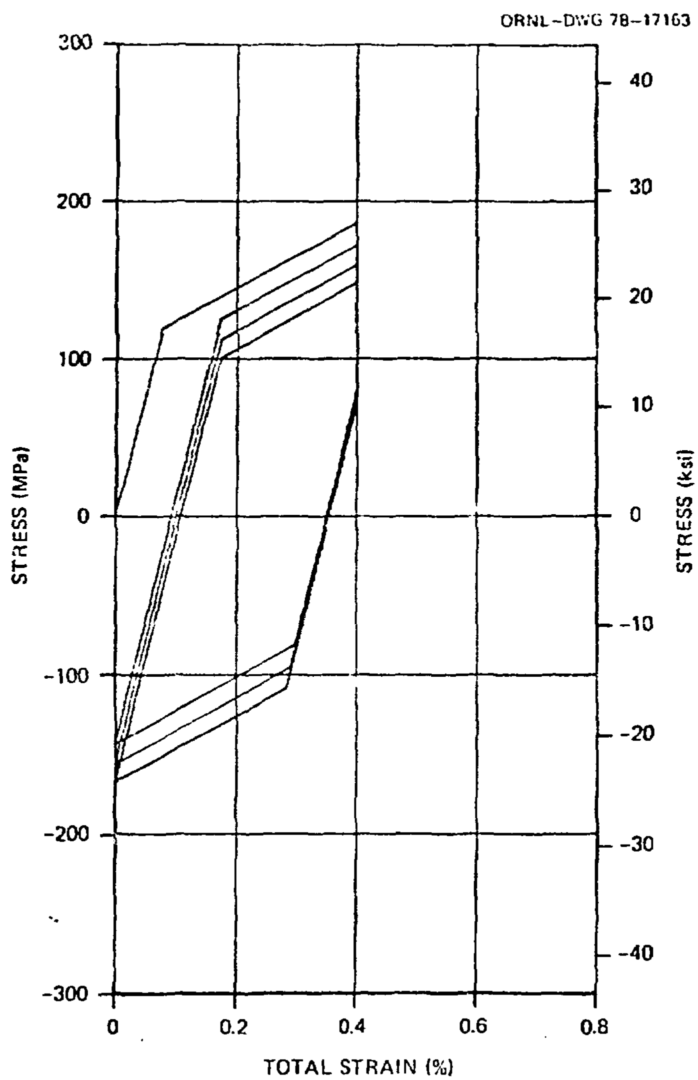

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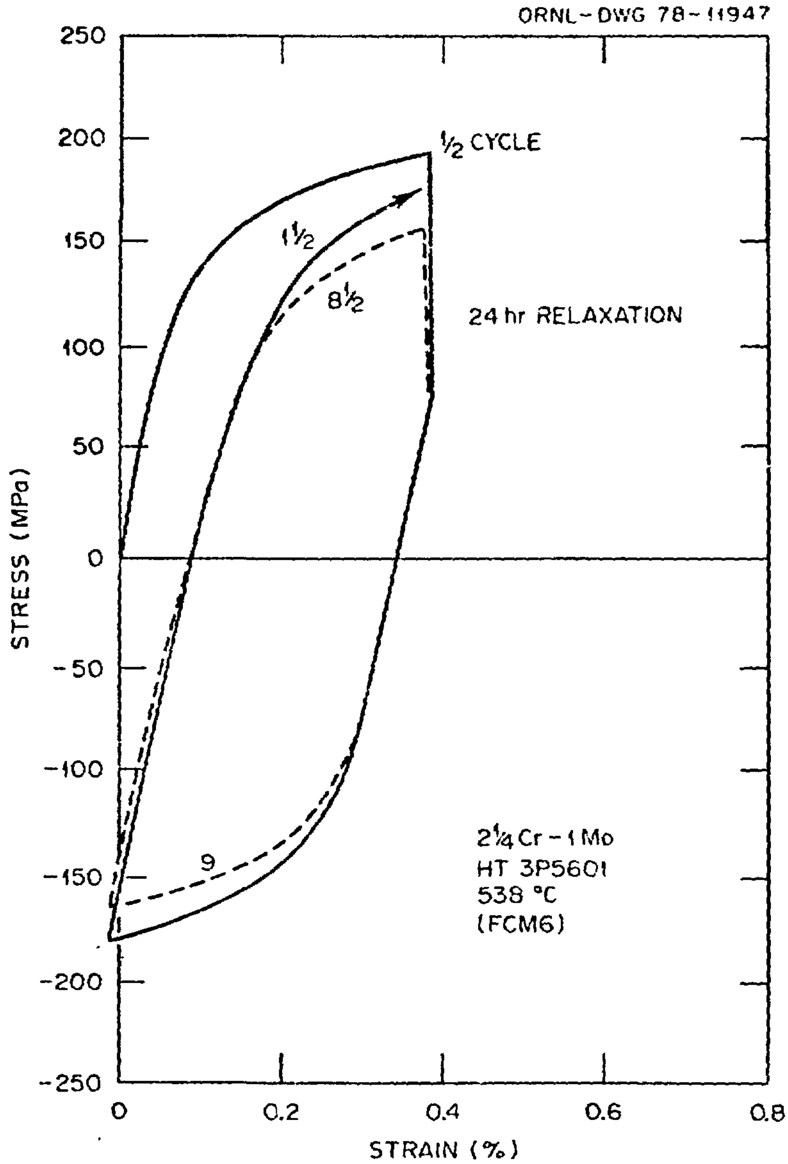

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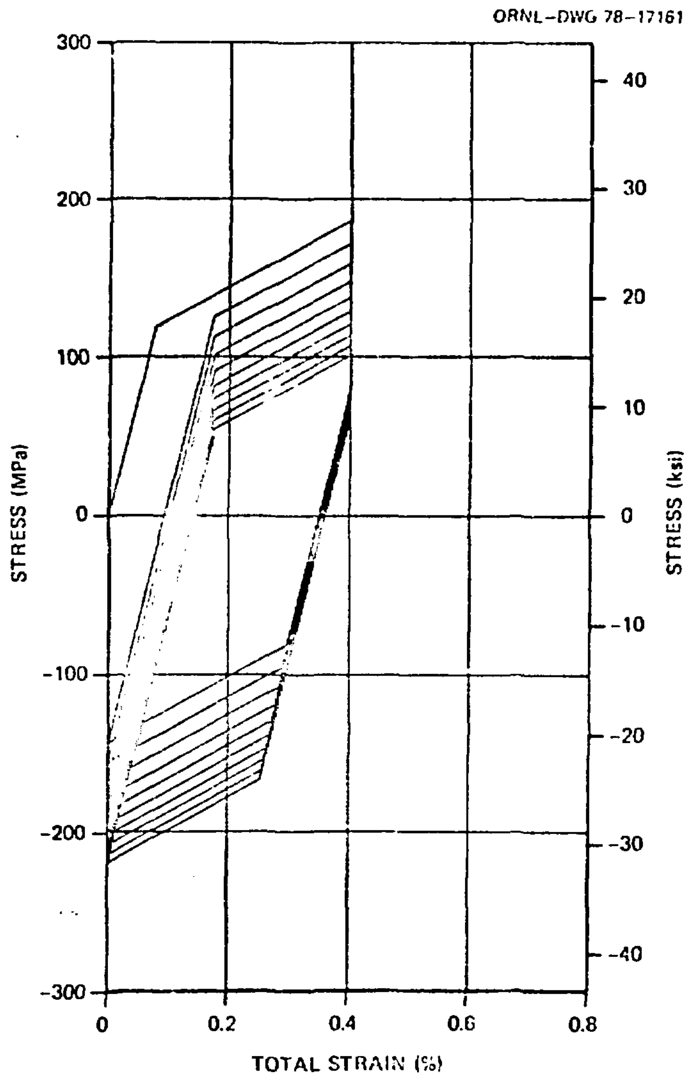

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