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Estimation Problems Associated with the Weibull Distribution

K. O. Bowman
L. R. Shenton

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ESTIMATION PROBLEMS ASSOCIATED WITH THE WEIBULL DISTRIBUTION

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF TABLES (Continued)

ABSTRACT

Series in descending powers of the sample size are developed for the moments of the coefficient of variation v^* for the Weibull distribution $F(t)=1-\exp[-(t/b)^c]$. A similar series for the moments of the estimator c^* of the shape parameter c are derived from these. Comparisons are made with basic asymptotic assessments for the means and variances.

From the first four moments, approximations are given to the distribution of v^* and c^*. In addition we give an almost unbiased estimator $\bar{c}$ of c when a sample is provided with the value of v^*. Comments are given on the validity of the asymptotically normal assessments of the distributions.

Key Words: Almost unbiased estimator, Confidence Intervals, Moment series.

1. INTRODUCTION

Moment estimators for the shape parameter c and scale parameter b are given by

$$\sqrt{\{\Gamma(1+2/c^*)/\Gamma^2(1+1/c^*) - 1\}} = \sqrt{m_2}/m_1' \qquad (1.1)$$

$$b^*\Gamma(1+1/c^*) = m_1' \qquad (1.2)$$

when a random sample of size n is given with mean m_1', and second central moment m_2; we prefer m_2 rather than the sample variance to simplify moment series developments, and in any case the choice has no effect on basic asymptotics. We thus use $v^* = \sqrt{m_2}/m_1'$ for the sample coefficient of variation. Our first objective is to set up an approximation to the distribution of v* by using four-moments. Previous work (Newby, 1980), has gone no further than to provide basic asymptotics for terms appearing in the covariance matrix. An impression of the forms of Weibull densities is given in Figure 1.

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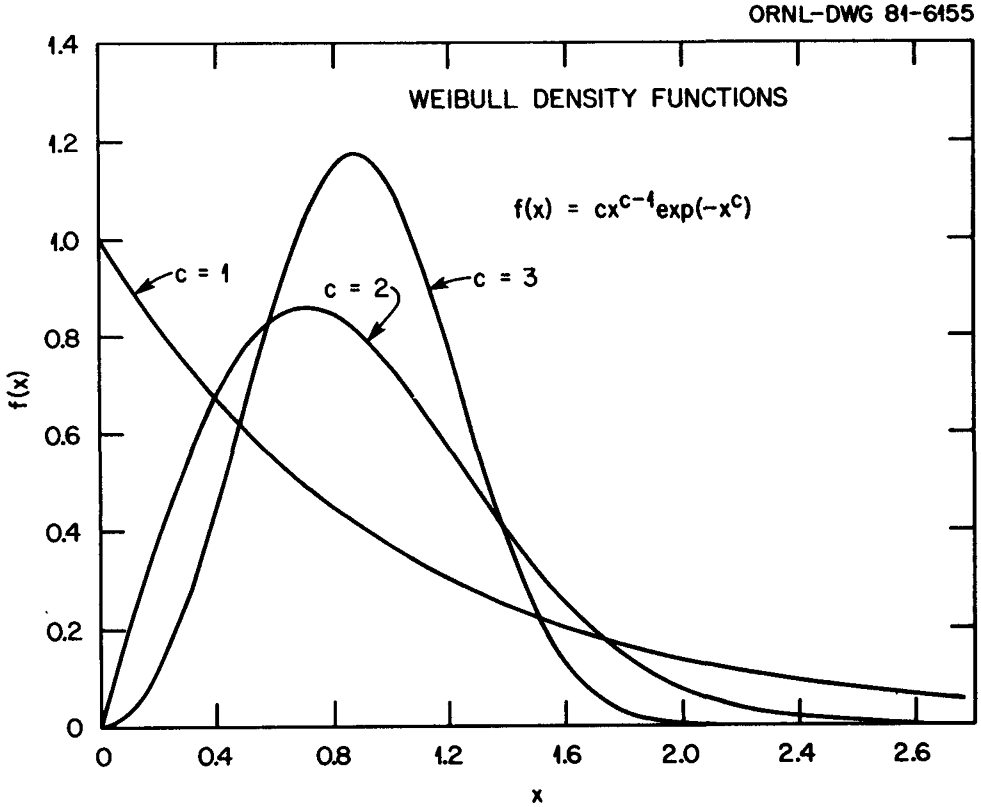

Figure 1. Weibull Density Functions

2. THE MOMENTS OF THE COEFFICIENT OF VARIATION

Fundamentally we construct a Taylor expansion for v^* in the arguments $\varepsilon_1 = m_1' - \mu_1'$, $\varepsilon_2 = m_2' - \mu_2'$ where m_2' is the sample second noncentral moment and $E(\varepsilon_1) = E(\varepsilon_2) = 0$, with

$$\mu_r' = b^r \Gamma(1+r/c) \quad , \quad r = 1, 2, \cdots \quad . \tag{2.1}$$

Central moments of the Weibull variate are derived from the usual correction formulas; for example

$$\mu_2 = \mu_2' - \mu_1'^2.$$

The technique and programming of expectations such as $E\left(\varepsilon_1^{r_1}\varepsilon_2^{r_2}\right)$ for positive integers r_1, r_2 has been described elsewhere (Shenton, et al 1971, Shenton and Bowman, 1975). We have for example a Taylor series for the mean

$$\mu_1'(v^*) = E(v^*) \sim v + v_1/n + \cdots + v_{24}/n^{24} \tag{2.2}$$

where n is the sample size, and v_s $(s = 1,2,\cdots)$ is a function of c only. Similar expansions have been constructed for the three central moments $\mu_2(v^*)$, $\mu_3(v^*)$, and $\mu_4(v^*)$, each up to the term n^{-24}. A tabulation (Table 1) has been constructed for c = 0.8(0.1)2.6(0.2)3.2, 3.5, 3.8, and 4.0. First order terms in the mean and variance of v^* are given in Appendix A; these give a glimpse of the complication to be expected for higher order moments.

Table 1. Moment Series for the Coefficient of Variation (v*)

Mean of v* ($\mu_1'(v^*)$)

s\c	0.8	exp	0.9	exp	1.0	exp	1.1	exp	1.2	exp
0	1.260512786807108	00	1.113029987157241	00	1.000000000000000	00	9.102155978248430	-01	8.369022119976303	-01
1	-3.186135824400678	00	-2.095198805124806	00	-1.500000000000000	00	-1.142895533137245	00	-9.127063247163171	-01
2	3.923596943322256	01	1.435493425359505	01	6.125000000000009	00	2.890698486926567	00	1.449366414310598	00
3	-2.625738463266615	03	-5.512314449592192	02	-1.513124999999992	02	-5.024113208779025	01	-1.917912603212864	01
4	4.401804281017673	05	5.091281171544579	04	8.642210937499948	03	1.925065290383985	03	5.227477831951734	02
5	-1.347179301010705	08	-8.306908444616155	06	-8.505428320312672	05	-1.246363592313010	05	-2.371358280735319	04
6	6.500345989094697	10	2.071586536386151	09	1.251222284423809	08	1.185955914814016	07	1.559743555428123	06
7	-4.551764290056750	13	-7.276477772356963	11	-2.538401905988445	10	-1.532284553319325	09	-1.376570158184262	08
8	4.383755140908904	16	3.415461476836423	14	6.743174152794177	12	2.554218846302653	11	1.550063826154326	10
9	-5.590059260807481	19	-2.064924221966277	17	-2.262406892236728	15	-5.301269872157670	13	-2.150162691111277	12
10	9.170370870635454	22	1.564581178028220	20	9.335091183857489	17	1.334600262767357	16	3.580738299855652	14
11	-1.891557369816764	26	-1.454330655073523	23	-4.640837914389187	20	-3.994552503591035	18	-7.019286969818116	16
12	4.814244075504368	29	1.630048663775322	26	2.734561982239502	23	1.399044549543169	21	1.594671778312624	19
13	-1.487980839237888	33	-2.171530858632594	29	-1.884275122746316	26	-5.660063271527936	23	-4.145970301334827	21
14	5.509107751933331	36	3.396595815550350	32	1.501227222116383	29	2.616700027901771	26	1.220721199844802	24
15	-2.414362500172714	40	-6.172104122806246	35	-1.369547748057011	32	-1.369783130757721	29	-4.034691761740833	26
16	1.239395393904510	44	1.290926289838932	39	1.418623118359454	35	8.055313418513408	31	1.485673913571678	29
17	-7.383673948068619	47	-3.082333639679746	42	-1.656091270620265	38	-5.284991115584740	34	-6.054842784460779	31
18	5.062873236313179	51	8.340371464281788	45	2.164488705901134	41	3.844885040937879	37	2.715447432650129	34
19	-3.966084583558087	55	-2.540756166442873	49	-3.148511719516817	44	-3.084832493367666	40	-1.333257711087846	37
20	3.525851548411277	59	8.662359671358378	52	5.070050970385084	47	2.716185574257837	43	7.133941387722537	39
21	-3.535690852366890	63	-3.287532009409473	56	-8.994383654014457	50	-2.613006596900501	46	-4.142783616859091	42
22	3.977422308217474	67	1.382097775917703	60	1.750110839792858	54	2.735402254164176	49	2.601197642270292	45
23	-4.994162485217707	71	-6.407669710633221	63	-3.719990760549686	57	-3.104554534567808	52	-1.759895270073822	48
24			3.262655127362382	67	8.605808038387258	60	3.807187511920443	55	1.279003825513046	51

s\c	1.3	exp	1.4	exp	1.5	exp	1.6	exp	1.7	exp
0	7.757191255905958	-01	7.237521513051114	-01	6.789686930973462	-01	6.399052963994529	-01	6.054795618121655	-01
1	-7.558025185405473	-01	-6.439742839794383	-01	-5.613201349285887	-01	-4.983564735578482	-01	-4.491561974913652	-01
2	7.435479698189682	-01	3.721261345038407	-01	1.655717684480837	-01	4.577163891856557	-02	-2.588843923782742	-02
3	-8.134985537471652	00	-3.747040937205332	00	-1.847462053672064	00	-9.683576234501915	-01	-5.398694515409572	-01
4	1.644149335298913	02	5.768723520967832	01	2.192258620089601	01	8.796028414057451	00	3.632090080934912	00
5	-5.477833710936661	03	-1.463508312141678	03	-4.358064523222984	02	-1.403665765475458	02	-4.761037561625504	01
6	2.615867960356166	05	5.264375690316561	04	1.214005399009297	04	3.090903637194482	03	8.410501345050868	02
7	-1.660431025080055	07	-2.497754141900497	06	-4.431927251015832	05	-8.869853615743112	04	-1.925270370845523	04
8	1.333087812031077	09	1.488485191440023	08	2.020357986806791	07	3.162845128938612	06	5.455627586783979	05
9	-1.307679440781435	11	-1.076747073637553	10	-1.112034682681322	09	-1.355551915292960	08	-1.850398646489909	07
10	1.527937360851153	13	9.220611015964353	11	7.209657473433931	10	6.814786299853632	09	7.335535327922167	08
11	-2.085590752026791	15	-9.169570471603355	13	-5.402445997147480	12	-3.944263279018107	11	-3.336756314301196	10
12	3.275014908456290	17	1.043064276932080	16	4.609548336980600	14	2.589718610892725	13	1.716422751767426	12
13	-5.843489315260924	19	-1.340713607762326	18	-4.424625661605893	16	-1.906058030545870	15	-9.867837192431894	13
14	1.172621125577913	22	1.927722115575248	20	4.730682023456777	18	1.557218917309768	17	6.279206363435675	15
15	-2.623798185176044	24	-3.074412201597502	22	-5.586987579397722	20	-1.400610225168476	19	-4.386738588063090	17
16	6.498295613664183	26	5.399594569185937	24	7.236994749327359	22	1.377208459152156	21	3.341429098727717	19
17	-1.770133725780086	29	-1.037875085701387	27	-1.021913166334475	25	-1.471574562508255	23	-2.758641890629547	21
18	5.273973489633881	31	2.171518539305070	29	1.564724879726098	27	1.699777782874756	25	2.455772325275397	23
19	-1.710269740113717	34	-4.922071909046155	31	-2.585854443253060	29	-2.112686875543274	27	-2.346624430440848	25
20	6.010188769106969	36	1.203544375501779	34	4.593225818803173	31	2.814150452231358	29	2.397303876081241	27
21	-2.279841182915260	39	-3.162740533550739	36	-8.737315845800662	33	-4.002736753860322	31	-2.609024848044394	29
22	9.301953182570813	41	8.901704892655380	38	1.773971402766864	36	6.059751954221292	33	3.015241191138177	31
23	-4.069085123981224	44	-2.675166406756157	41	-3.832868728219528	38	-9.735747656618453	35	-3.689806386763115	33
24	1.902799424499639	47	8.560033387009934	43	8.788793321981414	40	1.655569124495532	38	4.768595839978326	35

(the first columns for each value of c shows the subscript s for the coefficient of ther term is n^{-s})

$\mu_1^-(v^*)$

s\c	1.8		1.9		2.0		2.1		2.2	
0	5.748728379269565	-01	5.474527752162704	-01	5.227232008770633	-01	5.002885153206321	-01	4.798291239895961	-01
1	-4.098659758762409	-01	-3.778950171255416	-01	-3.514501786071399	-01	-3.292586952879362	-01	-3.103969548713979	-01
2	-6.964026054647190	-02	-9.661378275851690	-02	-1.131899247449153	-01	-1.231582719292188	-01	-1.288387837980988	-01
3	-3.225210720471004	-01	-2.088953695087506	-01	-1.481401128187685	-01	-1.150824655148252	-01	-9.679614843588610	-02
4	1.495377609997221	00	5.826882098080831	-01	1.891390979217576	-01	2.362883234555659	-02	-3.947497844886644	-02
5	-1.654325251525989	01	-5.677359541415991	00	-1.792654233140374	00	-4.062986369398400	-01	7.343631403118200	-02
6	2.369928551618551	02	6.683374020047622	01	1.816207541943433	01	4.678310819070297	00	1.391020880947071	00
7	-4.357758131007392	03	-9.797121958362158	02	-2.010397535925470	02	-2.892388789233817	01	3.121645965370004	00
8	9.912265896418992	04	1.798131327258898	04	2.994498798568059	03	3.712309073810552	02	-4.011072789810158	00
9	-2.686213219212307	06	-3.899946661568117	05	-5.139407138020691	04	-4.867173720925196	03	-5.950618343564495	01
10	8.482028749169343	07	9.829141439464158	06	1.020734296779746	06	6.947967259952438	04	-3.857128860243306	03
11	-3.064069966201532	09	-2.826767919849025	08	-2.314468693458788	07	-1.203010004523809	06	3.574310538076507	04
12	1.248249627518909	[illegible]	[illegible].143105115290659	09	5.875950615037040	08	2.247003212594304	07	-7.159543389202794	05
13	-5.668584161966677	12	-3.288842995418740	11	-1.655577172573338	10	-4.655584977571152	08	1.559006603768365	07
14	2.842238189994392	14	1.303246545164051	13	5.126851704009516	11	1.060104479059533	10	-2.560268430516770	08
15	-1.560899880123926	16	-5.644322343797464	14	-1.731361402329375	13	-2.608758863905702	11	5.706148647589176	09
16	9.325169447322388	17	2.653877283962620	16	6.334886536367213	14	6.937255989778236	12	-1.109282225841293	11
17	-6.025017209302132	19	-1.346875405768094	18	-2.496865378068563	16	-1.979202191757027	14	1.766894518819910	12
18	4.188578175357785	21	7.341088721107548	19	1.054944395185599	18	6.154516549425447	15	-3.226962866043460	13
19	-3.119177719075355	23	-4.278380958552785	21	-4.756681325287049	19	-1.953068698615227	17	-1.287053341206083	16
20	2.478359578568783	25	2.655611004522826	23	2.282284973571598	21	5.675242569669803	18	-2.228378689179806	18
21	-2.093677962871256	27	-1.749713910466322	25	-1.157752362060935	23	-1.646534240172574	20	-5.055572793244838	19
22	1.874605560144671	29	1.219874459014269	27	6.247490007129019	24	-2.808689545040477	21	3.064794230015497	22
23	-1.773933308629439	31	-8.967635390969300	28	-3.547774277631688	26	-2.631932582911197	24	3.618480642732044	24
24	1.769575968259164	33	6.950855217610325	30	1.935963660210449	28	1.179004908966799	26	-2.720212355438770	25

s\c	2.3		2.4		2.5		2.6		2.8	
0	4.610837421166577	-01	4.438364661556586	-01	4.279071708243759	-01	4.131442693700264	-01	3.866220527575303	-01
1	-2.941814802228577	-01	-2.800976167648178	-01	-2.677517472594534	-01	-2.568385734578668	-01	-2.384002763782784	-01
2	-1.316895102008845	-01	-1.326479488028436	-01	-1.323289849046587	-01	-1.311429493051200	-01	-1.271943860524912	-01
3	-8.644490687293320	-02	-8.033052685594840	-02	-7.642098851515860	-02	-7.359131516436340	-02	-6.897108439051820	-02
4	-5.628101529145515	-02	-5.247909084689426	-02	-4.001205735104412	-02	-2.445612416412450	-02	7.028404914768019	-03
5	2.285893670131119	-01	2.739998034761225	-01	2.873918764154751	-01	2.948084194855693	-01	3.124818381623297	-01
6	8.956292289436151	-01	1.020521970629624	00	1.171661763407665	00	1.220621365522714	00	1.027545825804611	00
7	5.513734046470894	00	3.255350872479340	00	1.114655748477728	00	-5.210036536772471	-01	-3.346203192241897	00
8	-2.664388922566173	01	-2.087331118340824	01	-2.494850911272805	01	-3.486964338882488	01	-5.363138682503487	01
9	-9.590617749185483	01	-2.856784530121110	02	-3.290067566496903	02	-3.138056301212995	02	-2.290004481570320	02
10	-2.536793289447193	03	-1.036031845702422	03	-8.425348552231876	02	-4.759122286748941	02	1.475399812367667	03
11	3.065873336179577	03	-1.748962193888344	03	1.080521938388099	04	2.171129192030009	04	3.786534071149209	04
12	1.153595316797978	05	2.377941937743532	05	2.345149720857312	05	2.795038622961875	05	2.773007279480426	05
13	8.450055533105785	05	1.675972454826021	06	2.294659410939863	06	1.494828422051080	06	-1.772748622087354	05
14	4.241310605175075	07	8.232739790668869	06	-8.718777973183467	05	-2.332961031097260	07	-6.933252132071771	07
15	-7.018530391458468	08	-6.358152924623945	07	-2.458346161248232	08	-6.948817608359736	08	-7.165203625660393	08
16	7.978628781857559	09	-7.801674029751454	09	-5.363071374066999	09	-7.734454327150390	09	6.977834968340613	09
17	9.443006197063769	10	-4.971102874680842	11	-1.266060830111162	12	2.267214375082640	11	-1.619435443281363	10
18	5.558597982242940	13	-1.213980864079632	13	-1.922798196811799	13	2.973163619859040	13	3.001478585405177	13
19	-3.436119791366436	15	-6.985806466197703	15	7.191543907382529	15	-3.630714411435815	15	-4.312902100299204	15
20	-7.893505422627584	17	7.899802595503447	16	7.214327903977333	16	-1.231078737546629	17	-2.105130193648432	16
21	-1.727018735061077	19	1.236147803267500	20	-5.425697797711954	19	2.505048290544235	19	1.811581120367471	19
22	1.929956377172678	21	8.008332515074875	21	-1.848760835542002	21	1.062452853265458	21	5.423077363438723	20
23	3.663362011552501	23	-4.953029798253095	23	7.535738905446888	23	-5.082358561837340	22	2.711366672481832	22
24	-7.401722481634196	24	-1.423972210925495	26	5.005590554306471	25	-2.328219800159354	25	4.700448522247526	24

s\c	3.0	3.2	3.5	3.8	4.0
0	3.634465032522934 -01	3.430023360261840 -01	3.164586446682611 -01	2.938514253453044 -01	2.805444748640734 -01
1	-2.233879550000342 -01	-2.108879484146718 -01	-1.955455752980300 -01	-1.831319331699070 -01	-1.760519153674434 -01
2	-1.221426137519103 -01	-1.166413906303405 -01	-1.082185839443030 -01	-9.999786157204490 -02	-9.472532981146350 -02
3	-6.422963219590310 -02	-5.885375700629701 -02	-4.973093047465780 -02	-3.993848108347188 -02	-3.335584462405140 -02
4	3.429094884328263 -02	5.622913475499490 -02	7.955762513597530 -02	9.217068325939320 -02	9.494861163601210 -02
5	3.271649589230083 -01	3.252220951710842 -01	2.763078271745338 -01	1.709115578373238 -01	7.453053045833700 -02
6	5.906831568365021 -01	2.602959806704393 -02	-9.152160339904037 -01	-1.802338528804464 00	-2.270069552873782 00
7	-6.342038236314093 00	-9.220832369452161 00	-1.182072152479311 01	-1.064499869376305 01	-7.232526996773013 00
8	-6.041232320356888 01	-5.215760117959231 01	-1.009296476434513 01	6.220192297736438 01	1.184952639837324 02
9	-5.082457050621886 01	2.409622260532542 02	7.856622948957092 02	1.171555648925734 03	1.154060811065649 03
10	4.097224071212084 03	6.338465949102035 03	6.688992302884825 03	6.694786183617975 02	-7.327366745222428 03
11	4.540952849082522 04	3.269406506866962 04	-3.984190823192532 04	-1.558577085520114 05	-2.187428014339201 05
12	1.169650449681317 04	-5.034080051157605 05	-1.404669706995017 06	-1.424900039513891 06	-3.262684012399649 05
13	-6.691600472357816 06	-1.132791442301872 07	-7.507836668834684 06	1.836599034667434 07	4.688081088931580 07
14	-9.764811436650307 07	-4.985899283519356 07	2.015280872784084 08	5.419429100392564 08	4.863615655677401 08
15	2.472322613108000 08	1.668716429037632 09	5.365130984049072 09	8.432312849600363 08	-1.045308882182529 10
16	2.027104358903691 10	4.869071132009804 10	6.103115372741787 09	5.071294990215871 09	8.365965680091540 10
17	-2.089923510318587 11	1.879108854442738 12	-3.660765478941362 12	-2.492981899366229 13	-3.499761264448463 13
18	-1.895289334812601 13	-1.318032408796385 14	3.891200772618120 13	1.489539522779028 15	2.368147392023344 15
19	-3.364478962878271 15	-1.143324962400478 15	-5.711082482730094 15	1.630990773812745 16	-9.813286437446624 16
20	7.214183714503772 16	6.884390735410886 17	2.238968800546681 18	-8.204979566684330 18	1.436338599620706 18
21	5.139384494613593 19	-1.542617579527219 19	-1.241306504591509 20	1.814822965765263 20	2.404592471784911 20
22	9.672786683089199 20	-7.212496913607385 21	-1.502042817358224 22	-6.824549479538167 21	-3.596605795209820 22
23	-2.468968021298490 23	2.020958171068996 22	9.946998069700945 23	3.559733532431792 24	6.590675363649559 24
24	-2.193071595751463 25	6.342122500387193 25	1.088625201557155 26	2.151887507694862 26	-7.232069644769035 26

Second moment of v* (μ_2(v*))

s\c	0.8	0.9	1.0	1.1	1.2
1	2.742710433754490 00	1.562582406371980 00	1.000000000000000 00	6.960721472711004 -01	5.157043449162944 -01
2	-9.201259881665373 01	-3.077980767834855 01	-1.250000000000001 01	-5.837953578492141 00	-3.023643537739777 00
3	6.785786080374245 03	1.273514568996598 03	3.189999999999985 02	9.813659586402005 01	3.508159895488919 01
4	-1.127406026660295 06	-1.158053212907677 05	-1.777387499999989 04	-3.628340720984640 03	-9.123534860094224 02
5	3.426333108738997 08	1.872065313467731 07	1.728863875000034 06	2.315831462881992 05	4.070122353009927 04
6	-1.647752257381798 11	-4.648049668949619 09	-2.529248459374963 08	-2.188807474849029 07	-2.655875155483380 06
7	1.151782439759352 14	1.628762977101047 12	5.115643930187339 10	2.817441089983220 09	2.333468904023310 08
8	-1.108116411361148 17	-7.634225673522139 14	-1.356436519339886 13	-4.685552414268583 11	-2.620202191520255 10
9	1.412102001532130 20	4.611197505872605 17	4.545406827420834 15	9.710034138923077 13	3.627730538231517 12
10	-2.315473630333409 23	-3.491613269449239 20	-1.873898629258839 18	-2.441834720876308 16	-6.033229004519154 14
11	4.774554856957178 26	3.244047612131084 23	9.309985169622261 20	7.302655135086070 18	1.181498135510099 17
12	-1.214896139747595 30	-3.634728017325047 26	-5.483169433576328 23	-2.556079032715912 21	-2.682096333002678 19
13	3.754320907808989 33	4.840832255104375 29	3.776814176008994 26	1.033598232286766 24	6.968873390503591 21
14	-1.389812222375478 37	-7.570174186535979 32	-3.008142395573900 29	-4.776545685659008 26	-2.050866112791615 24
15	6.090192109566600 40	1.375377136110991 36	2.743623192876359 32	2.499611495207931 29	6.775695430376794 26
16	-3.126090387405809 44	-2.876275711702201 39	-2.841373899324434 35	-1.469561180223933 32	-2.494129741802155 29
17	1.862234786449539 48	6.866887157497175 42	3.316455681025123 38	9.639457498738049 34	1.016186450369242 32
18	-1.276834784023626 52	-1.857912107307608 46	-4.333963508456870 41	-7.011477088010090 37	-4.556225281645355 34
19	1.000183272412463 56	5.659379552514853 49	6.303537648520966 44	5.624545303504139 40	2.236587638563189 37
20	-8.891293232022380 59	-1.929360322536969 53	-1.014957451947771 48	-4.951704824202079 43	-1.196524132663449 40
21	8.915816967134967 63	7.321880683363887 56	1.800401671674019 51	4.763025576654413 46	6.947271693771744 42
22	-1.002943918243713 68	-3.078012662057096 60	-3.502926304845161 54	-4.985600386137076 49	-4.361479633648663 45
23	1.259294865896459 72	1.426965814738656 64	7.445243031033138 57	5.657895843614584 52	2.950482719650349 48
24		-7.265577323154079 67	-1.722279776947100 61	-6.937835328304876 55	-2.144022505650880 51

s\c	1.3	1.4	1.5	1.6	1.7
1	4.006016379082361 -01	3.228042720727596 -01	2.677267398405026 -01	2.272232879980813 -01	1.964809130386898 -01
2	-1.692951217019321 00	-1.005834797929148 00	-6.252741338655030 -01	-4.022344191886298 -01	-2.653485951174719 -01
3	1.401797358237800 01	6.084510596824353 00	2.805419702075228 00	1.348398356496198 00	6.636767097946776 -01
4	-2.678993940305838 02	-8.834308878391112 01	-3.174030167302986 01	-1.211457348019081 01	-4.799016339951960 00
5	8.758738886243873 03	2.195339419654354 03	6.169821359184447 02	1.885263914980930 02	6.094055536975672 01
6	-4.144270003411691 05	-7.814437288635196 04	-1.698566057457510 04	-4.097588663056225 03	-1.061427477693965 03
7	2.616680031770954 07	3.684834034318829 06	6.156819494099406 05	1.166276477798145 05	2.407098400034691 04
8	-2.093807742063342 09	-2.187293245650938 08	-2.793885261690395 07	-4.136881754001981 06	-6.779704731767989 05
9	2.049242422040357 11	1.578009178491808 10	1.532965550211385 09	1.766539955634482 08	2.289791848930169 07
10	-2.390504275864236 13	-1.348693075180955 12	-9.916009922336329 10	-8.857280705154806 09	-9.049267646475445 08
11	3.258986126744108 15	1.339270658464075 14	7.417594821601848 12	5.116043410655412 11	4.106557285436774 10
12	-5.112764678859909 17	-1.521729383318542 16	-6.320422081592837 14	-3.353759940856435 13	-2.108481132158765 12
13	9.115659770457245 19	1.954199942915010 18	6.060326502127352 16	2.465258444977436 15	1.210366514829041 14
14	-1.828135155122038 22	-2.807739858385761 20	-6.473820032476090 18	-2.011976392502470 17	-7.692463031610196 15
15	4.088479460478750 24	4.475162074463664 22	7.640054848453776 20	1.808060294440578 19	5.368539965592260 17
16	-1.012155118195142 27	-7.855688382595592 24	-9.890283314765727 22	-1.776544627243811 21	-4.085721269697399 19
17	2.756117202013804 29	1.509307580931766 27	1.395839142824942 25	1.897079818810847 23	3.370604641322317 21
18	-8.209103022353712 31	-3.156692497860215 29	-2.136297117320892 27	-2.190076662729867 25	-2.998609654926412 23
19	2.661377544625657 34	7.152770033795801 31	3.529031474182933 29	2.720797397190129 27	2.863716972776951 25
20	-9.350370837955766 36	-1.748492051344693 34	-6.266398881628273 31	-3.622655690238657 29	-2.924108064320103 27
21	3.546144208229821 39	4.593619963430916 36	1.191638518765838 34	5.150816843307563 31	3.180948988883552 29
22	-1.446596052986955 42	-1.292608415607602 39	-2.418767198250694 36	-7.795260742885988 33	-3.674761961616571 31
23	6.327030132514728 44	3.883804327444544 41	5.224741820570659 38	1.252035383731643 36	4.495281012268389 33
24	-2.958240886893323 47	-1.242520870330272 44	-1.197772256076727 41	-2.128525027808678 38	-5.807708988625726 35

$\mu_2(v^*)$

s\c	1.8	1.9	2.0	2.1	2.2
1	1.725181764979709 -01	1.534125750656566 -01	1.378809605122418 -01	1.250409340830637 -01	1.142696184007132 -01
2	-1.781030509298652 -01	-1.207444806720710 -01	-8.206605877037910 -02	-5.544386833507809 -02	-3.682097699574050 -02
3	3.279274457370048 -01	1.581789616689751 -01	7.067560464623050 -02	2.521243051813685 -02	1.720462241839235 -03
4	-1.925492158949508 00	-7.593985793579152 -01	-2.818299423332384 -01	-9.128821140133840 -02	-2.245194912859514 -02
5	2.025611827442853 01	6.665789781019313 00	2.015483743147291 00	4.280050186391553 -01	-9.246582152521910 -02
6	-2.859625697060312 02	-7.739526862609875 01	-2.021080568420291 01	-4.935705582737343 00	-1.288247272605602 00
7	5.203086548215804 03	1.122256501421645 03	2.225408869449217 02	3.190156104579101 01	-2.130332349695712 00
8	-1.175181590005091 05	-2.041828079621423 04	-3.268492934128914 03	-3.895290896659497 02	6.095742968806347 00
9	3.169311582808827 06	4.404425816413099 05	5.579499401207821 04	5.108281994657187 03	5.558432155183871 01
10	-9.971371976351465 07	-1.105377990903663 07	-1.102639435272783 06	-7.264045896157327 04	3.664204915763097 03
11	3.592141772012112 09	3.168672130544384 08	2.490339924100481 07	1.248350177150943 06	-3.671139753015672 04
12	-1.460190929252356 11	-1.022278106047543 10	-6.303538858325174 08	-2.325926837013072 07	7.082536771399173 05
13	6.619394465094032 12	3.669577879322991 11	1.771629316912472 10	4.803445206207141 08	-1.539714286378450 07
14	-3.314168257234340 14	-1.451625688097122 13	-5.474964817177309 11	-1.090849399226195 10	2.551986280358816 08
15	1.817867132344436 16	6.277905925621542 14	1.845726375668665 13	2.678980694578802 11	-5.631010354162242 09
16	-1.084917137693616 18	-2.948167738097572 16	-6.743373944984178 14	-7.110549358548131 12	1.099325210480261 11
17	7.003482188288442 19	1.494656036687619 18	2.654491102606015 16	2.025409700102974 14	-1.763055421540411 12
18	-4.865069970771660 21	-8.139107187444786 19	-1.120300310974065 18	-6.286754318292584 15	3.203719284308344 13
19	3.620518915973320 23	4.739657995006591 21	5.046461930908183 19	1.994252979451611 17	1.233171501909844 16
20	-2.874998006510892 25	-2.939842986294117 23	-2.419210256919085 21	-5.805660623048000 18	2.130484057835591 18
21	2.427472570763359 27	1.935762912129996 25	1.226305801764128 23	1.684390080328360 20	4.712953482105569 19
22	-2.172449829375060 29	-1.348822059594726 27	-6.612292357673695 24	2.703235506529313 21	-2.944351199731482 22
23	2.054911440835182 31	9.910584271042830 28	3.752666384065070 26	2.631562484026318 24	-3.453492197514144 24
24	-2.049079143960823 33	-7.678078421808766 30	-2.048745669168105 28	-1.197024267000908 26	2.835896030310657 25

s\c	2.3	2.4	2.5	2.6	2.8
1	1.051170867663625 -01	9.725163163776350 -02	9.042419978001760 -02	8.444470831141040 -02	7.447225246410550 -02
2	-2.363408857130967 -02	-1.421863463488748 -02	-7.467126824408883 -03	-2.626656225694883 -03	3.254470234469329 -03
3	-1.014006507770728 -02	-1.582943361878475 -02	-1.828284646786408 -02	-1.909452819142796 -02	-1.886105570107147 -02
4	-4.990634156429778 -03	-8.354466005646322 -03	-1.918123329039962 -02	-3.169885769387101 -02	-5.379032288360587 -02
5	-2.448955057484292 -01	-2.767369228174791 -01	-2.740628969413097 -01	-2.648003552575486 -01	-2.492034898524620 -01
6	-6.946682789364400 -01	-7.559857217958692 -01	-8.508630582852521 -01	-8.568336330044271 -01	-6.400744152637400 -01
7	-4.507819906989540 00	-2.249823193855662 00	-2.505407757800010 -01	1.137963563384605 00	3.164076298466908 00
8	2.804700924712016 01	2.064037447279755 01	2.228678423965018 01	2.890032716260832 01	4.017821218027497 01
9	7.431377057236472 01	2.428909651278624 02	2.686624773388427 02	2.414097088230950 02	1.507741769305052 02
10	2.276826830173199 03	7.545630979085449 02	5.383832055543565 02	2.227449070268634 02	-1.264278135884331 03
11	-4.349338126250342 03	8.927879760921710 02	-9.789935558439434 03	-1.827247848046309 04	-2.864196404420986 04
12	-1.052672824009755 05	-2.123885997779826 05	-1.951921199898389 05	-2.199720512335308 05	-1.960214100163815 05
13	-7.110013285142174 05	-1.355158696755008 06	-1.8[illegible]5501791554049 06	-1.085953387342448 06	1.512864365482330 06
14	-3.858412742089199 07	-6.306222334117812 06	8.754351373725737 06	2.012160189237833 07	5.284170320776566 07
15	6.724248151544631 08	6.153547769489326 07	2.063651733421236 08	5.626242308258541 08	5.205733046770131 08
16	-7.759243114842897 09	6.892199603181852 09	4.456205959231229 09	6.028047553567945 09	-5.755124018416706 09
17	-8.256346395068345 10	4.368853846332410 11	1.080574686594688 12	-1.914961962221805 11	1.565714516925618 10
18	-5.120203995748837 13	1.049562009986213 13	1.577614303640121 13	-2.445258233446427 13	-2.321479960077043 13
19	3.201408823997109 15	6.194177495908988 15	-6.165258979438958 15	3.015348472747297 15	3.349234099720201 15
20	7.259063757646439 17	-7.404112614259287 16	-5.789543856278285 16	9.986544177311331 16	1.422903474627414 16
21	1.546068521571447 19	-1.096870955158807 20	4.647443312858860 19	-2.076311250761562 19	-1.401907451912919 19
22	-1.790112641806969 21	-7.039511850093466 21	1.553161427693625 21	-8.650575775162502 20	-4.107045822949171 20
23	-3.366925036434893 23	4.441860847040675 23	-6.459237088337635 23	4.254725611788161 22	-2.070230474848406 22
24	7.041671688512378 24	1.261268368817728 26	-4.243551797655136 25	1.921198880037889 25	-3.621525842959575 24

$\mu_2(v^*)$

s\c	3.0	3.2	3.5	3.8	4.0
1	6.649089432626640 -02	5.995844897696351 -02	5.211434956868685 -02	4.594380364082646 -02	4.251154765922463 -02
2	6.031808295146775 -03	7.068795021282001 -03	6.906655125829948 -03	5.764264271600073 -03	4.771633646187871 -03
3	-1.824944669137780 -02	-1.803109290237837 -02	-1.851137108571402 -02	-1.954268035623601 -02	-2.030913059310723 -02
4	-6.906064755470780 -02	-7.812657886899630 -02	-8.296466730103600 -02	-8.025920445531510 -02	-7.539414916749300 -02
5	-2.340670989186993 -01	-2.105552186679448 -01	-1.532895413042336 -01	-7.410751842326110 -02	-1.439757902181764 -02
6	-2.731336705206378 -01	1.329660595059682 -01	7.043744034262044 -01	1.140038186572599 00	1.317795082779661 00
7	4.963205901789160 00	6.422772093988196 00	7.193709203040933 00	5.639007002768913 00	3.280350181469126 00
8	4.126625571647766 01	3.193423008925826 01	1.589685383733001 00	-4.080927750054148 01	-6.946675477026141 01
9	8.451568039129965 00	-1.894868272740636 02	-5.039001421508010 02	-6.680542552685188 02	-6.073432426555028 02
10	-3.016686680907319 03	-4.259955025223297 03	-3.929607246949450 03	4.753951073772680 01	4.540045366530035 03
11	-3.119747965914910 04	-1.970379098621560 04	2.800406096808814 04	9.208507762726398 04	1.203824300616494 05
12	1.278806614773815 04	3.606486769356375 05	8.749911070698634 05	7.805491614694803 05	1.047076741191458 05
13	4.880973052914555 06	7.567057214874848 06	4.194386694530623 06	-1.134702728612913 07	-2.646138489010026 07
14	6.799855970278186 07	2.931112089962078 07	-1.307965013720305 08	-3.120724308415488 08	-2.564598906516431 08
15	-2.249778844764383 08	-1.168487224089850 09	-3.318629151232607 09	-2.934819719721749 08	6.045271811449271 09
16	-1.464917563981138 10	-3.271115896740430 10	-1.721364325661989 09	-2.561376657438818 09	-5.052574768265946 10
17	1.610199871604332 11	-1.268155729294195 12	2.320531968278635 12	1.465339146143195 13	1.966427249196129 13
18	1.368834868575728 13	9.122177118992522 13	-2.605795278155772 13	-8.845365501440548 14	-1.341049078233207 15
19	2.437100143629668 15	7.291787671876318 14	3.629068244027544 15	-9.044800506620216 15	5.588847702550344 16
20	-5.394722333857877 16	-4.727851809011895 17	-1.419307563827228 18	4.828359522945655 18	-8.400203730605120 17
21	-3.732641956261232 19	1.087251319750229 19	7.943884407256664 19	-1.096594650858478 20	-1.344317128545593 20
22	-6.801293751028772 20	4.941460670805037 21	9.458626309419164 21	4.075637667225332 21	2.026508942645964 22
23	1.799122798719052 23	-1.690944223913314 22	-5.354637049819152 23	-2.094529493900567 24	-3.710573235432004 24
24	1.583121940489080 25	-4.350041718923676 25	-6.851521655361912 25	-1.251645893697228 26	4.080972401820951 26

Third Moment of v* ($\mu_3(v^*)$)

s\c	0.8	0.9	1.0	1.1	1.2
2	6.449495801062287 01	1.891064457275639 01	7.000000000000023 00	3.074393622133652 00	1.536713530362650 00
3	-9.012187770939056 03	-1.505174469771329 03	-3.464999999999968 02	-1.006701446709427 02	-3.481486601050883 01
4	1.770453715633484 06	1.613811693952122 05	2.249262499999987 04	4.246497634564061 03	1.002203557069774 03
5	-5.688434503493147 08	-2.751472013021360 07	-2.294529562500049 06	-2.818031107850271 05	-4.595784804097634 04
6	2.813489314687625 11	7.018675666930308 09	3.441744566953075 08	2.722976635161040 07	3.055091769225173 06
7	-2.001836912222337 14	-2.502278717330255 12	-7.075834017577505 10	-3.558029647492483 09	-2.720344440989766 08
8	1.949949164988526 17	1.187156887964697 15	1.898108429377644 13	5.982058015031506 11	3.085156642625938 10
9	-2.507753363682650 20	-7.235484433930731 17	-6.416055464695293 15	-1.249901868046203 14	-4.304230517929452 12
10	4.141208518654037 23	5.516972175751205 20	2.662989115112059 18	3.163472236869052 16	7.201576904974935 14
11	-8.587362397957992 26	-5.154259960296601 23	-1.330176751927851 21	-9.509719898140640 18	-1.417173519427774 17
12	2.195096405045791 30	5.801110817405961 26	7.868658264778076 23	3.342685857473371 21	3.229999905337148 19
13	-6.809218000025341 33	-7.755121551625768 29	-5.439802066784131 26	-1.356447440450898 24	-8.420640304072296 21
14	2.528811572482400 37	1.216608335526079 33	4.346091456008931 29	6.287249174176992 26	2.485160191075918 24
15	-1.111181346411862 41	-2.216384859718041 36	-3.974437110557803 32	-3.298607011691917 29	-8.230607642341183 26
16	5.717273081829449 44	4.645950580862239 39	4.125501814603266 35	1.943612759073423 32	3.036118157327969 29
17	-3.412915273366616 48	-1.111467640126135 43	-4.824978409675943 38	-1.277376747595897 35	-1.239313398893904 32
18	2.344351670624670 52	3.012657866546272 46	6.316505209738441 41	9.307258011609394 37	5.565797709383132 34
19	-1.839398566325304 56	-9.191655324605881 49	-9.201554001387274 44	-7.477619778328900 40	-2.736178855073139 37
20	1.637550520372229 60	3.138086493974540 53	1.483672902103045 48	6.592122842418436 43	1.465718128107969 40
21	-1.644224070639439 64	-1.192444958918813 57	-2.635192143822779 51	-6.348765045919776 46	-8.520353818704553 42
22	1.851792196282676 68	5.018762482708242 60	5.133034557371991 54	6.652872380071385 49	5.354801927443010 45
23	-2.327618849781789 72	-2.329185545230097 64	-1.092137311496435 58	-7.557679955259307 52	-3.625995259835900 48
24		1.187092846289385 68	2.528820412691903 61	9.276015106743750 55	2.637253345773226 51

s\c	1.3	1.4	1.5	1.6	1.7
2	8.491762307683832 -01	5.080984930651881 -01	3.242088159640859 -01	2.181115027770043 -01	1.533701196328836 -01
3	-1.376261829064584 01	-6.040748010879664 00	-2.881280444174628 00	-1.469016566001903 00	-7.901603929507183 -01
4	2.795647288393865 02	8.845846922631348 01	3.074807079427086 01	1.142988045966905 01	4.430951309089160 00
5	-9.265526961542592 03	-2.192600859341698 03	-5.853872673639653 02	-1.707351318258891 02	-5.284865020248597 01
6	4.448866671441132 05	7.889117836118243 04	1.623403367666138 04	3.729726450677929 03	9.253635496760013 02
7	-2.841370774075291 07	-3.754881365932046 06	-5.925477269860761 05	-1.066178936244704 05	-2.101222084531233 04
8	2.293774988618074 09	2.245765222592262 08	2.705343827765824 07	3.798692756925685 06	5.933369630743145 05
9	-2.260556310649256 11	-1.630094736866731 10	-1.492029584860708 09	-1.628710668639398 08	-2.009604505344219 07
10	2.651644242753230 13	1.400131150477261 12	9.692666688091317 10	8.194971341522369 09	7.962835975813827 08
11	-3.631325246655212 15	-1.396037161908881 14	-7.276597164332618 12	-4.747796515313021 11	-3.622050012072049 10
12	5.718192646588135 17	1.591645930568332 16	6.219107455006479 14	3.120441632215063 13	1.863586434191808 12
13	-1.022711322489290 20	-2.049887721295960 18	-5.978622243332241 16	-2.298894374654032 15	-1.071746864071415 14
14	2.056521665082805 22	2.952496171142993 20	6.400781542485920 18	1.879856924686032 17	6.822480849749050 15
15	-4.609845278232702 24	-4.715934760288540 22	-7.568496400141973 20	-1.692212575955559 19	-4.768197835441682 17
16	1.143514474067883 27	8.293772800046474 24	9.814288204427568 22	1.665216117950130 21	3.633451045613165 19
17	-3.119295043494525 29	-1.596091351958638 27	-1.387191355473923 25	-1.780570325806813 23	-3.000905017930433 21
18	9.305313867509617 31	3.343058099537897 29	2.125894061162406 27	2.058021423350152 25	2.672440093290939 23
19	-3.020960351205297 34	-7.584902433475225 31	-3.516039212158390 29	-2.559476203363328 27	-2.554574060296563 25
20	1.062694970447505 37	1.856291694335436 34	6.250038825541476 31	3.411152265373608 29	2.610626693279119 27
21	-4.034824129460136 39	-4.881974407747815 36	-1.189683265702705 34	-4.854341883812464 31	-2.842094334048514 29
22	1.647625563950743 42	1.375064713993833 39	2.416933529649412 36	7.352436874725619 33	3.285587205996152 31
23	-7.212982486811090 44	-4.135163294835734 41	-5.224995990189238 38	-1.181773259741375 36	-4.021774149272202 33
24	3.375343605160985 47	1.323994774510246 44	1.198717130504956 41	2.010422394124357 38	5.199010475573369 35

$\mu_3(v^*)$

s\c	1.8	1.9	2.0	2.1	2.2
2	1.119681250096363 -01	8.441981652368950 -02	6.545679525109440 -02	5.201533762544492 -02	4.224131771776794 -02
3	-4.434869799707295 -01	-2.571872080542986 -01	-1.526249384778815 -01	-9.170307707678380 -02	-5.503771898050110 -02
4	1.742886013275127 00	6.702250148535036 -01	2.355458412654861 -01	6.238121969912347 -02	-1.369643371935655 -03
5	-1.684040184091811 01	-5.299862261282141 00	-1.516081764748940 00	-2.885720595502517 -01	8.300565939793870 -02
6	2.401911336345080 02	6.304791230402765 01	1.608477064812147 01	3.836346757646137 00	9.099684788393006 -01
7	-4.359647157917396 03	-9.079698557922414 02	-1.755467723003859 02	-2.547010792368622 01	6.321716396132756 -01
8	9.849443374673318 04	1.646870802411383 04	2.550229040836500 03	2.951904299585442 02	-5.472893022006502 00
9	-2.660065105753724 06	-3.552336306468675 05	-4.349516902862831 04	-3.886782387960880 03	-4.023189089637616 01
10	8.382373282627480 07	8.917920015254154 06	8.584196991305250 05	5.522457872421535 04	-2.450862879059653 03
11	-3.024451621509876 09	-2.557951649521903 08	-1.937260874032041 07	-9.433874610316107 05	2.722993982718613 04
12	1.231235424755812 11	8.258448064704043 09	4.902616419136347 08	1.757340582580473 07	-5.075543000115154 05
13	-5.588994776032125 12	-2.966661165912873 11	-1.377804809999685 10	-3.623852236780192 08	1.107548613754416 07
14	2.801684825620049 14	1.174416197492492 13	4.258215143447620 11	8.220683365667358 09	-1.863618204370621 08
15	-1.538455758789327 16	-5.082557775743151 14	-1.435742051176405 13	-2.017788316239670 11	4.075887023154462 09
16	9.190760301288131 17	2.388367980918725 16	5.246451064821356 14	5.352467289495283 12	-8.027301571847856 10
17	-5.938269190551571 19	-1.211581012304119 18	-2.065660852467372 16	-1.523983117156052 14	1.300474717861965 12
18	4.128475644173782 21	6.601329873905209 19	8.719774394141731 17	4.726142108065540 15	-2.369285057660647 13
19	-3.074640735142403 23	-3.846158283559028 21	-3.928732421456591 19	-1.500694098205298 17	-8.640009076735353 15
20	2.443182638715580 25	2.386787237936042 23	1.883760801451196 21	4.384641078483521 18	-1.487233257212722 18
21	-2.064157313855763 27	-1.572299063830603 25	-9.551217046321042 22	-1.272962333133965 20	-3.140023244431268 19
22	1.848359080452178 29	1.096018588277525 27	5.150474156542132 24	-1.876626696443452 21	2.078543453359663 22
23	-1.749275937869434 31	-8.056258901836750 28	-2.923810380420719 26	-1.928937336939950 24	2.412960802866032 24
24	1.745160279868763 33	6.243575091204530 30	1.598562873451309 28	9.039855135281815 25	-2.265878095982774 25

s\c	2.3	2.4	2.5	2.6	2.8
2	3.497282966993703 -02	2.945905939678234 -02	2.520146698365138 -02	2.186091065692479 -02	1.705650579531283 -02
3	-3.233739580617413 -02	-1.793068063133045 -02	-8.587849914754432 -03	-2.415295565756211 -03	4.539516929598222 -03
4	-1.940826915867334 -02	-1.904975040860837 -02	-1.210957622718750 -02	-3.635858540159379 -03	1.103000758975848 -02
5	1.723254363116553 -01	1.748672066670608 -01	1.558304241439144 -01	1.352704192774579 -01	1.040066870711868 -01
6	3.748548963350634 -01	3.566904643326545 -01	3.845983402873101 -01	3.675649039304097 -01	2.204922183258991 -01
7	2.513643742505041 00	9.806130205384501 -01	-3.216214229854256 -01	-1.139422456104311 00	-2.096277885421282 00
8	-2.128188917558577 01	-1.469255256138778 01	-1.408784416240773 01	-1.672519043869503 01	-2.081465330605813 01
9	-3.441971050535792 01	-1.430358294011827 02	-1.528878205075874 02	-1.282367786749412 02	-6.412767306513663 01
10	-1.464591972081741 03	-3.475351541368325 02	-1.820089690647165 02	1.497748834124543 00	7.871537929473951 02
11	4.348811027870333 03	-2.696220895143980 01	6.412106633249918 03	1.109087565922813 04	1.548673138249599 04
12	6.731028688178705 04	1.373647722602868 05	1.165182795955434 05	1.228367263034052 05	9.628401317731664 04
13	4.335809140907161 05	7.560396351606150 05	1.033756896583372 06	5.367871240958143 05	-9.496647125324259 05
14	2.520945234313732 07	3.320254129040240 06	-6.473272017017105 06	-1.275566656735608 07	-2.915401635010140 07
15	-4.718687880133952 08	-4.530299532441631 07	-1.254982155989713 08	-3.296078297883212 08	-2.695469030444783 08
16	5.575550133154178 09	-4.438927196981526 09	-2.704033835970994 09	-3.350770042561421 09	3.499200691023451 09
17	5.081934285223148 10	-2.793375013864887 11	-6.719162873749210 11	1.188028168833234 11	-1.084586598249759 10
18	3.436726314142022 13	-6.537396478945404 12	-9.191452930664340 12	1.464544248168281 13	1.311161681613451 13
19	-2.197741962845254 15	-4.009489681356559 15	3.868832082298435 15	-1.835076731941228 15	-1.904426508811812 15
20	-4.880831624842680 17	5.256807725293588 16	3.262253208352701 16	-5.867939250734650 16	-6.339607831773263 15
21	-9.945182458319410 18	7.134008508997589 19	-2.918484104505931 19	1.263451397955461 19	7.957160619463380 18
22	1.225232586790661 21	4.512983190573015 21	-9.461932751503560 20	5.129864202810835 20	2.262565593360331 20
23	2.270310389178482 23	-2.946317563316232 23	4.069875178438376 23	-2.635407847053926 22	1.153545874834163 22
24	-4.989904462121404 24	-8.205932432864125 25	2.630848060125006 25	-1.164518893344990 25	2.044891253644864 24

$\mu_3(v^*)$

s\c	3.0	3.2	3.5	3.8	4.0
2	1.385867725275520 -02	1.163505832386114 -02	9.387671594521128 -03	7.905194656477564 -03	7.174543873377518 -03
3	7.788045003172707 -03	9.301442301993220 -03	1.006861399557862 -02	1.000627872606547 -02	9.738231859542132 -03
4	2.034859759119160 -02	2.509369718659644 -02	2.652063184868760 -02	2.398643200904296 -02	2.110567743324280 -02
5	8.173871893469220 -02	6.149590090063451 -02	2.949023031913141 -02	-3.825368097173973 -03	-2.508719758585589 -02
6	2.042709049640197 -02	-1.696172670154871 -01	-3.901565647506286 -01	-5.139315573800575 -01	-5.400300573637393 -01
7	-2.743640169927067 00	-3.128871986304283 00	-3.004847732850238 00	-1.977442195998352 00	-8.831846945903851 -01
8	-1.928590497135448 01	-1.298786897550083 01	2.078430625460617 00	1.922755160920185 01	2.908132105494536 01
9	1.487797521294536 01	1.079262219963002 02	2.310751216383788 02	2.689947133050049 02	2.227920753245270 02
10	1.594724694986010 03	2.037799230624553 03	1.608906822524564 03	-2.519295009010122 02	-2.041178166288079 03
11	1.511656400224466 04	8.023373045026853 03	-1.436564243313754 04	-3.925838813681425 04	-4.752082364175534 04
12	-2.091437314070424 04	-1.884408357925968 05	-3.925456956437952 05	-3.021230241225772 05	-3.620560563253834 03
13	-2.585016226578563 06	-3.638283238370664 06	-1.608433389701320 06	5.130767421273151 06	1.085587750741663 07
14	-3.395507790303339 07	-1.156777113637758 07	6.211537561785977 07	1.301448705136434 08	9.664857316179200 07
15	1.491422311876821 08	5.998248778637978 08	1.487505648176101 09	2.053712451464209 07	-2.561437716331290 09
16	7.714816916483704 09	1.596477284467209 10	-4.424906805371456 08	6.211338413043441 08	2.201097105676584 10
17	-9.083249266999197 10	6.205760467056418 11	-1.074356596783244 12	-6.278042263732410 12	-8.050933950709481 12
18	-7.189375043015216 12	-4.624124718481129 13	1.284877162444936 13	3.843250165605915 14	5.554009166695791 14
19	-1.287571955503826 15	-3.285582193361870 14	-1.684743065714579 15	3.583375028309419 15	-2.333044505711332 16
20	2.995811457666215 16	2.376299746018768 17	6.585133071574952 17	-2.079772725192919 18	3.622538005900860 17
21	1.986058517761831 19	-5.670174491825548 18	-3.737822924819266 19	4.885972232620721 19	5.499825110621116 19
22	3.451856413308978 20	-2.481924968035839 21	-4.360670055473390 21	-1.785882288821276 21	-8.378860750334299 21
23	-9.641084007058661 22	1.065821303425024 22	2.989505121782058 23	9.050277515725040 23	1.534241902732829 24
24	-8.377306837201484 24	2.192794466124867 25	3.163295354861245 25	5.329498048335478 25	-1.693945986827668 26

Fourth Moment of v^* ($\mu_4(v^*)$)

s\c	0.8		0.9		1.0		1.1		1.2	
2	2.256738157027722	01	7.324991330109747	00	3.000000000000001	00	1.453549302619801	00	7.978529140966331	-01
3	3.432231320060289	03	4.313355995165085	02	7.499999999999536	01	1.629827426369728	01	4.120230385987859	00
4	-1.581195089398748	06	-1.232775359803976	05	-1.511025000000000	04	-2.569504887762686	03	-5.584264475034463	02
5	6.548333308602478	08	2.759365530671101	07	2.050059750000054	06	2.283532825403415	05	3.427455845709933	04
6	-3.610978804890386	11	-7.891592159163335	09	-3.457806986249944	08	-2.481938201255275	07	-2.556777453664141	06
7	2.726363531601481	14	2.993109734893749	12	7.573886675249731	10	3.456956203952031	09	2.425959260368009	08
8	-2.757407597060483	17	-1.476521016392320	15	-2.114514385557522	13	-6.051909194924684	11	-2.865106222519568	10
9	3.639735205606185	20	9.244861704867908	17	7.347384427303299	15	1.300347052284706	14	4.111288726121865	12
10	-6.126653917547813	23	-7.189736518743589	20	-3.111776208592170	18	-3.359317525664835	16	-7.022449530919874	14
11	1.289227624550112	27	6.819361505192768	23	1.578547739925903	21	1.025806257219469	19	1.403977409316750	17
12	-3.334031687971497	30	-7.767423892642233	26	-9.452515489639629	23	-3.650663615277707	21	-3.240215402033305	19
13	1.044022790838442	34	1.048478622759153	30	6.599647703960769	26	1.496355709663735	24	8.533341563699977	21
14	-3.907710922702780	37	-1.658058308317947	33	-5.315975052561298	29	-6.993455547743142	26	-2.539612759170339	24
15	1.728416326881260	41	3.041020041322203	36	4.894859384455707	32	3.694757984348138	29	8.470386150712775	26
16	-8.943184299091127	44	-6.411259780885752	39	-5.110714757451168	35	-2.189989713035657	32	-3.143373502152166	29
17	5.364591015763542	48	1.541411601016178	43	6.007464731472173	38	1.446682903043483	35	1.289750173153253	32
18	-3.700611137891687	52	-4.196128943155018	46	-7.899178607745355	41	-1.058794823385627	38	-5.818478294636554	34
19	2.914379845169624	56	1.285121179057460	50	1.155167171305789	45	8.539940438671708	40	2.871752409158799	37
20	-2.603166958553761	60	-4.402296279805995	53	-1.868999694416063	48	-7.554817427541818	43	-1.543750102314390	40
21	2.621513104786005	64	1.677875010111593	57	3.329741480774416	51	7.298479911573599	46	9.002093573955206	42
22	-2.960309090196894	68	-7.080953751084205	60	-6.503751428397245	54	-7.669359610426407	49	-5.673476723291737	45
23	3.729918505695468	72	3.294271339701116	64	1.387211446178504	58	8.734288891136381	52	3.851536878162182	48
24			-1.682682704009655	68	-3.219284516210091	61	-1.074455509207540	56	-2.807742123553089	51

s\c	1.3		1.4		1.5		1.6		1.7	
2	4.814450168842846	-01	3.126077942052726	-01	2.150328216768725	-01	1.548912678259770	-01	1.158142475655515	-01
3	1.148528630812271	00	3.394954135567535	-01	1.050250561566780	-01	3.629323840444131	-02	1.740104248326919	-02
4	-1.466690490568193	02	-4.477072619402122	01	-1.544845838914069	01	-5.907830034586893	00	-2.469348182764878	00
5	6.438583398835279	03	1.434870069820297	03	3.641532725445037	02	1.017793430355262	02	3.037998929370687	01
6	-3.451292921205505	05	-5.716808584872426	04	-1.105644504551854	04	-2.398956287870294	03	-5.640532071483903	02
7	2.346450058333313	07	2.892183527812793	06	4.282377688997828	05	7.265516443288603	04	1.356661634142809	04
8	-1.972215124475241	09	-1.800225678244181	08	-2.033498298811847	07	-2.690479748140295	06	-3.976623214659119	05
9	1.999139209351227	11	1.343835255647902	10	1.153092301803111	09	1.185625186327518	08	1.383600520903687	07
10	-2.394157336351802	13	-1.178439866603732	12	-7.647174360220822	10	-6.089174313958592	09	-5.594725163268909	08
11	3.331314646404128	15	1.193886655864256	14	5.833225654235743	12	3.584289091895283	11	2.585364443355078	10
12	-5.312264624466063	17	-1.378490669896391	16	-5.049031247397057	14	-2.385740489078043	13	-1.347089185825805	12
13	9.598640634714057	19	1.793682497800776	18	4.904029283016602	16	1.775841371405277	15	7.827342059425532	13
14	-1.946519091824725	22	-2.605525926459965	20	-5.295309302075206	18	-1.464626514621930	17	-5.025577835525780	15
15	4.394365452308554	24	4.191589882774624	22	6.306509281952498	20	1.327977245132656	19	3.537855210720967	17
16	-1.096681986281238	27	-7.416708731055670	24	-8.228146902336559	22	-1.314871377353241	21	-2.712641783538575	19
17	3.007206038673619	29	1.434834307467856	27	1.169174693072488	25	1.413463796309679	23	2.252420535704829	21
18	-9.011841770971550	31	-3.019113320324510	29	-1.800072764535589	27	-1.641320995829330	25	-2.015276644988035	23
19	2.937408371563859	34	6.877593738475952	31	2.989276765922495	29	2.049607834354816	27	1.934338413582444	25
20	-1.036970730167378	37	-1.689212499475606	34	-5.332841428768954	31	-2.741547140199780	29	-1.984013088769762	27
21	3.949616135721423	39	4.456745106273871	36	1.018363063527959	34	3.914091863664349	31	2.166978951359027	29
22	-1.617411527128210	42	-1.258890541883538	39	-2.074860000822484	36	-5.945587912122613	33	-2.512476158089358	31
23	7.098857925412561	44	3.795587257514571	41	4.497188049203901	38	9.581600404777280	35	3.083588411204265	33
24	-3.329659154548530	47	-1.218119348429386	44	-1.034184139051067	41	-1.633902300824849	38	-3.995794675221134	35

$\mu_4(v^*)$

s\c	1.8	1.9	2.0	2.1	2.2
2	8.928756366655510 -02	7.060625456482720 -02	5.703347781533512 -02	4.690570558909521 -02	3.917263706833386 -02
3	1.332375080059794 -02	1.316869914838979 -02	1.362043235881492 -02	1.379167858320903 -02	1.355392783474946 -02
4	-1.117529527689356 00	-5.444114780567185 -01	-2.846540664502414 -01	-1.596286521211077 -01	-9.605297320661710 -02
5	9.357601247868414 00	2.822597520383023 00	7.351233731895099 -01	7.565934959518160 -02	-1.151736683603006 -01
6	-1.389983149714256 02	-3.461528799977474 01	-8.338981521626192 00	-1.863350529466630 00	-4.272101419276934 -01
7	2.678495776214442 03	5.336530376966060 02	9.959219305426539 01	1.434720345885602 01	-4.185375756046443 -02
8	-6.270812078623221 04	-9.998658972175252 03	-1.483284624531710 03	-1.658958808954407 02	2.540106391444450 00
9	1.738512755802698 06	2.211565963173878 05	2.589635919699276 04	2.231321313012430 03	3.050372817718700 01
10	-5.588854687539865 07	-5.661221547066649 06	-5.207024738462674 05	-3.218687487557662 04	1.377388933944357 03
11	2.048262191371350 09	1.648919249847415 08	1.192728612314339 07	5.583974554016539 05	-1.464848988264731 04
12	-8.443516222869819 10	-5.389859598994950 09	-3.054873092428342 08	-1.051091132988673 07	2.857411141801620 05
13	3.872358178669118 12	1.955992684992880 11	8.671268270221690 09	2.188028813301310 08	-6.367571335541879 06
14	-1.957845478044485 14	-7.809395008160599 12	-2.702451370180781 11	-5.003774457137836 09	1.073078631292808 08
15	1.082900451900193 16	3.404175885487798 14	9.176987541350108 12	1.236464301377472 11	-2.384921183641219 09
16	-6.509510343569915 17	-1.609611705182334 16	-3.374065058231831 14	-3.299431875681444 12	4.724330921662670 10
17	4.228525470824652 19	8.209294234760834 17	1.335558103427933 16	9.442795376618318 13	-7.639971258030219 11
18	-2.953639239794594 21	-4.493941311772278 19	-5.664229720098053 17	-2.942542685609340 15	1.424228321918807 13
19	2.208813412421544 23	2.629213156052211 21	2.562612746682792 19	9.379320076722552 16	5.257335641786961 15
20	-1.761635988310106 25	-1.637625074015900 23	-1.233273994523926 21	-2.744519016257600 18	9.048260664704724 17
21	1.493238364827240 27	1.082354279387021 25	6.273601319239035 22	7.978895575944200 19	1.901229303707778 19
22	-1.341080469406304 29	-7.567327652222838 26	-3.393239078422380 24	1.232236093775852 21	-1.272915494712118 22
23	1.272581810189076 31	5.577271122502909 28	1.931637473454513 26	1.228547032842957 24	-1.480238558624456 24
24	-1.272668238742187 33	-4.332969556772676 30	-1.057729915659006 28	-5.816026300827411 25	1.389445287944811 25

s\c	2.3	2.4	2.5	2.6	2.8
2	3.314880579074095 -02	2.837363956862175 -02	2.452960771756962 -02	2.139272628539758 -02	1.663834916123239 -02
3	1.298808107708673 -02	1.220830255992742 -02	1.131159787194347 -02	1.035869157312722 -02	8.517082671672316 -03
4	-6.203594388032882 -02	-4.294234452592990 -02	-3.172309823227729 -02	-2.482871006436929 -02	-1.741341327730236 -02
5	-1.538198833230653 -01	-1.463702688428694 -01	-1.279486349607967 -01	-1.094231811125325 -01	-8.029563784060930 -02
6	-2.030186624526239 -01	-2.141532036651169 -01	-2.341226845987868 -01	-2.266186623317890 -01	-1.557964041151704 -01
7	-1.178298864244393 00	-5.038311973718616 -01	7.455976037253240 -02	4.313555624785359 -01	8.417063751291030 -01
8	1.122316196027379 01	7.843286025249874 00	7.434116843270765 00	8.527568529535096 00	1.002117641097103 01
9	2.414677367869279 01	7.913264877340100 01	8.263136432401806 01	6.929821272508752 01	3.692765542483369 01
10	8.440519358985174 02	2.416019226083150 02	1.432127827866672 02	3.956470035815685 01	-3.438632188209831 02
11	-2.161701792409827 03	1.093066404241750 02	-3.239073287200205 03	-5.562389644686867 03	-7.571872919493446 03
12	-3.743394002409336 04	-7.525013258495994 04	-6.394393942003136 04	-6.620011425857044 04	-5.084265010363285 04
13	-2.955582743698353 05	-4.557863568980553 05	-5.842451411449576 05	-3.167072443144668 05	4.239962013741753 05
14	-1.437178091171658 07	-2.188156346369759 06	3.114257215587122 06	6.432728150505536 06	1.437951358084639 07
15	2.620734027687792 08	2.589744482231946 07	6.966580352920086 07	1.748004960275015 08	1.403538385994541 08
16	-3.078502683468494 09	2.497092985468314 09	1.530434569644239 09	1.844670396227450 09	-1.611627467150416 09
17	-3.040457721854226 10	1.575649055902328 11	3.622257498595230 11	-6.088888668923914 10	3.926815742283933 09
18	-1.992319756824563 13	3.711475407637880 12	5.034267186873440 12	-7.645529451949308 12	-6.388409820224946 12
19	1.283520344548297 15	2.246623996463240 15	-2.093379342459795 15	9.557782913145508 14	9.276566008964013 14
20	2.858006836071496 17	-2.970376349118089 16	-1.828775156407530 16	3.099240602435503 16	3.446908208401026 15
21	5.865128735683632 18	-4.028273363361951 19	1.586592135419628 19	-6.628520241294060 18	-3.912802184853885 18
22	-7.227645978090119 20	-2.558381688192021 21	5.185280642288023 20	-2.723003131170330 20	-1.133536759552688 20
23	-1.340333319868091 23	1.670339994200316 23	-2.222766679386964 23	1.385848145768550 22	-5.731801710643210 21
24	2.904683103748281 24	4.667220554863826 25	-1.445654670629544 25	6.158184444020119 24	-1.010576268853162 24

$\mu_4(v^*)$

s\c	3.0	3.2	3.5	3.8	4.0
2	1.326311708492019 -02	1.078504681116941 -02	8.147716292901877 -03	6.332499278960457 -03	5.421695053147583 -03
3	6.853742093126406 -03	5.436142609019223 -03	3.754924625055242 -03	2.520842868085605 -03	1.893694950668143 -03
4	-1.374064415953209 -02	-1.151961664216127 -02	-9.218923626394815 -03	-7.385495575762074 -03	-6.283685642485556 -03
5	-5.991741909362035 -02	-4.436406541688025 -02	-2.571125914123355 -02	-1.069410373544439 -02	-2.433710716876422 -03
6	-6.494478409041310 -02	1.618825614656853 -02	1.040015758762039 -01	1.502776401079429 -01	1.605243737324320 -01
7	1.106169633536519 00	1.247346740573692 00	1.177645021019247 00	8.002067849201481 -01	4.324298144044426 -01
8	9.031648480643881 00	6.208531320525799 00	2.258217569599116 -01	-5.934623355877269 00	-9.221636306360811 00
9	1.557563682325594 -01	-3.961737526818234 01	-8.746289086470365 01	-1.000083103714314 02	-8.343950158961640 01
10	-7.012362831374701 02	-8.768012117601856 02	-6.886732229781598 02	3.836999662534380 00	6.217439787541587 02
11	-7.229561606297409 03	-4.019082112486176 03	5.035915603376026 03	1.417216750035245 04	1.693254994963114 04
12	4.396836863626471 03	7.698702421242626 04	1.576655744945277 05	1.218029596069757 05	1.600330151755073 04
13	1.173333242125540 06	1.609669986257204 06	7.562910435141081 05	-1.779547032071025 06	-3.784214644252995 06
14	1.624239448324569 07	6.117378008920785 06	-2.408090997983204 07	-4.942488970301183 07	-3.727968344629185 07
15	-5.693881179308468 07	-2.551590507805550 08	-6.105114953095346 08	-4.805229208779864 07	8.721236082592831 08
16	-3.577315308395574 09	-7.099784105631009 09	-2.705173776411342 08	-4.339160647550322 07	-6.669922384723406 09
17	3.850508258356292 10	-2.718920786660146 11	4.285773291201837 11	2.316689460458665 12	2.826597879437431 12
18	3.306770711967726 12	1.984701578618088 13	-4.792003758226045 12	-1.404747436779337 14	-1.939533348057836 14
19	5.920842521159903 14	1.542053553996210 14	6.679114221750487 14	-1.430736117821755 15	8.105497361653864 15
20	-1.364358275838057 16	-1.031278974831002 17	-2.634035710431927 17	7.731092233823333 17	-1.221665257014690 17
21	-9.180523174277850 18	2.419694450137992 18	1.486142227651569 19	-1.766820919568390 19	-1.968624986545343 19
22	-1.638529078064574 20	1.085172341859702 21	1.765809095807028 21	6.498692625980691 20	2.974012969288821 21
23	4.469492141962255 22	-4.121778162470410 21	-1.196341718471512 23	-3.387675010125543 23	-5.462526406243576 23
24	3.916492121298190 24	-9.620733668230785 24	-1.287522505125717 25	-2.022811392400344 25	6.028146577238111 25

2.1 Comments on the Moment Series. Briefly, apart from a slight irregularity initially in $\mu_4(v^*)$, the coefficients alternate in sign, but increase in magnitude rapidly, especially for small c. There is evidence of marked divergence; a stronger statement cannot be made since a general coefficient in the series is out of reach. Summation algorithms are therefore required, and an outline of some approaches is given in Bowman and Shenton (1976), and Shenton et al (1979). In the present situation we have used two approaches; c=1 produces an exponential density, so that v^* is distributed independently of the mean, and moments of v^* can be made to depend on those of m_2 and m_1'. However this property breaks down completely for $c \neq 1$.
An analysis of the moment series coefficients is given in Table 2. For small c (less than 2.0 approx.) the series alternate in sign; for $\mu_4(v^*)$ there is an anomaly initially. Disruption of this regular pattern sets in for $c > 2$, irregularities occuring first of all in the initial and (or) final coefficients. There is evidence of marked divergence for c small, and the magnitude of the 24th coefficient lies between 51! to 29! approximately for $0.9 < c < 4.0$. Summation algorithms used were Padé, Padé-Borel (2cB), Levin, and direct summation. For some details see Bowman and Shenton (1976), Shenton et al (1979), Shenton and Bowman (1977a); these are directed at statistical applications. A modern treatment of Padé methods is given in Baker (1975), and a general analysis of convergence accelerating techniques is to be found in Van Dyke (1974). Levin's (1973) recent algorithm is based on a modified Shanks' (1955) approach to the "slow-convergency" problem.

Since we shall use a three, and five point Shanks' formula, here is a brief outline. If there is a sequence $\{c_s\}$, slowly congergent, then

TABLE 2. Magnitude of Coefficients for "Coefficient of Variation" Moment Series

	Moment							
	$\mu_1'(v^*)$		$\mu_2(v^*)$		$\mu_3(v^*)$		$\mu_4(v^*)$	
	$\lvert v_{24}^{(1)}/v_0^{(1)}\rvert$		$\lvert v_{24}^{(2)}/v_1^{(2)}\rvert$		$\lvert v_{24}^{(3)}/v_2^{(3)}\rvert$		$\lvert v_{24}^{(4)}/v_2^{(4)}\rvert$	
0.9	2.9	67	4.6	67	6.3	66	2.3	67A
1.0	8.6	60	1.7	61	3.6	60	1.1	61A
1.1	4.2	55	1.0	56	3.0	55	7.4	55A
1.2	1.5	51	4.2	51	1.7	51	3.5	51A
1.3	2.5	47	7.4	47	4.0	47	6.9	47A
1.4	1.2	44	3.8	44	2.6	44	3.9	44A
1.5	1.3	41	4.5	41	3.7	41	4.8	41A
1.6	2.6	38	9.4	38	9.2	38	1.1	39A
1.7	7.9	35T	3.0	36	3.4	35	3.5	36A
1.8	3.1	33T	1.2	34	1.6	34	1.4	34A
1.9	1.3	31T	5.0	31	7.4	31	6.1	31A
2.0	3.7	28T	1.5	29	2.4	29	1.9	29A
2.1	2.4	26	9.6	26L	1.7	27L	1.2	27L
2.2	5.7	25B	2.5	26B	5.4	26B	3.5	26B
2.3	1.6	25B	6.7	25B	1.4	26B	8.8	25B
2.4	3.2	26B	1.3	27B	2.8	27B	1.6	27B
2.5	1.2	26B	4.7	26B	1.0	27B	5.9	26B
2.6	5.6	25B	2.3	26B	5.3	26B	2.9	26B
2.8	1.2	25B	4.9	25B	1.2	26B	6.1	25B
3.0	6.0	25B	2.4	26B	6.0	26B	3.0	26B
3.2	1.8	26B	7.3	26B	1.9	27B	8.9	26B
3.5	3.4	26B	1.3	27B	3.4	27B	1.6	27B
3.8	7.3	26B	2.7	27B	5.7	27B	3.2	27B
4.0	2.6	27B	9.6	27B	2.4	28B	1.1	28B

(Introduction of letters T, L, and B indicate disruption of alternating sign pattern. Disruption occurs at the top (T) of the series, bottom (L), and both top and bottom (B). A refers to a sign pattern with alternation except for the first two terms. In the moment columns each second column refers to the power of ten used as a multiplier).

we may consider the modified sequence $\{d_s\}$, where

$$d_s^{(3)} = \begin{vmatrix} c_s & c_{s+1} \\ \Delta c_s & \Delta c_{s+1} \end{vmatrix} \div \begin{vmatrix} 1 & 1 \\ \Delta c_s & \Delta c_{s+1} \end{vmatrix} \qquad (2.3)$$

where $\Delta c_r = c_{r+1} - c_r$.

This is derived from an assumption that

$$c_s \sim \alpha + \alpha_1 p_1^s, \qquad (2.4)$$

i.e. that sooner or later a geometric pattern dominates. Aitken (1925, 1950), surmising that power iterates to latent roots of matrices followed this pattern, used it extensively - indeed it is sometimes called Aitken's δ^2 - process. The expression (2.3) as s increases is expected to approach α.

A 5-point Shanks' accelerator is derived from the second - order version of (2.4), namely

$$c_s \sim \alpha + \alpha_1 p_1^s + \alpha_2 p_2^s \qquad (2.5)$$

leading to the modified sequence,

$$d_s^{(5)} = \begin{vmatrix} c_s & c_{s+1} & c_{s+2} \\ \Delta c_s & \Delta c_{s+1} & \Delta c_{s+2} \\ \Delta^2 c_s & \Delta^2 c_{s+1} & \Delta^2 c_{s+2} \end{vmatrix} \div \begin{vmatrix} 1 & 1 & 1 \\ \Delta c_s & \Delta c_{s+1} & \Delta c_{s+2} \\ \Delta^2 c_s & \Delta^2 c_{s+1} & \Delta^2 c_{s+2} \end{vmatrix} \qquad (2.6)$$

where $\Delta^2 c_r = \Delta(\Delta c_r)$.

There is also an iterated version (Van Dyke, 1974), which uses a 3 (or 5) point(s) formula on successive sequence sets. It should be noted that loss of accuracy in using Shanks' accelerators is always a possibility.

2.2 Summation for c=1 In sampling from the exponential (c=1), it is known (Hogg and Craig, 1970, p. 232) that a scale free statistic, such as v*, is distributed independently of the mean. Thus, in expectation,

$$E(v^* m_1') = E(\sqrt{m_2})$$

and

$$E(v^*) = E(\sqrt{m_2})/E(m_1'), \qquad (2.7)$$

with $Em_1' = 1$. Similarly

$$\mu_2'(v^*) = Em_2/Em_1'^2$$

$$= (n-1)/(n+1), \qquad (2.8)$$

so $\sigma^2(v^*) = (n-1)/(n+1) - (E\sqrt{m_2})^2$,

and $\mu_3'(v^*) = Em_2^{3/2}/(1+3/n+2/n^2), \qquad (2.9)$

$$\mu_4'(v^*) = (1+6/n-13/n^2+6/n^3)/(1+6/n+11/n^2+6/n^3). \qquad (2.10)$$

(Note that $Em_2^{3/2}$ can be found from $\mu_3(\sqrt{m_2})$, for

$$\mu_3(\sqrt{m_2}) = Em_2^{3/2} - 3(1-1/n)\ E\sqrt{m_2} + 2(E\sqrt{m_2})^3$$

holds in the present case since μ_2 for the exponential equals unity).

Clearly then for $c = 1$, we must assess from the moment series $\mu_1'(\sqrt{m_2})$ and $\mu_3(\sqrt{m_2})$; $\mu_2(\sqrt{m_2})$, and $\mu_4(\sqrt{m_2})$ are only of interest for checking purposes. Comments on the first and third moments are given in Shenton et al (1979).

Now a crucial test of a summation algorithm occurs when the argument n is small; it is not wise however to take n too small, for then we may be going to extremes. In the case of m_2 for example, $E\sqrt{m_2} = 0$ for n=1, but to use n=1 as a testing ground would be too stringent. We have studied in some detail the cases n = 2, 3, and 4 (exact results for n = 4 are due to Dr. H. K. Lam and are given in Appendix B), and here we state the situation for n = 4.

Illustration 1 When n=4, using Levin's algorithm with 17 terms,

$\mu_1'(\sqrt{m_2}) \sim 0.797175.$

For $\mu_3(\sqrt{m_2})$, Levin gave the approximants 0.1621 (11 terms) and 0.1602 16 terms). As a check the Borel-Padé 2cB scheme (Shenton & Bowman, 1977a) gave the approximants

n=4	a=5 tr=0	a=6 tr=1
r	$\mu_3(\sqrt{m_2})$	$\mu_3(\sqrt{m_2})$
18	0.173192	0.156105
19	0.171930	0.156255
20	0.170788	0.156403
21	0.169755	0.156548
22	0.168818	0.156688
23	0.167969	0.156822

(tr = truncation.)

A Shanks' 3-point formulas applied to sequence values at $r = 21, 22, 23$ gives the assessments 0.1597 and 0.1598. From (2.7) - (2.10) we now find $\mu_1'(v^*) \sim 0.727175$, $\mu_2'(v^*) \sim 0.6$, $\mu_3'(v^*) \sim 0.547684$, and $\mu_4'(v^*) \sim 0.542857$, leading to $\mu_2(v^*) \sim 0.0712165$, $\sqrt{\beta_1}(v^*) \sim 0.4107$, and $\beta_2(v^*) \sim 2.8773$. Exact values from Lam (appendix B) are: $\mu_1' = 0.72709$, $\mu_2 = 0.07135$, $\sqrt{\beta_1} = 0.3906$, $\beta_2 = 3.0316$. Using the Bowman - Shenton (1979) quick approximations to the 4-moment Pearson distribution, we find for the six standard percentage levels the values in Table 3.

Table 3

Percentage Points for v^* when $c = 1$, $n = 4$

%	1	5	10	90	95	99
Pbs approx.	0.2227	0.3271	0.3949	1.0891	1.2016	1.4073
True value*	0.1853	0.3169	0.3993	1.0862	1.2120	1.4231

(Pbs = Pearson - Bowman - Shenton value (see Bowman and Shenton, 1979); * given by Dr. H. K. Lam, and described in Appendix B.) The agreement is quite good, but poor at the 1% level, doubtless due to the rather abrupt tail near $v^* = 0$ (see Figure 2a). The corresponding density for c^* is shown in Figure 2b; note that this curve appears smoother than that for v^*.

Illustration 2 Since we do not know any exact results for $n > 4$, we consider the evaluation of moments. In particular take $n = 5$, and the case of $\mu_3(\sqrt{m_2})$. The Borel-Pade 2cB scheme leads to:

n=5	a=5 tr=0		a=6 tr=1	
r	$\mu_3(\sqrt{m_2})$	$\lvert\Delta\rvert$	$\mu_3(\sqrt{m_2})$	$\lvert\Delta\rvert$
18	0.133392		0.125493	
		655		66
19	0.132737		0.125559	
		566		65
20	0.132151		0.125624	
		525		63
21	0.131626		0.125687	
		471		60
22	0.131155		0.125747	
		423		57
23	0.130732		0.125804	

($\lvert\Delta\rvert$ = mod. first order differences)

Shanks' 3-point extrapolates on the last three members yields 0.126990 and 0.126840. We prefer the average, rounded to 0.1269, since the approximants are fairly tightly packed (differences near to 0.000060). For comparison the Levin values are 0.1280 (11 terms) and 0.1271 (16 terms) in fair agreement. As a further check, two simulations of 200,000 samples gave 0.1252 and 0.1302 with mean 0.1276. The uncertainty in these simulation values can be approximated as follows.

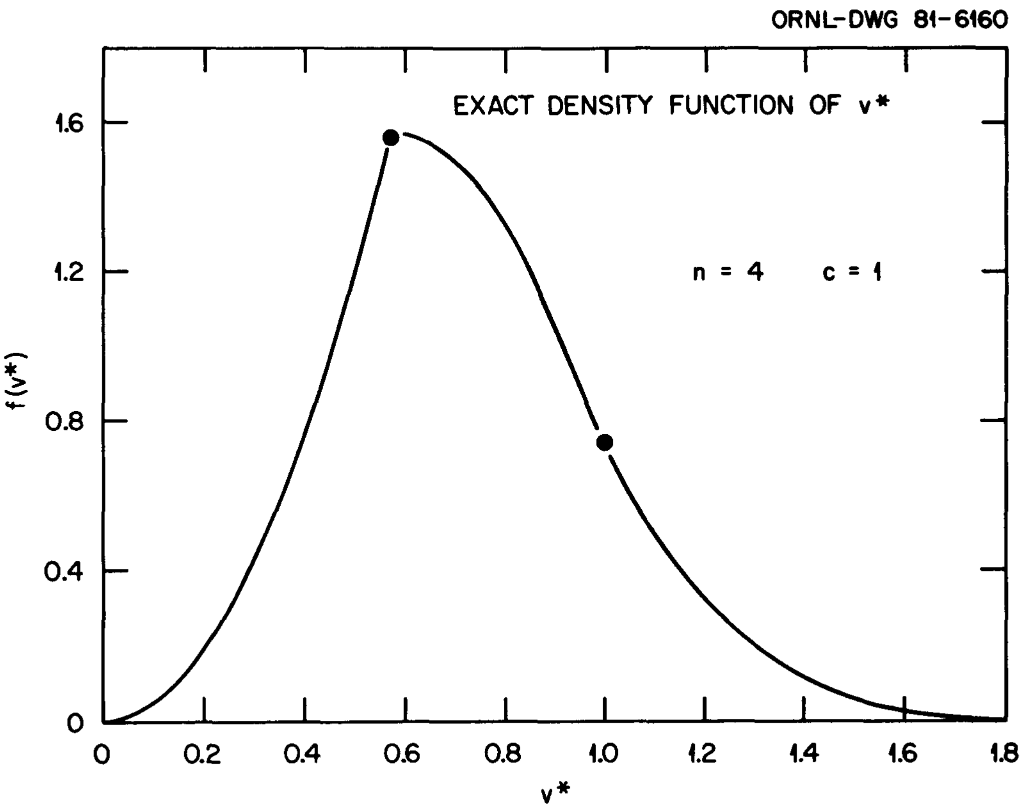

Figure 2a. Exact Density Function of v*, n = 4 c = 1

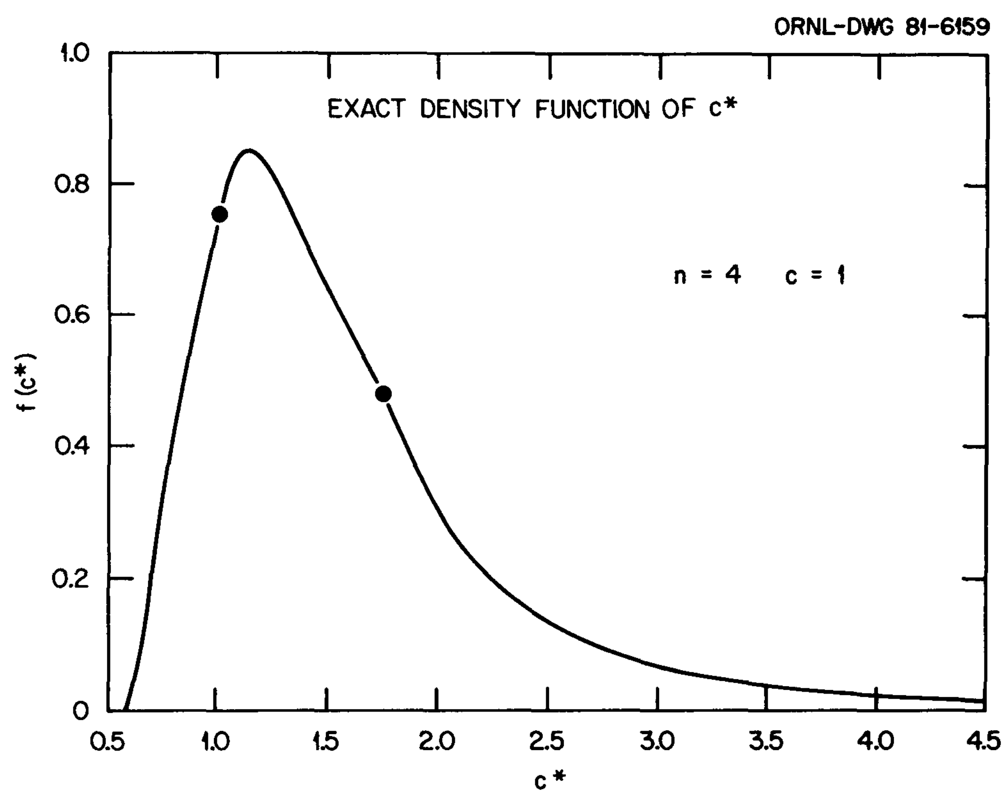

Figure 2b. Exact Density Function of c*, n = 4 c = 1

To a good approximation for a large simulation (N)

$$\mathrm{Var}\{m_3(\sqrt{m_2})\} \sim \frac{\mu_6(\sqrt{m_2}) - 6\mu_4(\sqrt{m_2})\mu_2(\sqrt{m_2}) - \mu_3^2(\sqrt{m_2}) + 9\mu_2^3(\sqrt{m_2})}{N} \tag{2.11}$$

so we need a value for $\mu_6(\sqrt{m_2})$. Take the sixth moment of the Pearson approximating distribution, for which from summatory techniques,

$\mu_1'(\sqrt{m_2}) \sim 0.777391$,

$\mu_2(\sqrt{m_2}) \sim 0.195663$,

$\mu_3(\sqrt{m_2}) \sim 0.126878$,

$\mu_4(\sqrt{m_2}) \sim 0.258763$,

or in standardized moments $\nu_s = \mu_s/\sigma^s$, $\nu_2 = 1$, $\nu_3 = 1.465961$, $\nu_4 = 6.759020$. Now the recursion for Pearson distribution moments is (Bowman & Shenton, 1973)

$$\nu_{s+1} = \frac{s\{14.306343\ \nu_s + 20.588955\nu_{s-1}\}}{21.659870 - 1.070915s} \tag{2.12}$$

from which

$\nu_5(\sqrt{m_2}) \sim 0.494617$,

$\nu_6(\sqrt{m_2}) \sim 1.279488$.

Hence there is the possibility

$\mathrm{Var}\{m_3(\sqrt{m_2})\} \sim 1.027025/400{,}000$

and $\sigma\{m_3(\sqrt{m_2})\} \sim 0.0016$, leading to the 95% interval for $m_3(\sqrt{m_2})$ based on $m_3(\sqrt{m_2})$ (which we have from the simulations)

$$0.1245 < \mu_3(\sqrt{m_2}) < 0.1307. \qquad (2.13)$$

Our assessments (Levin, 2cB) are consistent with this inequality. However (Pearson, 1963) it is known that 5th & 6th moments for different equivalent 4-moment systems may differ considerably so the confidence interval derived in this way is not too reliable.

Illustration 3 The case $n = 10$ and $\mu_3(\sqrt{m_2})$ For the Borel - Padé (2cB) we have:

n=10	a=5 tr=0	a=6 tr=1
r	$\mu_3(\sqrt{m_2})$	$\mu_3(\sqrt{m_2})$
18	0.0539077	0.0533528
19	0.0538433	0.0533568
20	0.0537878	0.0533606
21	0.0537400	0.0533642
22	0.0536986	0.0533674
23	0.0536627	0.0533704
Shanks on r=21-23	0.0534284	0.0534023

Since the algorithm for $a = 6$ $tr = 1$ has smaller first differences we prefer the value 0.05340 for our final assessment. For comparison, Levin's algorithm gave 0.05349 (11 terms) and 0.05342 (16 terms), where as two simulations runs gave 0.05299, and 0.05329 with mean 0.05314 and 95% confidence interval $0.0519 < \mu_3(\sqrt{m_2}) < 0.0543$. Our preferred assessment does lie in this interval, as also the other assessments.

2.3 Conclusions for summation of series for v^* when $c = 1$.

For $\mu_1'(v^*)$ use Levin's algorithm (17 terms).

For $\mu_3(v^*)$, use Levin (15 terms) on $\mu_3(\sqrt{m_2})$.

Derive $\mu_2(v^*)$ from (2.8), $\mu_3(v^*)$ from (2.9), (2.8), and (2.7). Similarly derive $\mu_4(v^*)$. With this procedure, we think the percentiles corresponding to $\alpha = 0.01$, 0.05, 0.10, 0.90, 0.95, and 0.99 will be in error by at most a few percent when a Pearson 4-moment approximant is used for samples exceeding 9. The least-reliable point will be for $\alpha = 0.01$.

As a check on these Levin approximants to the moments, simulation comparison (10^5 runs) are given in Table 4.

Table 4

Moments of v^* - Simulation and Levin c=1

n		μ_1'	μ_2	μ_3	μ_4
5	L	0.7745	0.0668	0.00945	0.01332
	S	0.7734	0.0666	0.00920	0.01499
10	L	0.8764	0.0501	0.00831	0.01014
	S	0.8761	0.0504	0.00855	0.01046
20	L	0.9337	0.03302	0.004679	0.004709
	S	0.9338	0.03302	0.004815	0.004823

2.4 Summation for $c > 1$ Two changes occur here, for (a) we lose the independence property of v^* and m_1', and (b) there is a marked drop in the magnitude of the coefficients especially for $c > 1.5$ approx., and an ultimate disruption of the sign pattern (Table 2). Judging from the performance of Levin's algorithm when $c = 1$, for which exact results are known for samples of 2, 3, and 4, we use this algorithm for $c > 1$ and stop at the 16th (using 17 terms) approximant for $\mu_1'(v^*)$, 11th for

$\mu_2(v^*)$, 14th for $\mu_3(v^*)$, and 11th for $\mu_4(v^*)$. A set of Levin sequences for a few values of c and samples sizes n = 10, 20, 50 are given in Table 5.

The Levin sequences are less smooth than the 2cB, so different strategies are needed. For Levin we have already mentioned the rules followed. Now 2cB sequences are quite smooth if a judicious choice of the Borel technique (choice of the parameter a) and truncation point is made. The reader will see that when c = 1.1 the 2cB sequences are ultimately monotonic (there may be anomalous behavior initially). So the final approximants can be found from the use of the Shanks' 5-point formula (2.6). Comparisons are shown in Table 6, and the agreement is quite satisfactory, except for $\mu_4(v^*)$ and the 2cB sequence corresponding to a=1, tr=0; here the associated successive differences are increasing rapidly, so it is not advisable to use the Shanks' formula. However for a=2, tr=0, the associated differences (K_s) are more stable and now the agreement with the Levin assessments is satisfactory.

2.5 Moments of v* These are given in Table 7 for n = 10(1)20, 22, 25(5)50(10)100 and c = 0.8(0.1)2.6(0.2)3.2, 3.5, 3.8, 4.0. It will be seen that judging from the four moments, the distribution of v* approaches nearer to the normal as c increases ($1 < c < 4$); i.e. the skewness decreases, and the kurtosis decreases towards three. However if we hold c constant and increase the sample size n, there is a move away from the normal point initially, especially for small c.

The probability intergral of v* can now be approximated using either a Pearson or Johnson 4-moment equivalent distribution, attention being confined to levels α between 0.01 and 0.99. Pearson approximate

Table 5. Levin Approximants for v* moments

C= 0.8

	N=10 μ_1'	N=10 μ_2	N=10 μ_3	N=10 μ_4	N=20 μ_1'	N=20 μ_2	N=20 μ_3	N=20 μ_4
3	1.2557203 00	4.4781000 -01	4.6084531 -01	-5.8245933 00	1.3188361 00	2.6914809 -01	1.7736090 -01	1.3520657 00
4	1.2191337 00	4.4400020 -01	4.7840105 -01	3.4660939 00	1.3022657 00	2.5873248 -01	1.6954379 -01	6.2031421 -01
5	1.1995832 00	4.0163208 -01	3.9098694 -01	2.1933764 00	1.2981945 00	2.4810725 -01	1.5152196 -01	5.0257271 -01
6	1.1975525 00	3.8600887 -01	3.6962711 -01	1.7001691 00	1.2988800 00	2.4749224 -01	1.5279541 -01	4.3328267 -01
7	1.2000837 00	3.8778621 -01	3.6376434 -01	1.4189995 00	1.2995657 00	2.4861161 -01	1.5277619 -01	3.9791629 -01
8	1.2017087 00	3.9222932 -01	3.8032780 -01	1.2514525 00	1.2996819 00	2.4899586 -01	1.5418982 -01	3.8070547 -01
9	1.2019111 00	3.9281346 -01	3.7314491 -01	1.1856731 00	1.2996055 00	2.4878891 -01	1.5305397 -01	3.7521158 -01
10	1.2016548 00	3.9228120 -01	3.6525607 -01	1.1605443 00	1.2995680 00	2.4872692 -01	1.5253738 -01	3.7293293 -01
11	1.2014983 00	3.9325580 -01	4.0863648 -01	1.0912420 00	1.2995730 00	2.4889716 -01	1.5570898 -01	3.6771494 -01
12	1.2013879 00	3.9116373 -01	3.4615269 -01	1.1623321 00	1.2995676 00	2.4870650 -01	1.5172783 -01	3.7286435 -01
13	1.2014168 00	3.9123102 -01	3.1007975 -01	1.2106865 00	1.2995697 00	2.4871830 -01	1.5030026 -01	3.7495433 -01
14	1.2015414 00	3.9816389 -01	5.7124985 -01	8.5544127 -01	1.2995795 00	2.4916844 -01	1.6141861 -01	3.5899934 -01
15	1.2012563 00	3.9004172 -01	4.1219390 -01	1.0765530 00	1.2995537 00	2.4866433 -01	1.5452547 -01	3.6903510 -01
16	1.2010741 00	3.7369985 -01	-4.7010098 -01	2.3412038 00	1.2995438 00	2.4793700 -01	1.2870812 -01	4.0753919 -01
17	1.2020985 00	4.1872861 -01	9.9969420 -01	1.9912731 -01	1.2996107 00	2.4986195 -01	1.6943226 -01	3.4592485 -01
18	1.2018628 00	4.4529928 -01	3.4422022 00	-3.4252888 00	1.2995908 00	2.5058110 -01	2.1840852 -01	2.7083960 -01
19	1.1983379 00	2.5474542 -01	-4.3402894 00	8.2703933 00	1.2994268 00	2.4476426 -01	6.6278677 -02	5.0673492 -01
20	1.2010278 00	2.5914202 -01	-9.8888239 00	1.6703126 01	1.2995536 00	2.4551845 -01	-5.6221263 -03	6.2081741 -01
21	1.2126295 00	1.0025605 00	2.5628269 01	-3.7988479 01	1.2999470 00	2.6202032 -01	5.0049944 -01	-1.7888221 -01
22	1.1985524 00	6.6833718 -01	2.6130138 01	-3.6706364 01	1.2994789 00	2.5369957 -01	4.7825841 -01	-1.5087335 -01
23	1.1625257 00	-1.7744589 00	-7.5489801 13	-2.6879150 13	1.2985395 00	2.1079857 -01	-7.5400917 11	-2.7497310 11
24	1.2040548 00	1.1196808 11	0.0	0.0	1.2998227 00	1.5544549 09	0.0	0.0
25	-2.2504456 08	0.0	0.0	0.0	-4.2844099 06	0.0	0.0	0.0

N=50

	μ_1'	μ_2	μ_3	μ_4
3	1.37379988480 00	1.24826499736 -01	4.26641692925 -02	9.97415813133 -02
4	1.37019741078 00	1.21595208732 -01	3.96786753363 -02	8.40260602226 -02
5	1.37010369111 00	1.20931709025 -01	3.87877374038 -02	7.71544027720 -02
6	1.37028140749 00	1.21168159458 -01	3.92714507531 -02	7.30900015068 -02
7	1.37030024789 00	1.21191651422 -01	3.91368088955 -02	7.18573817253 -02
8	1.37028887989 00	1.21175074579 -01	3.91667942372 -02	7.13401457240 -02
9	1.37028611595 00	1.21169384052 -01	3.91462498227 -02	7.11931640617 -02
10	1.37028748429 00	1.21173866654 -01	3.91409465929 -02	7.11566539002 -02
11	1.37028793538 00	1.21178723536 -01	3.91961791634 -02	7.10721248304 -02
12	1.37028715183 00	1.21171986342 -01	3.91319250878 -02	7.11633721905 -02
13	1.37028721663 00	1.21172925826 -01	3.91254272945 -02	7.11724865398 -02
14	1.37028758075 00	1.21180061505 -01	3.92222250665 -02	7.10257776702 -02
15	1.37028703174 00	1.21172944603 -01	3.91604736303 -02	7.11209275842 -02
16	1.37028693505 00	1.21166977327 -01	3.90346652790 -02	7.13168654132 -02
17	1.37028784697 00	1.21182539810 -01	3.92245783566 -02	7.10172969244 -02
18	1.37028749444 00	1.21184457510 -01	3.93550825003 -02	7.08100785130 -02
19	1.37028512650 00	1.21155990994 -01	3.89329805563 -02	7.14879891095 -02
20	1.37028722469 00	1.21163445160 -01	3.88481684104 -02	7.16237309222 -02
21	1.37028927020 00	1.21213795173 -01	3.97227730982 -02	7.01957634854 -02
22	1.37028683196 00	1.21185162029 -01	3.95567839158 -02	7.04370349567 -02
23	1.37028371452 00	1.21099546762 -01	-7.68984305921 08	-2.88250204574 08
24	1.37028824466 00	2.36755158977 06	0.0	0.0
25	-9.60115476954 03	0.0	0.0	0.0

C= 0.9   N=10

	μ_1'	μ_2	μ_3	μ_4
3	1.0371345 00	2.4196051 -01	1.5817360 -01	8.9186677 -01
4	1.0144468 00	2.2991672 -01	1.5016319 -01	5.1302331 -01
5	1.0078519 00	2.1614207 -01	1.3029368 -01	4.2211591 -01
6	1.0082981 00	2.1416064 -01	1.2851332 -01	3.6040462 -01
7	1.0092211 00	2.1536957 -01	1.2855277 -01	3.2115143 -01
8	1.0094373 00	2.1603365 -01	1.3022469 -01	2.9850484 -01
9	1.0093310 00	2.1592145 -01	1.2889120 -01	2.8919277 -01
10	1.0092440 00	2.1594937 -01	1.3017743 -01	2.8323345 -01
11	1.0092085 00	2.1597114 -01	1.3050654 -01	2.8101941 -01
12	1.0091890 00	2.1584375 -01	1.2682896 -01	2.8452684 -01
13	1.0091875 00	2.1616505 -01	1.3329027 -01	2.7729884 -01
14	1.0091643 00	2.1600393 -01	1.3231810 -01	2.7842296 -01
15	1.0091415 00	2.1539377 -01	1.1449763 -01	2.9948883 -01
16	1.0091853 00	2.1675110 -01	1.4347234 -01	2.6452693 -01
17	1.0091570 00	2.1679149 -01	1.6475369 -01	2.3844516 -01
18	1.0090447 00	2.1236864 -01	3.5617799 -02	3.9910775 -01
19	1.0092480 00	2.1735509 -01	1.1288103 -01	3.0177305 -01
20	1.0093829 00	2.2696870 -01	4.9402077 -01	-1.8317420 -01
21	1.0086048 00	2.0074882 -01	-1.7907925 -01	6.8893527 -01
22	1.0088130 00	1.9127830 -01	-9.8950571 -01	1.7508773 00
23	1.0111464 00	2.9537502 -01	2.3429350 00	-2.4740682 00
24	1.0082253 00	2.2700717 -01	0.0	0.0
25	1.0055589 00	0.0	0.0	0.0

N=20

	μ_1'	μ_2	μ_3	μ_4
3	1.0902026 00	1.3934019 -01	5.5879224 -02	1.4106924 -01
4	1.0824710 00	1.3374637 -01	5.1509958 -02	1.1542468 -01
5	1.0816668 00	1.3157395 -01	4.8939574 -02	1.0351947 -01
6	1.0820309 00	1.3183409 -01	4.9450520 -02	9.5096936 -02
7	1.0821441 00	1.3200112 -01	4.9378652 -02	9.0982214 -02
8	1.0821253 00	1.3199866 -01	4.9458037 -02	8.9081893 -02
9	1.0821062 00	1.3197332 -01	4.9352816 -02	8.8442264 -02
10	1.0821039 00	1.3198882 -01	4.9452434 -02	8.8082305 -02
11	1.0821032 00	1.3199216 -01	4.9459475 -02	8.8009099 -02
12	1.0821012 00	1.3198128 -01	4.9301071 -02	8.8180813 -02
13	1.0821010 00	1.3199890 -01	4.9541514 -02	8.7894782 -02
14	1.0820996 00	1.3199003 -01	4.9488476 -02	8.7957942 -02
15	1.0820987 00	1.3196804 -01	4.9069496 -02	8.8474417 -02
16	1.0821011 00	1.3201348 -01	4.9693198 -02	8.7692437 -02
17	1.0820996 00	1.3200940 -01	4.9971783 -02	8.7340256 -02
18	1.0820958 00	1.3191131 -01	4.8146259 -02	8.9685180 -02
19	1.0821023 00	1.3201608 -01	4.9264481 -02	8.8231487 -02
20	1.0821050 00	1.3215606 -01	5.2874062 -02	8.3510558 -02
21	1.0820882 00	1.3178943 -01	4.6869743 -02	9.1460550 -02
22	1.0820938 00	1.3172659 -01	4.1977082 -02	9.8053499 -02
23	1.0821286 00	1.3271816 -01	6.1294997 -02	7.1951088 -02
24	1.0820890 00	1.3220209 -01	0.0	0.0
25	1.0820386 00	0.0	0.0	0.0

N=50

	μ_1'	μ_2	μ_3	μ_4
3	1.13351850335 00	6.27630719655 -02	1.22192640824 -02	1.84145889252 -02
4	1.13227689064 00	6.18460594138 -02	1.15809243591 -02	1.73352214572 -02
5	1.13232715717 00	6.18154328223 -02	1.15615008864 -02	1.65965103194 -02
6	1.13235937873 00	6.18510738714 -02	1.15998469705 -02	1.62301538123 -02
7	1.13235632889 00	6.18458399112 -02	1.15818013538 -02	1.61302711080 -02
8	1.13235485620 00	6.18452652086 -02	1.15868592400 -02	1.60883844861 -02
9	1.13235493419 00	6.18456148833 -02	1.15854091065 -02	1.60801927078 -02
10	1.13235498246 00	6.18459842548 -02	1.15865906526 -02	1.60770191300 -02
11	1.13235488738 00	6.18458396981 -02	1.15864263379 -02	1.60768300299 -02
12	1.13235484595 00	6.18456903174 -02	1.15852125248 -02	1.60782111991 -02
13	1.13235486791 00	6.18459322174 -02	1.15868795726 -02	1.60761275492 -02
14	1.13235485025 00	6.18458233897 -02	1.15863691461 -02	1.60767863580 -02
15	1.13235484124 00	6.18456927623 -02	1.15849703837 -02	1.60785852599 -02
16	1.13235486350 00	6.18459378006 -02	1.15868815872 -02	1.60760851539 -02
17	1.13235485081 00	6.18458835911 -02	1.15871239074 -02	1.60757737889 -02
18	1.13235483293 00	6.18456059282 -02	1.15842048988 -02	1.60796568316 -02
19	1.13235486227 00	6.18458888941 -02	1.15860714876 -02	1.60771454594 -02
20	1.13235486608 00	6.18460876647 -02	1.15891618777 -02	1.60729809320 -02
21	1.13235482337 00	6.18455513317 -02	1.15842108217 -02	1.60797158428 -02
22	1.13235484157 00	6.18455734704 -02	1.15822458999 -02	1.60824148513 -02
23	1.13235489247 00	6.18463950645 -02	1.15911773251 -02	1.60701805247 -02
24	1.13235483808 00	6.18459643307 -02	0.0	0.0
25	1.13235478571 00	0.0	0.0	0.0

C= 1.0 N=10

	μ_1'	μ_2	μ_3	μ_4
3	8.9109790 -01	5.7677766 -02	1.6172191 -02	8.2257642 -02
4	8.7808771 -01	5.0743933 -02	7.8025111 -03	3.8391260 -02
5	8.7592781 -01	4.8129141 -02	4.9594854 -03	2.6105294 -02
6	8.7635194 -01	4.9182255 -02	7.5228687 -03	1.7245137 -02
7	8.7657982 -01	5.0015588 -02	8.7869823 -03	1.2764121 -02
8	8.7654774 -01	5.0211062 -02	8.9690529 -03	1.1671979 -02
9	8.7648350 -01	5.0164633 -02	8.4902218 -03	1.1873847 -02
10	8.7645276 -01	5.0143261 -02	8.4305283 -03	1.1400480 -02
11	8.7643586 -01	5.0107732 -02	8.1687970 -03	1.0816039 -02
12	8.7642456 -01	5.0098938 -02	8.1830449 -03	1.0189530 -02
13	8.7641656 -01	5.0112255 -02	8.5182239 -03	9.6428493 -03
14	8.7640889 -01	5.0072004 -02	7.9262171 -03	1.0300701 -02
15	8.7640793 -01	5.0108610 -02	8.3246680 -03	1.0052217 -02
16	8.7640659 -01	5.0146248 -02	9.2709506 -03	9.1861534 -03
17	8.7639915 -01	5.0000021 -02	6.6161702 -03	1.2067231 -02
18	8.7640649 -01	5.0138250 -02	8.1417295 -03	1.0422427 -02
19	8.7640960 -01	5.0339706 -02	1.3754026 -02	4.2615757 -03
20	8.7638236 -01	4.9660595 -02	8.4691971 -04	1.8480034 -02
21	8.7640726 -01	4.9992987 -02	8.6905971 -04	1.8438103 -02
22	8.7644998 -01	5.1613385 -02	4.1018784 -02	-2.6679732 -02
23	8.7632524 -01	4.8791796 -02	-5.9632626 -03	2.6439246 -02
24	8.7631431 -01	4.6496098 -02	0.0	0.0
25	8.7675932 -01	0.0	0.0	0.0

N=20

	μ_1'	μ_2	μ_3	μ_4
3	9.3728763 -01	3.4988485 -02	6.2359157 -03	1.1492038 -02
4	9.3367387 -01	3.2926464 -02	4.4692999 -03	8.5674388 -03
5	9.3356383 -01	3.2755923 -02	4.3624944 -03	6.7514531 -03
6	9.3368509 -01	3.2975967 -02	4.6713134 -03	5.4917348 -03
7	9.3369058 -01	3.3024407 -02	4.6984236 -03	4.9809044 -03
8	9.3367877 -01	3.3024417 -02	4.6974235 -03	4.8313057 -03
9	9.3367452 -01	3.3021710 -02	4.6769711 -03	4.7968759 -03
10	9.3367346 -01	3.3022291 -02	4.6797877 -03	4.7580656 -03
11	9.3367235 -01	3.3020284 -02	4.6703813 -03	4.7405918 -03
12	9.3367170 -01	3.3020037 -02	4.6718602 -03	4.7264657 -03
13	9.3367143 -01	3.3020700 -02	4.6800061 -03	4.7156852 -03
14	9.3367117 -01	3.3019390 -02	4.6670854 -03	4.7310281 -03
15	9.3367120 -01	3.3020471 -02	4.6749925 -03	4.7245442 -03
16	9.3367117 -01	3.3021140 -02	4.6866395 -03	4.7129438 -03
17	9.3367097 -01	3.3018496 -02	4.6562917 -03	4.7467076 -03
18	9.3367117 -01	3.3020797 -02	4.6736686 -03	4.7272510 -03
19	9.3367121 -01	3.3022929 -02	4.7130094 -03	4.6828146 -03
20	9.3367077 -01	3.3015966 -02	4.6292493 -03	4.7777050 -03
21	9.3367115 -01	3.3019525 -02	4.6398827 -03	4.7653775 -03
22	9.3367157 -01	3.3030203 -02	4.8083840 -03	4.5713315 -03
23	9.3367039 -01	3.3012705 -02	4.6166435 -03	4.7921363 -03
24	9.3367048 -01	3.3005747 -02	0.0	0.0
25	9.3367301 -01	0.0	0.0	0.0

N=50

	μ_1'	μ_2	μ_3	μ_4
3	9.72229870787 -01	1.66045035982 -02	1.54294966500 -03	1.44660379234 -03
4	9.71761969225 -01	1.63726369730 -02	1.40833024573 -03	1.29970271225 -03
5	9.71791554041 -01	1.63935674239 -02	1.42575674419 -03	1.18210696047 -03
6	9.71795867092 -01	1.64006914909 -02	1.43125225548 -03	1.13083578162 -03
7	9.71794496978 -01	1.63999968325 -02	1.42985046089 -03	1.11806775203 -03
8	9.71794357769 -01	1.64002112334 -02	1.43035676492 -03	1.11449237889 -03
9	9.71794337860 -01	1.64002024102 -02	1.43009757828 -03	1.11399949310 -03
10	9.71794308884 -01	1.64001931869 -02	1.43014266070 -03	1.11368351019 -03
11	9.71794291761 -01	1.64001718771 -02	1.43008873015 -03	1.11362798283 -03
12	9.71794289466 -01	1.64001771495 -02	1.43010490959 -03	1.11358286936 -03
13	9.71794288524 -01	1.64001811031 -02	1.43012978672 -03	1.11355566464 -03
14	9.71794286763 -01	1.64001737599 -02	1.43009174620 -03	1.11360110205 -03
15	9.71794287333 -01	1.64001786784 -02	1.43011321603 -03	1.11357899995 -03
16	9.71794287188 -01	1.64001796840 -02	1.43012589781 -03	1.11356540042 -03
17	9.71794286548 -01	1.64001743692 -02	1.43009207210 -03	1.11360418441 -03
18	9.71794287177 -01	1.64001785900 -02	1.43011156899 -03	1.11358157246 -03
19	9.71794287159 -01	1.64001801878 -02	1.43013093004 -03	1.11355905871 -03
20	9.71794286509 -01	1.64001742003 -02	1.43009081656 -03	1.11360592504 -03
21	9.71794287053 -01	1.64001774495 -02	1.43010162533 -03	1.11359317051 -03
22	9.71794287334 -01	1.64001819295 -02	1.43014207751 -03	1.11354533975 -03
23	9.71794286490 -01	1.64001748109 -02	1.43009770445 -03	1.11359813005 -03
24	9.71794286704 -01	1.64001743127 -02	0.0	0.0
25	9.71794287576 -01	0.0	0.0	0.0

C= 1.1 N=10

	μ_1'	μ_2	μ_3	μ_4
3	7.8635991 -01	4.5062794 -02	9.1719668 -03	2.0856628 -02
4	7.7895683 -01	4.1212846 -02	5.5781482 -03	1.5852161 -02
5	7.7813928 -01	4.0423162 -02	4.8502804 -03	1.2486281 -02
6	7.7826156 -01	4.0864789 -02	5.5311941 -03	9.6040993 -03
7	7.7825180 -01	4.1080808 -02	5.7723523 -03	7.8941847 -03
8	7.7819801 -01	4.1131271 -02	5.8333371 -03	7.1951988 -03
9	7.7816687 -01	4.1143899 -02	5.7997839 -03	7.0211126 -03
10	7.7815311 -01	4.1154009 -02	5.7999830 -03	6.9340989 -03
11	7.7814413 -01	4.1152088 -02	5.7561729 -03	6.8881835 -03
12	7.7813799 -01	4.1153015 -02	5.7710097 -03	6.8034945 -03
13	7.7813367 -01	4.1150941 -02	5.7606902 -03	6.7674678 -03
14	7.7813091 -01	4.1148524 -02	5.7338780 -03	6.7725066 -03
15	7.7812954 -01	4.1153240 -02	5.7950904 -03	6.7097194 -03
16	7.7812807 -01	4.1147802 -02	5.7376019 -03	6.7694692 -03
17	7.7812752 -01	4.1147800 -02	5.7086708 -03	6.8038298 -03
18	7.7812765 -01	4.1160124 -02	5.9015512 -03	6.6150050 -03
19	7.7812617 -01	4.1139671 -02	5.6253770 -03	6.8970444 -03
20	7.7812672 -01	4.1146978 -02	5.5800864 -03	6.9449105 -03
21	7.7812799 -01	4.1192706 -02	6.3234884 -03	6.1789403 -03
22	7.7812294 -01	4.1121154 -02	5.3356557 -03	7.1995017 -03
23	7.7812329 -01	4.1120872 -02	7.4511929 -03	4.9772273 -03
24	7.7812963 -01	4.0906190 -02	0.0	0.0
25	7.7816289 -01	0.0	0.0	0.0

N=20

μ_1'	μ_2	μ_3	μ_4
8.2743695 -01	2.6455727 -02	3.3680699 -03	4.3701751 -03
8.2567201 -01	2.5551216 -02	2.7507146 -03	3.8042407 -03
8.2565923 -01	2.5549855 -02	2.7593942 -03	3.2848905 -03
8.2568161 -01	2.5608526 -02	2.8162659 -03	2.9180213 -03
8.2567234 -01	2.5615399 -02	2.8177256 -03	2.7582649 -03
8.2566727 -01	2.5617253 -02	2.8206833 -03	2.7047849 -03
8.2566585 -01	2.5618260 -02	2.8195249 -03	2.6925140 -03
8.2566519 -01	2.5618625 -02	2.8196703 -03	2.6882419 -03
8.2566473 -01	2.5618402 -02	2.8184927 -03	2.6870931 -03
8.2566452 -01	2.5618467 -02	2.8190177 -03	2.6853721 -03
8.2566440 -01	2.5618421 -02	2.8188015 -03	2.6850455 -03
8.2566434 -01	2.5618373 -02	2.8184399 -03	2.6852357 -03
8.2566432 -01	2.5618464 -02	2.8191784 -03	2.6844856 -03
8.2566429 -01	2.5618375 -02	2.8185662 -03	2.6851365 -03
8.2566429 -01	2.5618385 -02	2.8184297 -03	2.6853075 -03
8.2566429 -01	2.5618501 -02	2.8195810 -03	2.6841377 -03
8.2566428 -01	2.5618329 -02	2.8181234 -03	2.6856613 -03
8.2566428 -01	2.5618395 -02	2.8181338 -03	2.6856535 -03
8.2566429 -01	2.5618620 -02	2.8204860 -03	2.6831710 -03
8.2566426 -01	2.5618289 -02	2.8176488 -03	2.6861691 -03
8.2566427 -01	2.5618323 -02	2.8221492 -03	2.6812980 -03
8.2566430 -01	2.5617798 -02	0.0	0.0
8.2566440 -01	0.0	0.0	0.0

N=50

	μ_1'	μ_2	μ_3	μ_4
3	8.56512647886 -01	1.21513274449 -02	7.82884882077 -04	6.35244812867 -04
4	8.56316379266 -01	1.20747133222 -02	7.48327813832 -04	6.00623566874 -04
5	8.56327061423 -01	1.20826362507 -02	7.53097325656 -04	5.71519624788 -04
6	8.56326713433 -01	1.20834123949 -02	7.53351622537 -04	5.60599036136 -04
7	8.56326257944 -01	1.20833481162 -02	7.53266752891 -04	5.57896525837 -04
8	8.56326224585 -01	1.20834286340 -02	7.53350165952 -04	5.57295470931 -04
9	8.56326204975 -01	1.20834271246 -02	7.53322178503 -04	5.57235053829 -04
10	8.56326196324 -01	1.20834276199 -02	7.53326913471 -04	5.57211459804 -04
11	8.56326193385 -01	1.20834266197 -02	7.53322922819 -04	5.57209118331 -04
12	8.56326192644 -01	1.20834272725 -02	7.53324904124 -04	5.57205253728 -04
13	8.56326192190 -01	1.20834269649 -02	7.53324156318 -04	5.57205475613 -04
14	8.56326192040 -01	1.20834268802 -02	7.53323749354 -04	5.57205785283 -04
15	8.56326192027 -01	1.20834270851 -02	7.53324605571 -04	5.57204905972 -04
16	8.56326191976 -01	1.20834269230 -02	7.53323988271 -04	5.57205571542 -04
17	8.56326191980 -01	1.20834269597 -02	7.53324024256 -04	5.57205542186 -04
18	8.56326191987 -01	1.20834270395 -02	7.53324469078 -04	5.57205072624 -04
19	8.56326191969 -01	1.20834269325 -02	7.53323974073 -04	5.57205603862 -04
20	8.56326191978 -01	1.20834269768 -02	7.53324067844 -04	5.57205502808 -04
21	8.56326191982 -01	1.20834270336 -02	7.53324412440 -04	5.57205129787 -04
22	8.56326191968 -01	1.20834269515 -02	7.53324025475 -04	5.57205549105 -04
23	8.56326191973 -01	1.20834269715 -02	7.53324426770 -04	5.57205106311 -04
24	8.56326191979 -01	1.20834269201 -02	0.0	0.0
25	8.56326191991 -01	0.0	0.0	0.0

C= 1.2 N=10

	μ_1'	μ_2	μ_3	μ_4
3	7.0832710 -01	3.6453581 -02	5.6380467 -03	9.3367822 -03
4	7.0416711 -01	3.4364934 -02	4.0190010 -03	8.1526581 -03
5	7.0379124 -01	3.4137113 -02	3.8087229 -03	6.9685543 -03
6	7.0376981 -01	3.4306348 -02	3.9850233 -03	5.8418431 -03
7	7.0371149 -01	3.4368418 -02	4.0398997 -03	5.0985503 -03
8	7.0367046 -01	3.4391916 -02	4.0694481 -03	4.7558196 -03
9	7.0365139 -01	3.4405579 -02	4.0759927 -03	4.6584082 -03
10	7.0364185 -01	3.4412860 -02	4.0784746 -03	4.6407833 -03
11	7.0363586 -01	3.4414652 -02	4.0725133 -03	4.6402977 -03
12	7.0363215 -01	3.4415604 -02	4.0735872 -03	4.6315838 -03
13	7.0362980 -01	3.4415240 -02	4.0688832 -03	4.6273737 -03
14	7.0362839 -01	3.4415289 -02	4.0692332 -03	4.6207668 -03
15	7.0362751 -01	3.4415323 -02	4.0706745 -03	4.6157685 -03
16	7.0362693 -01	3.4414763 -02	4.0655326 -03	4.6189082 -03
17	7.0362666 -01	3.4415401 -02	4.0719874 -03	4.6126570 -03
18	7.0362643 -01	3.4415027 -02	4.0703309 -03	4.6145041 -03
19	7.0362630 -01	3.4414339 -02	4.0620166 -03	4.6228442 -03
20	7.0362637 -01	3.4415871 -02	4.0877238 -03	4.5988273 -03
21	7.0362626 -01	3.4412746 -02	4.0809492 -03	4.6056735 -03
22	7.0362652 -01	3.4408340 -02	4.1310043 -03	4.5579521 -03
23	7.0362767 -01	3.4395743 -02	4.6114123 -03	4.0933732 -03
24	7.0363079 -01	3.4286470 -02	0.0	0.0
25	7.0364997 -01	0.0	0.0	0.0

N=20

	μ_1'	μ_2	μ_3	μ_4
3	7.4493688 -01	2.0856290 -02	1.9819662 -03	2.1722001 -03
4	7.4405066 -01	2.0454563 -02	1.7535458 -03	2.0114911 -03
5	7.4404258 -01	2.0469484 -02	1.7632765 -03	1.8317700 -03
6	7.4403792 -01	2.0484599 -02	1.7732238 -03	1.7052689 -03
7	7.4403034 -01	2.0486203 -02	1.7737044 -03	1.6501497 -03
8	7.4402779 -01	2.0487510 -02	1.7750548 -03	1.6329306 -03
9	7.4402692 -01	2.0488063 -02	1.77[illegible]008 -03	1.6295845 -03
10	7.4402651 -01	2.0488220 -02	1.7751226 -03	1.6290449 -03
11	7.4402630 -01	2.0488236 -02	1.7750082 -03	1.6289816 -03
12	7.4402621 -01	2.0488258 -02	1.7750489 -03	1.6288278 -03
13	7.4402616 -01	2.0488248 -02	1.7749870 -03	1.6288028 -03
14	7.4402614 -01	2.0488250 -02	1.7749990 -03	1.6287470 -03
15	7.4402613 -01	2.0488251 -02	1.7750104 -03	1.6287172 -03
16	7.4402612 -01	2.0488245 -02	1.7749765 -03	1.6287436 -03
17	7.4402612 -01	2.0488251 -02	1.7750128 -03	1.6287038 -03
18	7.4402612 -01	2.0488248 -02	1.7750015 -03	1.6287207 -03
19	7.4402611 -01	2.0488245 -02	1.7749767 -03	1.6287459 -03
20	7.4402612 -01	2.0488251 -02	1.7750433 -03	1.6286819 -03
21	7.4402611 -01	2.0488241 -02	1.7750195 -03	1.6287058 -03
22	7.4402612 -01	2.0488231 -02	1.7750961 -03	1.6286310 -03
23	7.4402612 -01	2.0488208 -02	1.7756742 -03	1.6280607 -03
24	7.4402613 -01	2.0488036 -02	0.0	0.0
25	7.4402617 -01	0.0	0.0	0.0

N=50

	μ_1'	μ_2	μ_3	μ_4
3	7.69543202768 -01	9.32581253394 -03	4.35687418674 -04	3.33019987785 -04
4	7.69454618257 -01	9.29866522537 -03	4.25945794029 -04	3.22521905776 -04
5	7.69457686643 -01	9.30134623456 -03	4.27154188265 -04	3.14014792555 -04
6	7.69456999149 -01	9.30138051776 -03	4.27100108328 -04	3.11240128450 -04
7	7.69456827062 -01	9.30140137645 -03	4.27113418057 -04	3.10614537367 -04
8	7.69456802947 -01	9.30142101205 -03	4.27124810959 -04	3.10507115295 -04
9	7.69456792970 -01	9.30142149259 -03	4.27121990967 -04	3.10497492771 -04
10	7.69456790167 -01	9.30142218090 -03	4.27122691945 -04	3.10495393205 -04
11	7.69456789301 -01	9.30142219725 -03	4.27122369117 -04	3.10495359563 -04
12	7.69456789026 -01	9.30142224879 -03	4.27122474469 -04	3.10495125236 -04
13	7.69456788917 -01	9.30142221624 -03	4.27122391363 -04	3.10495143595 -04
14	7.69456788887 -01	9.30142222575 -03	4.27122416244 -04	3.10495099423 -04
15	7.69456788876 -01	9.30142222556 -03	4.27122419401 -04	3.10495091122 -04
16	7.69456788871 -01	9.30142222101 -03	4.27122404368 -04	3.10495104283 -04
17	7.69456788871 -01	9.30142222511 -03	4.27122417721 -04	3.10495091853 -04
18	7.69456788870 -01	9.30142222321 -03	4.27122412936 -04	3.10495096756 -04
19	7.69456788870 -01	9.30142222270 -03	4.27122410189 -04	3.10495099589 -04
20	7.69456788870 -01	9.30142222413 -03	4.27122417367 -04	3.10495092463 -04
21	7.69456788870 -01	9.30142222244 -03	4.27122414088 -04	3.10495095777 -04
22	7.69456788870 -01	9.30142222193 -03	4.27122418081 -04	3.10495091777 -04
23	7.69456788871 -01	9.30142222069 -03	4.27122436916 -04	3.10495072790 -04
24	7.69456788871 -01	9.30142221297 -03	0.0	0.0
25	7.69456788874 -01	0.0	0.0	0.0

C= 1.3   N=10

	μ_1'	μ_2	μ_3	μ_4
3	6.4763953 -01	3.0332382 -02	3.6889728 -03	5.1693588 -03
4	6.4532422 -01	2.9208007 -02	2.9279286 -03	4.7945765 -03
5	6.4509474 -01	2.9147833 -02	2.8560632 -03	4.3116256 -03
6	6.4502004 -01	2.9211138 -02	2.8987745 -03	3.8088777 -03
7	6.4496011 -01	2.9233321 -02	2.9144899 -03	3.4585354 -03
8	6.4492938 -01	2.9246577 -02	2.9279248 -03	3.2991452 -03
9	6.4491538 -01	2.9254737 -02	2.9332659 -03	3.2563061 -03
10	6.4490820 -01	2.9258704 -02	2.9351413 -03	3.2518310 -03
11	6.4490415 -01	2.9260201 -02	2.9347825 -03	3.2539891 -03
12	6.4490184 -01	2.9260833 -02	2.9347635 -03	3.2542451 -03
13	6.4490050 -01	2.9260912 -02	2.9339021 -03	3.2538509 -03
14	6.4489972 -01	2.9260944 -02	2.9339502 -03	3.2524172 -03
15	6.4489927 -01	2.9260858 -02	2.9336240 -03	3.2515799 -03
16	6.4489901 -01	2.9260794 -02	2.9335045 -03	3.2509899 -03
17	6.4489888 -01	2.9260785 -02	2.9338582 -03	3.2503374 -03
18	6.4489881 -01	2.9260682 -02	2.9333297 -03	3.2506847 -03
19	6.4489879 -01	2.9260715 -02	2.9336061 -03	3.2504472 -03
20	6.4489878 -01	2.9260748 -02	2.9324578 -03	3.2515286 -03
21	6.4489877 -01	2.9261048 -02	2.9226932 -03	3.2603855 -03
22	6.4489869 -01	2.9263930 -02	2.8735812 -03	3.3049912 -03
23	6.4489807 -01	2.9279606 -02	2.5974412 -03	3.5569227 -03
24	6.4489449 -01	2.9364592 -02	0.0	0.0
25	6.4487529 -01	0.0	0.0	0.0

N=20

μ_1'	μ_2	μ_3	μ_4
6.8041969 -01	1.6983952 -02	1.2468930 -03	1.2521166 -03
6.7996396 -01	1.6801497 -02	1.1577808 -03	1.1959623 -03
6.7994955 -01	1.6811263 -02	1.1618220 -03	1.1248146 -03
6.7993937 -01	1.6815179 -02	1.1632869 -03	1.0752879 -03
6.7993415 -01	1.6815931 -02	1.1636274 -03	1.0549293 -03
6.7993255 -01	1.6816572 -02	1.1640848 -03	1.0493456 -03
6.7993196 -01	1.6816803 -02	1.1641463 -03	1.0484141 -03
6.7993172 -01	1.6816878 -02	1.1641684 -03	1.0483349 -03
6.7993161 -01	1.6816900 -02	1.1641615 -03	1.0483517 -03
6.7993157 -01	1.6816908 -02	1.1641628 -03	1.0483474 -03
6.7993155 -01	1.6816908 -02	1.1641550 -03	1.0483445 -03
6.7993154 -01	1.6816908 -02	1.1641567 -03	1.0483352 -03
6.7993153 -01	1.6816907 -02	1.1641548 -03	1.0483324 -03
6.7993153 -01	1.6816907 -02	1.1641545 -03	1.0483306 -03
6.7993153 -01	1.6816907 -02	1.1641558 -03	1.0483287 -03
6.7993153 -01	1.6816907 -02	1.1641542 -03	1.0483300 -03
6.7993153 -01	1.6816907 -02	1.1641550 -03	1.0483293 -03
6.7993153 -01	1.6816907 -02	1.1641532 -03	1.0483311 -03
6.7993153 -01	1.6816907 -02	1.1641416 -03	1.0483418 -03
6.7993153 -01	1.6816912 -02	1.1640965 -03	1.0483836 -03
6.7993153 -01	1.6816931 -02	1.1638943 -03	1.0485715 -03
6.7993152 -01	1.6817012 -02	0.0	0.0
6.7993150 -01	0.0	0.0	0.0

N=50

	μ_1'	μ_2	μ_3	μ_4
3	7.01611891481 -01	7.42894121619 -03	2.60828250739 -04	1.96475261349 -04
4	7.01569439711 -01	7.41864152452 -03	2.57822074382 -04	1.92727648046 -04
5	7.01569744085 -01	7.41953969062 -03	2.58126265829 -04	1.89866092889 -04
6	7.01569239114 -01	7.41952200276 -03	2.58099015853 -04	1.89052220362 -04
7	7.01569153548 -01	7.41953706694 -03	2.58106511960 -04	1.88893689180 -04
8	7.01569137225 -01	7.41954187520 -03	2.58108060828 -04	1.88871962464 -04
9	7.01569132597 -01	7.41954238654 -03	2.58107900963 -04	1.88870140777 -04
10	7.01569131491 -01	7.41954263043 -03	2.58108013216 -04	1.88869955587 -04
11	7.01569131174 -01	7.41954265480 -03	2.58107983528 -04	1.88869984784 -04
12	7.01569131083 -01	7.41954266302 -03	2.58107989026 -04	1.88869974266 -04
13	7.01569131054 -01	7.41954266119 -03	2.58107983406 -04	1.88869974037 -04
14	7.01569131046 -01	7.41954266195 -03	2.58107985238 -04	1.88869970019 -04
15	7.01569131044 -01	7.41954266140 -03	2.58107984348 -04	1.88869969981 -04
16	7.01569131043 -01	7.41954266138 -03	2.58107984424 -04	1.88869969669 -04
17	7.01569131043 -01	7.41954266141 -03	2.58107984623 -04	1.88869969433 -04
18	7.01569131043 -01	7.41954266132 -03	2.58107984410 -04	1.88869969625 -04
19	7.01569131043 -01	7.41954266136 -03	2.58107984512 -04	1.88869969533 -04
20	7.01569131043 -01	7.41954266136 -03	2.58107984409 -04	1.88869969632 -04
21	7.01569131043 -01	7.41954266139 -03	2.58107984056 -04	1.88869969965 -04
22	7.01569131043 -01	7.41954266158 -03	2.58107983137 -04	1.88869970835 -04
23	7.01569131043 -01	7.41954266210 -03	2.58107980195 -04	1.88869973625 -04
24	7.01569131043 -01	7.41954266370 -03	0.0	0.0
25	7.01569131042 -01	0.0	0.0	0.0

C= 1.4 N=10

	μ_1'	μ_2	μ_3	μ_4
3	5.9880845 -01	2.5814945 -02	2.5364467 -03	3.2272526 -03
4	5.9755114 -01	2.5210500 -02	2.1656219 -03	3.0889367 -03
5	5.9737205 -01	2.5197950 -02	2.1357619 -03	2.8716088 -03
6	5.9728066 -01	2.5222207 -02	2.1442122 -03	2.6243695 -03
7	5.9722660 -01	2.5232496 -02	2.1497884 -03	2.4495957 -03
8	5.9720198 -01	2.5240054 -02	2.1556379 -03	2.3755126 -03
9	5.9719100 -01	2.5244435 -02	2.1583552 -03	2.3573350 -03
10	5.9718568 -01	2.5246513 -02	2.1595138 -03	2.3559724 -03
11	5.9718297 -01	2.5247376 -02	2.1598312 -03	2.3572435 -03
12	5.9718154 -01	2.5247697 -02	2.1599175 -03	2.3579324 -03
13	5.9718079 -01	2.5247749 -02	2.1598203 -03	2.3580601 -03
14	5.9718039 -01	2.5247720 -02	2.1598162 -03	2.3578174 -03
15	5.9718019 -01	2.5247650 -02	2.1597601 -03	2.3575773 -03
16	5.9718010 -01	2.5247589 -02	2.1597848 -03	2.3573367 -03
17	5.9718007 -01	2.5247531 -02	2.1598092 -03	2.3571786 -03
18	5.9718007 -01	2.5247476 -02	2.1598574 -03	2.3570675 -03
19	5.9718008 -01	2.5247422 -02	2.1600087 -03	2.3569149 -03
20	5.9718011 -01	2.5247334 -02	2.1598660 -03	2.3570388 -03
21	5.9718015 -01	2.5247306 -02	2.1569745 -03	2.3595337 -03
22	5.9718019 -01	2.5248210 -02	2.1352304 -03	2.3782333 -03
23	5.9718001 -01	2.5255801 -02	2.0136260 -03	2.4829521 -03
24	5.9717811 -01	2.5299849 -02	0.0	0.0
25	5.9716655 -01	0.0	0.0	0.0

N=20

	μ_1'	μ_2	μ_3	μ_4
3	6.2835586 -01	1.4189452 -02	8.2741783 -04	7.9555024 -04
4	6.2811975 -01	1.4104553 -02	7.9089666 -04	7.7335943 -04
5	6.2810078 -01	1.4109501 -02	7.9219288 -04	7.4214423 -04
6	6.2809041 -01	1.4110589 -02	7.9226736 -04	7.2077091 -04
7	6.2808663 -01	1.4111023 -02	7.9244612 -04	7.1281316 -04
8	6.2808550 -01	1.4111311 -02	7.9259346 -04	7.1092869 -04
9	6.2808510 -01	1.4111409 -02	7.9262501 -04	7.1065281 -04
10	6.2808496 -01	1.4111444 -02	7.9263794 -04	7.1064099 -04
11	6.2808491 -01	1.4111454 -02	7.9263997 -04	7.1065204 -04
12	6.2808488 -01	1.4111457 -02	7.9264039 -04	7.1065570 -04
13	6.2808488 -01	1.4111457 -02	7.9263970 -04	7.1065594 -04
14	6.2808487 -01	1.4111457 -02	7.9263981 -04	7.1065469 -04
15	6.2808487 -01	1.4111456 -02	7.9263959 -04	7.1065400 -04
16	6.2808487 -01	1.4111456 -02	7.9263969 -04	7.1065342 -04
17	6.2808487 -01	1.4111456 -02	7.9263973 -04	7.1065316 -04
18	6.2808487 -01	1.4111456 -02	7.9263980 -04	7.1065302 -04
19	6.2808487 -01	1.4111456 -02	7.9263999 -04	7.1065284 -04
20	6.2808487 -01	1.4111456 -02	7.9263981 -04	7.1065300 -04
21	6.2808487 -01	1.4111456 -02	7.9263753 -04	7.1065500 -04
22	6.2808487 -01	1.4111456 -02	7.9262449 -04	7.1066642 -04
23	6.2808487 -01	1.4111462 -02	7.9256824 -04	7.1071571 -04
24	6.2808487 -01	1.4111489 -02	0.0	0.0
25	6.2808485 -01	0.0	0.0	0.0

N=50

	μ_1'	μ_2	μ_3	μ_4
3	6.46912651807 -01	6.09644453982 -03	1.65718420328 -04	1.26177898193 -04
4	6.46891456924 -01	6.09229312900 -03	1.64710767367 -04	1.24677863363 -04
5	6.46890781201 -01	6.09260057453 -03	1.64787563321 -04	1.23590206479 -04
6	6.46890428440 -01	6.09259225737 -03	1.64778716857 -04	1.23321371975 -04
7	6.46890375552 -01	6.09259902545 -03	1.64781151043 -04	1.23277340840 -04
8	6.46890365353 -01	6.09260039099 -03	1.64781404112 -04	1.23272345064 -04
9	6.46890363122 -01	6.09260064264 -03	1.64781439058 -04	1.23271959258 -04
10	6.46890362640 -01	6.09260071499 -03	1.64781459673 -04	1.23271948750 -04
11	6.46890362520 -01	6.09260072402 -03	1.64781457962 -04	1.23271958252 -04
12	6.46890362490 -01	6.09260072583 -03	1.64781458585 -04	1.23271958877 -04
13	6.46890362482 -01	6.09260072565 -03	1.64781458233 -04	1.23271958892 -04
14	6.46890362480 -01	6.09260072557 -03	1.64781458324 -04	1.23271958566 -04
15	6.46890362480 -01	6.09260072546 -03	1.64781458267 -04	1.23271958505 -04
16	6.46890362479 -01	6.09260072543 -03	1.64781458290 -04	1.23271958448 -04
17	6.46890362479 -01	6.09260072541 -03	1.64781458289 -04	1.23271958438 -04
18	6.46890362479 -01	6.09260072540 -03	1.64781458292 -04	1.23271958433 -04
19	6.46890362479 -01	6.09260072540 -03	1.64781458298 -04	1.23271958427 -04
20	6.46890362479 -01	6.09260072539 -03	1.64781458292 -04	1.23271958433 -04
21	6.46890362479 -01	6.09260072539 -03	1.64781458257 -04	1.23271958465 -04
22	6.46890362479 -01	6.09260072541 -03	1.64781458121 -04	1.23271958586 -04
23	6.46890362479 -01	6.09260072550 -03	1.64781457729 -04	1.23271958936 -04
24	6.46890362479 -01	6.09260072574 -03	0.0	0.0
25	6.46890362479 -01	0.0	0.0	0.0

C= 1.5

	N=10				N=20			
	μ_1'	μ_2	μ_3	μ_4	μ_1'	μ_2	μ_3	μ_4
3	5.5844405 -01	2.2371453 -02	1.8159536 -03	2.1807595 -03	5.8528184 -01	1.2101722 -02	5.7359174 -04	5.4179860 -04
4	5.5781362 -01	2.2046581 -02	1.6301017 -03	2.1245983 -03	5.8516417 -01	1.2061366 -02	5.5798619 -04	5.3228475 -04
5	5.5765271 -01	2.2046261 -02	1.6154933 -03	2.0192603 -03	5.8514196 -01	1.2063682 -02	5.5829219 -04	5.1745001 -04
6	5.5755256 -01	2.2056310 -02	1.6159898 -03	1.8888405 -03	5.8513179 -01	1.2064047 -02	5.5821438 -04	5.0759000 -04
7	5.5750089 -01	2.2062023 -02	1.6183237 -03	1.7986258 -03	5.8512866 -01	1.2064294 -02	5.5829571 -04	5.0436503 -04
8	5.5747915 -01	2.2066259 -02	1.6209209 -03	1.7641473 -03	5.8512778 -01	1.2064422 -02	5.5834593 -04	5.0370870 -04
9	5.5747014 -01	2.2068535 -02	1.6222850 -03	1.7565346 -03	5.8512751 -01	1.2064465 -02	5.5836124 -04	5.0362378 -04
10	5.5746622 -01	2.2069566 -02	1.6229738 -03	1.7560969 -03	5.8512743 -01	1.2064479 -02	5.5836767 -04	5.0362263 -04
11	5.5746443 -01	2.2069960 -02	1.6232768 -03	1.7566805 -03	5.8512740 -01	1.2064482 -02	5.5836951 -04	5.0362702 -04
12	5.5746359 -01	2.2070057 -02	1.6234076 -03	1.7570385 -03	5.8512739 -01	1.2064483 -02	5.5837011 -04	5.0362869 -04
13	5.5746320 -01	2.2070027 -02	1.6234555 -03	1.7571383 -03	5.8512739 -01	1.2064483 -02	5.5837026 -04	5.0362894 -04
14	5.5746304 -01	2.2069957 -02	1.6234881 -03	1.7570937 -03	5.8512738 -01	1.2064482 -02	5.5837036 -04	5.0362874 -04
15	5.5746298 -01	2.2069881 -02	1.6235051 -03	1.7570131 -03	5.8512738 -01	1.2064482 -02	5.5837039 -04	5.0362855 -04
16	5.5746298 -01	2.2069815 -02	1.6235256 -03	1.7569319 -03	5.8512738 -01	1.2064482 -02	5.5837043 -04	5.0362841 -04
17	5.5746301 -01	2.2069760 -02	1.6235331 -03	1.7568789 -03	5.8512738 -01	1.2064482 -02	5.5837043 -04	5.0362835 -04
18	5.5746304 -01	2.2069722 -02	1.6235154 -03	1.7568645 -03	5.8512738 -01	1.2064482 -02	5.5837041 -04	5.0362835 -04
19	5.5746307 -01	2.2069706 -02	1.6234261 -03	1.7569209 -03	5.8512738 -01	1.2064482 -02	5.5837033 -04	5.0362840 -04
20	5.5746309 -01	2.2069735 -02	1.6232908 -03	1.7570217 -03	5.8512738 -01	1.2064482 -02	5.5837025 -04	5.0362846 -04
21	5.5746310 -01	2.2069819 -02	1.6237689 -03	1.7566266 -03	5.8512738 -01	1.2064482 -02	5.5837052 -04	5.0362824 -04
22	5.5746307 -01	2.2069716 -02	1.6274907 -03	1.7535845 -03	5.8512738 -01	1.2064482 -02	5.5837197 -04	5.0362703 -04
23	5.5746308 -01	2.2068246 -02	1.6363392 -03	1.7463105 -03	5.8512738 -01	1.2064481 -02	5.5837444 -04	5.0362497 -04
24	5.5746348 -01	2.2063677 -02	0.0	0.0	5.8512738 -01	1.2064479 -02	0.0	0.0
25	5.5746506 -01	0.0	0.0	0.0	5.8512739 -01	0.0	0.0	0.0

N=50

	μ_1'	μ_2	μ_3	μ_4
3	6.01775499711 -01	5.12475693090 -03	1.10650456051 -04	8.63518526856 -05
4	6.01764853716 -01	5.12299862176 -03	1.10287578722 -04	8.56990817436 -05
5	6.01763744279 -01	5.12310732357 -03	1.10306616084 -04	8.52435957654 -05
6	6.01763465147 -01	5.12310525850 -03	1.10303984567 -04	8.51469340648 -05
7	6.01763426296 -01	5.12310803384 -03	1.10304701243 -04	8.51337864980 -05
8	6.01763419754 -01	5.12310849031 -03	1.10304763500 -04	8.51325222018 -05
9	6.01763418594 -01	5.12310858936 -03	1.10304784919 -04	8.51324358994 -05
10	6.01763418375 -01	5.12310860945 -03	1.10304790060 -04	8.51324374594 -05
11	6.01763418329 -01	5.12310861176 -03	1.10304790581 -04	8.51324398445 -05
12	6.01763418320 -01	5.12310861189 -03	1.10304790794 -04	8.51324402136 -05
13	6.01763418318 -01	5.12310861172 -03	1.10304790806 -04	8.51324402390 -05
14	6.01763418317 -01	5.12310861163 -03	1.10304790821 -04	8.51324402041 -05
15	6.01763418317 -01	5.12310861159 -03	1.10304790822 -04	8.51324401895 -05
16	6.01763418317 -01	5.12310861157 -03	1.10304790825 -04	8.51324401822 -05
17	6.01763418317 -01	5.12310861156 -03	1.10304790824 -04	8.51324401807 -05
18	6.01763418317 -01	5.12310861156 -03	1.10304790824 -04	8.51324401807 -05
19	6.01763418317 -01	5.12310861156 -03	1.10304790822 -04	8.51324401818 -05
20	6.01763418317 -01	5.12310861156 -03	1.10304790822 -04	8.51324401823 -05
21	6.01763418317 -01	5.12310861156 -03	1.10304790824 -04	8.51324401803 -05
22	6.01763418317 -01	5.12310861155 -03	1.10304790832 -04	8.51324401738 -05
23	6.01763418317 -01	5.12310861156 -03	1.10304790839 -04	8.51324401674 -05
24	6.01763418317 -01	5.12310861155 -03	0.0	0.0
25	6.01763418317 -01	0.0	0.0	0.0

C=0.9 V=1.1130

S	K	E(V*) N=10	E(V*) N=20	E(V*) N=50	K	VAR(V*) N=10	VAR(V*) N=20	VAR(V*) N=50
4	-1.0589640 +00	9.7104450 -01	1.0292651 +00	1.0749075 +00				
5	-1.3708620 +00	9.6582073 -01	1.0274136 +00	1.0745943 +00	-5.5523740 +01	2.0157960 -01	1.2912967 -01	6.1621368 -02
6	-1.8975417 +00	9.6194396 -01	1.0262245 +00	1.0744388 +00	8.0794931 +01	2.3236708 -01	1.3458586 -01	6.1990479 -02
7	-2.1358582 +00	9.5940031 -01	1.0255365 +00	1.0743668 +00	-1.3117713 +02	2.0556695 -01	1.3047560 -01	6.1775406 -02
8	-2.6044093 +00	9.5747820 -01	1.0250718 +00	1.0743270 +00	1.7658188 +02	2.2659656 -01	1.3331938 -01	6.1894349 -02
9	-2.7839521 +00	9.5614376 -01	1.0247802 +00	1.0743062 +00	-2.5515908 +02	2.0776530 -01	1.3104275 -01	6.1816338 -02
10	-3.2372824 +00	9.5509767 -01	1.0245719 +00	1.0742935 +00	3.4498791 +02	2.2430164 -01	1.3284931 -01	6.1868033 -02
11	-3.3517129 +00	9.5434498 -01	1.0244343 +00	1.0742863 +00	-4.8856794 +02	2.0851408 -01	1.3127735 -01	6.1829890 -02
12	-3.8299200 +00	9.5373192 -01	1.0243309 +00	1.0742817 +00	6.7630025 +02	2.2370167 -01	1.3266528 -01	6.1858809 -02
13	-3.8548006 +00	9.5328255 -01	1.0242605 +00	1.0742789 +00	-9.6328874 +02	2.0828227 -01	1.3136440 -01	6.1835288 -02
14	-4.4121308 +00	9.5290100 -01	1.0242049 +00	1.0742769 +00	1.3678561 +03	2.2422786 -01	1.3261253 -01	6.1855046 -02
15	-4.2921852 +00	9.5262117 -01	1.0241667 +00	1.0742758 +00	-1.9796712 +03	2.0710783 -01	1.3136391 -01	6.1837607 -02
16	-5.0212315 +00	9.5237088 -01	1.0241347 +00	1.0742749 +00	2.8810385 +03	2.2589035 -01	1.3264508 -01	6.1853500 -02
17	-4.6395197 +00	9.5219182 -01	1.0241131 +00	1.0742743 +00	-4.2531522 +03	2.0468969 -01	1.3128815 -01	6.1838460 -02
18	-5.7197876 +00	9.5201898 -01	1.0240934 +00	1.0742739 +00	6.3350647 +03	2.2913927 -01	1.3276080 -01	6.1853121 -02
19	-4.8270776 +00	9.5190361 -01	1.0240810 +00	1.0742736 +00	-9.5496402 +03	2.0028163 -01	1.3112081 -01	6.1838388 -02
20	-6.6335972 +00	9.5177706 -01	1.0240681 +00	1.0742734 +00	1.4539433 +04	2.3503206 -01	1.3298855 -01	6.1853592 -02
21	-4.6833078 +00	9.5170514 -01	1.0240612 +00	1.0742733 +00	-2.2375674 +04	1.9234466 -01	1.3081406 -01	6.1837491 -02
22	-8.0424411 +00	9.5160499 -01	1.0240520 +00	1.0742731 +00	3.4781047 +04	2.4575116 -01	1.3339748 -01	6.1854954 -02
23	-3.7928338 +00	9.5156641 -01	1.0240486 +00	1.0742731 +00	-5.4607253 +04	1.7774643 -01	1.3026812 -01	6.1835580 -02
24	-1.0607861 +01	9.5147772 -01	1.0240412 +00	1.0742730 +00	8.6566057 +04	2.6579355 -01	1.3412874 -01	6.1857531 -02
25	-1.1315860 +00	9.5146990 -01	1.0240405 +00	1.0742730 +00	-1.3852783 +05	1.4997192 -01	1.2928242 -01	6.1832178 -02
S3		9.5146914 -01	1.0240405 +00	1.0742730 +00		2.1577234 -01	1.3197990 -01	6.1845766 -02
S5		9.5118248 -01	1.0240228 +00	1.0742728 +00		2.1596138 -01	1.3198882 -01	6.1845819 -02

S	K	THIRD MOMENT(V*) N=10	THIRD MOMENT(V*) N=20	THIRD MOMENT(V*) N=50	K	FOURTH MOMENT(V*) N=10	FOURTH MOMENT(V*) N=20	FOURTH MOMENT(V*) N=50
6	-2.3647492 +04	-9.9321218 -01	-5.9134774 -02	8.2183382 -03	2.8582452 +04	1.7086147 +00	2.2639885 -01	2.0389063 -02
7	6.7999376 +04	1.5979525 +00	1.7046975 -01	1.4431435 -02	-8.9128102 +04	-1.6876752 +00	-7.4548252 -02	1.2245437 -02
8	-1.5988630 +05	-1.6686032 +00	-8.0021393 -02	9.1885661 -03	2.1852732 +05	2.7769456 +00	2.6781479 -01	1.9411217 -02
9	3.2977943 +05	2.2588290 +00	1.8552654 -01	1.3631278 -02	-4.6277415 +05	-2.7343568 +00	-1.0482434 -01	1.3176831 -02
10	-6.2704414 +05	-2.3688850 +00	-9.4209929 -02	9.7970854 -03	8.9502739 +05	3.8711285 +00	2.9446461 -01	1.8649664 -02
11	1.1352589 +06	3.0727631 +00	2.0303373 -01	1.3199340 -02	-1.6403091 +06	-3.9913816 +00	-1.3501581 -01	1.3733825 -02
12	-1.9945597 +06	-3.3724500 +00	-1.1783840 -01	1.0084981 -02	2.9081038 +06	5.4058542 +00	3.3282150 -01	1.8274617 -02
13	3.4391344 +06	4.3507714 +00	2.3505662 -01	1.3026150 -02	-5.0499513 +06	-5.9347586 +00	-1.8536198 -01	1.3955867 -02
14	-5.8614885 +06	-5.0317352 +00	-1.6072704 -01	1.0163746 -02	8.6558641 +06	7.9207157 +00	3.9910552 -01	1.8182880 -02
15	9.9233638 +06	6.5362885 +00	2.9201155 -01	1.3030528 -02	-1.4722634 +07	-9.2419901 +00	-2.7259255 -01	1.3929626 -02
16	-1.6744629 +07	-7.9443210 +00	-2.3604811 -01	1.0080393 -02	2.4940066 +07	1.2325962 +01	5.1391890 -01	1.8323664 -02
17	2.8230268 +07	1.0459995 +01	3.9163935 -01	1.3195133 -02	-4.2186835 +07	-1.5177138 +01	-4.2408664 -01	1.3669045 -02
18	-4.7636359 +07	-1.3285271 +01	-3.6825734 -01	9.8260443 -03	7.1390763 +07	2.0408971 +01	7.1474105 -01	1.8718185 -02
19	8.0557996 +07	1.7805325 +01	5.6806830 -01	1.3554598 -02	-1.2102980 +08	-2.6301333 +01	-6.9198831 -01	1.3116520 -02
20	-1.3665932 +08	-2.3491153 +01	-6.0538207 -01	9.3380582 -03	2.0576594 +08	3.5878170 +01	1.0748587 +00	1.9465229 -02
21	2.3272329 +08	3.2131605 +01	8.8940477 -01	1.4205242 -02	-3.5108828 +08	-4.8034782 +01	-1.1801891 +00	1.2122351 -02
22	-3.9805221 +08	-4.3807181 +01	-1.0447487 +00	8.4761620 -03	6.0154646 +08	6.6725809 +01	1.7427519 +00	2.0780084 -02
23	6.8408619 +08	6.1234643 +01	1.4958329 +00	1.5344021 -02	-1.0354193 +09	-9.2263419 +01	-2.1026232 +00	1.0384196 -02
24	-1.1816205 +09	-8.5917556 +01	-1.8899046 +00	6.9643791 -03	1.7909979 +09	1.3077714 +02	3.0291857 +00	2.3089148 -02
25	2.0517891 +09	1.2277176 +02	2.6853154 +00	1.7368396 -02	-3.1139065 +09	-1.8594115 +02	-3.9144208 +00	7.2987793 -03
S3		3.8235735 -01	5.5903312 -02	1.1605751 -02		-9.8017949 -02	7.8292491 -02	1.6048838 -02
S5		1.3085750 -01	4.9456259 -02	1.1586094 -02		2.7955225 -01	8.7983354 -02	1.6077179 -02

C=1.0 V=1.0000

E(V*)

S	K	N=10	N=20	N=50
4	-6.9453125 -01	8.9302530 -01	9.3796172 -01	9.7230519 -01
5	-1.0950345 +00	8.8885259 -01	9.3648273 -01	9.7205505 -01
6	-1.4621632 +00	8.8586532 -01	9.3556644 -01	9.7193519 -01
7	-1.8049979 +00	8.8371570 -01	9.3498506 -01	9.7187440 -01
8	-2.1289686 +00	8.8214448 -01	9.3460515 -01	9.7184185 -01
9	-2.4378299 +00	8.8097595 -01	9.3434983 -01	9.7182359 -01
10	-2.7344478 +00	8.8009234 -01	9.3417387 -01	9.7181291 -01
11	-3.0211123 +00	8.7941390 -01	9.3404987 -01	9.7180645 -01
12	-3.2996958 +00	8.7888571 -01	9.3396075 -01	9.7180242 -01
13	-3.5717578 +00	8.7846934 -01	9.3389557 -01	9.7179984 -01
14	-3.8386213 +00	8.7813738 -01	9.3384715 -01	9.7179815 -01
15	-4.1014305 +00	8.7786999 -01	9.3381067 -01	9.7179702 -01
16	-4.3611912 +00	8.7765260 -01	9.3378284 -01	9.7179625 -01
17	-4.6188017 +00	8.7747434 -01	9.3376137 -01	9.7179572 -01
18	-4.8750741 +00	8.7732702 -01	9.3374462 -01	9.7179534 -01
19	-5.1307506 +00	8.7720440 -01	9.3373144 -01	9.7179507 -01
20	-5.3865164 +00	8.7710163 -01	9.3372097 -01	9.7179488 -01
21	-5.6430091 +00	8.7701498 -01	9.3371259 -01	9.7179474 -01
22	-5.9008268 +00	8.7694150 -01	9.3370583 -01	9.7179463 -01
23	-6.1605350 +00	8.7687884 -01	9.3370033 -01	9.7179455 -01
24	-6.4226710 +00	8.7682514 -01	9.3369584 -01	9.7179450 -01
25	-6.6877491 +00	8.7677890 -01	9.3369214 -01	9.7179445 -01
S3		8.7649226 -01	9.3367496 -01	9.7179431 -01
S5		8.7643847 -01	9.3367233 -01	9.7179450 -01

VAR(V*)

S	K	N=10	N=20	N=50
5	-2.5609375 +00	4.9323187 -02	3.2942816 -02	1.6397976 -02
6	-1.1518167 -01	4.9279296 -02	3.2935038 -02	1.6397450 -02
7	5.0082612 -01	4.9381617 -02	3.2950730 -02	1.6398271 -02
8	9.3698694 -01	4.9493205 -02	3.2965820 -02	1.6398902 -02
9	1.4246318 +00	4.9598346 -02	3.2978531 -02	1.6399337 -02
10	1.9379219 +00	4.9691237 -02	3.2988679 -02	1.6399628 -02
11	2.4251190 +00	4.9769602 -02	3.2996482 -02	1.6399817 -02
12	2.8620548 +00	4.9833875 -02	3.3002356 -02	1.6399940 -02
13	3.2462415 +00	4.9885838 -02	3.3006740 -02	1.6400019 -02
14	3.5828425 +00	4.9927604 -02	3.3010009 -02	1.6400071 -02
15	3.8768261 +00	4.9961130 -02	3.3012454 -02	1.6400105 -02
16	4.1309671 +00	4.9988062 -02	3.3014291 -02	1.6400127 -02
17	4.3464824 +00	5.0009728 -02	3.3015678 -02	1.6400143 -02
18	4.5241018 +00	5.0027188 -02	3.3016729 -02	1.6400153 -02
19	4.6647363 +00	5.0041284 -02	3.3017531 -02	1.6400161 -02
20	4.7696853 +00	5.0052684 -02	3.3018143 -02	1.6400166 -02
21	4.8405754 +00	5.0061919 -02	3.3018614 -02	1.6400169 -02
22	4.8792182 +00	5.0069411 -02	3.3018976 -02	1.6400171 -02
23	4.8874888 +00	5.0075497 -02	3.3019256 -02	1.6400173 -02
24	4.8672590 +00	5.0080448 -02	3.3019473 -02	1.6400174 -02
25	4.8203752 +00	5.0084478 -02	3.3019642 -02	1.6400175 -02
S3		5.0102128 -02	3.3020229 -02	1.6400177 -02
S5		5.0102332 -02	3.3020246 -02	1.6400178 -02

THIRD MOMENT(V*)

S	K	N=10	N=20	N=50
6	-8.8801849 +02	-1.5159079 -02	2.5315192 -03	1.3700133 -03
7	7.2584263 +02	1.2499672 -02	4.9823762 -03	1.4363335 -03
8	-1.7452657 +02	8.9340084 -03	4.7089484 -03	1.4306105 -03
9	-3.8710808 +01	8.4729909 -03	4.6777773 -03	1.4300890 -03
10	4.5782435 +00	8.5067793 -03	4.6798198 -03	1.4301170 -03
11	1.3586753 +01	8.5719048 -03	4.6833772 -03	1.4301577 -03
12	2.9553982 +00	8.5814548 -03	4.6838526 -03	1.4301624 -03
13	-1.1881071 +01	8.5547737 -03	4.6826335 -03	1.4301522 -03
14	-2.3239005 +01	8.5175749 -03	4.6810643 -03	1.4301409 -03
15	-3.0257226 +01	8.4823030 -03	4.6796839 -03	1.4301321 -03
16	-3.4858501 +01	8.4521576 -03	4.6785846 -03	1.4301260 -03
17	-3.8907869 +01	8.4267922 -03	4.6777195 -03	1.4301217 -03
18	-4.3253141 +01	8.4052318 -03	4.6770295 -03	1.4301186 -03
19	-4.7886073 +01	8.3867507 -03	4.6764729 -03	1.4301164 -03
20	-5.2419563 +01	8.3709102 -03	4.6760228 -03	1.4301148 -03
21	-5.6457445 +01	8.3574164 -03	4.6756602 -03	1.4301136 -03
22	-5.9756684 +01	8.3460163 -03	4.6753698 -03	1.4301127 -03
23	-6.2236852 +01	8.3364598 -03	4.6751387 -03	1.4301121 -03
24	-6.3922931 +01	8.3284992 -03	4.6749555 -03	1.4301117 -03
25	-6.4882626 +01	8.3218999 -03	4.6748108 -03	1.4301113 -03
S3		8.2899087 -03	4.6742669 -03	1.4301105 -03
S5		8.2917489 -03	4.6742799 -03	1.4301105 -03

FOURTH MOMENT(V*)

S	K	N=10	N=20	N=50
6	1.0740240 +03	3.0446207 -02	6.6724143 -03	1.1723416 -03
7	-8.1125185 +02	-4.6712354 -04	3.9331672 -03	1.0982175 -03
8	3.5966495 +02	6.8810085 -03	4.4966481 -03	1.1100114 -03
9	1.7694991 +02	8.9883527 -03	4.6391333 -03	1.1123952 -03
10	1.1704089 +02	9.8521384 -03	4.6913475 -03	1.1131109 -03
11	9.4924663 +01	1.0307142 -02	4.7162015 -03	1.1133954 -03
12	8.8876221 +01	1.0594336 -02	4.7304994 -03	1.1135342 -03
13	7.9218662 +01	1.0772236 -02	4.7386281 -03	1.1136019 -03
14	5.7607473 +01	1.0864449 -02	4.7425180 -03	1.1136300 -03
15	2.4682331 +01	1.0893222 -02	4.7436440 -03	1.1136372 -03
16	-1.5583341 +01	1.0879746 -02	4.7431526 -03	1.1136344 -03
17	-5.9573091 +01	1.0840908 -02	4.7418280 -03	1.1136279 -03
18	-1.0515807 +02	1.0788490 -02	4.7401505 -03	1.1136204 -03
19	-1.5139447 +02	1.0730060 -02	4.7383909 -03	1.1136134 -03
20	-1.9786333 +02	1.0670269 -02	4.7366919 -03	1.1136073 -03
21	-2.4418750 +02	1.0611906 -02	4.7351235 -03	1.1136022 -03
22	-2.8983404 +02	1.0556613 -02	4.7337152 -03	1.1135980 -03
23	-3.3411586 +02	1.0505309 -02	4.7324743 -03	1.1135947 -03
24	-3.7627496 +02	1.0458450 -02	4.7313962 -03	1.1135920 -03
25	-4.1556671 +02	1.0416182 -02	4.7304695 -03	1.1135899 -03
S3		1.0027079 -02	4.7248013 -03	1.1135819 -03
S5		1.0140062 -02	4.7257496 -03	1.1135822 -03

C=1.1 V=0.9102

S	K	E(V*) N=10	E(V*) N=20	E(V*) N=50	K	VAR(V*) N=10	VAR(V*) N=20	VAR(V*) N=50
4	-5.1256990 -01	8.2471696 -01	8.6129566 -01	8.8869289 -01				
5	-8.8427378 -01	8.2134737 -01	8.6010134 -01	8.8849090 -01	-8.3314585 -01	4.1459811 -02	2.5691609 -02	1.2088606 -02
6	-1.2359824 +00	8.1882220 -01	8.5932678 -01	8.8838958 -01	-8.5950487 -01	4.1132290 -02	2.5633565 -02	1.2084679 -02
7	-1.5725213 +00	8.1694944 -01	8.5882028 -01	8.8833662 -01	-7.1580318 -01	4.0986048 -02	2.5611137 -02	1.2083505 -02
8	-1.8973107 +00	8.1554919 -01	8.5848171 -01	8.8830762 -01	-4.2391061 -01	4.0935564 -02	2.5604310 -02	1.2083220 -02
9	-2.2129559 +00	8.1448845 -01	8.5824995 -01	8.8829104 -01	-8.6450070 -02	4.0929183 -02	2.5603539 -02	1.2083193 -02
10	-2.5214660 +00	8.1367366 -01	8.5808769 -01	8.8828119 -01	2.4365648 -01	4.0940863 -02	2.5604814 -02	1.2083230 -02
11	-2.8244069 +00	8.1303939 -01	8.5797176 -01	8.8827515 -01	5.5988158 -01	4.0958955 -02	2.5606616 -02	1.2083274 -02
12	-3.1230265 +00	8.1253949 -01	8.5788741 -01	8.8827134 -01	8.7266558 -01	4.0978552 -02	2.5608407 -02	1.2083311 -02
13	-3.4183499 +00	8.1214100 -01	8.5782503 -01	8.8826887 -01	1.1913220 +00	4.0997622 -02	2.5610016 -02	1.2083340 -02
14	-3.7112449 +00	8.1182005 -01	8.5777822 -01	8.8826724 -01	1.5190745 +00	4.1015330 -02	2.5611402 -02	1.2083362 -02
15	-4.0024653 +00	8.1155912 -01	8.5774262 -01	8.8826613 -01	1.8545380 +00	4.1031368 -02	2.5612571 -02	1.2083378 -02
16	-4.2926789 +00	8.1134514 -01	8.5771523 -01	8.8826538 -01	2.1945158 +00	4.1045675 -02	2.5613547 -02	1.2083390 -02
17	-4.5824869 +00	8.1116828 -01	8.5769392 -01	8.8826485 -01	2.5359475 +00	4.1058316 -02	2.5614356 -02	1.2083399 -02
18	-4.8724374 +00	8.1102104 -01	8.5767719 -01	8.8826447 -01	2.8766956 +00	4.1069418 -02	2.5615025 -02	1.2083406 -02
19	-5.1630353 +00	8.1089764 -01	8.5766392 -01	8.8826420 -01	3.2155579 +00	4.1079135 -02	2.5615577 -02	1.2083411 -02
20	-5.4547509 +00	8.1079358 -01	8.5765332 -01	8.8826400 -01	3.5519580 +00	4.1087624 -02	2.5616034 -02	1.2083415 -02
21	-5.7480264 +00	8.1070532 -01	8.5764478 -01	8.8826386 -01	3.8856078 +00	4.1095037 -02	2.5616411 -02	1.2083418 -02
22	-6.0432811 +00	8.1063006 -01	8.5763786 -01	8.8826375 -01	4.2162699 +00	4.1101511 -02	2.5616724 -02	1.2083420 -02
23	-6.3409160 +00	8.1056557 -01	8.5763220 -01	8.8826367 -01	4.5436408 +00	4.1107170 -02	2.5616985 -02	1.2083421 -02
24	-6.6413176 +00	8.1051004 -01	8.5762755 -01	8.8826361 -01	4.8673238 +00	4.1112120 -02	2.5617202 -02	1.2083423 -02
25	-6.9448605 +00	8.1046202 -01	8.5762371 -01	8.8826356 -01	5.1868523 +00	4.1116457 -02	2.5617383 -02	1.2083423 -02
S3		8.1015500 -01	8.5760544 -01	8.8826342 -01		4.1147090 -02	2.5618308 -02	1.2083427 -02
S5		8.1010019 -01	8.5760275 -01	8.8826343 -01		4.1138678 -02	2.5618430 -02	1.2083426 -02

S	K	THIRD MOMENT(V*) N=10	THIRD MOMENT(V*) N=20	THIRD MOMENT(V*) N=50	K	FOURTH MOMENT(V*) N=10	FOURTH MOMENT(V*) N=20	FOURTH MOMENT(V*) N=50
6	-8.5118285 +00	6.1906620 -03	2.8708164 -03	7.5517364 -04	2.1870187 +01	3.6414794 -03	2.4533349 -03	5.5198611 -04
7	-2.6872897 +01	5.1666511 -03	2.7800783 -03	7.5271826 -04	4.4743154 +01	5.3464492 -03	2.6044132 -03	5.5607429 -04
8	3.0505009 +00	5.2289743 -03	2.7848575 -03	7.5281829 -04	1.7989675 +01	5.7139871 -03	2.6325973 -03	5.5666419 -04
9	2.3256390 +01	5.5059410 -03	2.8035842 -03	7.5313160 -04	6.5778717 +00	5.7923247 -03	2.6378940 -03	5.5675280 -04
10	2.2439964 +01	5.6715525 -03	2.8135951 -03	7.5326881 -04	1.9117061 +01	5.9334125 -03	2.6464225 -03	5.5686970 -04
11	1.2857628 +01	5.7331831 -03	2.8169616 -03	7.5330735 -04	3.9381955 +01	6.1221825 -03	2.6567338 -03	5.5698772 -04
12	5.4391588 +00	5.7507592 -03	2.8178366 -03	7.5331584 -04	5.4672988 +01	6.2988526 -03	2.6655293 -03	5.5707309 -04
13	3.2620840 +00	5.7580848 -03	2.8181713 -03	7.5331863 -04	6.2018575 +01	6.4381269 -03	2.6718931 -03	5.5712613 -04
14	4.4954193 +00	5.7652806 -03	2.8184749 -03	7.5332082 -04	6.3991343 +01	6.5405581 -03	2.6762140 -03	5.5715738 -04
15	6.3509271 +00	5.7726841 -03	2.8187646 -03	7.5332266 -04	6.3956419 +01	6.6151144 -03	2.6791319 -03	5.5717586 -04
16	7.0874531 +00	5.7788133 -03	2.8189881 -03	7.5332391 -04	6.3852655 +01	6.6703336 -03	2.6811455 -03	5.5718711 -04
17	6.2711924 +00	5.7829017 -03	2.8191276 -03	7.5332460 -04	6.4036229 +01	6.7120811 -03	2.6825694 -03	5.5719417 -04
18	4.2669736 +00	5.7850287 -03	2.8191956 -03	7.5332490 -04	6.3962810 +01	6.7439646 -03	2.6835897 -03	5.5719870 -04
19	1.6685656 +00	5.7856726 -03	2.8192150 -03	7.5332498 -04	6.2880111 +01	6.7682326 -03	2.6843206 -03	5.5720161 -04
20	-1.0490229 +00	5.7853556 -03	2.8192060 -03	7.5332495 -04	6.0225871 +01	6.7864320 -03	2.6848377 -03	5.5720346 -04
21	-3.6433034 +00	5.7844849 -03	2.8191826 -03	7.5332487 -04	5.5741001 +01	6.7997545 -03	2.6851957 -03	5.5720463 -04
22	-6.0733619 +00	5.7833262 -03	2.8191531 -03	7.5332478 -04	4.9415727 +01	6.8091819 -03	2.6854358 -03	5.5720534 -04
23	-8.4153588 +00	5.7820340 -03	2.8191218 -03	7.5332470 -04	4.1379998 +01	6.8155358 -03	2.6855895 -03	5.5720576 -04
24	-1.0782105 +01	5.7806913 -03	2.8190909 -03	7.5332462 -04	3.1805171 +01	6.8194966 -03	2.6856806 -03	5.5720598 -04
25	-1.3270490 +01	5.7793415 -03	2.8190614 -03	7.5332455 -04	2.0842545 +01	6.8216165 -03	2.6857271 -03	5.5720609 -04
S3		6.0391261 -03	2.8183893 -03	7.5332407 -04		6.8240577 -03	2.6857755 -03	5.5720618 -04
S5		5.7344822 -03	2.8186702 -03	7.5332415 -04		6.7853087 -03	2.6851952 -03	5.5720559 -04

C=1.2 V=0.8369

E(V*)

S	K	N=10	N=20	N=50
4	-3.7762415 -01	7.6621233 -01	7.9688072 -01	8.1948495 -01
5	-7.1975456 -01	7.6346966 -01	7.9590860 -01	8.1932054 -01
6	-1.0490476 +00	7.6132640 -01	7.9525119 -01	8.1923454 -01
7	-1.3695944 +00	7.5969532 -01	7.9481006 -01	8.1918841 -01
8	-1.6839356 +00	7.5845254 -01	7.9450956 -01	8.1916267 -01
9	-1.9938242 +00	7.5749683 -01	7.9430075 -01	8.1914773 -01
10	-2.3005820 +00	7.5675343 -01	7.9415270 -01	8.1913875 -01
11	-2.6052665 +00	7.5616836 -01	7.9404577 -01	8.1913318 -01
12	-2.9087546 +00	7.5570276 -01	7.9396721 -01	8.1912963 -01
13	-3.2117875 +00	7.5532835 -01	7.9390860 -01	8.1912731 -01
14	-3.5150018 +00	7.5502437 -01	7.9386426 -01	8.1912576 -01
15	-3.8189511 +00	7.5477540 -01	7.9383029 -01	8.1912471 -01
16	-4.1241241 +00	7.5456983 -01	7.9380398 -01	8.1912398 -01
17	-4.4309586 +00	7.5439882 -01	7.9378338 -01	8.1912347 -01
18	-4.7398534 +00	7.5425559 -01	7.9376710 -01	8.1912310 -01
19	-5.0511768 +00	7.5413486 -01	7.9375412 -01	8.1912284 -01
20	-5.3652736 +00	7.5403250 -01	7.9374369 -01	8.1912265 -01
21	-5.6824709 +00	7.5394525 -01	7.9373525 -01	8.1912250 -01
22	-6.0030815 +00	7.5387049 -01	7.9372837 -01	8.1912240 -01
23	-6.3274073 +00	7.5380613 -01	7.9372272 -01	8.1912232 -01
24	-6.6557420 +00	7.5375049 -01	7.9371807 -01	8.1912226 -01
25	-6.9883727 +00	7.5370217 -01	7.9371420 -01	8.1912221 -01
S3		7.5338364 -01	7.9369537 -01	8.1912206 -01
S5		7.5332738 -01	7.9369264 -01	8.1912208 -01

VAR(V*)

S	K	N=10	N=20	N=50
4				
5	-9.0171871 -01	3.4989395 -02	2.0586721 -02	9.3072341 -03
6	-8.2035852 -01	3.4676792 -02	2.0531321 -02	9.3034863 -03
7	-7.8226105 -01	3.4516972 -02	2.0506809 -02	9.3022037 -03
8	-7.3473042 -01	3.4429471 -02	2.0494977 -02	9.3017088 -03
9	-6.5037862 -01	3.4381472 -02	2.0489174 -02	9.3015099 -03
10	-5.2583924 -01	3.4356267 -02	2.0486420 -02	9.3014311 -03
11	-3.6907328 -01	3.4344340 -02	2.0485233 -02	9.3014023 -03
12	-1.8980364 -01	3.4340078 -02	2.0484843 -02	9.3013942 -03
13	4.8970434 -03	3.4340156 -02	2.0484850 -02	9.3013943 -03
14	2.1145381 -01	3.4342621 -02	2.0485043 -02	9.3013974 -03
15	4.2896310 -01	3.4346331 -02	2.0485314 -02	9.3014012 -03
16	6.5796443 -01	3.4350621 -02	2.0485606 -02	9.3014048 -03
17	8.9943062 -01	3.4355104 -02	2.0485893 -02	9.3014080 -03
18	1.1541595 +00	3.4359558 -02	2.0486161 -02	9.3014106 -03
19	1.4225257 +00	3.4363857 -02	2.0486406 -02	9.3014128 -03
20	1.7044721 +00	3.4367931 -02	2.0486625 -02	9.3014146 -03
21	1.9996219 +00	3.4371746 -02	2.0486819 -02	9.3014161 -03
22	2.3074224 +00	3.4375289 -02	2.0486990 -02	9.3014172 -03
23	2.6272696 +00	3.4378561 -02	2.0487141 -02	9.3014182 -03
24	2.9585940 +00	3.4381570 -02	2.0487273 -02	9.3014189 -03
25	3.3009038 +00	3.4384330 -02	2.0487388 -02	9.3014195 -03
S3		3.4414875 -02	2.0488198 -02	9.3014220 -03
S5		3.4412933 -02	2.0488189 -02	9.3014220 -03

THIRD MOMENT(V*)

S	K	N=10	N=20	N=50
6	1.0759515 +01	3.8856529 -03	1.7638239 -03	4.2689915 -04
7	2.9076180 +00	3.9964498 -03	1.7736417 -03	4.2716482 -04
8	-2.3288551 +00	3.9488701 -03	1.7699931 -03	4.2708846 -04
9	-1.9550084 +00	3.9255874 -03	1.7684189 -03	4.2706212 -04
10	1.7504423 +00	3.9385060 -03	1.7691998 -03	4.2707282 -04
11	5.7955391 +00	3.9662858 -03	1.7707172 -03	4.2709019 -04
12	8.4495126 +00	3.9935896 -03	1.7720765 -03	4.2710339 -04
13	9.3888046 +00	4.0146739 -03	1.7730399 -03	4.2711141 -04
14	9.1239215 +00	4.0292786 -03	1.7736560 -03	4.2711587 -04
15	8.3923271 +00	4.0390618 -03	1.7740389 -03	4.2711829 -04
16	7.7795752 +00	4.0457895 -03	1.7742842 -03	4.2711967 -04
17	7.5834703 +00	4.0507335 -03	1.7744528 -03	4.2712050 -04
18	7.8397481 +00	4.0546413 -03	1.7745779 -03	4.2712106 -04
19	8.4185202 +00	4.0578904 -03	1.7746757 -03	4.2712145 -04
20	9.1269077 +00	4.0606484 -03	1.7747541 -03	4.2712173 -04
21	9.7849512 +00	4.0629871 -03	1.7748170 -03	4.2712193 -04
22	1.0266096 +01	4.0649456 -03	1.7748668 -03	4.2712208 -04
23	1.0507461 +01	4.0665590 -03	1.7749059 -03	4.2712219 -04
24	1.0500507 +01	4.0678667 -03	1.7749359 -03	4.2712226 -04
25	1.0272685 +01	4.0689116 -03	1.7749589 -03	4.2712231 -04
S3		4.0730652 -03	1.7750319 -03	4.2712243 -04
S5		4.0722631 -03	1.7750223 -03	4.2712243 -04

FOURTH MOMENT(V*)

S	K	N=10	N=20	N=50
6	-1.5221554 +01	4.2659529 -03	1.6161818 -03	3.1049296 -04
7	-3.5662238 +00	4.1300594 -03	1.6041402 -03	3.1016712 -04
8	4.2485142 +00	4.2168586 -03	1.6107963 -03	3.1030643 -04
9	6.3344704 +00	4.2922975 -03	1.6158970 -03	3.1039177 -04
10	5.6672858 +00	4.3341233 -03	1.6184253 -03	3.1042642 -04
11	5.4119885 +00	4.3600646 -03	1.6198423 -03	3.1044264 -04
12	7.1511654 +00	4.3831728 -03	1.6209927 -03	3.1045381 -04
13	1.0953105 +01	4.4077701 -03	1.6221166 -03	3.1046318 -04
14	1.6077669 +01	4.4335057 -03	1.6232023 -03	3.1047103 -04
15	2.1634722 +01	4.4587261 -03	1.6241893 -03	3.1047728 -04
16	2.6961989 +01	4.4820426 -03	1.6250396 -03	3.1048203 -04
17	3.1736664 +01	4.5027328 -03	1.6257452 -03	3.1048553 -04
18	3.5922020 +01	4.5206388 -03	1.6263183 -03	3.1048807 -04
19	3.9649585 +01	4.5359412 -03	1.6267791 -03	3.1048990 -04
20	4.3104274 +01	4.5489667 -03	1.6271492 -03	3.1049123 -04
21	4.6444221 +01	4.5600673 -03	1.6274475 -03	3.1049221 -04
22	4.9761381 +01	4.5695605 -03	1.6276893 -03	3.1049292 -04
23	5.3075013 +01	4.5777102 -03	1.6278865 -03	3.1049346 -04
24	5.6345377 +01	4.5847272 -03	1.6280479 -03	3.1049386 -04
25	5.9495823 +01	4.5907785 -03	1.6281806 -03	3.1049416 -04
S3		4.6287039 -03	1.6287921 -03	3.1049509 -04
S5		4.6291675 -03	1.6288098 -03	3.1049521 -04

C=1.3 V=0.7757

S	E(V*) K	E(V*) N=10	E(V*) N=20	E(V*) N=50	VAR(V*) K	VAR(V*) N=10	VAR(V*) N=20	VAR(V*) N=50
4	-2.7633975 -01	7.1563220 -01	7.4198719 -01	7.6116138 -01				
5	-5.9111553 -01	7.1337971 -01	7.4118882 -01	7.6102636 -01	-7.5866098 -01	2.9893222 -02	1.6916407 -02	7.4251017 -03
6	-8.9808221 -01	7.1154489 -01	7.4062601 -01	7.6095273 -01	-7.5190674 -01	2.9606702 -02	1.6865630 -02	7.4216666 -03
7	-1.2002877 +00	7.1011543 -01	7.4023941 -01	7.6091231 -01	-7.3983083 -01	2.9455551 -02	1.6842448 -02	7.4204536 -03
8	-1.4997827 +00	7.0900856 -01	7.3997178 -01	7.6088938 -01	-7.3081099 -01	2.9368517 -02	1.6830679 -02	7.4199613 -03
9	-1.7980003 +00	7.0814672 -01	7.3978347 -01	7.6087591 -01	-7.1874338 -01	2.9315472 -02	1.6824266 -02	7.4197416 -03
10	-2.0959884 +00	7.0746943 -01	7.3964860 -01	7.6086773 -01	-6.9562253 -01	2.9282129 -02	1.6820623 -02	7.4196373 -03
11	-2.3945530 +00	7.0693168 -01	7.3955031 -01	7.6086261 -01	-6.5589683 -01	2.9260934 -02	1.6818513 -02	7.4195861 -03
12	-2.6943438 +00	7.0650040 -01	7.3947754 -01	7.6085932 -01	-5.9709623 -01	2.9247525 -02	1.6817287 -02	7.4195606 -03
13	-2.9959063 +00	7.0615116 -01	7.3942287 -01	7.6085715 -01	-5.1907155 -01	2.9239216 -02	1.6816586 -02	7.4195479 -03
14	-3.2997133 +00	7.0586580 -01	7.3938124 -01	7.6085570 -01	-4.2296136 -01	2.9234286 -02	1.6816201 -02	7.4195418 -03
15	-3.6061846 +00	7.0563070 -01	7.3934917 -01	7.6085471 -01	-3.1034056 -01	2.9231602 -02	1.6816005 -02	7.4195391 -03
16	-3.9156994 +00	7.0543551 -01	7.3932418 -01	7.6085401 -01	-1.8267252 -01	2.9230411 -02	1.6815924 -02	7.4195381 -03
17	-4.2286054 +00	7.0527232 -01	7.3930452 -01	7.6085352 -01	-4.1038157 -02	2.9230207 -02	1.6815910 -02	7.4195379 -03
18	-4.5452244 +00	7.0513497 -01	7.3928891 -01	7.6085317 -01	1.1393499 -01	2.9230646 -02	1.6815937 -02	7.4195382 -03
19	-4.8658579 +00	7.0501867 -01	7.3927641 -01	7.6085292 -01	2.8202391 -01	2.9231499 -02	1.6815985 -02	7.4195386 -03
20	-5.1907903 +00	7.0491964 -01	7.3926632 -01	7.6085273 -01	4.6330587 -01	2.9232606 -02	1.6816045 -02	7.4195391 -03
21	-5.5202931 +00	7.0483488 -01	7.3925812 -01	7.6085259 -01	6.5804281 -01	2.9233861 -02	1.6816109 -02	7.4195396 -03
22	-5.8546268 +00	7.0476197 -01	7.3925141 -01	7.6085249 -01	8.6657952 -01	2.9235192 -02	1.6816173 -02	7.4195400 -03
23	-6.1940441 +00	7.0469897 -01	7.3924589 -01	7.6085241 -01	1.0892672 +00	2.9236548 -02	1.6816236 -02	7.4195404 -03
24	-6.5387915 +00	7.0464429 -01	7.3924131 -01	7.6085235 -01	1.3264143 +00	2.9237898 -02	1.6816295 -02	7.4195407 -03
25	-6.8891110 +00	7.0459666 -01	7.3923750 -01	7.6085231 -01	1.5782619 +00	2.9239217 -02	1.6816350 -02	7.4195410 -03
S3		7.0427430 -01	7.3921856 -01	7.6085215 -01		2.9298162 -02	1.6817124 -02	7.4195428 -03
S5		7.0421689 -01	7.3921577 -01	7.6085297 -01		2.9260754 -02	1.6816891 -02	7.4195421 -03

S	THIRD MOMENT(V*) K	THIRD MOMENT(V*) N=10	THIRD MOMENT(V*) N=20	THIRD MOMENT(V*) N=50	FOURTH MOMENT(V*) K	FOURTH MOMENT(V*) N=10	FOURTH MOMENT(V*) N=20	FOURTH MOMENT(V*) N=50
6	2.2820525 +00	2.7502482 -03	1.1508201 -03	2.5779402 -04	-7.1869162 +00	3.5750415 -03	1.0793195 -03	1.8967128 -04
7	2.7738676 +00	2.8559484 -03	1.1601863 -03	2.5804747 -04	-6.6786065 +00	3.3205484 -03	1.0567687 -03	1.8906106 -04
8	1.5794811 +00	2.8882180 -03	1.1626608 -03	2.5809927 -04	-4.6671518 +00	3.2251962 -03	1.0494567 -03	1.8890802 -04
9	2.3028210 -01	2.8909605 -03	1.1628463 -03	2.5810237 -04	-2.6081215 +00	3.1941354 -03	1.0473566 -03	1.8887288 -04
10	-4.2566525 -01	2.8878190 -03	1.1626564 -03	2.5809977 -04	-1.1771526 +00	3.1854478 -03	1.0468315 -03	1.8886568 -04
11	-1.9943502 -01	2.8868630 -03	1.1626041 -03	2.5809917 -04	-4.2059528 -01	3.1834317 -03	1.0467213 -03	1.8886442 -04
12	7.0738268 -01	2.8891488 -03	1.1627179 -03	2.5810027 -04	-5.6541718 -02	3.1832490 -03	1.0467122 -03	1.8886433 -04
13	1.9582392 +00	2.8935464 -03	1.1629189 -03	2.5810195 -04	2.7670406 -01	3.1838704 -03	1.0467406 -03	1.8886457 -04
14	3.2465323 +00	2.8987432 -03	1.1631381 -03	2.5810353 -04	8.7468965 -01	3.1852705 -03	1.0467997 -03	1.8886500 -04
15	4.3702888 +00	2.9038378 -03	1.1633375 -03	2.5810480 -04	1.9105158 +00	3.1874977 -03	1.0468868 -03	1.8886555 -04
16	5.2432532 +00	2.9083721 -03	1.1635028 -03	2.5810572 -04	3.4371164 +00	3.1904701 -03	1.0469952 -03	1.8886616 -04
17	5.8713748 +00	2.9121998 -03	1.1636334 -03	2.5810637 -04	5.4185908 +00	3.1940027 -03	1.0471157 -03	1.8886675 -04
18	6.3170368 +00	2.9153487 -03	1.1637341 -03	2.5810681 -04	7.7693139 +00	3.1978754 -03	1.0472397 -03	1.8886730 -04
19	6.6649457 +00	2.9179210 -03	1.1638116 -03	2.5810712 -04	1.0388383 +01	3.2018847 -03	1.0473604 -03	1.8886778 -04
20	6.9964579 +00	2.9200352 -03	1.1638717 -03	2.5810734 -04	1.3183968 +01	3.2058687 -03	1.0474736 -03	1.8886819 -04
21	7.3741500 +00	2.9217977 -03	1.1639191 -03	2.5810749 -04	1.6086759 +01	3.2097136 -03	1.0475769 -03	1.8886853 -04
22	7.8355481 +00	2.9232925 -03	1.1639571 -03	2.5810760 -04	1.9054249 +01	3.2133487 -03	1.0476695 -03	1.8886880 -04
23	8.3936236 +00	2.9245814 -03	1.1639883 -03	2.5810769 -04	2.2068591 +01	3.2167373 -03	1.0477515 -03	1.8886902 -04
24	9.0414117 +00	2.9257073 -03	1.1640142 -03	2.5810775 -04	2.5130820 +01	3.2198670 -03	1.0478235 -03	1.8886920 -04
25	9.7584481 +00	2.9266999 -03	1.1640360 -03	2.5810780 -04	2.8253727 +01	3.2227407 -03	1.0478865 -03	1.8886934 -04
S3		2.9340833 -03	1.1641502 -03	2.5810797 -04		3.2550075 -03	1.0483272 -03	1.8886993 -04
S5		2.9351105 -03	1.1641641 -03	2.5810798 -04		3.2518704 -03	1.0483116 -03	1.8886999 -04

C=1.4 V=0.7238

S	K	E(V*) N=10	E(V*) N=20	E(V*) N=50	K	VAR(V*) N=10	VAR(V*) N=20	VAR(V*) N=50
4	-2.0089773 -01	6.7150791 -01	6.9462423 -01	7.1126410 -01				
5	-4.9055104 -01	6.6963863 -01	6.9396168 -01	7.1115204 -01	-5.4808283 -01	2.5820335 -02	1.4197577 -02	6.0972564 -03
6	-7.7642287 -01	6.6805236 -01	6.9347512 -01	7.1108839 -01	-6.0408471 -01	2.5590144 -02	1.4156782 -02	6.0944966 -03
7	-1.0605252 +00	6.6678935 -01	6.9313353 -01	7.1105268 -01	-6.3285230 -01	2.5460849 -02	1.4136953 -02	6.0934590 -03
8	-1.3443483 +00	6.6579719 -01	6.9289363 -01	7.1103213 -01	-6.5065138 -01	2.5383361 -02	1.4126474 -02	6.0930207 -03
9	-1.6290127 +00	6.6501635 -01	6.9272302 -01	7.1101992 -01	-6.6333407 -01	2.5334406 -02	1.4120556 -02	6.0928179 -03
10	-1.9153786 +00	6.6439742 -01	6.9259977 -01	7.1101244 -01	-6.7156790 -01	2.5302216 -02	1.4117039 -02	6.0927173 -03
11	-2.2041225 +00	6.6390244 -01	6.9250930 -01	7.1100773 -01	-6.7384706 -01	2.5280441 -02	1.4114871 -02	6.0926647 -03
12	-2.4957922 +00	6.6350294 -01	6.9244189 -01	7.1100468 -01	-6.6810934 -01	2.5265437 -02	1.4113500 -02	6.0926361 -03
13	-2.7908442 +00	6.6317760 -01	6.9239096 -01	7.1100267 -01	-6.5250787 -01	2.5254993 -02	1.4112619 -02	6.0926202 -03
14	-3.0896708 +00	6.6291041 -01	6.9235199 -01	7.1100131 -01	-6.2569664 -01	2.5247699 -02	1.4112048 -02	6.0926112 -03
15	-3.3926186 +00	6.6268923 -01	6.9232182 -01	7.1100037 -01	-5.8685688 -01	2.5242624 -02	1.4111678 -02	6.0926060 -03
16	-3.7000009 +00	6.6250480 -01	6.9229821 -01	7.1099972 -01	-5.3560369 -01	2.5239132 -02	1.4111439 -02	6.0926030 -03
17	-4.0121067 +00	6.6234996 -01	6.9227955 -01	7.1099925 -01	-4.7185331 -01	2.5236780 -02	1.4111289 -02	6.0926014 -03
18	-4.3292072 +00	6.6221913 -01	6.9226468 -01	7.1099892 -01	-3.9569436 -01	2.5235253 -02	1.4111197 -02	6.0926004 -03
19	-4.6515601 +00	6.6210796 -01	6.9225273 -01	7.1099868 -01	-3.0728268 -01	2.5234324 -02	1.4111144 -02	6.0926000 -03
20	-4.9794124 +00	6.6201296 -01	6.9224305 -01	7.1099850 -01	-2.0676617 -01	2.5233830 -02	1.4111118 -02	6.0925998 -03
21	-5.3130035 +00	6.6193138 -01	6.9223516 -01	7.1099836 -01	-9.4238114 -02	2.5233650 -02	1.4111108 -02	6.0925997 -03
22	-5.6525661 +00	6.6186099 -01	6.9222868 -01	7.1099826 -01	3.0285776 -02	2.5233697 -02	1.4111111 -02	6.0925997 -03
23	-5.9983285 +00	6.6179998 -01	6.9222333 -01	7.1099819 -01	1.6687261 -01	2.5233904 -02	1.4111120 -02	6.0925998 -03
24	-6.3505154 +00	6.6174688 -01	6.9221889 -01	7.1099813 -01	3.1566566 -01	2.5234226 -02	1.4111134 -02	6.0925998 -03
25	-6.7093489 +00	6.6170049 -01	6.9221518 -01	7.1099809 -01	4.7687082 -01	2.5234624 -02	1.4111151 -02	6.0925999 -03
S3		6.6137963 -01	6.9219642 -01	7.1099794 -01		2.5232577 -02	1.4111044 -02	6.0925989 -03
S5		6.6132207 -01	6.9219364 -01	7.1099793 -01		2.5250018 -02	1.4111474 -02	6.0926006 -03

S	K	THIRD MOMENT(V*) N=10	THIRD MOMENT(V*) N=20	THIRD MOMENT(V*) N=50	K	FOURTH MOMENT(V*) N=10	FOURTH MOMENT(V*) N=20	FOURTH MOMENT(V*) N=50
6	-5.4099384 -01	2.1189680 -03	7.9017362 -04	1.6472938 -04	-2.7816148 +00	2.6649239 -03	7.3502404 -04	1.2381819 -04
7	3.1673034 -01	2.1310373 -03	7.9124308 -04	1.6475832 -04	-3.6474418 +00	2.5259355 -03	7.2270820 -04	1.2348493 -04
8	5.5952186 -01	2.1424686 -03	7.9211968 -04	1.6477666 -04	-3.8566601 +00	2.4471421 -03	7.1666604 -04	1.2335846 -04
9	3.7659009 -01	2.1469535 -03	7.9242292 -04	1.6478174 -04	-3.6547762 +00	2.4036164 -03	7.1372311 -04	1.2330922 -04
10	2.9081775 -02	2.1471681 -03	7.9243589 -04	1.6478192 -04	-3.2874699 +00	2.3793542 -03	7.1225651 -04	1.2328912 -04
11	-2.7216795 -01	2.1458635 -03	7.9236463 -04	1.6478110 -04	-2.9249508 +00	2.3653340 -03	7.1149067 -04	1.2328036 -04
12	-4.0204166 -01	2.1445644 -03	7.9229995 -04	1.6478047 -04	-2.6500239 +00	2.3567707 -03	7.1106435 -04	1.2327622 -04
13	-3.1480308 -01	2.1438574 -03	7.9226765 -04	1.6478020 -04	-2.4744059 +00	2.3512139 -03	7.1081045 -04	1.2327410 -04
14	-2.1810909 -02	2.1438225 -03	7.9226618 -04	1.6478019 -04	-2.3636236 +00	2.3474305 -03	7.1065085 -04	1.2327295 -04
15	4.3305648 -01	2.1443274 -03	7.9228594 -04	1.6478032 -04	-2.2605853 +00	2.3447952 -03	7.1054771 -04	1.2327230 -04
16	9.9394588 -01	2.1451869 -03	7.9231728 -04	1.6478049 -04	-2.1037060 +00	2.3429760 -03	7.1048137 -04	1.2327193 -04
17	1.6074341 +00	2.1462349 -03	7.9235302 -04	1.6478067 -04	-1.8386733 +00	2.3417773 -03	7.1044049 -04	1.2327172 -04
18	2.2309063 +00	2.1473469 -03	7.9238861 -04	1.6478083 -04	-1.4245502 +00	2.3410672 -03	7.1041776 -04	1.2327162 -04
19	2.8360271 +00	2.1484414 -03	7.9242157 -04	1.6478096 -04	-8.3554924 -01	2.3407447 -03	7.1040805 -04	1.2327158 -04
20	3.4087294 +00	2.1494715 -03	7.9245084 -04	1.6478106 -04	-5.9914523 -02	2.3407266 -03	7.1040754 -04	1.2327158 -04
21	3.9470668 +00	2.1504149 -03	7.9247619 -04	1.6478115 -04	9.0282375 -01	2.3409424 -03	7.1041334 -04	1.2327160 -04
22	4.4580366 +00	2.1512654 -03	7.9249786 -04	1.6478121 -04	2.0452397 +00	2.3413326 -03	7.1042327 -04	1.2327163 -04
23	4.9541795 +00	2.1520261 -03	7.9251625 -04	1.6478126 -04	3.3553619 +00	2.3418478 -03	7.1043574 -04	1.2327166 -04
24	5.4504831 +00	2.1527049 -03	7.9253187 -04	1.6478130 -04	4.8195628 +00	2.3424480 -03	7.1044954 -04	1.2327170 -04
25	5.9618800 +00	2.1533112 -03	7.9254517 -04	1.6478133 -04	6.4247798 +00	2.3431015 -03	7.1046387 -04	1.2327173 -04
S3		2.1583912 -03	7.9262124 -04	1.6478144 -04		2.3350852 -03	7.1006668 -04	1.2327239 -04
S5		2.1583714 -03	7.9261991 -04	1.6478143 -04		2.3536442 -03	7.1062507 -04	1.2327194 -04

C=1.5 V=0.6790

S	K	E(V*) N=10	E(V*) N=20	E(V*) N=50	K	VAR(V*) N=10	VAR(V*) N=20	VAR(V*) N=50
4	-1.4488096 -01	6.3269326 -01	6.5331418 -01	6.6802800 -01				
5	-4.1199860 -01	6.3112331 -01	6.5275772 -01	6.6793389 -01	-3.6359075 -01	2.2536941 -02	1.2132353 -02	5.1266746 -03
6	-6.7840412 -01	6.2973730 -01	6.5233258 -01	6.6787827 -01	-4.4249948 -01	2.2368324 -02	1.2102471 -02	5.1246531 -03
7	-9.4536726 -01	6.2861144 -01	6.5202809 -01	6.6784643 -01	-4.9554135 -01	2.2267082 -02	1.2086944 -02	5.1238406 -03
8	-1.2139033 +00	6.2771555 -01	6.5181147 -01	6.6782787 -01	-5.3365076 -01	2.2203528 -02	1.2078349 -02	5.1234811 -03
9	-1.4848317 +00	6.2700383 -01	6.5165596 -01	6.6781675 -01	-5.6296677 -01	2.2161980 -02	1.2073326 -02	5.1233090 -03
10	-1.7588211 +00	6.2643548 -01	6.5154278 -01	6.6780988 -01	-5.8654605 -01	2.2133865 -02	1.2070255 -02	5.1232211 -03
11	-2.0364254 +00	6.2597816 -01	6.5145920 -01	6.6780553 -01	-6.0554763 -01	2.2114298 -02	1.2068307 -02	5.1231738 -03
12	-2.3181105 +00	6.2560710 -01	6.5139659 -01	6.6780270 -01	-6.2003343 -01	2.2100374 -02	1.2067034 -02	5.1231473 -03
13	-2.6042760 +00	6.2530351 -01	6.5134906 -01	6.6780082 -01	-6.2949343 -01	2.2090297 -02	1.2066184 -02	5.1231320 -03
14	-2.8952709 +00	6.2505313 -01	6.5131254 -01	6.6779954 -01	-6.3317666 -01	2.2082916 -02	1.2065606 -02	5.1231228 -03
15	-3.1914066 +00	6.2484507 -01	6.5128416 -01	6.6779866 -01	-6.3028805 -01	2.2077465 -02	1.2065209 -02	5.1231173 -03
16	-3.4929658 +00	6.2467096 -01	6.5126187 -01	6.6779805 -01	-6.2009510 -01	2.2073423 -02	1.2064933 -02	5.1231138 -03
17	-3.8002103 +00	6.2452429 -01	6.5124420 -01	6.6779761 -01	-6.0197592 -01	2.2070422 -02	1.2064741 -02	5.1231117 -03
18	-4.1133860 +00	6.2439999 -01	6.5123007 -01	6.6779729 -01	-5.7543095 -01	2.2068201 -02	1.2064607 -02	5.1231104 -03
19	-4.4327270 +00	6.2429405 -01	6.5121868 -01	6.6779706 -01	-5.4007389 -01	2.2066569 -02	1.2064514 -02	5.1231096 -03
20	-4.7584593 +00	6.2420327 -01	6.5120943 -01	6.6779689 -01	-4.9561216 -01	2.2065385 -02	1.2064451 -02	5.1231090 -03
21	-5.0908024 +00	6.2412510 -01	6.5120187 -01	6.6779676 -01	-4.4182368 -01	2.2064542 -02	1.2064408 -02	5.1231087 -03
22	-5.4299719 +00	6.2405748 -01	6.5119565 -01	6.6779666 -01	-3.7853407 -01	2.2063961 -02	1.2064380 -02	5.1231085 -03
23	-5.7761802 +00	6.2399873 -01	6.5119050 -01	6.6779659 -01	-3.0559656 -01	2.2063580 -02	1.2064362 -02	5.1231084 -03
24	-6.1296378 +00	6.2394748 -01	6.5118621 -01	6.6779653 -01	-2.2287575 -01	2.2063353 -02	1.2064352 -02	5.1231084 -03
25	-6.4905543 +00	6.2390260 -01	6.5118262 -01	6.6779649 -01	-1.3023539 -01	2.2063244 -02	1.2064348 -02	5.1231083 -03
S3		6.2358659 -01	6.5116422 -01	6.6779634 -01		2.2063144 -02	1.2064344 -02	5.1231083 -03
S5		6.2352942 -01	6.5116149 -01	6.6779619 -01		2.2083510 -02	1.2064594 -02	5.1231084 -03

S	K	THIRD MOMENT(V*) N=10	THIRD MOMENT(V*) N=20	THIRD MOMENT(V*) N=50	K	FOURTH MOMENT(V*) N=10	FOURTH MOMENT(V*) N=20	FOURTH MOMENT(V*) N=50
6	-9.6724087 -01	1.6596381 -03	5.6150126 -04	1.1037889 -04	-1.0527185 +00	1.9577614 -03	5.1796713 -04	8.5425443 -05
7	-6.0085606 -01	1.6367421 -03	5.5947243 -04	1.1032399 -04	-1.6881753 +00	1.8934323 -03	5.1226689 -04	8.5271194 -05
8	-3.4781609 -01	1.6296360 -03	5.5892752 -04	1.1031258 -04	-2.1546539 +00	1.8494116 -03	5.0889123 -04	8.5200540 -05
9	-2.4666145 -01	1.6266985 -03	5.5872890 -04	1.1030926 -04	-2.4574584 +00	1.8201451 -03	5.0691242 -04	8.5167434 -05
10	-2.7085186 -01	1.6246995 -03	5.5860806 -04	1.1030760 -04	-2.6362092 +00	1.8006893 -03	5.0573635 -04	8.5151314 -05
11	-3.6933169 -01	1.6229292 -03	5.5851136 -04	1.1030650 -04	-2.7381658 +00	1.7875644 -03	5.0501942 -04	8.5143108 -05
12	-4.8919928 -01	1.6213484 -03	5.5843266 -04	1.1030573 -04	-2.8043743 +00	1.7785024 -03	5.0456827 -04	8.5138730 -05
13	-5.8685141 -01	1.6200305 -03	5.5837245 -04	1.1030523 -04	-2.8638549 +00	1.7720711 -03	5.0427441 -04	8.5136280 -05
14	-6.3203690 -01	1.6190188 -03	5.5832977 -04	1.1030492 -04	-2.9325388 +00	1.7673769 -03	5.0407639 -04	8.5134848 -05
15	-6.0783631 -01	1.6183102 -03	5.5830204 -04	1.1030475 -04	-3.0147605 +00	1.7638625 -03	5.0393885 -04	8.5133977 -05
16	-5.0856004 -01	1.6178704 -03	5.5828600 -04	1.1030466 -04	-3.1058850 +00	1.7611766 -03	5.0384090 -04	8.5133430 -05
17	-3.3683028 -01	1.6176508 -03	5.5827851 -04	1.1030462 -04	-3.1952008 +00	1.7590935 -03	5.0376986 -04	8.5133078 -05
18	-1.0060437 -01	1.6176007 -03	5.5827690 -04	1.1030461 -04	-3.2685850 +00	1.7574642 -03	5.0371772 -04	8.5132847 -05
19	1.8944698 -01	1.6176738 -03	5.5827911 -04	1.1030462 -04	-3.3106951 +00	1.7561865 -03	5.0367924 -04	8.5132693 -05
20	5.2202021 -01	1.6178316 -03	5.5828359 -04	1.1030464 -04	-3.3066073 +00	1.7551873 -03	5.0365085 -04	8.5132591 -05
21	8.8667430 -01	1.6180435 -03	5.5828928 -04	1.1030466 -04	-3.2429136 +00	1.7544122 -03	5.0363002 -04	8.5132523 -05
22	1.2747808 +00	1.6182867 -03	5.5829548 -04	1.1030468 -04	-3.1083460 +00	1.7538192 -03	5.0361491 -04	8.5132479 -05
23	1.6800115 +00	1.6185446 -03	5.5830172 -04	1.1030469 -04	-2.8940198 +00	1.7533748 -03	5.0360417 -04	8.5132450 -05
24	2.0984643 +00	1.6188060 -03	5.5830773 -04	1.1030471 -04	-2.5933917 +00	1.7530519 -03	5.0359673 -04	8.5132431 -05
25	2.5285266 +00	1.6190632 -03	5.5831337 -04	1.1030472 -04	-2.2020238 +00	1.7528279 -03	5.0359182 -04	8.5132420 -05
S3		1.6349938 -03	5.5839825 -04	1.1030480 -04		1.7523212 -03	5.0358226 -04	8.5132403 -05
S5		1.6223830 -03	5.5836103 -04	1.1030478 -04		1.7604946 -03	5.0368532 -04	8.5132460 -05

Table 7. Theoretical Moments Assessments of v^* using Levin Algorithm

n \ c	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
v	1.2605	1.1130	1.0000	0.9102	0.8369	0.7757	0.7238	0.6790	0.6399	0.6055	0.5749	0.5475
10.	1.0494	0.9548	0.8764	0.8101	0.7533	0.7042	0.6613	0.6235	0.5899	0.5599	0.5329	0.5085
	0.2759	0.2489	0.2238	0.2029	0.1855	0.1711	0.1589	0.1486	0.1397	0.1320	0.1253	0.1194
	—	—	0.7424	0.6942	0.6376	0.5861	0.5384	0.4952	0.4569	0.4235	0.3947	0.3700
	—	—	4.0597	4.0172	3.9104	3.8008	3.6990	3.6072	3.5260	3.4552	3.3940	3.3415
11.	1.0644	0.9670	0.8864	0.8185	0.7605	0.7105	0.6669	0.6285	0.5945	0.5642	0.5368	0.5122
	0.2736	0.2442	0.2183	0.1970	0.1795	0.1650	0.1530	0.1427	0.1340	0.1265	0.1199	0.1142
	—	0.2760	0.7557	0.6999	0.6396	0.5842	0.5335	0.4881	0.4482	0.4137	0.3840	0.3588
	—	3.9114	4.1268	4.0470	3.9241	3.8029	3.6925	3.5950	3.5105	3.4382	3.3768	3.3247
12.	1.0773	0.9773	0.8948	0.8255	0.7666	0.7158	0.6716	0.6327	0.5983	0.5676	0.5401	0.5152
	0.2709	0.2397	0.2131	0.1916	0.1740	0.1596	0.1476	0.1376	0.1290	0.1216	0.1152	0.1096
	—	0.4672	0.7654	0.7032	0.6392	0.5806	0.5275	0.4805	0.4395	0.4042	0.3740	0.3484
	—	4.0836	4.1794	4.0693	3.9313	3.7997	3.6822	3.5804	3.4938	3.4209	3.3598	3.3086
13.	1.0885	0.9863	0.9021	0.8316	0.7717	0.7202	0.6755	0.6363	0.6016	0.5706	0.5428	0.5177
	0.2679	0.2353	0.2083	0.1866	0.1690	0.1547	0.1429	0.1329	0.1245	0.1173	0.1110	0.1056
	—	0.5905	0.7722	0.7045	0.6372	0.5759	0.5209	0.4726	0.4308	0.3950	0.3645	0.3387
	—	4.2103	4.2206	4.0851	3.9334	3.7927	3.6696	3.5648	3.4770	3.4040	3.3436	3.2935
14.	1.0984	0.9940	0.9084	0.8368	0.7761	0.7241	0.6789	0.6393	0.6043	0.5731	0.5451	0.5199
	0.2649	0.2312	0.2038	0.1820	0.1645	0.1502	0.1385	0.1287	0.1204	0.1134	0.1073	0.1019
	1.3182	0.6728	0.7768	0.7044	0.6341	0.5705	0.5140	0.4647	0.4224	0.3862	0.3555	0.3297
	—	4.3064	4.2530	4.0955	3.9316	3.7832	3.6557	3.5487	3.4604	3.3878	3.3283	3.2794
15.	1.1071	1.0009	0.9139	0.8414	0.7800	0.7274	0.6819	0.6419	0.6067	0.5753	0.5472	0.5218
	0.2618	0.2272	0.1995	0.1777	0.1602	0.1461	0.1345	0.1249	0.1168	0.1098	0.1038	0.0986
	1.2203	0.7293	0.7797	0.7031	0.6302	0.5646	0.5069	0.4570	0.4142	0.3778	0.3471	0.3213
	2.8307	4.3810	4.2781	4.1016	3.9269	3.7721	3.6411	3.5328	3.4443	3.3724	3.3140	3.2663
16.	1.1149	1.0070	0.9188	0.8454	0.7834	0.7304	0.6844	0.6442	0.6088	0.5772	0.5489	0.5234
	0.2587	0.2234	0.1956	0.1737	0.1563	0.1423	0.1309	0.1214	0.1134	0.1066	0.1007	0.0956
	1.1552	0.7690	0.7813	0.7010	0.6257	0.5586	0.4999	0.4494	0.4063	0.3699	0.3392	0.3135
	3.2715	4.4400	4.2974	4.1042	3.9200	3.7599	3.6263	3.5172	3.4290	3.3580	3.3007	3.2543
17.	1.1220	1.0124	0.9231	0.8489	0.7864	0.7330	0.6867	0.6463	0.6106	0.5789	0.5505	0.5249
	0.2557	0.2198	0.1918	0.1699	0.1527	0.1388	0.1275	0.1182	0.1103	0.1036	0.0979	0.0929
	1.1113	0.7974	0.7819	0.6983	0.6209	0.5524	0.4929	0.4420	0.3988	0.3624	0.3318	0.3061
	3.6104	4.4873	4.3120	4.1040	3.9115	3.7470	3.6115	3.5020	3.4144	3.3444	3.2883	3.2431
18.	1.1283	1.0173	0.9270	0.8521	0.7891	0.7353	0.6887	0.6481	0.6122	0.5804	0.5518	0.5261
	0.2527	0.2163	0.1882	0.1664	0.1493	0.1356	0.1244	0.1152	0.1075	0.1009	0.0953	0.0904
	1.0812	0.8179	0.7816	0.6950	0.6158	0.5462	0.4861	0.4349	0.3916	0.3553	0.3248	0.2993
	3.8756	4.5256	4.3227	4.1016	3.9018	3.7337	3.5969	3.4875	3.4006	3.3317	3.2768	3.2329
19.	1.1341	1.0217	0.9305	0.8550	0.7915	0.7373	0.6906	0.6497	0.6137	0.5817	0.5531	0.5273
	0.2498	0.2131	0.1849	0.1632	0.1461	0.1325	0.1215	0.1124	0.1048	0.0984	0.0929	0.0881
	1.0604	0.8330	0.7806	0.6914	0.6106	0.5400	0.4794	0.4280	0.3847	0.3485	0.3182	0.2929
	4.0865	4.5568	4.3301	4.0973	3.8914	3.7203	3.5826	3.4735	3.3875	3.3198	3.2661	3.2234
20.	1.1393	1.0257	0.9337	0.8576	0.7937	0.7392	0.6922	0.6512	0.6150	0.5829	0.5542	0.5283
	0.2469	0.2099	0.1817	0.1601	0.1431	0.1297	0.1188	0.1098	0.1024	0.0960	0.0906	0.0859
	1.0458	0.8440	0.7791	0.6875	0.6053	0.5338	0.4728	0.4214	0.3782	0.3421	0.3119	0.2868
	4.2566	4.5823	4.3349	4.0916	3.8803	3.7069	3.5687	3.4601	3.3752	3.3086	3.2562	3.2146
22.	1.1486	1.0328	0.9392	0.8621	0.7975	0.7425	0.6950	0.6537	0.6173	0.5850	0.5561	0.5301
	0.2414	0.2040	0.1758	0.1544	0.1377	0.1245	0.1139	0.1052	0.0979	0.0918	0.0866	0.0821
	1.0283	0.8580	0.7749	0.6792	0.5945	0.5218	0.4604	0.4088	0.3659	0.3301	0.3004	0.2757
	4.5099	4.6202	4.3382	4.0772	3.8572	3.6806	3.5423	3.4352	3.3525	3.2883	3.2382	3.1938

The entries in the table for each c and n refer to μ_1', σ, $\sqrt{\beta_1}$ and β_2.

n	c	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
	v	1.2605	1.1130	1.0000	0.9102	0.8369	0.7757	0.7238	0.6790	0.6399	0.6055	0.5749	0.5475
25.		1.1600	1.0414	0.9460	0.8676	0.8020	0.7464	0.6984	0.6567	0.6200	0.5874	0.5583	0.5322
		0.2338	0.1960	0.1680	0.1469	0.1306	0.1178	0.1076	0.0992	0.0923	0.0864	0.0814	0.0771
		1.0165	0.8668	0.7665	0.6660	0.5787	0.5048	0.4430	0.3918	0.3493	0.3143	0.2852	0.2612
		4.7531	4.6540	4.3319	4.0505	3.8218	3.6430	3.5062	3.4020	3.3228	3.2622	3.2154	3.1790
30.		1.1745	1.0522	0.9544	0.8744	0.8077	0.7512	0.7026	0.6604	0.6233	0.5905	0.5611	0.5347
		0.2224	0.1845	0.1570	0.1365	0.1209	0.1087	0.0990	0.0911	0.0847	0.0792	0.0746	0.0706
		1.0094	0.8663	0.7498	0.6437	0.5538	0.4791	0.4177	0.3673	0.3260	0.2921	0.2643	0.2414
		4.9706	4.6735	4.3042	4.0008	3.7648	3.5869	3.4545	3.3562	3.2828	3.2276	3.1856	3.1533
35.		1.1853	1.0601	0.9605	0.8793	0.8117	0.7546	0.7056	0.6630	0.6257	0.5926	0.5631	0.5366
		0.2126	0.1749	0.1479	0.1281	0.1131	0.1014	0.0922	0.0848	0.0787	0.0736	0.0692	0.0655
		1.0048	0.8579	0.7320	0.6224	0.5312	0.4566	0.3960	0.3467	0.3067	0.2740	0.2473	0.2254
		5.0765	4.6672	4.2661	3.9503	3.7130	3.5388	3.4119	3.3194	3.2514	3.2010	3.1630	3.1340
40.		1.1936	1.0662	0.9652	0.8830	0.8148	0.7572	0.7078	0.6650	0.6275	0.5942	0.5646	0.5379
		0.2039	0.1667	0.1403	0.1211	0.1066	0.0955	0.0867	0.0796	0.0738	0.0690	0.0649	0.0613
		0.9992	0.8462	0.7143	0.6025	0.5109	0.4369	0.3773	0.3292	0.2904	0.2588	0.2332	0.2122
		5.1285	4.6472	4.2243	3.9023	3.6667	3.4975	3.3764	3.2894	3.2263	3.1798	3.1452	3.1190
45.		1.2002	1.0710	0.9688	0.8859	0.8172	0.7592	0.7096	0.6666	0.6288	0.5955	0.5657	0.5390
		0.1962	0.1595	0.1338	0.1151	0.1011	0.0904	0.0820	0.0753	0.0697	0.0651	0.0612	0.0579
		0.9923	0.8333	0.6972	0.5841	0.4927	0.4194	0.3609	0.3141	0.2764	0.2459	0.2212	0.2011
		5.1512	4.6197	4.1818	3.8575	3.6255	3.4618	3.3465	3.2646	3.2057	3.1627	3.1309	3.1070
50.		1.2056	1.0749	0.9718	0.8883	0.8191	0.7609	0.7110	0.6678	0.6299	0.5965	0.5666	0.5399
		0.1894	0.1532	0.1281	0.1099	0.0964	0.0861	0.0781	0.0716	0.0663	0.0619	0.0582	0.0550
		0.9843	0.8200	0.6809	0.5671	0.4761	0.4039	0.3465	0.3008	0.2642	0.2348	0.2109	0.1915
		5.1563	4.5881	4.1402	3.8162	3.5889	3.4309	3.3209	3.2436	3.1885	3.1486	3.1192	3.0971
60.		1.2139	1.0809	0.9763	0.8918	0.8220	0.7633	0.7131	0.6697	0.6316	0.5980	0.5680	0.5411
		0.1777	0.1427	0.1186	0.1014	0.0887	0.0791	0.0716	0.0656	0.0607	0.0566	0.0532	0.0502
		0.9664	0.7937	0.6510	0.5370	0.4474	0.3773	0.3222	0.2787	0.2441	0.2164	0.1940	0.1759
		5.1374	4.5201	4.0622	3.7436	3.5268	3.3799	3.2797	3.2103	3.1615	3.1266	3.1011	3.0821
70.		1.2200	1.0852	0.9796	0.8944	0.8241	0.7651	0.7146	0.6710	0.6328	0.5991	0.5690	0.5420
		0.1679	0.1340	0.1110	0.0946	0.0826	0.0736	0.0665	0.0609	0.0563	0.0525	0.0493	0.0466
		0.9474	0.7688	0.6244	0.5111	0.4233	0.3553	0.3023	0.2608	0.2279	0.2017	0.1806	0.1636
		5.0983	4.4516	3.9923	3.6823	3.4765	3.3397	3.2478	3.1850	3.1413	3.1102	3.0877	3.0710
80.		1.2247	1.0885	0.9820	0.8963	0.8257	0.7664	0.7158	0.6720	0.6337	0.5999	0.5697	0.5427
		0.1597	0.1268	0.1046	0.0890	0.0776	0.0690	0.0624	0.0570	0.0527	0.0492	0.0461	0.0436
		0.9283	0.7456	0.6006	0.4886	0.4027	0.3368	0.2857	0.2459	0.2146	0.1896	0.1696	0.1535
		5.0501	4.3860	3.9301	3.6301	3.4349	3.3072	3.2225	3.1652	3.1256	3.0976	3.0775	3.0626
90.		1.2284	1.0911	0.9840	0.8978	0.8269	0.7674	0.7166	0.6727	0.6344	0.6005	0.5703	0.5432
		0.1525	0.1207	0.0993	0.0843	0.0735	0.0653	0.0589	0.0539	0.0498	0.0464	0.0435	0.0411
		0.9097	0.7242	0.5794	0.4688	0.3849	0.3209	0.2716	0.2334	0.2033	0.1795	0.1604	0.1451
		4.9982	4.3245	3.8748	3.5853	3.4000	3.2804	3.2019	3.1492	3.1130	3.0876	3.0694	3.0560
100.		1.2313	1.0932	0.9855	0.8990	0.8279	0.7682	0.7173	0.6734	0.6349	0.6010	0.5708	0.5437
		0.1463	0.1153	0.0947	0.0803	0.0699	0.0620	0.0560	0.0512	0.0473	0.0440	0.0413	0.0390
		0.8917	0.7044	0.5602	0.4513	0.3692	0.3070	0.2594	0.2226	0.1937	0.1709	0.1526	0.1379
		4.9452	4.2673	3.8256	3.5464	3.3703	3.2579	3.1848	3.1360	3.1027	3.0795	3.0628	3.0506

n \ c	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.8	3.0	3.2	3.5	3.8	4.0
v	0.5227	0.5003	0.4798	0.4611	0.4438	0.4279	0.4131	0.3866	0.3634	0.3430	0.3165	0.2939	0.2805
10.	0.4863	0.4660	0.4474	0.4303	0.4144	0.3997	0.3861	0.3614	0.3398	0.3207	0.2958	0.2745	0.2620
	0.1141	0.1094	0.1051	0.1013	0.0978	0.0946	0.0916	0.0863	0.0817	0.0777	0.0725	0.0680	0.0654
	0.3492	0.3317	0.3173	0.3054	0.2958	0.2884	0.2824	0.2755	0.2728	0.2734	0.2783	0.2915	0.2957
	3.2964	3.2579	3.2249	3.1971	3.1722	3.1513	3.1329	3.1033	3.0824	3.0649	3.0552	3.0429	3.0494
11.	0.4897	0.4693	0.4505	0.4332	0.4172	0.4024	0.3887	0.3638	0.3421	0.3228	0.2978	0.2764	0.2637
	0.1091	0.1045	0.1004	0.0967	0.0933	0.0902	0.0874	0.0823	0.0779	0.0741	0.0691	0.0648	0.0623
	0.3375	0.3197	0.3049	0.2927	0.2828	0.2751	0.2689	0.2612	0.2580	0.2581	0.2628	0.2729	0.2790
	3.2806	3.2432	3.2115	3.1845	3.1610	3.1411	3.1238	3.0960	3.0759	3.0601	3.0489	3.0389	3.0412
12.	0.4926	0.4719	0.4530	0.4356	0.4195	0.4046	0.3908	0.3658	0.3439	0.3246	0.2994	0.2779	0.2652
	0.1047	0.1002	0.0963	0.0927	0.0894	0.0864	0.0837	0.0788	0.0746	0.0709	0.0662	0.0621	0.0597
	0.3268	0.3088	0.2938	0.2815	0.2715	0.2635	0.2572	0.2491	0.2455	0.2452	0.2494	0.2581	0.2644
	3.2657	3.2296	3.1991	3.1732	3.1510	3.1321	3.1158	3.0896	3.0705	3.0559	3.0442	3.0354	3.0357
13.	0.4950	0.4742	0.4551	0.4376	0.4215	0.4065	0.3926	0.3675	0.3455	0.3261	0.3008	0.2792	0.2664
	0.1007	0.0964	0.0926	0.0891	0.0860	0.0831	0.0804	0.0757	0.0717	0.0681	0.0635	0.0596	0.0573
	0.3170	0.2989	0.2838	0.2715	0.2614	0.2533	0.2470	0.2386	0.2346	0.2341	0.2377	0.2455	0.2518
	3.2518	3.2170	3.1878	3.1631	3.1420	3.1241	3.1087	3.0840	3.0660	3.0523	3.0406	3.0324	3.0317
14.	0.4970	0.4761	0.4570	0.4394	0.4231	0.4081	0.3941	0.3689	0.3468	0.3273	0.3019	0.2802	0.2675
	0.0972	0.0930	0.0893	0.0859	0.0829	0.0801	0.0775	0.0730	0.0691	0.0656	0.0612	0.0575	0.0552
	0.3080	0.2899	0.2748	0.2625	0.2524	0.2443	0.2379	0.2293	0.2251	0.2243	0.2276	0.2347	0.2408
	3.2390	3.2055	3.1775	3.1539	3.1339	3.1169	3.1024	3.0791	3.0620	3.0491	3.0377	3.0299	3.0286
15.	0.4987	0.4778	0.4585	0.4409	0.4245	0.4094	0.3954	0.3701	0.3480	0.3284	0.3029	0.2812	0.2684
	0.0940	0.0900	0.0863	0.0831	0.0801	0.0774	0.0749	0.0705	0.0667	0.0634	0.0591	0.0555	0.0534
	0.2996	0.2816	0.2666	0.2543	0.2443	0.2362	0.2297	0.2211	0.2167	0.2157	0.2186	0.2253	0.2310
	3.2272	3.1949	3.1681	3.1456	3.1266	3.1105	3.0967	3.0747	3.0584	3.0463	3.0356	3.0278	3.0262
16.	0.5003	0.4792	0.4599	0.4422	0.4258	0.4106	0.3966	0.3712	0.3490	0.3294	0.3038	0.2820	0.2692
	0.0912	0.0872	0.0837	0.0805	0.0776	0.0749	0.0725	0.0683	0.0646	0.0614	0.0573	0.0537	0.0517
	0.2919	0.2740	0.2591	0.2469	0.2369	0.2288	0.2224	0.2137	0.2092	0.2080	0.2106	0.2168	0.2224
	3.2164	3.1853	3.1596	3.1381	3.1200	3.1046	3.0916	3.0707	3.0553	3.0438	3.0396	3.0259	3.0242
17.	0.5016	0.4805	0.4611	0.4433	0.4269	0.4117	0.3976	0.3721	0.3499	0.3302	0.3046	0.2827	0.2699
	0.0885	0.0847	0.0812	0.0781	0.0753	0.0727	0.0704	0.0662	0.0627	0.0596	0.0555	0.0521	0.0501
	0.2847	0.2669	0.2522	0.2400	0.2301	0.2221	0.2157	0.2070	0.2025	0.2011	0.2034	0.2093	0.2146
	3.2065	3.1765	3.1518	3.1313	3.1140	3.0994	3.0869	3.0671	3.0525	3.0415	3.0299	3.0243	3.0225
18.	0.5028	0.4816	0.4622	0.4443	0.4279	0.4126	0.3985	0.3730	0.3506	0.3309	0.3053	0.2834	0.2705
	0.0861	0.0823	0.0790	0.0759	0.0732	0.0707	0.0684	0.0644	0.0609	0.0579	0.0540	0.0507	0.0487
	0.2781	0.2604	0.2458	0.2337	0.2239	0.2160	0.2096	0.2009	0.1963	0.1949	0.1969	0.2025	0.2076
	3.1974	3.1685	3.1447	3.1250	3.1085	3.0946	3.0827	3.0639	3.0499	3.0395	3.0287	3.0229	3.0210
19.	0.5039	0.4826	0.4631	0.4452	0.4287	0.4134	0.3993	0.3737	0.3513	0.3316	0.3059	0.2839	0.2710
	0.0839	0.0802	0.0769	0.0739	0.0713	0.0688	0.0666	0.0627	0.0593	0.0563	0.0525	0.0493	0.0474
	0.2718	0.2543	0.2398	0.2279	0.2182	0.2103	0.2040	0.1953	0.1907	0.1892	0.1911	0.1963	0.2012
	3.1890	3.1611	3.1383	3.1193	3.1035	3.0902	3.0789	3.0609	3.0476	3.0377	3.0274	3.0217	3.0197
20.	0.5049	0.4835	0.4640	0.4460	0.4295	0.4142	0.4000	0.3744	0.3520	0.3322	0.3064	0.2844	0.2715
	0.0818	0.0782	0.0750	0.0721	0.0695	0.0671	0.0649	0.0611	0.0578	0.0549	0.0512	0.0480	0.0462
	0.2659	0.2486	0.2343	0.2225	0.2129	0.2051	0.1988	0.1902	0.1856	0.1840	0.1856	0.1907	0.1954
	3.1813	3.1543	3.1323	3.1141	3.0990	3.0862	3.0754	3.0582	3.0455	3.0360	3.0262	3.0206	3.0186
22.	0.5065	0.4851	0.4654	0.4474	0.4308	0.4155	0.4012	0.3755	0.3530	0.3332	0.3073	0.2853	0.2723
	0.0781	0.0746	0.0715	0.0688	0.0663	0.0640	0.0619	0.0582	0.0551	0.0523	0.0488	0.0458	0.0440
	0.2553	0.2383	0.2243	0.2128	0.2033	0.1957	0.1896	0.1811	0.1764	0.1747	0.1761	0.1807	0.1851
	3.1675	3.1423	3.1218	3.1049	3.0909	3.0791	3.0692	3.0535	3.0418	3.0331	3.0240	3.0187	3.0168

n \ c	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.8	3.0	3.2	3.5	3.8	4.0
v	0.5227	0.5003	0.4798	0.4611	0.4438	0.4279	0.4131	0.3866	0.3634	0.3430	0.3165	0.2939	0.2805
25.	0.5085	0.4869	0.4672	0.4491	0.4324	0.4170	0.4027	0.3769	0.3543	0.3344	0.3085	0.2864	0.2733
	0.0734	0.0701	0.0672	0.0645	0.0622	0.0600	0.0581	0.0546	0.0517	0.0491	0.0458	0.0430	0.0413
	0.2414	0.2250	0.2114	0.2003	0.1912	0.1838	0.1779	0.1696	0.1650	0.1631	0.1642	0.1684	0.1724
	3.1502	3.1272	3.1087	3.0935	3.0809	3.0704	3.0616	3.0476	3.0372	3.0295	3.0214	3.0165	3.0147
30.	0.5109	0.4892	0.4693	0.4511	0.4343	0.4188	0.4044	0.3785	0.3559	0.3358	0.3098	0.2876	0.2746
	0.0671	0.0641	0.0614	0.0590	0.0568	0.0548	0.0530	0.0499	0.0471	0.0448	0.0418	0.0392	0.0377
	0.2225	0.2069	0.1941	0.1836	0.1750	0.1680	0.1624	0.1545	0.1500	0.1481	0.1488	0.1524	0.1559
	3.1281	3.1081	3.0921	3.0791	3.0684	3.0595	3.0520	3.0402	3.0315	3.0249	3.0181	3.0138	3.0121
35.	0.5126	0.4908	0.4709	0.4526	0.4357	0.4201	0.4057	0.3797	0.3570	0.3369	0.3108	0.2885	0.2754
	0.0622	0.0594	0.0569	0.0546	0.0526	0.0508	0.0491	0.0462	0.0436	0.0415	0.0387	0.0363	0.0349
	0.2074	0.1926	0.1805	0.1705	0.1623	0.1557	0.1504	0.1428	0.1385	0.1366	0.1371	0.1402	0.1434
	3.1115	3.0939	3.0798	3.0685	3.0591	3.0514	3.0449	3.0348	3.0272	3.0216	3.0157	3.0119	3.0104
40.	0.5139	0.4920	0.4720	0.4536	0.4367	0.4211	0.4066	0.3806	0.3578	0.3377	0.3115	0.2892	0.2761
	0.0583	0.0556	0.0532	0.0511	0.0492	0.0475	0.0459	0.0432	0.0408	0.0388	0.0362	0.0339	0.0326
	0.1950	0.1809	0.1693	0.1598	0.1521	0.1457	0.1407	0.1335	0.1293	0.1275	0.1277	0.1306	0.1335
	3.0988	3.0830	3.0704	3.0603	3.0521	3.0453	3.0396	3.0306	3.0240	3.0191	3.0138	3.0105	3.0091
45.	0.5149	0.4929	0.4729	0.4545	0.4375	0.4219	0.4074	0.3813	0.3584	0.3383	0.3121	0.2897	0.2766
	0.0550	0.0525	0.0502	0.0482	0.0464	0.0448	0.0433	0.0407	0.0385	0.0365	0.0341	0.0320	0.0308
	0.1846	0.1711	0.1600	0.1509	0.1435	0.1375	0.1326	0.1257	0.1218	0.1199	0.1201	0.1227	0.1254
	3.0886	3.0743	3.0630	3.0539	3.0465	3.0404	3.0353	3.0273	3.0215	3.0170	3.0124	3.0094	3.0081
50.	0.5156	0.4937	0.4736	0.4551	0.4382	0.4225	0.4080	0.3818	0.3589	0.3387	0.3125	0.2901	0.2770
	0.0522	0.0498	0.0477	0.0457	0.0440	0.0425	0.0411	0.0386	0.0365	0.0347	0.0323	0.0303	0.0292
	0.1757	0.1627	0.1521	0.1434	0.1363	0.1305	0.1258	0.1192	0.1154	0.1136	0.1137	0.1161	0.1187
	3.0803	3.0673	3.0570	3.0487	3.0420	3.0365	3.0319	3.0247	3.0194	3.0154	3.0112	3.0085	3.0073
60.	0.5168	0.4948	0.4746	0.4561	0.4391	0.4234	0.4088	0.3826	0.3597	0.3395	0.3132	0.2908	0.2776
	0.0477	0.0455	0.0435	0.0418	0.0402	0.0388	0.0375	0.0352	0.0333	0.0316	0.0295	0.0277	0.0266
	0.1611	0.1491	0.1392	0.1312	0.1246	0.1192	0.1149	0.1087	0.1051	0.1034	0.1034	0.1056	0.1079
	3.0677	3.0566	3.0478	3.0409	3.0352	3.0306	3.0267	3.0207	3.0163	3.0129	3.0094	3.0071	3.0061
70.	0.5177	0.4956	0.4754	0.4569	0.4398	0.4241	0.4094	0.3832	0.3602	0.3400	0.3136	0.2912	0.2780
	0.0442	0.0421	0.0403	0.0387	0.0372	0.0359	0.0347	0.0326	0.0308	0.0293	0.0273	0.0256	0.0247
	0.1497	0.1384	0.1292	0.1216	0.1154	0.1104	0.1064	0.1006	0.0972	0.0956	0.0956	0.0975	0.0996
	3.0585	3.0488	3.0412	3.0352	3.0303	3.0263	3.0230	3.0178	3.0140	3.0112	3.0081	3.0061	3.0052
80.	0.5183	0.4962	0.4759	0.4574	0.4403	0.4245	0.4099	0.3836	0.3606	0.3403	0.3140	0.2915	0.2783
	0.0414	0.0394	0.0377	0.0362	0.0348	0.0336	0.0325	0.0305	0.0288	0.0274	0.0255	0.0240	0.0231
	0.1404	0.1297	0.1210	0.1139	0.1081	0.1033	0.0995	0.0941	0.0909	0.0893	0.0892	0.0910	0.0929
	3.0515	3.0429	3.0362	3.0309	3.0266	3.0231	3.0202	3.0156	3.0123	3.0098	3.0071	3.0054	3.0046
90.	0.5188	0.4966	0.4764	0.4578	0.4407	0.4249	0.4103	0.3840	0.3609	0.3406	0.3143	0.2918	0.2786
	0.0390	0.0372	0.0356	0.0341	0.0328	0.0317	0.0306	0.0288	0.0272	0.0258	0.0241	0.0226	0.0217
	0.1326	0.1225	0.1142	0.1075	0.1019	0.0975	0.0938	0.0887	0.0856	0.0841	0.0840	0.0857	0.0875
	3.0459	3.0383	3.0323	3.0275	3.0237	3.0206	3.0180	3.0139	3.0110	3.0087	3.0063	3.0048	3.0041
100.	0.5192	0.4970	0.4767	0.4581	0.4410	0.4252	0.4106	0.3842	0.3612	0.3409	0.3145	0.2920	0.2788
	0.0370	0.0353	0.0337	0.0324	0.0312	0.0301	0.0291	0.0273	0.0258	0.0245	0.0228	0.0214	0.0206
	0.1260	0.1163	0.1085	0.1020	0.0968	0.0925	0.0890	0.0841	0.0812	0.0798	0.0796	0.0812	0.0829
	3.0415	3.0345	3.0291	3.0248	3.0214	3.0185	3.0162	3.0126	3.0099	3.0079	3.0057	3.0043	3.0037

percentage points are given in Table 15a; the simulation comparisons are in excellent agreement.

Simulation results for the moments of v* are given in Table 8, showing very satisfactory agreement. The moments of v*, using a 4-moment Pearson approximant, can be used (when c is known) to set up an approximate distribution of c*, using the inverse of expression (1.1).

TABLE 8

MOMENTS OF V* BY SERIES (LEVIN) AND SIMULATION (10^5 RUNS)

			Moment Parameter			
c	n		$\mu_1'(v^*)$	$\sigma(v^*)$	$\sqrt{\beta_1}(v^*)$	$\beta_2(v^*)$
0.8	20	L	1.1393	0.2469	1.0458	4.2566
		S	1.1377(.0008)	0.2469	0.9607	4.7411
1.0	20	L	0.9337	0.1817	0.7791	4.3349
		S	0.9338(.0006)	0.1817	0.8026	4.4245
1.5	18	L	0.6481	0.1152	0.4349	3.4875
		S	0.6477(.0004)	0.1153	0.4333	3.4639
	20	L	0.6512	0.1098	0.4214	3.4601
		S	0.6512(.0003)	0.1099	0.4389	3.5175
2.0	20	L	0.5049	0.0818	0.2659	3.1813
		S_1	0.5048(.0003)	0.0819	0.2770	3.2002
		S_2	0.5044(.0003)	0.0817	0.2569	3.1581
	25	L	0.5085	0.0734	0.2414	3.1502
		S	0.5085(.0002)	0.0734	0.2426	3.1423
2.5	20	L	0.4142	0.0671	0.2051	3.0862
		S	0.4141(.0002)	0.0671	0.2121	3.0820
3.0	20	L	0.3520	0.0578	0.1856	3.0455
		S	0.3519(.0002)	0.0578	0.1903	3.0293

(S_1, S_2 refer to Simulations, the parenthetic entries being the approximate standard deviation.)

Illustrations of the density of the Weibull variate, v^*, and c^* are given in Figures 1, 3a and 3b. In Figure 4 we give the loci of $(\sqrt{\beta_1}, \beta_2)$ points for the density of v^* for several values of c and samples between 10 and 1000. The initial drift away from normality for c=1 as n increases from 10 to about 25 is noteworthy.

2.6 An Unbiassed Estimator for c based on v^*. Another important feature of the distribution of v^* is its bias; the moment estimator (as far as the tabulation goes) always underestimates v, especially for c small. We therefore set up an expression for c as a rational fraction in v^* and n, using linearized least squares on the tabulated values. We find the approximation

$$\bar{c} \sim \{1 + b_{01}/n + b_{02}/n^2 + v^*(b_{10} + b_{11}/n) + v^{*2}b_{20}\}/$$
$$\{a_{00} + a_{01}/n + a_{02}/n^2 + v^*(a_{10} + a_{11}/n) + v^{*2}a_{20}\} \quad (2.14)$$
$$(0 < v^* < 1)$$

where

$a_{00} = -0.02586359654$ $\qquad b_{00} = 1.0$

$a_{01} = 0.1368685508$ $\qquad b_{01} = -0.03224982824$

$a_{02} = 0.1941632032$ $\qquad b_{02} = -0.05540306547$

$a_{10} = 0.9555907114$ $\qquad b_{10} = -0.4528195583$

$a_{11} = -0.2097962308$ $\qquad b_{11} = -0.9542316494$

$a_{20} = -0.2918002479$ $\qquad b_{20} = 0.08948942647$,

the errors being numerically less than 0.5%. The grid of values used was n = 10(1)20,22,25(5)50(10)100 and $0.8 < c < 4$ involving 575 points.

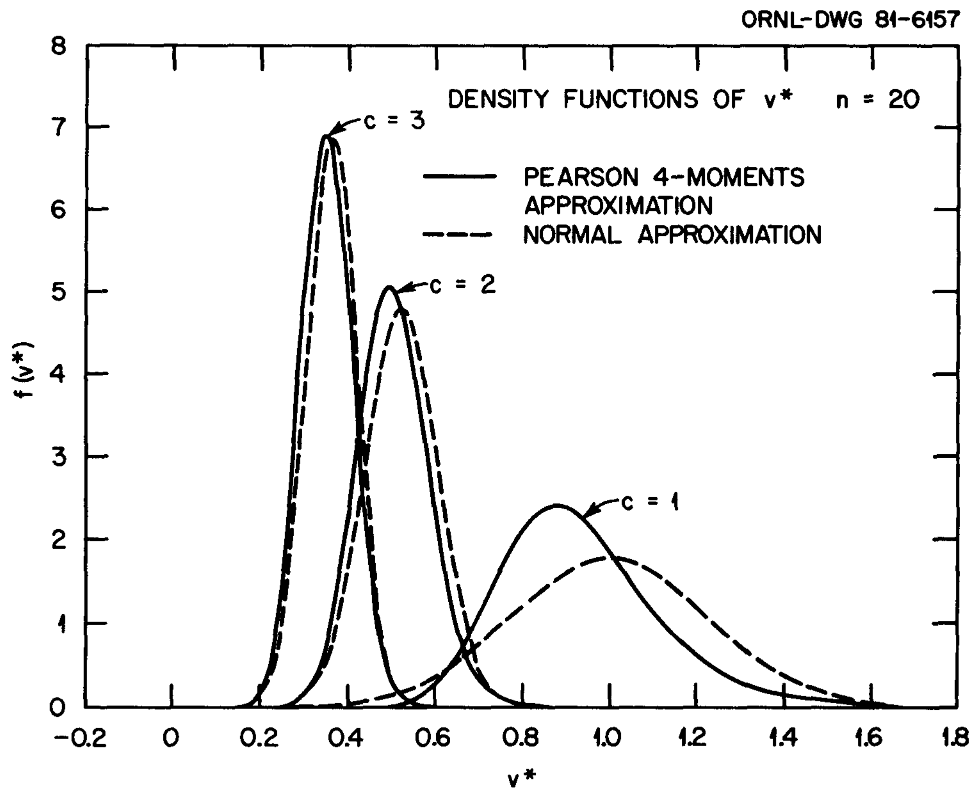

Figure 3a. Density Functions of v*

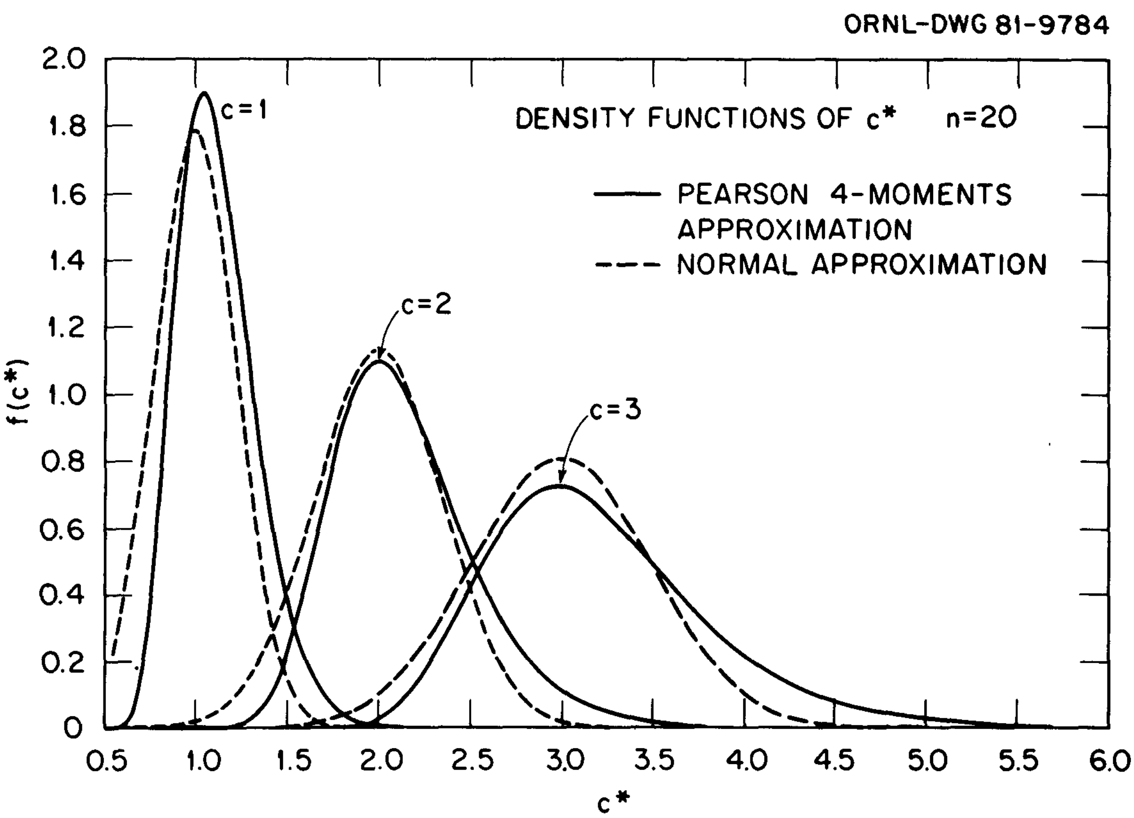

Figure 3b. Density Function of c*

ORNL-DWG 81-9785

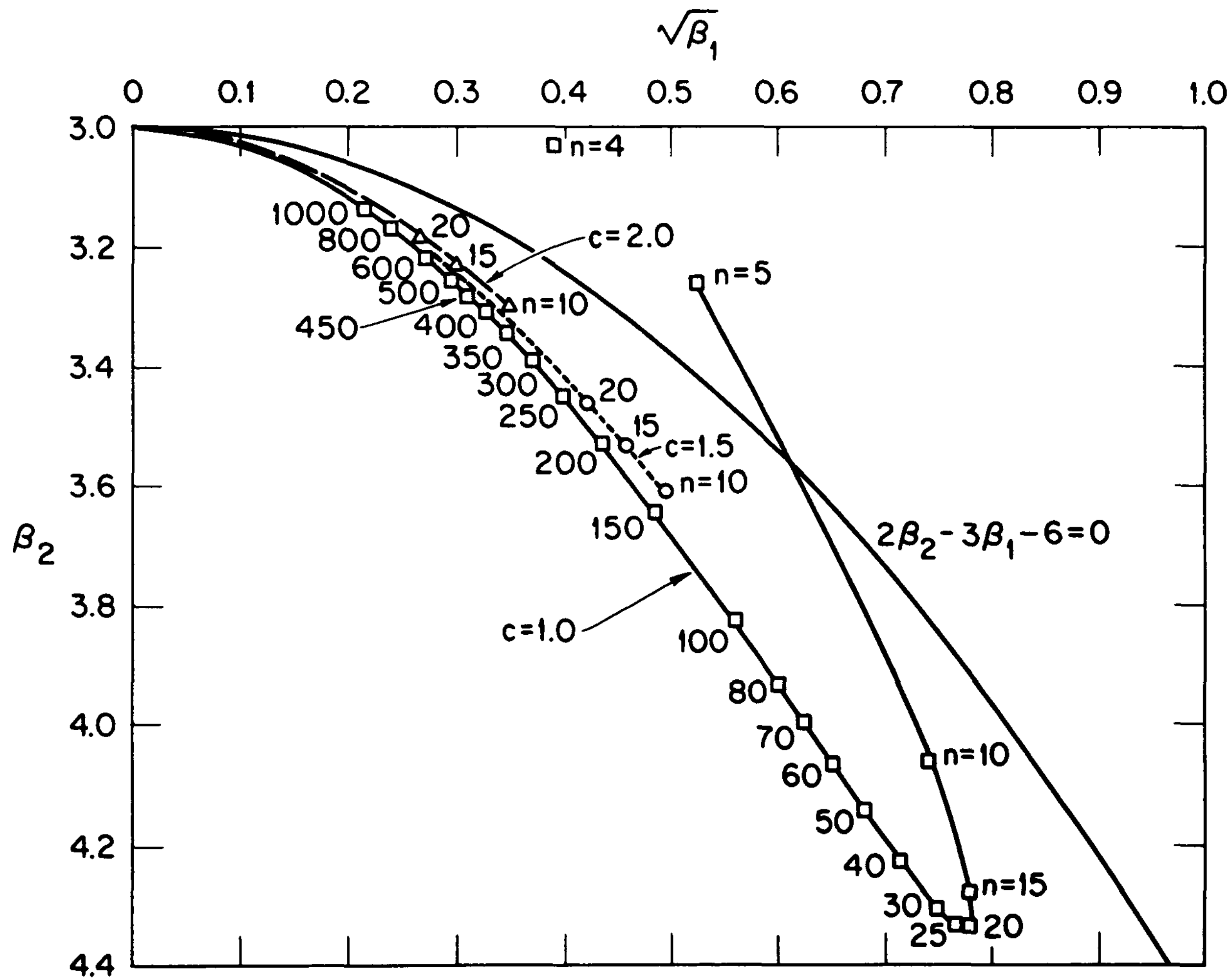

Figure 4. Skewness and Kurtosis Plot for the Distribiton of v^*
c=1, 1.5, 2, and Various Sample Sizes.

A similar formula for $0.6 \leq c < 1$ has been considered but here the moment series are strongly divergent and require further study to discover an acceptable summation algorithm. In any event coefficients of variation in this range perhaps should be avoided by resorting to a root transformation of the observations. This type of transformation for a Weibull variate produces a Weibull variate.

In practical situations it is thought that the use of (2.14) should improve the assessment of the true shape parameter c. In addition we have developed a formula to determine c given v for the equation

$$\frac{\Gamma(1 + 2/c)}{\Gamma^2(1 + 1/c)} = 1 + v^2, \qquad (2.15)$$

in the form

$$1/c = \frac{c_1 v + c_2 v^2 + c_3 v^3 + c_4 v^4,}{1 + d_1 v + d_2 v^2 + d_3 v^3} \qquad (2.16)$$

where

$c_1 = 0.779960622$

$c_2 = 0.587095391$

$c_3 = 0.471569800$

$c_4 = -0.0382146209$

$d_1 = 0.188028602$

$d_2 = 0.609555293$

$d_3 = 0.00282363508$

The error in the approximation to c is 0.0004% or less for $0.6 < c < 6.6$ $(0.178 < v < 1.758)$.

3. THE DISTRIBUTION OF c*

3.1 Moment Series It will be evident from the equation for c* that the Taylor series for its moments will be more complicated. However we use a two-stage process, expressing c* in terms of v*, and v* in terms of the moments m_1' and m_2. Thus we set

$$c^* = c + c_1(v^*-v) + c_2(v^*-v)^2/2! + \cdots$$

with

$$c_s = \left.\frac{d^s c^*}{dv^{*s}}\right|_{c^* = c,\ v^* = v} . \tag{3.1}$$

If we wish to carry the series (3.1), and similar ones for higher moments, so that in expectation all terms are included contributing to say n^{-12} then we need all derivatives up to c_{24}. These can be found using Faà di Bruno's formula for a derivative of a function of a function (see for example Shenton & Bowman, pp. 14, 130, 169, (1977b); for several generalizations see Good (1961)).

3.2 Derivatives of c* with respect to v*

From (1.1)

$$\Gamma(1+2/c^*)/\Gamma^2(1+1/c^*) = 1+v^{*2} \tag{3.2}$$

so that taking logarithmic derivatives

$$\frac{1}{c^{*2}}\{\psi(1+\frac{1}{c^*}) - \Psi(1+\frac{2}{c^*})\}\frac{\partial c^*}{\partial v^*} = \frac{1}{2}\left\{\frac{1}{v^*+i} + \frac{1}{v^*-i}\right\} \tag{3.3}$$

where $\Psi(x) = d\ell n\, \Gamma(x)/dx$, $i = \surd(-1)$. Clearly we can drop the astericks and replace them when necessary. We write c_r for $\partial^r c/\partial v^r$, the modified (3.3) in the form

$$c_1\, J(c) = \nu_0. \tag{3.4}$$

Using the formula of Leibniz for the sth derivative of a product,

$$\frac{\partial^s J(c)}{\partial c^s} = J^{(s)}(c)$$
$$= \sum_{r=0}^{s} \binom{s}{r} \frac{(-1)^r}{c^{r+2}} (r+1)!\, H^{(s-r)}(c) \tag{3.5}$$

where

$$H^{(o)}(c) = H(c) = \psi\,(1 + \tfrac{1}{c}) - \psi(1 + \tfrac{2}{c}), \quad H^{(m)}(c) = d^m H(c)/dc^m.$$

From (3.4)

$$\begin{aligned} &Jc_2 + J^{(1)} c_1^2 = \nu_1 = \partial\nu_0/\partial v, \\ &Jc_3 + 3\, J^{(1)} c_1 c_2 + J^{(2)} c_1{}^3 = \nu_2, \\ &Jc_4 + 4\, J^{(1)} c_1 c_3 + 3J^{(1)} c_2^2 + 6\, J^{(2)} c_1^2\, c_2 + J^{(3)} c_1^4 = \nu_3 \end{aligned} \tag{3.6}$$

and so on, where

$$\nu_s = \frac{\partial^s \nu_0}{\partial v^s} = \frac{(-1)^s}{2}\, s!\left(\frac{1}{(v+i)^{s+1}} + \frac{1}{(v-i)^{s+1}}\right)$$

Now the structure of these formulas is the same (Luckacs, 1955) as occurs in the expression of non-central moments (μ'_r) in terms of cumulants. For example,

$$\kappa_2 + \kappa_1^2 = \mu'_2,$$

$$\kappa_3 + 3\,\kappa_2\kappa_1 + \kappa_1^3 = \mu'_3, \tag{3.7}$$

$$\kappa_4 + 4\,\kappa_3\kappa_1 + 3\,\kappa_2^2 + 6\,\kappa_2\kappa_1^2 + \kappa_1^4 = \mu'_4.$$

But these formulas are equivalent to

$$\mu'_r = \sum_{s=0}^{r-1} \binom{r-1}{s} \kappa_{r-s}\, \mu'_s \qquad (r = 2,3,\ldots) \tag{3.8}$$

giving μ'_r in terms of $\mu'_{r-1}, \mu'_{r-2}, \ldots, \mu'_0$ (note $\kappa_1 = \mu'_1$, $\mu'_0 = 1$). Hence the left side members of (3.7), and the generalization, can be set up recursively from previous members, and awkward combinatorial problems avoided, a distinct advantage in digital implementation.

3.3 Moment Series for c* A tabulation is given in Table 9 for the first four moments for c = 0.8(0.1)2.6(0.2)3.2, 3.5, 3.8, 4.0. The sign pattern for c = 1 (apart from one anomoly in $\mu_4(c^*)$) is alternating. As c increases this regular pattern is disrupted and the plus signs start to predominate, especially for the higher moments. As for magnitude, very approximately, the coefficient of n^{-12} decreases from (24)!, for c = 1, towards (12)! for c = 3 for $\mu_1'(c^*)$, with slight increases for the higher moments. See Table 10 for further details.

TABLE 10. Magnitude of Coefficients for C* Moment Series

	Moment							
	$\mu_1'(c^*)$		$\mu_2(c^*)$		$\mu_3(c^*)$		$\mu_4(c^*)$	
	$\lvert c_{12}^{(1)}/c_0^{(1)}\rvert$		$\lvert c_{12}^{(2)}/c_1^{(2)}\rvert$		$\lvert c_{12}^{(3)}/c_2^{(3)}\rvert$		$\lvert c_{12}^{(4)}/c_2^{(4)}\rvert$	
0.8	1.1	31	3.4	31	1.2	31	2.5	31
0.9	4.3	27	1.7	28	1.8	28	1.6	28
1.0	7.9	24	3.7	25	6.7	26	4.4	25
1.1	4.4	22	2.3	23	6.1	23	3.2	23
1.2	5.3	20	3.1	21	5.0	21	4.7	21
1.3	1.2	19	7.3	19	9.3	19	1.2	20
1.4	3.9	17	2.6	18	2.9	18	4.3	18
1.5	1.8	16	1.2	17	1.3	17	2.1	17
1.6	1.0	15	7.2	15	7.1	15	1.2	16
1.7	7.2	13	5.0	14	4.6	14	8.2	14
1.8	5.3	12	3.7	13	3.3	13	5.9	13
1.9	4.0	11	2.8	12	2.3	12	4.2	12
2.0	2.6	10	1.8	11	1.4	11	2.5	11
2.1	1.0	09	6.7	09	5.1	09	3.4	09
2.2	4.1	07	3.9	08	4.4	08	1.3	09
2.3	5.3	06	1.9	08	3.9	08	1.8	09
2.4	3.6	06	1.4	08	4.2	08	2.4	09
2.5	1.0	07	7.9	07	4.2	08	2.9	09
2.6	2.2	07	4.3	07	3.3	08	3.2	09
2.8	4.7	07	3.8	08	5.5	07	2.7	09
3.0	5.5	07	6.9	08	6.3	08	6.8	08
3.2	2.9	07	7.6	08	1.1	09	2.4	09
3.5	9.5	07	6.5	07	1.0	09	6.0	09
3.8	2.6	08	1.9	09	7.6	08	3.7	09
4.0	3.0	08	3.1	09	2.6	09	2.3	09

(In the moment columns each second column refers to the power of ten used as a multiplier.)

Table 9. Moments Series for c*

Mean of c* ($\mu_1'(c^*)$)

s\c	0.8	0.9	1.0	1.1	1.2
0	8.000000000000000 -01	9.000000000000000 -01	1.000000000000000 +00	1.100000000000000 +00	1.200000000000000 +00
1	3.342846071515215 +00	2.901123960342889 +00	2.644934066848226 +00	2.496081450342826 +00	2.414723268274164 +00
2	-7.604008463723208 +01	-3.429760019266803 +01	-1.711237632182688 +01	-8.875349967346315 +00	-4.436135051856997 +00
3	9.447839180288620 +03	2.523659338538721 +03	8.531805653680439 +02	3.415385649929241 +02	1.554404017452981 +02
4	-2.311498757069305 +06	-3.400877257028193 +05	-7.082363897516906 +04	-1.884015023610159 +04	-5.979249935918503 +03
5	9.314634171314171 +08	7.304409401780578 +07	9.169191139869852 +06	1.603517572585727 +06	3.565951792660971 +05
6	-5.577865171185695 +11	-2.259778356657433 +10	-1.672421433278949 +09	-1.890727071895130 +08	-2.904793330670826 +07
7	4.663914270810639 +14	9.475104762067066 +12	4.048786015613954 +11	2.914250307788106 +10	3.057660031028213 +09
8	-5.220230018964119 +17	-5.167839141041526 +15	-1.249575120074503 +14	-5.643316057318772 +12	-3.999464093296250 +11
9	7.582998416488933 +20	3.559011733404036 +18	4.775696760514957 +16	1.334246089237793 +15	6.320055134330638 +13
10	-1.395494009398496 +24	-3.025322954261554 +21	-2.210967686499977 +19	-3.769248026387996 +17	-1.181177648209620 +16
11	3.190172538943057 +27	3.117135130327109 +24	1.218597388494825 +22	1.250967575807782 +20	2.567870107359608 +18
12	-8.910868869125806 +30	-3.835105697517927 +27	-7.883825983130659 +24	-4.811547235047940 +22	-6.407672961105291 +20

s\c	1.3	1.4	1.5	1.6	1.7
0	1.300000000000000 +00	1.400000000000000 +00	1.500000000000000 +00	1.600000000000000 +00	1.700000000000000 +00
1	2.378690872133493 +00	2.374764286518997 +00	2.394604637049735 +00	2.432701287054720 +00	2.485266439634239 +00
2	-1.803287931136735 +00	-1.080816825972225 -01	1.065996016221097 +00	1.934336472504487 +00	2.615402871242541 +00
3	7.862415352896470 +01	4.381995088961512 +01	2.697130177697455 +01	1.847262933686909 +01	1.413144463421430 +01
4	-2.156362679023748 +03	-8.504192588496146 +02	-3.537212157421571 +02	-1.481956185538115 +02	-5.720363814657952 +01
5	9.468786096923717 +04	2.869204730203715 +04	9.594166831266352 +03	3.454338917885300 +03	1.319222248691344 +03
6	-5.595868531706391 +06	-1.275425037878143 +06	-3.290372389768705 +05	-9.266021135764909 +04	-2.755489979873472 +04
7	4.235722213632370 +08	7.215565672552363 +07	1.432441089058159 +07	3.173655336302205 +06	7.555014925285458 +05
8	-3.950121724048175 +10	-4.994535019478585 +09	-7.584226145737180 +08	-1.314133423391159 +08	-2.484864066998026 +07
9	4.414383363175902 +12	4.116213342401120 +11	4.755992201007089 +10	6.416562759895045 +09	9.600307927267919 +08
10	-5.789050495159947 +14	-3.956478943717080 +13	-3.461164627487290 +12	-3.620970580669340 +11	-4.271849654510707 +10
11	8.764353077331536 +16	4.364424823729129 +15	2.877088280119400 +14	2.324888499729232 +13	2.155531874868719 +12
12	-1.511859111768918 +19	-5.454316327904547 +17	-2.697127943527571 +16	-1.677185602382278 +15	-1.218248281142987 +14

s\c	1.8	1.9	2.0	2.1	2.2
0	1.800000000000000 +00	1.900000000000000 +00	2.000000000000000 +00	2.100000000000000 +00	2.200000000000000 +00
1	2.549606975788820 +00	2.623750871363565 +00	2.706216190943829 +00	2.795863341410286 +00	2.891797743794941 +00
2	3.177742384183311 +00	3.662727174299173 +00	4.096220306669345 +00	4.494868377559933 +00	4.869646288193082 +00
3	1.197897151458507 +01	1.103241345243356 +01	1.077505758001807 +01	1.092281751428141 +01	1.131325338473866 +01
4	-1.458131134912682 +01	6.421636066064623 +00	1.731397191726435 +01	2.330205801303114 +01	2.683643830556392 +01
5	5.346176810596800 +02	2.367642570257817 +02	1.228611992587433 +02	8.038524941735564 +01	6.565873320910957 +01
6	-8.343012301729654 +03	-2.437761116590477 +03	-6.096436738358027 +02	-7.043571009037965 +01	6.015498402001445 +01
7	1.860432207559851 +05	4.521225580497668 +04	9.963099994572504 +03	1.460713173472494 +03	-3.624321177831969 +02
8	-4.906698689095599 +06	-9.611072356268987 +05	-1.736427025909631 +05	-2.595707486897112 +04	-3.697920287794827 +03
9	1.513848458237575 +08	2.366496854852774 +07	3.319045211266513 +06	3.179558169530553 +05	-1.103419451572150 +04
10	-5.365118968023834 +09	-6.697325986013577 +08	-7.441751867733587 +07	-5.406647665046420 +06	3.787101686347024 +05
11	2.149623673099344 +11	2.135513856253243 +10	1.869620293893464 +09	1.053696178274878 +08	-3.651467995350914 +05
12	-9.620631154361564 +12	-7.586460286380094 +11	-5.207200281598111 +10	-2.103730993335457 +09	9.116694205740128 +07

$\mu_1^*(c^*)$

s\c	2.3	2.4	2.5	2.6	2.8
0	2.300000000000000 +00	2.400000000000000 +00	2.500000000000000 +00	2.600000000000000 +00	2.800000000000000 +00
1	2.993304023560711 +00	3.099800466478725 +00	3.210806835414755 +00	3.325921195956940 +00	3.567160158375866 +00
2	5.227931118170919 +00	5.574756510833321 +00	5.913594083487444 +00	6.246853146519800 +00	6.902816055625931 +00
3	1.185016103085331 +01	1.247470234234709 +01	1.314979362143295 +01	1.385138283721483 +01	1.527496295906659 +01
4	2.910539042304001 +01	3.069666493953278 +01	3.190365937153182 +01	3.287239251398665 +01	3.434305721248891 +01
5	6.123742368902122 +01	6.008429640359439 +01	5.941397960987385 +01	5.820989093614784 +01	5.340107211297222 +01
6	6.632702906720348 +01	3.944578276832171 +01	7.692552019514396 +00	-2.106094143128009 +01	-6.359403693546399 +01
7	-6.537555278728011 +02	-6.706940361056393 +02	-6.670922601192025 +02	-6.578207665032075 +02	-5.160191386151021 +02
8	-2.425432933988257 +03	-2.698618549701953 +03	-1.903689177833028 +03	-1.965596709932167 +02	4.764667680430839 +03
9	-2.598952731016689 +03	2.228222428022857 +04	4.027569074852912 +04	5.694156885070493 +04	9.039437196431081 +04
10	4.070254745016277 +05	4.170896788401855 +05	5.342319388564549 +05	6.161162103013902 +05	5.167379848398131 +05
11	3.112040933732522 +06	4.079375764306322 +06	2.954579134538508 +06	1.247432183150636 +06	-4.581265330711302 +06
12	1.221094458982430 +07	-8.671923367769027 +06	-2.614589981080574 +07	-5.805368088655494 +07	-1.313647075253196 +08

s\c	3.0	3.2	3.5	3.8	4.0
0	3.000000000000000 +00	3.200000000000000 +00	3.500000000000000 +00	3.800000000000000 +00	4.000000000000000 +00
1	3.821321874780198 +00	4.086734060795148 +00	4.503138758343224 +00	4.938504391896484 +00	5.237961927728282 +00
2	7.550690846002486 +00	8.194049769698747 +00	9.152944443984572 +00	1.010365085678189 +01	1.073143495909880 +01
3	1.666777440548605 +01	1.799021011543080 +01	1.979688625642553 +01	2.136035733451712 +01	2.226027798435711 +01
4	3.533375683655083 +01	3.589360853285601 +01	3.599776919231509 +01	3.540824943839283 +01	3.479057081113513 +01
5	4.612933542350450 +01	3.799261301533483 +01	2.781887521422620 +01	2.470204150317555 +01	2.877845726142008 +01
6	-8.305469429134837 +01	-7.345634764644122 +01	1.082815774372908 +01	1.902052925910968 +02	3.591932084690246 +02
7	-9.584121969600138 +01	6.270625417457432 +02	2.121774177628624 +03	3.658315534305217 +03	4.356347374051102 +03
8	1.072371211813033 +04	1.644239047176342 +04	2.056007381924659 +04	1.302108236762610 +04	-1.194725497021433 +03
9	1.085255242401343 +05	8.891823677077069 +04	-4.393952331961554 +04	-2.951683606797123 +05	-4.853525295812820 +05
10	-5.321380224636974 +03	-9.738228964165411 +05	-2.862124063925947 +06	-3.799281365591091 +06	-2.726870627446004 +06
11	-1.354790354813795 +07	-2.202528702196422 +07	-1.963649978416871 +07	2.048581591015229 +07	7.082670556955600 +07
12	-1.661276167519973 +08	-9.402319912383780 +07	3.312582354775654 +08	9.845374519822000 +08	1.208302071831469 +09

$(\mu_2^{-}(c^*))$

s\c	0.8	0.9	1.0	1.1	1.2
1	9.459218162031611 -01	9.557697447604388 -01	1.000000000000000 +00	1.069693422203836 +00	1.160314905714114 +00
2	-4.460239391231447 +01	-2.219285766458683 +01	-1.162176225503480 +01	-5.778682994533397 +00	-2.087621063420190 +00
3	8.989176994072629 +03	2.750471647360025 +03	1.046997724004699 +03	4.676566366274851 +02	2.376474038635794 +02
4	-2.993523061910937 +06	-5.064396498694257 +05	-1.186829845083503 +05	-3.492433576353363 +04	-1.207814499973814 +04
5	1.514145204592005 +09	1.369708881505232 +08	1.939127967420604 +07	3.759140354977837 +06	9.141606707416135 +05
6	-1.083027268935133 +12	-5.071079973867167 +10	-4.236910277711713 +09	-5.312368446210853 +08	-8.925889295391165 +07
7	1.046495838803849 +15	2.459782181329311 +13	1.187095739200548 +12	9.477345148442705 +10	1.087460152869620 +10
8	-1.322273423198953 +18	-1.515282938040432 +16	-4.138311720205306 +14	-2.072709737939244 +13	-1.606167684818381 +12
9	2.130977919553621 +21	1.157962288544643 +19	1.754767677422695 +17	5.435745998981207 +15	2.814651377408007 +14
10	-4.293278656017861 +24	-1.077542023668982 +22	-8.891154268499079 +19	-1.680153080837426 +18	-5.754129741643691 +16
11	1.063257998147768 +28	1.202520171204765 +25	5.306190491431647 +22	6.036168249605213 +20	1.353792535644683 +19
12	-3.190303773154598 +31	-1.588814329348084 +28	-3.685379917166499 +25	-2.491730127439963 +23	-3.624781185971469 +21

s\c	1.3	1.4	1.5	1.6	1.7
1	1.269435094815821 +00	1.395722122824563 +00	1.538452757982729 +00	1.697257599098593 +00	1.871980278980348 +00
2	5.331121215600171 -01	2.592301560945741 +00	4.351867337582610 +00	5.956986889458666 +00	7.493271433158083 +00
3	1.359780683825198 +02	8.795879695102334 +01	6.482154515779689 +01	5.428032050052429 +01	5.063229541959477 +01
4	-4.671837356188167 +03	-1.932152154745879 +03	-8.064476882756389 +02	-3.004768455331490 +02	-5.255157553534476 +01
5	2.626080229108651 +05	8.542829375744838 +04	3.058797216032082 +04	1.189704483474448 +04	5.083770286551824 +03
6	-1.859139473351798 +07	-4.537117816571582 +06	-1.242153694774538 +06	-3.675535675705168 +05	-1.129780232383271 +05
7	1.628707454228477 +09	2.971151059056031 +08	6.265188825319860 +07	1.464343101487319 +07	3.659312517723919 +06
8	-1.714803149999925 +11	-2.321489662874168 +10	-3.743615775981356 +09	-6.838967477149091 +08	-1.354459368543555 +08
9	2.124716274106786 +13	2.120952508380297 +12	2.602180373316126 +11	3.701240762655829 +10	5.800657198591368 +09
10	-3.047274813227866 +15	-2.229218305559070 +14	-2.070557624989550 +13	-2.283547999221596 +12	-2.821778344075357 +11
11	4.991800506782737 +17	2.660399396843333 +16	1.861907553904994 +15	1.586008277042466 +14	1.540148487805841 +13
12	-9.237985509600313 +19	-3.566465156026829 +18	-1.872202758386563 +17	-1.227194950970490 +16	-9.336049223422849 +14

s\c	1.8	1.9	2.0	2.1	2.2
1	2.062595865331671 +00	2.269161680097931 +00	2.491786687833855 +00	2.730611934947397 +00	2.985797784728197 +00
2	9.013946001761574 +00	1.055364550019826 +01	1.213581717250786 +01	1.377688671792281 +01	1.548870263967094 +01
3	5.104284649474503 +01	5.401990826402084 +01	5.873856162948459 +01	6.472245301946164 +01	7.168517691337032 +01
4	8.114817523964615 +01	1.620831238704892 +02	2.180876153864081 +02	2.623537795674875 +02	3.013428815251574 +02
5	2.509854749213842 +03	1.548210661693903 +03	1.228746931396156 +03	1.172455583516213 +03	1.224596245270390 +03
6	-3.385392747673647 +04	-8.199023385632939 +03	3.010321574275961 +02	3.132321020078229 +03	4.076833352161236 +03
7	9.462642135045286 +05	2.464528016181057 +05	6.477996585472037 +04	1.953059624635830 +04	9.389162677130561 +03
8	-2.783291350716620 +07	-5.628873492915683 +06	-1.033835967661209 +06	-1.493070830142345 +05	-1.918601071281889 +04
9	9.524628046360638 +08	1.541412526685071 +08	2.216905398387491 +07	2.015433103450200 +06	-2.993483169652006 +05
10	-3.690147038807594 +10	-4.771437511052003 +09	-5.479195344767157 +08	-4.248341357447181 +07	1.651225645399940 +06
11	1.599156171157037 +12	1.644839196044746 +11	1.483406320293580 +10	8.646187029625584 +08	1.424110760870084 +07
12	-7.676096860461626 +13	-6.266903281178073 +12	-4.430068169053591 +11	-1.819401858821266 +10	1.157499549284045 +09

$\mu_2^c(c^*)$

s\c	2.3	2.4	2.5	2.6	2.8
1	3.257515450977020 +00	3.545941317477409 +00	3.851253100961467 +00	4.173627255883246 +00	4.870252297893815 +00
2	1.728002414826140 +01	1.915745723162460 +01	2.112606147202390 +01	2.318975443379334 +01	2.761394303728450 +01
3	7.944754997041713 +01	8.789270591760768 +01	9.694086727440076 +01	1.065347494699643 +02	1.271944918426988 +02
4	3.382773014668345 +02	3.747599782169147 +02	4.115584221868900 +02	4.489989162799040 +02	5.260403444336718 +02
5	1.319850325783336 +03	1.429980602348129 +03	1.542602941670439 +03	1.652279433224142 +03	1.854982709776654 +03
6	4.402165668836595 +03	4.528786629044862 +03	4.589540316406540 +03	4.622759036894611 +03	4.646532580763060 +03
7	7.348732356854213 +03	6.599176385329899 +03	5.774123714791939 +03	4.881573965879368 +03	4.061748532829571 +03
8	-1.965270044532789 +04	-2.936787928579902 +04	-3.005886111158726 +04	-2.098490556738459 +04	2.540930717786655 +04
9	-2.609023787355073 +05	-5.809132830541066 +04	1.551045730004594 +05	4.273344133681881 +05	1.189243825138380 +06
10	2.934340822009499 +06	4.559463162691995 +06	7.528856011158675 +06	1.064186384830920 +07	1.510447671332741 +07
11	5.783354533672277 +07	8.438902522388161 +07	9.343433394035628 +07	9.329390950782541 +07	3.910147231014879 +07
12	6.126585523583362 +08	5.002176306801027 +08	3.055716513902534 +08	-1.788288981364826 +08	-1.847350334602955 +09

s\c	3.0	3.2	3.5	3.8	4.0
1	5.637149820250038 +00	6.475568920928392 +00	7.869809345035926 +00	9.431090406505318 +00	1.056629685804164 +01
2	3.244717500609405 +01	3.769949828575374 +01	4.637417709870081 +01	5.600644541023304 +01	6.295632782476253 +01
3	1.496095159362520 +02	1.735779109851736 +02	2.120585311853578 +02	2.530831777212671 +02	2.815890732934564 +02
4	6.054304088812400 +02	6.861048776400615 +02	8.072390578897879 +02	9.263172120135144 +02	1.004033581795601 +03
5	2.033145816789931 +03	2.190373953578009 +03	2.406029733513791 +03	2.638497942418278 +03	2.832866089287613 +03
6	4.673979371325142 +03	4.832217716140870 +03	5.710800685584065 +03	7.944680058015955 +03	1.046251766999019 +04
7	6.355948638968088 +03	1.342881554484019 +04	3.496610100181690 +04	6.870638658350475 +04	9.488336434077224 +04
8	1.116915433896901 +05	2.345331248573838 +05	4.429297349195814 +05	5.611905196379312 +05	5.032081062621762 +05
9	2.065963441378622 +06	2.684654677695284 +06	2.08615[illegible]267004335 +06	-1.590700184306859 +06	-5.916361576798741 +06
10	1.396513117305715 +07	2.889934755232620 +06	-3.653498639179512 +07	-8.891805674692473 +07	-1.082923378224645 +08
11	-1.189704291695588 +08	-3.671063988082241 +08	-6.874412043406963 +08	-3.665789348525460 +08	5.491469840255346 +08
12	-3.899685686644897 +09	-4.898641883587975 +09	5.085969811805090 +08	1.808731515106764 +10	3.312387507462685 +10

$\mu_3^s(c^*)$

s\c	0.8	0.9	1.0	1.1	1.2
2	-4.710052752794365 +00	-1.806386321727491 +00	-1.303955989106630 -01	1.070099801555730 +00	2.093602297894440 +00
3	2.805412215043343 +03	9.595112591261273 +02	4.045375768259426 +02	2.022712862096456 +02	1.194969349131602 +02
4	-1.497389840655442 +06	-2.865477463868689 +05	-7.429380335945079 +04	-2.371980948047978 +04	-8.708319030345913 +03
5	1.036202907180798 +09	1.070121001208324 +08	1.691611750365374 +07	3.598670571326909 +06	9.472692817245141 +05
6	-9.340512289367754 +11	-5.020704080659743 +10	-4.706210733931175 +09	-6.503190403908220 +08	-1.186951771927860 +08
7	1.082376315914933 +15	2.931298251220714 +13	1.591832547095180 +12	1.404287136470454 +11	1.754764935058025 +10
8	-1.586882046234394 +18	-2.100507550286566 +16	-6.467476498817294 +14	-3.585214008083745 +13	-3.030131215568206 +12
9	2.898840204920249 +21	1.822608658037781 +19	3.117776600210929 +17	1.070033248024353 +16	6.048814738899732 +14
10	-6.505868798574500 +24	-1.891535349969709 +22	-1.763276435113096 +20	-3.694095335444406 +18	-1.382005648045743 +17
11	1.770966300932248 +28	2.322036735638096 +25	1.158160214718756 +23	1.461242164954037 +21	3.581393202011327 +19
12	-5.778999110520944 +31	-3.338233958799320 +28	-8.755341740626216 +25	-6.567115045175944 +23	-1.044241090131540 +22

s\c	1.3	1.4	1.5	1.6	1.7
2	3.079938177910506 +00	4.105256051600928 +00	5.217380401742011 +00	6.450526849707391 +00	7.831993646605232 +00
3	8.453467634870441 +01	7.162657127119529 +01	7.037475018612553 +01	7.621346310006138 +01	8.702784825159593 +01
4	-3.452863796614386 +03	-1.349425660682167 +03	-3.956587377287625 +02	1.015643125176861 +02	4.091243962850075 +02
5	2.916514041572143 +05	1.013951982194775 +05	3.935873435554824 +04	1.752812297191270 +04	9.669178730661211 +03
6	-2.652905487935438 +07	-6.869853151901245 +06	-1.970613281584054 +06	-5.975304087016733 +05	-1.764584868414004 +05
7	2.827724058069002 +09	5.493328846069633 +08	1.222829445857965 +08	2.998078106897056 +07	7.867228189468520 +06
8	-3.485942030587832 +11	-5.032940220078982 +10	-8.577221700655794 +09	-1.642161791745344 +09	-3.379048148083984 +08
9	4.924901302156124 +13	5.248257509768526 +12	6.812479040366614 +11	1.016942583995691 +11	1.660439990238046 +10
10	-7.898850047208507 +15	-6.172858849864586 +14	-6.070534760934115 +13	-7.031952381537756 +12	-9.059530049879230 +11
11	1.425802646249418 +18	8.121465032787664 +16	6.021213471001216 +15	5.390383260226574 +14	5.461149945276633 +13
12	-2.874218595267024 +20	-1.186343426509888 +19	-6.599867941372071 +17	-4.548656681326575 +16	-3.612139143901120 +15

s\c	1.8	1.9	2.0	2.1	2.2
2	9.385427090482731 +00	1.113250712321034 +01	1.309385635781992 +01	1.528954637574261 +01	1.773938556771753 +01
3	1.018219405846500 +02	1.201467747949443 +02	1.418363901224444 +02	1.668781486732910 +02	1.953454677566357 +02
4	6.392621793860572 +02	8.429205035231856 +02	1.044882712691595 +03	1.258041912426905 +03	1.489576441721930 +03
5	7.099771850582116 +03	6.698882381065906 +03	7.269665736349328 +03	8.330031395065400 +03	9.678722254270224 +03
6	-3.716403029526765 +04	1.314462157084819 +04	3.431724908546053 +04	4.598262881205285 +04	5.479032689657161 +04
7	2.201345562425232 +06	7.103241535248057 +05	3.339133303338067 +05	2.605525377113072 +05	2.673612799267014 +05
8	-7.118241804146193 +07	-1.413976702167234 +07	-1.929665732408436 +06	5.326971065838685 +05	9.535051647291830 +05
9	2.822571032660925 +09	4.716936173863876 +08	7.155021798826619 +07	8.821419858860903 +06	1.208323734099190 +06
10	-1.226615169218354 +11	-1.631226199954645 +10	-1.918436363274103 +09	-1.583080623980647 +08	-2.447271716822121 +06
11	5.874977339347256 +12	6.218384079903537 +11	5.726631156494940 +10	3.336479812480778 +09	6.695938341923593 +07
12	-3.078824032514578 +14	-2.588556081201472 +13	-1.872601235343921 +12	-7.742605486061881 +10	7.809977396682196 +09

$\mu_3'(c^*)$

s\c	2.3	2.4	2.5	2.6	2.8
2	2.046308395446080 +01	2.348034657802892 +01	2.681092441239410 +01	3.047463957245410 +01	3.888117349910726 +01
3	2.273611888217921 +02	2.630767284350314 +02	3.026597190987529 +02	3.462864021807478 +02	4.463914420118657 +02
4	1.743770456814147 +03	2.023362867348347 +03	2.330218781539440 +03	2.665676032048243 +03	3.426139056783611 +03
5	1.122956301517793 +04	1.294483710979076 +04	1.480743107842953 +04	1.680888026052396 +04	2.120973436943716 +04
6	6.299891555424020 +04	7.133841694912377 +04	8.000568585976248 +04	8.902129455553212 +04	1.080004526457461 +05
7	2.929740762953979 +05	3.213561321782153 +05	3.491420491672529 +05	3.764127112597935 +05	4.338699946221582 +05
8	1.004034373113495 +06	1.019775346636724 +06	1.058944246687358 +06	1.135334088416799 +06	1.463562802722889 +06
9	1.048645126783067 +06	1.604860201012518 +06	2.569363522967514 +06	4.298860375227178 +06	1.096170685590701 +07
10	5.385847189729451 +06	2.014395242843917 +07	4.714010515528877 +07	8.327677027411862 +07	1.748737461310563 +08
11	3.808683893230143 +08	7.057556946402229 +08	1.030715839184867 +09	1.362265655294937 +09	1.735675474767159 +09
12	7.917032559302506 +09	9.977493609836852 +09	1.128545248462003 +10	1.019800973502237 +10	-2.133849321971507 +09

s\c	3.0	3.2	3.5	3.8	4.0
2	4.886009869352272 +01	6.057259739237936 +01	8.174595408204640 +01	1.077312591242222 +02	1.279978880728384 +02
3	5.648266326119142 +02	7.029817680069083 +02	9.500420336302122 +02	1.248469119246619 +03	1.477747379848664 +03
4	4.310191150873190 +03	5.320917327605150 +03	7.077572528081648 +03	9.123714401866082 +03	1.064831151885445 +04
5	2.611907394353488 +04	3.151607073062324 +04	4.050867631654718 +04	5.063518403067319 +04	5.811176953442251 +04
6	1.282125205549627 +05	1.500211031942875 +05	1.878331771445017 +05	2.365927244542005 +05	2.787391980646194 +05
7	5.057473059092345 +05	6.083736140696865 +05	8.632574881699074 +05	1.296632720483547 +06	1.704846379742203 +06
8	2.171463133061174 +06	3.426240479749521 +06	6.478902618535622 +06	1.050902444869278 +07	1.305657378537200 +07
9	2.225546100079989 +07	3.693511045254878 +07	5.670024149794354 +07	5.372140459114849 +07	2.653375694700712 +07
10	2.649253619416373 +08	2.944279693993272 +08	6.302584814341221 +07	-6.241885321254200 +08	-1.245408049806848 +09
11	1.080654235116075 +09	-1.253804011482865 +09	-7.703075324187372 +09	-1.247749458622326 +10	-8.775827008105137 +09
12	-3.081745262528032 +10	-6.892433751974388 +10	-8.313240872217296 +10	8.206917988120633 +10	3.381131948021957 +11

$\tilde{\mu}_4(c^*)$

s\c	0.8	0.9	1.0	1.1	1.2
2	2.684304247107260 +00	2.740487414998303 +00	3.000000000000001 +00	3.432732052518460 +00	4.038992041267062 +00
3	1.920109251125982 +01	-2.423224072757342 +01	-1.949369494452693 +01	-4.825781893659697 +00	1.297350073229591 +01
4	-2.712098904459001 +05	-5.209859128710272 +04	-1.263709788732350 +04	-3.315903973022107 +03	-6.320495614262000 +02
5	3.401938602703180 +08	3.850167262063357 +07	6.472757578609086 +06	1.425932635843227 +06	3.800362921259436 +05
6	-4.325873233252653 +11	-2.613713892061976 +10	-2.683019821516475 +09	-3.972366948270479 +08	-7.617456386573408 +07
7	6.392478827055314 +14	1.969533941214514 +13	1.186496115101858 +12	1.137818392829079 +11	1.519132605607607 +10
8	-1.130936443843692 +18	-1.714971909784953 +16	-5.899336373204022 +14	-3.581902469386183 +13	-3.261783811069121 +12
9	2.407149302165587 +21	1.741653410520566 +19	3.343031060969482 +17	1.262349664059372 +16	7.726224246429326 +14
10	-6.143694363805711 +24	-2.061870120625298 +22	-2.162929969885338 +20	-5.000071288516321 +18	-2.031587829701268 +17
11	1.868281408259014 +28	2.833773164276786 +25	1.593669338849707 +23	2.223047520820597 +21	5.929745189190658 +19
12	-6.718456509812518 +31	-4.496492031643986 +28	-1.331584790749423 +26	-1.105761600788832 +24	-1.916337418607012 +22

s\c	1.3	1.4	1.5	1.6	1.7
2	4.834396379850156 +00	5.844120732425711 +00	7.100510665633992 +00	8.642050073093763 +00	1.051293049467403 +01
3	3.318625799211612 +01	5.644403294783048 +01	8.371318269972107 +01	1.160706681925937 +02	1.546642870102648 +02
4	3.090875725843628 +02	7.574272086350867 +02	1.087244727507627 +03	1.428407757789396 +03	1.834898007453275 +03
5	1.175523136748529 +05	4.304412130363045 +04	2.118538902038171 +04	1.612669588958221 +04	1.725273732490963 +04
6	-1.754171121404509 +07	-4.566157683598795 +06	-1.251923906742946 +06	-3.015382176505415 +05	7.951056019201018 +03
7	2.575800747602787 +09	5.192097463211854 +08	1.185574795426751 +08	2.980899972586896 +07	8.448342487885711 +06
8	-3.985764451033327 +11	-6.033616112007543 +10	-1.064739681104673 +10	-2.082072824625490 +09	-4.278107706150780 +08
9	6.718302746849400 +13	7.553551020700375 +12	1.022877894044947 +12	1.575935880516005 +11	2.629803442411661 +10
10	-1.244341815361358 +16	-1.029945592712834 +15	-1.061341686613900 +14	-1.275226276204354 +13	-1.686911019774967 +12
11	2.535625785104386 +18	1.533670650675640 +17	1.194995082938280 +16	1.113469429675587 +15	1.163097616083917 +14
12	-5.674463516035181 +20	-2.491559862791146 +19	-1.459758681664252 +18	-1.049662588443758 +17	-8.618342100558535 +15

s\c	1.8	1.9	2.0	2.1	2.2
2	1.276290511104991 +01	1.544728419127459 +01	1.862700269299804 +01	2.236872461783150 +01	2.674496523386342 +01
3	2.007159434018468 +02	2.555328764706422 +02	3.205183199435348 +02	3.971799532565641 +02	4.871362208580300 +02
4	2.335625135171944 +03	2.951255022055022 +03	3.700185404291369 +03	4.600764446002328 +03	5.672121621525878 +03
5	2.119759183711262 +04	2.689218783502697 +04	3.405377200078040 +04	4.268582622771883 +04	5.290117442376722 +04
6	1.339413275351921 +05	2.094049615949131 +05	2.759313931458338 +05	3.474635269729432 +05	4.291383505290867 +05
7	3.200994995433239 +06	2.105713075154908 +06	2.152571425909584 +06	2.548906287581907 +06	3.084003891406162 +06
8	-8.323150311070601 +07	-7.761730050550874 +06	1.010606052214240 +07	1.580801387389184 +07	1.926790916202381 +07
9	4.553092483226592 +09	8.101720922542401 +08	1.838084876466354 +08	9.766646814552492 +07	9.873082767615528 +07
10	-2.318845072034350 +11	-3.072421089017954 +10	-3.292541723806543 +09	6.057616234236192 +07	3.791885867768111 +08
11	1.277760928671454 +13	1.367572200165802 +12	1.264721369720922 +11	8.001938943872696 +09	1.276521390247449 +09
12	-7.526213230298639 +14	-6.421156625817876 +13	-4.665564643587314 +12	-1.881530514414925 +11	3.460246468736547 +10

$\mu_4^*(c^*)$

s\c	2.3	2.4	2.5	2.6	2.8
2	3.183422074006205 +01	3.772109948098026 +01	4.449645134299595 +01	5.225749341315455 +01	7.115807233541996 +01
3	5.921209283216930 +02	7.139865048349711 +02	8.547062361783491 +02	1.016375693024369 +03	1.411561704564195 +03
4	6.934458979212783 +03	8.409124113567532 +03	1.011859244361906 +04	1.208641151874138 +04	1.689622100183033 +04
5	6.485590452647890 +04	7.872347965451750 +04	9.468410213461813 +04	1.129200975724356 +05	1.569463965151300 +05
6	5.231915024809579 +05	6.308834578961063 +05	7.531422735227207 +05	8.907807173811332 +05	1.215285888102569 +06
7	3.704537366775448 +06	4.399584219002354 +06	5.169178821325197 +06	6.016160795440182 +06	7.959803147672497 +06
8	2.257758998814103 +07	2.613417836274040 +07	3.001614507418266 +07	3.428144396905198 +07	4.438872167722245 +07
9	1.116879221539967 +08	1.269325544153598 +08	1.451776981703724 +08	1.684742801187144 +08	2.396688117550772 +08
10	4.428372180581423 +08	5.541281606545690 +08	7.422593692283843 +08	1.025907738948651 +09	1.958639965329260 +09
11	2.801088651040360 +09	5.093570718437277 +09	8.350490822133557 +09	1.268664597966726 +10	2.356314526407175 +10
12	5.588635820440172 +10	9.066691488117262 +10	1.307995037178961 +11	1.682334926825565 +11	1.941613243256618 +11

s\c	3.0	3.2	3.5	3.8	4.0
2	9.533237428783509 +01	1.257989785490811 +02	1.858016973816444 +02	2.668363987670299 +02	3.349398878767819 +02
3	1.918772563392190 +03	2.559196947925720 +03	3.820703268182633 +03	5.520354310768940 +03	6.943472230876385 +03
4	2.304557760228949 +04	3.075389287306174 +04	4.574390100169106 +04	6.556788626232775 +04	8.188252574642672 +04
5	2.122454878744041 +05	2.802348768597267 +05	4.090701045021264 +05	5.743548605232276 +05	7.072547352128756 +05
6	1.610652063629136 +06	2.083786325966754 +06	2.958489891364662 +06	4.068464289797075 +06	4.968463298051151 +06
7	1.028271885683447 +07	1.307748428288873 +07	1.849418010333633 +07	2.607276982913376 +07	3.281427886924036 +07
8	5.775782335988008 +07	7.634185616049046 +07	1.192273289687985 +08	1.866193577439371 +08	2.467274387484324 +08
9	3.608520023247074 +08	5.502891152034520 +08	9.711356487318537 +08	1.475518711186378 +09	1.749167646224705 +09
10	3.416527875328572 +09	5.202726991859219 +09	7.188075282048998 +09	5.524996909269744 +09	1.002355017471150 +09
11	3.306468185677523 +10	3.252570132090106 +10	-8.328296140813247 +09	-9.955615403180578 +10	-1.539485230614988 +11
12	6.472142416809467 +10	-3.019018874990913 +11	-1.109334934415034 +12	-9.918132660969378 +11	7.671900648028759 +11

These properties suggest that $E(c^*)$ will exceed c, and $Var(c^*)$ will exceed the asymptotic variance $(Var_1(c^*))$ for c in the region of 1.5 or more. Numerical evidence for $8.0 < c < 4.0$ and $10 < n < 100$ suggests $E(c^*-c) > 0$, and $Var(c^*)/Var_1(c^*) > 1$. For example when $c = 1.9$, $n = 10$, $E(c^*-c) = 0.2$, and the variance ratio is 1.4; similarly when $n = 10$, $c = 1.5$, $E(c^*-c) = 0.8$, and the variance ratio is 1.7.

3.4 Summation of the c* Series The diversified structure of the series' coefficients arising from the 100 cases tabulated (25 values of c, for four moments) makes it imperative to diminish the labor involved in a detailed study; so we confine attention generally to samples in the region of 25 or more. This makes less stringent demands on the summatory algorithms chosen.

Again since magnitudes decrease and sign patterns become irregular as c increases, summatory algorithms successful for small c may fail for large c $(1.6 < c < 4)$.

For c in the vicinity of unity, we use Levin's t-algorithm or its truncated versions; some illustrations being given in Table 11. We look first of all for monotonicity, and secondly smallness of first differences.

A word on notation - we use S_3, and S_5 to denote the Shanks' approximant based on *the last three, and last five values computed. For c*, terms up to the coefficient of* n^{-12} *are always used.*

In addition L (tr=s, S_r) means a Levin algorithm with s initial terms truncated, with a Shanks' smoothing formula applied to the last r (3 or 5) terms. Similarly 2cB ($a=\alpha$, tr=s, S_r) and 1cB ($a=\alpha$, tr=s, S_r)

Table 11. Levin Approximants for c* moments

C= 1.0	N=25 μ_1'	μ_2	μ_3	μ_4	N=30 μ_1'	μ_2	μ_3	μ_4
3	1.08602524				1.07376016			
4	1.09220336	0.033567548			1.07820109	0.028541019		
5	1.09368189	0.034721688	0.000075122	0.004533784	1.07909713	0.029272347	0.000962018	0.003180959
6	1.09425699	0.035809459	0.000375307	0.004895037	1.07944959	0.029997145	0.000309809	0.003414627
7	1.09464478	0.036579332	0.001061848	0.005341132	1.07957847	0.030453958	0.000859171	0.003745219
8	1.09486239	0.036990305	0.002302849	0.005227298	1.07979415	0.030674234	0.001717512	0.004626877
9	1.09497281	0.037234382	0.003356607	0.034744501	1.07984848	0.030798216	0.002300588	-0.000779532
10	1.09502755	0.037352617	0.003743369	0.003305360	1.07987388	0.030852739	0.002477689	0.002550477
11	1.09505620	0.037422590	0.003941368	0.004316355	1.07988642	0.030883491	0.002565972	0.002989497
12	1.09507022	0.037453463	0.003970366	0.004681540	1.07989212	0.030895954	0.002574902	0.003166370
13	1.09507727	0.037474618	0.004035306	0.004911231	1.07989482	0.030904125	0.002601242	0.003273502
S3	1.09508439	0.037520671	0.003917975	0.005300603	1.07989726	0.030920832	0.002561391	0.003438072
S5	1.09508426	0.037493691	0.004031647	0.005285620	1.07989718	0.030910390	0.002598322	0.003464833

C= 1.1	N=25 μ_1'	μ_2	μ_3	μ_4	N=30 μ_1'	μ_2	μ_3	μ_4
3	1.18822552				1.17488020			
4	1.19210373	0.039323787			1.17755938	0.033093929		
5	1.19302927	0.040440343	-0.005553423	0.005422654	1.17912257	0.033836674	-0.007447137	0.003774482
6	1.19351925	0.041618102	0.007346055	0.005518846	1.17841936	0.034596845	0.004289741	0.003836346
7	1.19381936	0.042397350	0.004996108	0.005610336	1.17858530	0.035037188	0.003203613	0.003904656
8	1.19396897	0.042806552	0.004886525	0.005550039	1.17865978	0.035245575	0.003230323	0.003958425
9	1.19403962	0.043014857	0.005014405	0.001315561	1.17869239	0.035343349	0.003293725	0.004133559
10	1.19407267	0.043110299	0.005115995	0.006396023	1.17870559	0.035384223	0.003338575	0.004185982
11	1.19408829	0.043158597	0.005205614	0.006499187	1.17871283	0.035403690	0.003375935	0.004286511
12	1.19409553	0.043181935	0.005245530	0.006657932	1.17871553	0.035412394	0.003390352	0.004372569
13	1.19409911	0.043195512	0.005280801	0.006810568	1.17871678	0.035417186	0.003403040	0.004440648
S3	1.19410260	0.043214396	0.005548704	0.010624210	1.17871788	0.035423054	0.003495250	0.004698444
S5	1.19410209	0.043206633	0.005329524	0.011159724	1.17871768	0.035420610	0.003416340	0.004676590

C= 1.2	N=25 μ_1'	μ_2	μ_3	μ_4	N=30 μ_1'	μ_2	μ_3	μ_4
3	1.29028662				1.27603005			
4	1.29255420	0.045241370			1.27756024	0.037820883		
5	1.29325361	0.046279249	0.009000970	0.006852052	1.27799677	0.038552962	0.005294537	0.004725111
6	1.29370361	0.047626409	0.006314566	0.006925228	1.27826280	0.039428084	0.004155598	0.004774628
7	1.29395391	0.048606745	0.006132726	0.007110040	1.27839409	0.039971437	0.004080226	0.004879466
8	1.29406781	0.049098230	0.006354484	0.007359950	1.27844770	0.040209142	0.004196291	0.005018654
9	1.29411694	0.049305641	0.006542822	0.007655089	1.27846893	0.040299398	0.004281476	0.005182478
10	1.29413783	0.049388634	0.005663131	0.008006455	1.27847725	0.040332448	0.004331455	0.005373261
11	1.29414684	0.049425233	0.006740836	0.008402513	1.27848058	0.040346058	0.004351125	0.005574103
12	1.29415082	0.049442345	0.006782456	0.008781974	1.27848194	0.040351954	0.004375496	0.005745576
13	1.29415267	0.049451155	0.006808609	0.009077414	1.27848254	0.040354785	0.004383962	0.005861592
S3	1.29415428	0.049460507	0.006852827	0.010115245	1.27848299	0.040357398	0.004396170	0.006104299
S5	1.29414944	0.049480243	0.006843783	0.007966763	1.27848396	0.040435822	0.004393140	0.005631572

C= 1.[illegible]

	N=25				N=30			
	μ_1'	μ_2	μ_3	μ_4	μ_1'	μ_2	μ_3	μ_4
3	1.39244023				1.37738982			
4	1.39356024	0.050940630			1.37814181	0.042420014		
5	1.39415453	0.049275593	0.008166268	0.011196665	1.37852886	0.040844223	0.005318644	0.007178765
6	1.39462249	0.079810180	0.007726745	0.010213655	1.37880331	0.050118426	0.005146168	0.006764984
7	1.39487390	0.057301318	0.007924555	0.009808469	1.37893041	0.046136462	0.005254171	0.006552124
8	1.39497800	0.056390551	0.008207962	0.009217534	1.37897692	0.045793014	0.005390151	0.006172911
9	1.39501731	0.056264562	0.008399354	0.007250697	1.37899284	0.045748117	0.005472414	0.004141002
10	1.39503195	0.056263634	0.008518423	0.026512299	1.37899826	0.045749104	0.005519181	0.009632241
11	1.39503760	0.056277287	0.008589306	0.013685376	1.37900020	0.045754161	0.005544416	0.008133084
12	1.39503992	0.056287040	0.008628293	0.012727880	1.37900094	0.045757343	0.005557017	0.007875519
13	1.39504092	0.056292574	0.008650310	0.012487318	1.37900123	0.045758990	0.005563562	0.007806917
S3	1.39504170	0.056299833	0.008678879	0.012406600	1.37900143	0.045760757	0.005570637	0.007782012
S5	1.39504185	0.056298652	0.008678390	0.012446634	1.37900147	0.045760542	0.005570405	0.007784475

C= 1.4

	N=25				N=30			
3	1.49482195				1.47904122			
4	1.49491698	0.052902159			1.47910717	0.042821888		
5	1.49508670	0.065627585	0.009703521	0.017742669	1.47923816	0.052426247	0.006431409	0.010432415
6	1.49549110	0.063787602	0.009830813	0.013467986	1.47953809	0.051571850	0.006523901	0.008740541
7	1.49605328	0.063618864	0.010129835	0.000561581	1.47988270	0.051605917	0.006673253	0.033396466
8	1.49642563	0.063656399	0.010405400	0.018187987	1.48005997	0.051625350	0.006800343	0.010658839
9	1.49656298	0.063703907	0.010595874	0.016518805	1.48011363	0.051644868	0.006879557	0.010169828
10	1.49660129	0.063733893	0.010716783	0.016131596	1.48012675	0.051655833	0.006924712	0.010044382
11	1.49661120	0.063750058	0.010785845	0.016044875	1.48012983	0.051661178	0.006947786	0.010018567
12	1.49661387	0.063758206	0.010822500	0.016064615	1.48013059	0.051663621	0.006958844	0.010026875
13	1.49661468	0.063762222	0.010841733	0.016115587	1.48013081	0.051664722	0.006964130	0.010042996
S3	1.49661504	0.063766133	0.010862966	0.016032397	1.48013089	0.051665627	0.006968969	0.010009732
S5	1.49661625	0.063766080	0.010862875	0.016412389	1.48013097	0.051665621	0.006968968	0.010087798

C= 1.5

	N=25				N=30			
3	1.59749341				1.59100675			
4	1.59502707	0.086005306			1.57860963	0.062693348		
5	1.59941351	0.071930148	0.012192178	0.022749051	1.58203014	0.058108258	0.008063683	0.013422674
6	1.59883413	0.071669203	0.012415220	0.040944451	1.58179397	0.058004867	0.008182599	0.018071301
7	1.59875991	0.071726241	0.012656337	0.021366866	1.58176397	0.058035029	0.008302395	0.013040108
8	1.59875263	0.071800630	0.012914522	0.020650411	1.58176162	0.058067528	0.008423491	0.012803377
9	1.59875531	0.071849047	0.013137104	0.020506572	1.58176276	0.058086239	0.008515972	0.012758538
10	1.59875782	0.071875233	0.013286271	0.020539425	1.58176364	0.058095280	0.008569647	0.012774080
11	1.59875920	0.071888188	0.013365999	0.020630029	1.58176406	0.058099299	0.008594728	0.012806980
12	1.59875984	0.071894297	0.013403998	0.020728420	1.58176424	0.058101011	0.008605405	0.012839499
13	1.59876012	0.071897130	0.013421769	0.020809547	1.58176431	0.058101732	0.008609928	0.012861822
S3	1.59876035	0.071899583	0.013437383	0.021190786	1.58176436	0.058102258	0.008613252	0.012928187
S5	1.59876033	0.071899552	0.013437275	0.020918862	1.58176436	0.058102256	0.008613250	0.012884896

C= 1.6	μ_1'	μ_2 (N=25)	μ_3	μ_4	μ_1'	μ_2	(N=30) μ_3	μ_4
3	1.70047240				1.68327939			
4	1.70358225	0.083656635			1.68467347	0.066377672		
5	1.70145475	0.080462983	0.015432196	0.028207612	1.68384765	0.064976127	0.010105045	0.016818545
6	1.70137423	0.080480854	0.015287194	0.027743539	1.68381382	0.064992849	0.010031366	0.016684091
7	1.70137371	0.080567482	0.014934837	0.026222557	1.68381431	0.065034016	0.009805046	0.016193410
8	1.70138051	0.080641245	-0.006094494	0.026117171	1.68381720	0.065065395	0.011920441	0.016163750
9	1.70138505	0.080687480	0.016843090	0.026184615	1.68381886	0.065082673	0.010628014	0.016191301
10	1.70138730	0.080711425	0.016491274	0.026295364	1.68381959	0.065090543	0.010540764	0.016231182
11	1.70138828	0.080722422	0.016434026	0.026411792	1.68381988	0.065093755	0.010526582	0.016270159
12	1.70138869	0.080727184	0.016430004	0.026517120	1.68381998	0.065095004	0.010526026	0.016302398
13	1.70138885	0.080729212	0.016435072	0.026600759	1.68382002	0.065095485	0.010527365	0.016325300
S3	1.70138896	0.080730716	0.016432246	0.026923287	1.68382004	0.065095785	0.010526419	0.016381478
S5	1.70138895	0.080730701	0.016498695	0.026579061	1.68382004	0.065095784	0.010530596	0.016319147
C= 1.7		N=25					N=30	
3	1.80375394				1.78583949			
4	1.80496477	0.091558481			1.78647777	0.073250003		
5	1.80441634	0.090006279	0.019249013	0.034921425	1.78622754	0.072548616	0.012471773	0.020935819
6	1.80440848	0.090038212	0.017256774	0.033920914	1.78622524	0.072565294	0.010921017	0.020620230
7	1.80441777	0.090093383	0.020198805	0.033205774	1.78622956	0.072592947	0.012821556	0.020392705
8	1.80442513	0.090165013	0.019884296	0.033199707	1.78623252	0.072626377	0.012730897	0.020390279
9	1.80442904	0.090229162	0.019856322	0.033187679	1.78623389	0.072650895	0.012724142	0.020385710
10	1.80443076	0.090264544	0.019872816	0.033165416	1.78623442	0.072662313	0.012730011	0.020376902
11	1.80443144	0.090279194	0.019892861	0.033119157	1.78623461	0.072666365	0.012735910	0.020354729
12	1.80443169	0.090284456	0.019907282	0.032979103	1.78623467	0.072667650	0.012739538	0.020269711
13	1.80443179	0.090286279	0.019915772	0.032081971	1.78623469	0.072668050	0.012741581	0.024048554
S3	1.80443184	0.090287245	0.019927928	0.033145066	1.78623470	0.072668230	0.012743695	0.020352858
S5	1.80443183	0.090287228	0.019923367	0.033206995	1.78623470	0.072668229	0.012743000	0.020391426
C= 1.8		N=25					N=30	
3	1.90732246				1.88856341			
4	1.90805493	0.101251843			1.88906227	0.081143209		
5	1.90780729	0.100386874	0.023528742	0.043173286	1.88894720	0.080754032	0.015129593	0.025936801
6	1.90781185	0.100225714	0.025027186	0.042276252	1.88894966	0.080662247	0.015577806	0.025645846
7	1.90782019	0.100846124	0.023837465	0.042051445	1.88895355	0.080901165	0.015257271	0.025578964
8	1.90782739	0.100619862	0.023821539	0.041521569	1.88895650	0.080846141	0.015253870	0.025370145
9	1.90783147	0.100594971	0.023841422	0.042946116	1.88895791	0.080839209	0.015261071	0.025765872
10	1.90783317	0.100591492	0.023866777	0.042405314	1.88895842	0.080838371	0.015269300	0.025573229
11	1.90783376	0.100592010	0.023889127	0.042328495	1.88895857	0.080838552	0.015275728	0.025657465
12	1.90783395	0.100592834	0.023904440	0.042314989	1.88895861	0.080838757	0.015279545	0.025655227
13	1.90783400	0.100593331	0.023912930	0.042320439	1.88895863	0.080838864	0.015281381	0.025655803
S3	1.90783402	0.100594092	0.023923493	0.042318872	1.88895863	0.080838981	0.015283083	0.025655152
S5	1.90783402	0.100593703	0.023913375	0.042314673	1.88895863	0.080838928	0.015281886	0.025654035

C= 1.9	μ_1'	μ_2 (N=25)	μ_3	μ_4	μ_1'	μ_2	(N=30) μ_3	μ_4
3	2.01115834				1.99172744			
4	2.01165744	0.112036323			1.99200234	0.089793369		
5	2.01153125	0.111547316	0.028274350	0.053188488	1.99194393	0.089575206	0.018098043	0.031962510
6	2.01152291	0.112044987	0.028729782	0.052524035	1.99193940	0.089716481	0.018252877	0.031742097
7	2.01162292	0.111674767	0.028387212	0.052555079	1.99196454	0.089625779	0.018150454	0.031749604
8	2.01155624	0.111665234	0.028368237	0.053121481	1.99195355	0.089623086	0.018143002	0.031857493
9	2.01155227	0.111666272	0.028204880	0.052569198	1.99195248	0.089623527	0.017978293	0.031757694
10	2.01155148	0.111668487	0.028557130	0.052558121	1.99195228	0.089624205	0.018189521	0.031755670
11	2.01155132	0.111670127	0.028490509	0.052562948	1.99195224	0.089624638	0.018178618	0.031757069
12	2.01155130	0.111671009	0.028478986	0.052572460	1.99195223	0.089624840	0.018176488	0.031759489
13	2.01155129	0.111671396	0.028476407	0.052584480	1.99195223	0.089624917	0.018176046	0.031762359
S3	2.01155129	0.111671696	0.028475662	0.052526871	1.99195223	0.089624965	0.018175929	0.031744062
S5	2.01155129	0.111671547	0.028475687	0.052503449	1.99195223	0.089624946	0.018175930	0.031728908

C= 2.0		N=25					N=30	
3	2.11524121				2.09500956			
4	2.11560140	0.123748098			2.09520886	0.099136711		
5	2.11553669	0.123477880	0.033550392	0.065205303	2.09517910	0.099016893	0.021418012	0.039159719
6	2.11557024	0.123593409	0.033730302	0.064796625	2.09518958	0.099057934	0.021480288	0.039018061
7	2.11554860	0.123532994	0.033598677	0.064876695	2.09518382	0.099039408	0.021441425	0.039039120
8	2.11554764	0.123532836	0.033815690	0.064922929	2.09518354	0.099039336	0.021479220	0.039049478
9	2.11554754	0.123531926	0.033640828	0.064816714	2.09518351	0.099038962	0.021452787	0.039028188
10	2.11554756	0.123527758	0.033636307	0.064776804	2.09518352	0.099036320	0.021451878	0.039015931
11	2.11554758	0.123565733	0.033636351	0.064922468	2.09518352	0.099043735	0.021451930	0.039044080
12	2.11554758	0.123542570	0.033637176	0.064870569	2.09518352	0.099041895	0.021452122	0.039037978
13	2.11554758	0.123540785	0.033637967	0.064862328	2.09518352	0.099041620	0.021452281	0.039036818
S3	2.11554758	0.123540636	0.033655906	0.064860773	2.09518352	0.099041572	0.021453050	0.039036545
S5	2.11554758	0.123546658	0.033640314	0.064872002	2.09518352	0.099041841	0.021452566	0.039037561

C= 2.1		N=25					N=30	
3	2.21955153				2.19848997			
4	2.21982074	0.136337239			2.19863919	0.109156542		
5	2.21978902	0.136193640	0.039431134	0.079484488	2.19862465	0.109093102	0.025131586	0.047686621
6	2.21979569	0.136232545	0.039494390	0.079321891	2.19862714	0.109107276	0.025153330	0.047619920
7	2.21979308	0.136213829	0.039445081	0.079369837	2.19862630	0.109101518	0.025138890	0.047632145
8	2.21979307	0.136221930	0.039463580	0.079346834	2.19862630	0.109103355	0.025143355	0.047626896
9	2.21979309	0.136217306	0.039450772	0.079324372	2.19862631	0.109102458	0.025140644	0.047622491
10	2.21979310	0.136216905	0.039448830	0.079341055	2.19862631	0.109102371	0.025140015	0.047625155
11	2.21979311	0.136216812	0.039456359	0.079328801	2.19862631	0.109102352	0.025141503	0.047623374
12	2.21979310	0.136216781	0.039453658	0.079328840	2.19862631	0.109102347	0.025141181	0.047623375
13	2.21979310	0.136216766	0.039453199	0.079328637	2.19862631	0.109102344	0.025141115	0.047623335
S3	2.21979309	0.136216752	0.039453105	0.079328807	2.19862631	0.109102342	0.025141098	0.047623374
S5	2.21979310	0.136216742	0.039453723	0.079328630	2.19862631	0.109102341	0.025141156	0.047623332

C= 2.2	μ_1'	μ_2	N=25 μ_3	μ_4	μ_1'	μ_2	N=30 μ_3	μ_4
3	2.32407122				2.30215117			
4	2.32427717	0.149789184			2.30226535	0.119851299		
5	2.32426291	0.149716445	0.045983742	0.096309780	2.30225881	0.119819254	0.029277119	0.057714103
6	2.32426359	0.149722521	0.045991485	0.096374588	2.30225907	0.119821481	0.029279337	0.057715107
7	2.32426304	0.149716805	0.045968839	0.096371410	2.30225890	0.119819727	0.029272689	0.057713240
8	2.32426303	0.149716832	0.045966240	0.096336473	2.30225889	0.119819731	0.029272036	0.057705164
9	2.32426306	0.149716672	0.045964426	0.096327412	2.30225891	0.119819692	0.029271648	0.057703361
10	2.32426299	0.149716720	0.045964311	0.096325808	2.30225888	0.119819707	0.029271624	0.057703078
11	2.32426298	0.149716493	0.045964314	0.096325947	2.30225888	0.119819662	0.029271625	0.057703095
12	2.32426298	0.149716517	0.045964317	0.096326204	2.30225888	0.119819661	0.029271625	0.057703126
13	2.32426298	0.149716636	0.045964317	0.096325943	2.30225888	0.119819677	0.029271625	0.057703093
S3	2.32426298	0.149716487	0.045964318	0.096326074	2.30225888	0.119819662	0.029271625	0.057703110
S5	2.32426298	0.149716577	0.045964318	0.096326052	2.30225888	0.119819672	0.029271625	0.057703108
C= 2.3			N=25				N=30	
3	2.42878389				2.40597774			
4	2.42894373	0.164102624			2.40606631	0.131225025		
5	2.42893817	0.164066563	0.053267403	0.115987200	2.40606375	0.131209202	0.033889542	0.069426027
6	2.42893643	0.164056930	0.053245639	0.116260753	2.40606309	0.131205663	0.033881361	0.069487171
7	2.42893631	0.164054589	0.053227055	0.116206842	2.40606305	0.131204940	0.033875871	0.069471193
8	2.42893627	0.164053627	0.053219861	0.116167760	2.40606304	0.131204682	0.033874073	0.069462062
9	2.42893598	0.164053583	0.053219168	0.116157331	2.40606300	0.131204670	0.033873918	0.069459970
10	2.42893619	0.164053645	0.053218882	0.116155692	2.40606302	0.131204690	0.033873863	0.069459677
11	2.42893619	0.164053401	0.053219100	0.116156515	2.40606302	0.131204641	0.033873890	0.069459794
12	2.42893619	0.164053465	0.053218971	0.116159274	2.40606302	0.131204648	0.033873877	0.069460042
13	2.42893619	0.164053485	0.053218958	0.116157043	2.40606302	0.131204651	0.033873876	0.069459849
S3	2.42893619	0.164053495	0.053218956	0.116158040	2.40606302	0.131204656	0.033873875	0.069459934
S5	2.42893619	0.164053472	0.053219009	0.116156925	2.40606302	0.131204650	0.033873881	0.069459852
C= 2.4			N=25				N=30	
3	2.53367476				2.50995615			
4	2.53379954	0.179281665			2.51002523	0.143283902		
5	2.53379771	0.179260834	0.061335719	0.138844374	2.51002437	0.143274815	0.039002030	0.083019257
6	2.53379502	0.179243296	0.061294484	0.139311142	2.51002334	0.143268363	0.038986711	0.083133719
7	2.53379490	0.179241046	0.061271674	0.139210351	2.51002330	0.143267661	0.038979942	0.083104527
8	2.53379487	0.179240111	0.061263728	0.139164873	2.51002329	0.143267409	0.038977945	0.083093805
9	2.53379570	0.179240040	0.061262775	0.139152761	2.51002318	0.143267390	0.038977732	0.083091356
10	2.53379477	0.179239928	0.061262779	0.139152514	2.51002327	0.143267369	0.038977732	0.083091297
11	2.53379478	0.179239955	0.061262459	0.139154296	2.51002327	0.143267373	0.038977644	0.083091549
12	2.53379478	0.179239958	0.061264371	0.139139064	2.51002327	0.143267373	0.038977843	0.083095063
13	2.53379479	0.179239967	0.061263152	0.139157830	2.51002327	0.143267375	0.038977786	0.083091875
S3	2.53379477	0.179239953	0.061263627	0.139147472	2.51002327	0.143267373	0.038977799	0.083093392
S5	2.53379477	0.179239925	0.061262768	0.139152731	2.51002327	0.143267371	0.038977721	0.083091363

C= 2.5	μ_1'	μ_2	N=25 μ_3	μ_4	μ_1'	μ_2	N=30 μ_3	μ_4
3	2.63873059				2.61407451			
4	2.63882756	0.195332606			2.61412810	0.156034806		
5	2.63882659	0.195313067	0.070238784	0.165229998	2.61412764	0.156026293	0.044645844	0.098703625
6	2.63882359	0.195291128	0.070180303	0.165878379	2.61412649	0.156018209	0.044624211	0.098865589
7	2.63882340	0.195288054	0.070150610	0.165733250	2.61412643	0.156017244	0.044615368	0.098823697
8	2.63882339	0.195287287	0.070142557	0.165678654	2.61412642	0.156017035	0.044613329	0.098810732
9	2.63882346	0.195287214	0.070141585	0.165666700	2.61412646	0.156017016	0.044613112	0.098808288
10	2.63882318	0.195287181	0.070141951	0.165668298	2.61412638	0.156017008	0.044613178	0.098808542
11	2.63882326	0.195287167	0.070141553	0.165672685	2.61412639	0.156017005	0.044613093	0.098809138
12	2.63882328	0.195287168	0.070141338	0.165663330	2.61412639	0.156017006	0.044613058	0.098806534
13	2.63882329	0.195286894	0.070146890	0.165643380	2.61412640	0.156017112	0.044613442	0.098802085
S3	2.63882331	0.195287167	0.070141545	0.165680944	2.61412640	0.156017005	0.044613090	0.098812816
S5	2.63882331	0.195287157	0.070141744	0.165667704	2.61412640	0.156017004	0.044613132	0.098808377
C= 3.0			N=25				N=30	
3	3.16610786				3.13643533			
4	3.16611029	0.288908480			3.13643630	0.230401314		
5	3.16609888	0.288795279	0.128907033	0.363718661	3.13643106	0.230351177	0.081895779	0.216737565
6	3.16609592	0.288759709	0.128716506	0.365239398	3.13642991	0.230337913	0.081825293	0.217084965
7	3.16609563	0.288754552	0.128656545	0.364865039	3.13642980	0.230336264	0.081807095	0.216974165
8	3.16609553	0.288754791	0.128653210	0.364790531	3.13642978	0.230336326	0.081806206	0.216955796
9	3.16609554	0.288754130	0.128660422	0.364822194	3.13642978	0.230336105	0.081807703	0.216962036
10	3.16609554	0.288754064	0.128577017	0.364872874	3.13642978	0.230336158	0.081814960	0.216969930
11	3.16609564	0.288756699	0.128655744	0.365049555	3.13642980	0.230336664	0.081807661	0.216978784
12	3.16609564	0.288756168	0.128664972	0.364847616	3.13642980	0.230336615	0.081808520	0.216958148
13	3.16609564	0.288756161	0.128664898	0.364869462	3.13642980	0.230336615	0.081808516	0.216969906
S3	3.16609564	0.288756161	0.128664899	0.364867329	3.13642980	0.230336615	0.081808516	0.216969656
S5	3.16609564	0.288756181	0.128664758	0.364920222	3.13642980	0.230336616	0.081808505	0.216971743
C= 4.0			N=25				N=30	
3	4.22842815				4.18751208			
4	4.22824027	0.544770088			4.18740729	0.434168544		
5	4.22820616	0.544321868	0.334313886	1.282774408	4.18739143	0.433967353	0.213079078	0.766682601
6	4.22820545	0.544292128	0.333848562	1.288490577	4.18739114	0.433955999	0.212901027	0.767902732
7	4.22820127	0.544318924	0.333973711	1.288908214	4.18738726	0.433964212	0.212938296	0.767997464
8	4.22820536	0.544388930	0.334152934	1.290081319	4.18739135	0.433973713	0.212980472	0.768273124
9	4.22820762	0.544322506	0.334142551	1.290752309	4.18739183	0.433965598	0.212978086	0.768391604
10	4.22820762	0.544324994	0.334092740	1.290165049	4.18739183	0.433966016	0.212969736	0.768304736
11	4.22820762	0.544325396	0.334093784	1.290136883	4.18739183	0.433966106	0.212969860	0.768297604
12	4.22820760	0.544316376	0.334096945	1.290137010	4.18739182	0.433965354	0.212970621	0.768297582
13	4.22820758	0.544322678	0.334172581	1.290130775	4.18739182	0.433965652	0.212959900	0.768296456
S3	4.22820764	0.544320086	0.334093646	1.290136885	4.18739184	0.433965567	0.212969911	0.768297604
S5	4.22820760	0.544323445	0.334093628	1.290135773	4.18739183	0.433965830	0.212969907	0.768297183

refer to the Borel-Padé type algorithm with $\int_0^\infty e^{-t} t^{\alpha-1} dt$ as the basic kernel, with s terms truncated followed by a Shanks' formula.

3.5 Analysis of Algorithmic Results Guidelines for the analysis of the algorithms used are now considered.

(1) c=1. From Table 11 for c=1, n=25 we see that the Levin algorithm yields monotonic increasing, decreasing, and increasing sequences for $\mu_1'(c^*)$, $\mu_2(c^*)$, and $\mu_3(c^*)$; the μ_4 sequence is mixed, but the last three values are increasing. Now the differences for the last four $\mu_1'(c^*)$ values are 2.6E-05, 1.7E-05, and 7.1E-06 and are quite small. Apply Shanks to find the approximant S_5 = 1.0950843. For a Borel-Padé (2cB) algorithm with a=1, tr=1, we find for S_5 the value 1.09506. Our preferred assessment of $\mu_1'(c^*)$ is therefore 1.0951, with error of at most a digit or so in the 4th d.p.. Similarly for $\mu_2(c^*)$ we have L(tr=0, S_5) = 0.037494, and 2cB(a=3, tr=1; S_5) = 0.0375240, with preferred value 0.0375, and $\sigma(c^*)$ = 0.1936. Similarly

$\mu_3(c^*)$: L(tr=0, S_5) = 0.004032

2cB (a=4, tr=1, S_5) = 0.004052

The preferred value is 0.00404 leading to $\sqrt{\beta_1} \sim 0.5568$.

$\mu_4(c^*)$: L(tr=0, S_3) = 0.005301,

2cB(a=5, tr=1, S_5) = 0.005319.

Preferred Values $\mu_4(c^*) \sim 0.00531$, $\beta_2(c^*) \sim 3.7798$.

Without going into detail for n=30 we have

n=30 c=1.0

$\mu_1'(c^*)$: $L(tr=0, S_5)$ = 1.079897 (c.p.)

$2cB(a=1, tr=1, S_5)$ = 1.079883

Preferred value 1.0799

$\mu_2(c^*)$: $L(tr=0, S_5)$ = 0.0309104

$2cB(a=3, tr=1, S_3)$ = 0.0309210 (c.p.)

Preferred value 0.03091 ($\sigma \sim 0.1758$)

$\mu_3(c^*)$: $L(tr=0, S_5)$ = 0.00259832

$2cB(a=4, tr=1, S_5)$ = 0.00260685 (c.p.)

Preferred value 0.002607 ($\sqrt{\beta_1} \sim 0.4798$)

$\mu_4(c^*)$: $L(tr=0, S_3)$ = 0.00343807

$2cB(a=5, tr=1, S_5)$ = 0.00334097 (c.p.)

Preferred value 0.00334 ($\beta_2 \sim 3.4968$)

(c.p. means 'closer packed'.)

(2) c=1.3

Similarly here we have:

	c=1.3, n=25	c=1.3, n=30
$\mu_1'(c^*)$;	$L(tr=0, S_5) = 1.395042$(c.p.)	$L(tr=0, S_5) = 1.379002$(c.p.)
	$2cB(a=1, tr=1, S_5) = 1.395031$	$2cB(a=1, tr=1, S_5) = 1.378998$
	Preferred 1.39504 (1.3950)	1.37900 (1.3790)
$\mu_2(c^*)$;	$L(tr=0, S_3) = 0.0562998$	$L(tr=0, S_5) = 0.0457605$ (c.p)
	$2cB(a=1, tr=1, S_3) = 0.0562816$	$2cB(a=1, tr=1, S_3) = 0.0457548$
	Preferred 0.05628	0.04576
	$\sigma \sim 0.2372$ (0.2373)	$\sigma \sim 0.2139$ (0.2139)
$\mu_3(c^*)$;	$L(tr=0, S_5) = 0.00867839$	$L(tr=0, S_5) = 0.0055704$
	$2cB(a=1, tr=1, S_3) = 0.0086752$ (c.p.)	$2cB(a=1, tr=1, S_5) = 0.0055716$ (c.p.)
	Preferred 0.008675	0.005563
	$\sqrt{\beta_1} \sim 0.6500$ (0.6477)	$\sqrt{\beta_1} \sim 0.5684$ (0.5684)
$\mu_4(c^*)$;	$L(tr=0, S_5) = 0.0124466$	$L(tr=0, S_3) = 0.00778201$
	$2cB(a=4, tr=1, S_3) = 0.0124674$ (c.p.)	$2cB(a=4, tr=1, S_3) = 0.00779953$ (c.p.)
	Preferred 0.01247	0.007800
	$\beta_2 \sim 3.9369$ (3.9406)	$\beta_2 \sim 3.7250$ (3.7284)

(the entries in parentheses are from the Table 14).

(3) <u>c=2</u>

For c > 1.7 or so, the magnitude of the coefficients has decreased considersably, and a string of positive terms occurs initially. As an example here, take c=2.

c=2

	n=25	n=30
$\mu_1'(c^*)$;	DS(10) = 2.115547	DS(11) = 2.095184
	Levin(+) = 2.115548	Levin(+) = 2.095184
	Preferred 2.115548(2.115548)	2.095184(2.095184)
$\mu_2(c^*)$;	DS(6) = 0.123533	DS(6) = 0.0990395
	L(tr=0, S_3) = 0.123541	L(tr=0, S_3) = 0.0990416
	Preferred 0.12354	0.099040
	$\sigma \sim$ 0.3515 (0.3515)	$\sigma \sim$ 0.3147 (0.3147)
$\mu_3(c^*)$;	DS(7) = 0.0336296	DS(6) = 0.0214505
	Levin(+) = 0.0336380	L(tr=0, S_3) = 0.0214531
	Preferred 0.03363	0.021451
	$\sqrt{\beta_1} \sim$ 0.7745 (.7747)	$\sqrt{\beta_1} \sim$ 0.6882(.6883)
$\mu_4(c^*)$;	DS(10) - 0.064839	DS(10) = 0.0390333
	L(tr=0, S_3) = 0.064861	L(tr=0, S_3) = 0.0390365
	Preferred 0.06485	0.390350
	$\beta_2 \sim$ 4.2491 (4.2498)	$\beta_2 \sim$ 3.9811 (3.9796)

(DS = direct sum to numerically smallest term, and the entries in parenthsis are from the Table 14; + refers to Levin's algorithm using all terms.)

(4) c=1.5. For $\mu_1'(c^*)$ the sign pattern is ++++-+-+-+-+- so we truncate three terms and consider

$$\mu_1'(c^*) \sim c_0^{(1)} + c_1^{(1)}/n + c_2^{(1)}/n^2 + f_1(n)/n^3.$$

Now, approximately $|c_s^{(1)}/c_{s-1}^{(1)}| = 9(s-2)$, so this suggests a 1cB algorithm. In particular

$$f_1(n) = \int_0^\infty e^{-t}\, t^3 \left\{ d_0^{(1)} - \frac{9d_1^{(1)}t}{n} + \frac{9^2 d_2^{(1)} t^2}{n^2} - \ldots \right\} dt$$

where

s	$d_s^{(1)}$		$K_s^{(1)}$
0	4.495216963	00	4.495216963
1	1.637598221	00	2.857618742
2	9.870542007	-01	2.207074722
3	6.268808851	-01	1.916704017
4	4.331877790	-01	1.792813522
5	3.185504359	-01	1.756347473
6	2.466170192	-01	1.770954034
7	1.994167609	-01	1.818252138
8	1.674392979	-01	1.888321120
9	1.453389437	-01	1.975536048

and $K_s^{(1)} = d_0^{(1)} - \binom{s}{1} d_1^{(1)} + \binom{s}{2} d_2^{(1)} - \ldots$. The Borel dilution almost stabalizes the series in the sense of producing modified coefficients $\left(d_s^{(1)}\right)$ approximately polynomial in form. Approximants to $\mu_1'(c^*)$ are now found from.

$$F_r^{(1)}(n) = c_0^{(1)} + c_1^{(1)}/n + c_2^{(1)}/n + G_r^{(1)}(n)/n^3$$

where, with N = n/9,

$$G_2^{(1)}(N) = Na_1^{(2)} + (B_0^{(2)} - NB_1^{(2)})W_3(N),$$

$$G_3^{(2)}(N) = Na_1^{(3)} + N^2a_2^{(3)} + (B_0^{(3)} - NB_1^{(3)} + \frac{N^2B_2^{(3)}}{2!})W_3(N)$$

and so on, the unknown $a_1^{(2)}, B_0^{(2)}, B_1^{(2)}$ for example being determined by the equivalence of powers of N in the expressions and $F_r^{(1)}(n)$, with

$$W_3(N) = \int_0^\infty \frac{e^{-t}\, t^3\, dt}{1 + t/N} .$$

For example

$B_0^{(3)} = 37.996439145$ $\qquad a_1^{(3)} = -56.873057438$

$B_1^{(3)} = -13.892992349$ $\qquad a_2^{(3)} = -6.621224165;$

$B_2^{(3)} = 2.207074722.$

The coefficients in the approximations are independent of n (and N). Examples are given in Table 12a. We conclude with the preferred values $\mu_1'(c^*) \sim 1.639528$, and 1.624777 for n = 18 and 20 respectively, the preferences depending on the smallness of the later differences.

For $\mu_2(c^*)$, the sign pattern is +++-+-+-+-+- so it is suggested to use Levin in several truncated versions (Table 12b) to avoid the consecutive plus signs. $L(tr=3, S_5)$ is clearly preferred, giving $\sigma(c^*) \sim 0.3286$. If truncation is pursued to four terms, smoothness in the approximation sequences is lost, although the final term is consistent with $L(tr=3, S_5)$.

For $\mu_3(c^*)$ the sign pattern is ++-+-+-+-+- (Table 12c) and from the Levin approximants we have some confidence in the assessments 0.03153 to 0.03157. L(tr=2) has smaller differences so we opt for $\mu_3(c^*) \sim 0.03157$, with $\sqrt{\beta_1} \sim 0.8895$. The tabulated value (Table 14) is 0.8830, which was computed from the last term in the sequence for L(tr=0).

In Table (12d) approximants to $\mu_4(c^*)$ are given for n=18. Since the differences for 1cB (the second column) are monotonic and relatively small we prefer the assessment 0.05460 ($\beta_2 \sim 4.6812$); Table 14 gives $\beta_2 \sim 4.5722$.

TABLE 12a

APPROXIMANTS TO $\mu_1'(c^*)$, c=1.5

r	n=18 $F_r^{(1)}(n)$	Δ	n=20 $F_r^{(1)}(n)$	Δ	r	n=18 L(tr=0)	Δ	n=20 L(tr=0)	Δ
3	1.638984	171	1.624422	119	6	1.639905	-374	1.625000	-222
4	1.639155	106	1.624541	82	7	1.639531	- 49	1.624778	- 28
5	1.639261	69	1.624613	46	8	1.639482	10	1.624750	7
6	1.639330	48	1.624659	31	9	1.639492	14	1.624757	9
7	1.639378	35	1.624690	21	10	1.639506	10	1.624766	5
8	1.639413	25	1.624711	16	11	1.639516	6	1.624771	3
9	1.639438	19	1.624727	11	12	1.639522	3	1.624774	1
10	1.639457		1.624738		13	1.639525		1.624775	
S_3	1.639513		1.624769			1.639528		1.624777	
S_5	1.639522		1.624774			1.639528		1.624777	

($F_r^{(1)}(n)$ refers to 1cB (a=4, tr=3, N=n/9))

TABLE 12b

APPROXIMANTS TO $\mu_2(c^*)$, c=1.5, n=18

r	$F_r^{(1)}(n)$	Δ	r	L(tr=0)	Δ	L(tr=2)	Δ	L(tr=3)	Δ
3	.105506	595	5	.106934	145				
4	.106101	422	6	.107079	303	.107332	351	.107405	468
5	.106523	310	7	.107382	242	.107683	144	.107873	-3
6	.106833	233	8	.107624	159	.107827	85	.107870	86
7	.107066	180	9	.107783	95	.107912	41	.107956	13
8	.107246	139	10	.107878	53	.107953	20	.107969	13
9	.107385	109	11	.107931	29	.107973	10	.107982	7
10	.107494		12	.107960		.107983		.107989	
S_3	.107899			.107995		.107995		.107995	
S_5	.107956			.107994		.107987		.107997	

($F_r^{(1)}(n)$ refers to 1cB (a=4, tr=2, N=n/9))

TABLE 12c

APPROXIMANTS TO 100 $\mu_3(c^*)$, c=1.5, n=18

r	$F_r^{(3)}(n)$	Δ	r	L(tr=0)	Δ	L(t =1)	Δ	L(tr=2)	Δ
3	2.448	131	3	2.669	82	2.697	96		
4	2.579	102	4	2.751	94	2.793	115	2.848	149
5	2.681	80	5	2.845	97	2.908	99	2.997	69
6	2.761	65	6	2.942	83	3.007	68	3.066	46
7	2.826	52	7	3.025	56	3.075	37	3.112	18
8	2.878	42	8	3.081	33	3.112	19	3.130	8
9	2.920	35	9	3.114	18	3.131	11	3.142	6
10	2.955		10	3.132		3.142		3.148	
S_3	3.117			3.155		3.156		3.157	
S_5	3.145			3.154		3.154		3.157	
$\sqrt{\beta_1}$~	0.8866			0.8887		0.8888		0.8895	

($F_r^{(3)}(n)$ refers to 1cB (a=4, tr=1, N=n/9))

TABLE 12d

APPROXIMANTS TO 100 $\mu_4(c^*)$, c=1.5, n=18

r	$F_r^{(4)}(n)$	Δ	r	L(tr=1)	Δ	L(tr=2)	Δ	L(tr=3)	Δ
3	5.019	67	6	5.145	-40	5.060	47	5.048	117
4	5.086	58	7	5.105	54	5.107	107	5.165	146
5	5.144	49	8	5.159	81	5.214	92	5.311	59
6	5.193	42	9	5.240	78	5.306	72	5.370	63
7	5.235	35	10	5.318	64	5.378	51	5.433	31
8	5.270		11	5.382		5.429		5.464	
S_3	5.473			5.674		5.547		5.493	
S_5	5.460								
β_2~	4.6849							4.8813	

($F_r^{(4)}(n)$ refers to lcB (a=6, tr=3, N=n/9))

(5) c=2.5. For this case with n=20 we have:

(a) μ_1': 2cB(a=1, tr=1) 2cB(a=1, tr=0) 2cB(a=2, tr=0)

r	2cB(a=1, tr=1) F_r	Δ	2cB(a=1, tr=0) F_r	Δ	2cB(a=2, tr=0) F_r	Δ
9	2.676077		2.668530		2.658901	
		235		1766		3744
10	2.676312		2.670296		2.662645	
		172		1314		2785
11	2.676484		2.671610		2.665430	
		129		999		2117
12	2.676613		2.672609		2.667547	
		100		775		1641
13	2.676713		2.673384		2.669188	
S_5	2.67712		2.67864		2.67606	

The preferred value is 2.67712 *because of the small differences in* 2cB (a=1, tr=1)*: this agrees with the direct sum assessment* DS(6).

	Direct Sum Term
0	2.5
1	0.160540342
2	0.014783985
3	0.001643724
4	0.000199398
5	0.000018567
6	0.000000120
7	-0.000000521**
DS(6) = 2.677187	

(** stop at previous term.)

(b) $\mu_2(c^*)$: <u>2cB(a=2, tr=0)</u> <u>2cB(a=1, tr=1)</u>

	2cB(a=2, tr=0)		2cB(a=1, tr=1)	
r	F_r	Δ	F_r	Δ
9	0.2493811	17514	0.2587758	3567
10	0.2511325	14470	0.2591325	2663
11	0.2525795	11530	0.2593988	2042
12	0.2537325	9340	0.2596630	1600
13	0.2546665		0.2597630	
S_5	0.259590		0.260476	

$\sigma \sim 0.5095$ $\sigma \sim 0.5104$

DS(9) = 0.260625 (n^{-10} term 0.0000007)

$\sigma \sim 0.5104$

The preferred value is $\sigma \sim 0.5104$; *Table 14 gives* $\sigma \sim 0.5105$.

(c) $\mu_3(c^*)$: <u>2cB(a=1, tr=0)</u> <u>2cB(a=1, tr=1)</u>

	2cB(a=1, tr=0)		2cB(a=1, tr=1)	
r	F_r	Δ	F_r	Δ
9	0.1140509	17471	0.1214065	7610
10	0.1157980	13752	0.1221675	5711
11	0.1171732	11058	0.1227386	4420
12	0.1182790	9049	0.1231806	3496
13	0.1191839		0.1235302	
S_5	0.124287		0.125184	

$\sqrt{\beta_1} \sim 0.9347$ $\sqrt{\beta_1} \sim 0.9415$

DS(10) = 0.125625 $\sqrt{\beta_1} \sim 0.9442$

The preferred value is $\sqrt{\beta_1} \sim 0.9415$; *Table 14 gives* 0.9440

(c) $\mu_4(c^*)$: 2cB(a=1, tr=0) 2cB(a=1, tr=1)

r	F_r (a=1, tr=0)	Δ	F_r (a=1, tr=1)	Δ
9	0.2630947		0.2966641	
		76628		45608
10	0.2707575		0.3012249	
		62896		35777
11	0.2770471		0.3048026	
		52531		28778
12	0.2823002		0.3070804	
		44498		23608
13	0.2867500		0.3100412	
S_5	0.318118		0.323717	

$\beta_2 \sim 4.6887$ $\beta_2 \sim 4.7712$

DS(12) = 0.3283169 (n^{-12} term 0.00003)

$\beta_2 \sim 4.8390$

The preferred value is $\beta_2 \sim 4.7712$; *Table 14 gives* 4.8307.

Comment We can see what difference the preferred values given here make on the 4-moment Pearson percentage points, compared to those derived from Table 14 (which are based on Levin's algorithm).

	Percentile 1	5	10	90	95	99
From 2cB	1.7857	1.9787	2.0953	3.3444	3.6117	4.2070
From Table 14	1.7804	1.9776	2.0956	3.3430	3.6103	4.2086

The differences are quite small. Note that the moment series in this case are predominantly one-signed and this creates problems with the choice of the summatory algorithm. Further comments on this are given in Appendix C.

3.6 Summary comments on the moments of c* Simulation results for the moments of c* using (2.16) are given in Table 13, showing very satisfactory agreement to the values in Table 14. The first four moment parameters $(\mu_1'(c^*), \sigma(c^*), \sqrt{\beta_1}(c^*), \beta_2(c^*))$ for c = 0.9(0.1)2.6(0.2) 3.2, 3.5, 3.8, 4.0, and n = 13(1)50(5)100 are given in Table 14. These were calculated using the last term in the Levin (t-algorithm) sequence for the moments μ_1, μ_2, μ_3, and μ_4. Since at this time we do not have a computer program to decide an optimum strategy over the algorithmic choices Levin, Borel, Padé, direct-summation (each, except the last, capable of several versions through truncation devices), we have used Levin's t-algorithm involving all the coefficients available (i.e. up to the n^{-12} term). In addition we have looked at the comparison (Table 15a) of 4-moment approximations to the percentage points of v*, and those derived from c* using transformation (1.1), and the converse structure (Table 15b) using (2.15). For samples of about 20 or more the agreement is good, but deteriorates especially at the 1% level of c* for smaller n ($10 \leq n < 20$). Simulation results (10^5 runs) provide strong evidence of satisfactory series summation procedures.

Altogether we think Pearson 4-moment fits in Tables 15a, 15b for $n \geq 25$ are correct to within a few digits in the last decimal place. For $13 < n < 25$ we would guess a few digits error in the third decimal place for $\mu_1'(c^*)$, and $\sigma(c^*)$; for $\sqrt{\beta_1}(c^*)$ and $\beta_2(c^*)$ accuracy within a digit or so is expected in the second decimal place.

TABLE 13

MOMENTS OF C* BY SERIES (LEVIN) AND SIMULATION (10^5 RUNS)

c	n		$\mu_1'(c^*)$	$\sigma(c^*)$	$\sqrt{\beta_1}(c^*)$	$\beta_2(c^*)$
1.0	20	L	1.1177	0.2188	0.6645	3.5796
		S	1.1174	0.2183	0.6751	4.1027
1.5	20	L	1.6248	0.3072	0.8166	4.3727
		S	1.6247	0.3068	0.8240	4.6079
2.0	20	L	2.1470	0.4050	0.9035	4.7211
		S	2.1472	0.4048	0.9106	4.8945
	25	L	2.1155	0.3515	0.7747	4.2498
		S	2.1156	0.3511	0.7705	4.2502
2.5	20	L	2.6772	0.5105	0.9440	4.8307
		S	2.6777	0.5105	0.9512	5.0155
3.0	20	L	3.2123	0.6214	0.9601	4.8627
		S	3.2130	0.6215	0.9669	5.0429

Table 14. Theoretical moment assessments of c* Using Levins's algorithm

$\mu_1'(c^*)$

n\c	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
13	1.0791	1.1790	1.2805	1.3834	1.4875	1.5926	1.6985	1.8052	1.9126	2.0206	2.1292	2.2382
14	1.0670	1.1664	1.2674	1.3698	1.4733	1.5777	1.6830	1.7890	1.8956	2.0029	2.1106	2.2188
15	1.0565	1.1556	1.2562	1.3581	1.4611	1.5650	1.6697	1.7751	1.8812	1.9877	2.0948	2.2023
16	1.0473	1.1461	1.2464	1.3479	1.4505	1.5540	1.6583	1.7632	1.8687	1.9748	2.0812	2.1881
17	1.0392	1.1377	1.2378	1.3390	1.4413	1.5444	1.6483	1.7528	1.8579	1.9635	2.0695	2.1758
18	1.0320	1.1303	1.2301	1.3311	1.4331	1.5360	1.6395	1.7437	1.8484	1.9535	2.0591	2.1651
19	1.0255	1.1237	1.2233	1.3241	1.4259	1.5285	1.6317	1.7356	1.8399	1.9448	2.0500	2.1555
20	1.0197	1.1177	1.2172	1.3178	1.4194	1.5218	1.6248	1.7284	1.8324	1.9369	2.0418	2.1470
21	1.0144	1.1124	1.2117	1.3122	1.4136	1.5157	1.6185	1.7219	1.8257	1.9299	2.0345	2.1394
22	1.0096	1.1075	1.2067	1.3070	1.4083	1.5103	1.6129	1.7160	1.8196	1.9236	2.0279	2.1326
23	1.0051	1.1030	1.2021	1.3024	1.4035	1.5053	1.6077	1.7107	1.8141	1.9178	2.0220	2.1264
24	1.0011	1.0989	1.1979	1.2981	1.3991	1.5008	1.6031	1.7058	1.8090	1.9126	2.0165	2.1207
25	0.9973	1.0951	1.1941	1.2942	1.3950	1.4966	1.5988	1.7014	1.8044	1.9079	2.0116	2.1155
26	0.9939	1.0916	1.1905	1.2905	1.3913	1.4928	1.5948	1.6973	1.8002	1.9034	2.0070	2.1108
27	0.9907	1.0883	1.1873	1.2872	1.3879	1.4893	1.5912	1.6935	1.7963	1.8994	2.0028	2.1064
28	0.9877	1.0853	1.1842	1.2841	1.3847	1.4860	1.5878	1.6901	1.7927	1.8957	1.9989	2.1024
29	0.9849	1.0825	1.1814	1.2812	1.3818	1.4830	1.5847	1.6868	1.7893	1.8922	1.9953	2.0987
30	0.9823	1.0799	1.1787	1.2785	1.3790	1.4801	1.5818	1.6838	1.7862	1.8890	1.9920	2.0952
31	0.9798	1.0774	1.1762	1.2760	1.3764	1.4775	1.5790	1.6810	1.7833	1.8860	1.9888	2.0919
32	0.9775	1.0751	1.1739	1.2736	1.3740	1.4750	1.5765	1.6784	1.7806	1.8831	1.9859	2.0889
33	0.9754	1.0730	1.1717	1.2714	1.3718	1.4727	1.5741	1.6759	1.7781	1.8805	1.9832	2.0861
34	0.9733	1.0709	1.1697	1.2693	1.3696	1.4705	1.5719	1.6736	1.7757	1.8780	1.9806	2.0834
35	0.9714	1.0690	1.1677	1.2673	1.3676	1.4685	1.5698	1.6715	1.7735	1.8757	1.9782	2.0809
36	0.9695	1.0672	1.1659	1.2655	1.3657	1.4665	1.5678	1.6694	1.7713	1.8735	1.9760	2.0786
37	0.9678	1.0654	1.1641	1.2637	1.3640	1.4647	1.5659	1.6675	1.7693	1.8715	1.9738	2.0764
38	0.9662	1.0638	1.1625	1.2621	1.3623	1.4630	1.5641	1.6657	1.7675	1.8695	1.9718	2.0743
39	0.9646	1.0622	1.1609	1.2605	1.3607	1.4614	1.5625	1.6639	1.7657	1.8677	1.9699	2.0723
40	0.9631	1.0607	1.1595	1.2590	1.3591	1.4598	1.5609	1.6623	1.7640	1.8659	1.9681	2.0704
41	0.9617	1.0593	1.1580	1.2576	1.3577	1.4583	1.5594	1.6607	1.7624	1.8642	1.9663	2.0686
42	0.9603	1.0580	1.1567	1.2562	1.3563	1.4569	1.5579	1.6592	1.7608	1.8627	1.9647	2.0669
43	0.9590	1.0567	1.1554	1.2549	1.3550	1.4556	1.5565	1.6578	1.7594	1.8612	1.9631	2.0653
44	0.9578	1.0555	1.1542	1.2537	1.3537	1.4543	1.5552	1.6565	1.7580	1.8597	1.9617	2.0638
45	0.9566	1.0543	1.1530	1.2525	1.3525	1.4531	1.5540	1.6552	1.7567	1.8584	1.9602	2.0623
46	0.9554	1.0532	1.1519	1.2514	1.3514	1.4519	1.5528	1.6540	1.7554	1.8570	1.9589	2.0609
47	0.9543	1.0521	1.1508	1.2503	1.3503	1.4508	1.5516	1.6528	1.7542	1.8558	1.9576	2.0595
48	0.9533	1.0511	1.1498	1.2492	1.3493	1.4497	1.5505	1.6517	1.7530	1.8546	1.9564	2.0583
49	0.9523	1.0501	1.1488	1.2482	1.3483	1.4487	1.5495	1.6506	1.7519	1.8535	1.9552	2.0570
50	0.9513	1.0491	1.1478	1.2473	1.3473	1.4477	1.5485	1.6496	1.7509	1.8524	1.9540	2.0559
55	0.9470	1.0448	1.1436	1.2430	1.3430	1.4433	1.5440	1.6450	1.7461	1.8475	1.9490	2.0506
60	0.9433	1.0412	1.1400	1.2395	1.3394	1.4397	1.5403	1.6412	1.7422	1.8434	1.9448	2.0463
65	0.9402	1.0382	1.1370	1.2365	1.3364	1.4366	1.5372	1.6379	1.7389	1.8400	1.9413	2.0426
70	0.9375	1.0356	1.1345	1.2339	1.3338	1.4340	1.5345	1.6352	1.7361	1.8371	1.9383	2.0395
75	0.9352	1.0333	1.1322	1.2317	1.3315	1.4317	1.5322	1.6328	1.7336	1.8346	1.9357	2.0368
80	0.9331	1.0313	1.1302	1.2297	1.3296	1.4297	1.5301	1.6307	1.7315	1.8324	1.9334	2.0345
85	0.9313	1.0295	1.1285	1.2280	1.3278	1.4280	1.5284	1.6289	1.7296	1.8305	1.9314	2.0324
90	0.9296	1.0279	1.1269	1.2264	1.3263	1.4264	1.5268	1.6273	1.7280	1.8287	1.9296	2.0306
95	0.9282	1.0265	1.1256	1.2251	1.3249	1.4250	1.5254	1.6258	1.7265	1.8272	1.9280	2.0290
100	0.9268	1.0252	1.1243	1.2238	1.3237	1.4238	1.5241	1.6245	1.7251	1.8258	1.9266	2.0275

$\mu_1^*(c^*)$

n\c	2.1	2.2	2.3	2.4	2.5	2.6	2.8	3.0	3.2	3.5	3.8	4.0
13	2.3477	2.4575	2.5678	2.6783	2.7892	2.9004	3.1235	3.3476	3.5724	3.9110	4.2510	4.4780
14	2.3274	2.4363	2.5457	2.6553	2.7652	2.8754	3.0966	3.3186	3.5413	3.8766	4.2138	4.4380
15	2.3102	2.4184	2.5270	2.6358	2.7449	2.8543	3.0738	3.2940	3.5150	3.8475	4.1812	4.4043
16	2.2954	2.4030	2.5109	2.6191	2.7275	2.8362	3.0542	3.2730	3.4924	3.8226	4.1539	4.3753
17	2.2826	2.3896	2.4970	2.6046	2.7124	2.8205	3.0373	3.2548	3.4729	3.8010	4.1303	4.3502
18	2.2713	2.3779	2.4848	2.5919	2.6992	2.8068	3.0225	3.2388	3.4558	3.7822	4.1096	4.3283
19	2.2614	2.3676	2.4740	2.5807	2.6876	2.7946	3.0094	3.2248	3.4407	3.7655	4.0913	4.3089
20	2.2526	2.3584	2.4644	2.5707	2.6772	2.7839	2.9978	3.2123	3.4273	3.7508	4.0751	4.2917
21	2.2446	2.3501	2.4558	2.5618	2.6679	2.7742	2.9874	3.2011	3.4153	3.7375	4.0605	4.2764
22	2.2375	2.3427	2.4481	2.5537	2.6595	2.7655	2.9780	3.1910	3.4045	3.7256	4.0475	4.2625
23	2.2310	2.3360	2.4411	2.5465	2.6520	2.7577	2.9695	3.1819	3.3948	3.7149	4.0357	4.2500
24	2.2252	2.3299	2.4348	2.5398	2.6451	2.7505	2.9618	3.1736	3.3859	3.7051	4.0250	4.2386
25	2.2198	2.3243	2.4289	2.5338	2.6388	2.7440	2.9548	3.1661	3.3778	3.6961	4.0152	4.2282
26	2.2149	2.3191	2.4236	2.5283	2.6331	2.7380	2.9484	3.1592	3.3704	3.6879	4.0062	4.2187
27	2.2103	2.3144	2.4187	2.5232	2.6278	2.7325	2.9424	3.1528	3.3636	3.6804	3.9979	4.2099
28	2.2061	2.3101	2.4142	2.5184	2.6229	2.7274	2.9370	3.1469	3.3573	3.6735	3.9903	4.2018
29	2.2022	2.3060	2.4100	2.5141	2.6183	2.7227	2.9319	3.1415	3.3515	3.6670	3.9832	4.1943
30	2.1986	2.3023	2.4061	2.5100	2.6141	2.7184	2.9272	3.1364	3.3460	3.6611	3.9767	4.1874
31	2.1953	2.2988	2.4024	2.5062	2.6102	2.7143	2.9228	3.1317	3.3410	3.6555	3.9706	4.1809
32	2.1921	2.2955	2.3990	2.5027	2.6065	2.7105	2.9187	3.1273	3.3363	3.6503	3.9649	4.1749
33	2.1892	2.2924	2.3959	2.4994	2.6031	2.7069	2.9149	3.1232	3.3319	3.6454	3.9596	4.1692
34	2.1864	2.2896	2.3929	2.4963	2.5999	2.7036	2.9113	3.1194	3.3278	3.6409	3.9546	4.1639
35	2.1838	2.2869	2.3901	2.4934	2.5969	2.7005	2.9079	3.1158	3.3239	3.6366	3.9499	4.1590
36	2.1814	2.2843	2.3875	2.4907	2.5941	2.6975	2.9048	3.1124	3.3203	3.6326	3.9455	4.1543
37	2.1791	2.2820	2.3850	2.4881	2.5914	2.6947	2.9018	3.1091	3.3168	3.6288	3.9413	4.1499
38	2.1769	2.2797	2.3826	2.4857	2.5888	2.6921	2.8989	3.1061	3.3136	3.6252	3.9374	4.1457
39	2.1748	2.2776	2.3804	2.4834	2.5865	2.6896	2.8963	3.1032	3.3105	3.6218	3.9336	4.1418
40	2.1729	2.2755	2.3783	2.4812	2.5842	2.6873	2.8937	3.1005	3.3076	3.6186	3.9301	4.1380
41	2.1710	2.2736	2.3763	2.4791	2.5820	2.6850	2.8913	3.0979	3.3048	3.6156	3.9268	4.1345
42	2.1693	2.2718	2.3744	2.4771	2.5800	2.6829	2.8891	3.0955	3.3022	3.6127	3.9236	4.1311
43	2.1676	2.2700	2.3726	2.4753	2.5780	2.6809	2.8869	3.0932	3.2997	3.6099	3.9206	4.1279
44	2.1660	2.2684	2.3709	2.4735	2.5762	2.6790	2.8848	3.0910	3.2973	3.6073	3.9177	4.1249
45	2.1645	2.2668	2.3692	2.4718	2.5744	2.6772	2.8829	3.0888	3.2951	3.6048	3.9150	4.1220
46	2.1630	2.2653	2.3677	2.4702	2.5727	2.6754	2.8810	3.0868	3.2929	3.6024	3.9124	4.1192
47	2.1616	2.2638	2.3662	2.4686	2.5711	2.6737	2.8792	3.0849	3.2908	3.6002	3.9099	4.1165
48	2.1603	2.2625	2.3647	2.4671	2.5696	2.6721	2.8775	3.0830	3.2889	3.5980	3.9075	4.1140
49	2.1590	2.2611	2.3634	2.4657	2.5681	2.6706	2.8758	3.0813	3.2870	3.5959	3.9052	4.1116
50	2.1578	2.2599	2.3621	2.4643	2.5667	2.6691	2.8742	3.0796	3.2852	3.5939	3.9030	4.1092
55	2.1524	2.2543	2.3562	2.4583	2.5604	2.6626	2.8672	3.0721	3.2771	3.5850	3.8933	4.0989
60	2.1479	2.2496	2.3514	2.4533	2.5552	2.6572	2.8614	3.0659	3.2705	3.5777	3.8852	4.0904
65	2.1441	2.2457	2.3473	2.4491	2.5508	2.6527	2.8566	3.0606	3.2649	3.5715	3.8784	4.0832
70	2.1409	2.2423	2.3439	2.4455	2.5471	2.6488	2.8524	3.0562	3.2601	3.5663	3.8727	4.0771
75	2.1381	2.2395	2.3409	2.4424	2.5439	2.6455	2.8488	3.0523	3.2560	3.5617	3.8677	4.0718
80	2.1357	2.2369	2.3383	2.4396	2.5411	2.6426	2.8457	3.0490	3.2524	3.5578	3.8634	4.0672
85	2.1335	2.2347	2.3360	2.4373	2.5386	2.6400	2.8429	3.0460	3.2492	3.5543	3.8595	4.0631
90	2.1316	2.2327	2.3339	2.4351	2.5364	2.6377	2.8405	3.0434	3.2464	3.5512	3.8561	4.0596
95	2.1299	2.2310	2.3321	2.4333	2.5345	2.6357	2.8383	3.0411	3.2439	3.5484	3.8531	4.0564
100	2.1284	2.2294	2.3305	2.4316	2.5327	2.6339	2.8364	3.0390	3.2417	3.5460	3.8504	4.0535

$\sigma(c^*)$

n\c	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
13	0.2580	0.2830	0.3078	0.3328	0.3583	0.3843	0.4107	0.4376	0.4649	0.4927	0.5208	0.5498
14	0.2469	0.2701	0.2932	0.3166	0.3404	0.3647	0.3894	0.4145	0.4401	0.4661	0.4925	0.5195
15	0.2371	0.2589	0.2806	0.3026	0.3250	0.3478	0.3711	0.3948	0.4189	0.4434	0.4683	0.4937
16	0.2285	0.2490	0.2695	0.2903	0.3115	0.3331	0.3551	0.3776	0.4004	0.4237	0.4473	0.4714
17	0.2208	0.2403	0.2597	0.2795	0.2996	0.3201	0.3411	0.3624	0.3842	0.4064	0.4289	0.4518
18	0.2139	0.2324	0.2509	0.2698	0.2890	0.3086	0.3286	0.3490	0.3698	0.3910	0.4126	0.4345
19	0.2076	0.2253	0.2430	0.2610	0.2794	0.2982	0.3174	0.3369	0.3569	0.3773	0.3980	0.4190
20	0.2019	0.2188	0.2358	0.2531	0.2708	0.2888	0.3072	0.3261	0.3453	0.3649	0.3848	0.4050
21	0.1967	0.2129	0.2293	0.2459	0.2629	0.2803	0.2980	0.3162	0.3347	0.3536	0.3728	0.3924
22	0.1919	0.2075	0.2233	0.2393	0.2557	0.2725	0.2896	0.3072	0.3251	0.3433	0.3619	0.3808
23	0.1874	0.2025	0.2177	0.2332	0.2491	0.2653	0.2819	0.2989	0.3162	0.3339	0.3519	0.3703
24	0.1833	0.1979	0.2126	0.2276	0.2429	0.2587	0.2748	0.2912	0.3080	0.3252	0.3427	0.3605
25	0.1795	0.1936	0.2078	0.2224	0.2373	0.2525	0.2681	0.2841	0.3005	0.3172	0.3342	0.3515
26	0.1759	0.1896	0.2034	0.2175	0.2320	0.2468	0.2620	0.2775	0.2934	0.3097	0.3262	0.3431
27	0.1725	0.1858	0.1992	0.2130	0.2270	0.2414	0.2562	0.2714	0.2869	0.3027	0.3189	0.3353
28	0.1694	0.1823	0.1953	0.2087	0.2224	0.2364	0.2509	0.2656	0.2807	0.2962	0.3119	0.3280
29	0.1664	0.1789	0.1917	0.2047	0.2180	0.2317	0.2458	0.2602	0.2750	0.2901	0.3055	0.3211
30	0.1636	0.1758	0.1882	0.2009	0.2139	0.2273	0.2410	0.2551	0.2696	0.2843	0.2994	0.3147
31	0.1609	0.1728	0.1849	0.1973	0.2100	0.2231	0.2366	0.2503	0.2645	0.2789	0.2936	0.3086
32	0.1584	0.1700	0.1818	0.1939	0.2064	0.2192	0.2323	0.2458	0.2596	0.2738	0.2882	0.3029
33	0.1560	0.1673	0.1789	0.1907	0.2029	0.2154	0.2283	0.2415	0.2551	0.2689	0.2831	0.2975
34	0.1537	0.1648	0.1761	0.1877	0.1996	0.2119	0.2245	0.2374	0.2507	0.2643	0.2782	0.2924
35	0.1515	0.1624	0.1734	0.1848	0.1964	0.2085	0.2208	0.2336	0.2466	0.2599	0.2736	0.2875
36	0.1494	0.1601	0.1709	0.1820	0.1934	0.2052	0.2174	0.2299	0.2427	0.2558	0.2692	0.2828
37	0.1475	0.1578	0.1684	0.1793	0.1906	0.2022	0.2141	0.2263	0.2389	0.2518	0.2650	0.2784
38	0.1455	0.1557	0.1661	0.1768	0.1878	0.1992	0.2109	0.2230	0.2354	0.2480	0.2610	0.2742
39	0.1437	0.1537	0.1639	0.1744	0.1852	0.1964	0.2079	0.2198	0.2319	0.2444	0.2571	0.2701
40	0.1420	0.1517	0.1618	0.1721	0.1827	0.1937	0.2050	0.2167	0.2287	0.2409	0.2535	0.2663
41	0.1403	0.1499	0.1597	0.1698	0.1803	0.1911	0.2023	0.2137	0.2255	0.2376	0.2500	0.2626
42	0.1387	0.1481	0.1577	0.1677	0.1780	0.1886	0.1996	0.2109	0.2225	0.2344	0.2466	0.2590
43	0.1371	0.1463	0.1558	0.1656	0.1757	0.1862	0.1970	0.2082	0.2196	0.2313	0.2433	0.2556
44	0.1356	0.1447	0.1540	0.1636	0.1736	0.1839	0.1946	0.2056	0.2168	0.2284	0.2402	0.2523
45	0.1341	0.1431	0.1522	0.1617	0.1715	0.1817	0.1922	0.2030	0.2142	0.2256	0.2372	0.2491
46	0.1327	0.1415	0.1505	0.1599	0.1695	0.1796	0.1899	0.2006	0.2116	0.2228	0.2343	0.2461
47	0.1314	0.1400	0.1489	0.1581	0.1676	0.1775	0.1877	0.1982	0.2091	0.2202	0.2316	0.2432
48	0.1300	0.1385	0.1473	0.1564	0.1658	0.1755	0.1856	0.1960	0.2067	0.2176	0.2289	0.2403
49	0.1288	0.1371	0.1458	0.1547	0.1640	0.1736	0.1835	0.1938	0.2043	0.2152	0.2263	0.2376
50	0.1275	0.1358	0.1443	0.1531	0.1622	0.1717	0.1815	0.1917	0.2021	0.2128	0.2238	0.2349
55	0.1219	0.1295	0.1375	0.1457	0.1543	0.1632	0.1725	0.1820	0.1919	0.2020	0.2123	0.2229
60	0.1170	0.1241	0.1316	0.1393	0.1474	0.1559	0.1647	0.1737	0.1831	0.1927	0.2025	0.2126
65	0.1126	0.1194	0.1264	0.1337	0.1414	0.1495	0.1578	0.1665	0.1754	0.1845	0.1939	0.2036
70	0.1088	0.1151	0.1218	0.1288	0.1361	0.1438	0.1518	0.1600	0.1686	0.1774	0.1864	0.1956
75	0.1053	0.1113	0.1177	0.1243	0.1314	0.1387	0.1464	0.1543	0.1625	0.1710	0.1796	0.1885
80	0.1022	0.1079	0.1140	0.1203	0.1271	0.1341	0.1415	0.1492	0.1571	0.1652	0.1736	0.1821
85	0.0993	0.1048	0.1106	0.1167	0.1232	0.1300	0.1371	0.1445	0.1521	0.1600	0.1681	0.1763
90	0.0967	0.1019	0.1075	0.1134	0.1196	0.1262	0.1331	0.1402	0.1476	0.1552	0.1631	0.1711
95	0.0943	0.0993	0.1046	0.1103	0.1164	0.1227	0.1294	0.1363	0.1435	0.1509	0.1585	0.1663
100	0.0921	0.0969	0.1020	0.1075	0.1134	0.1195	0.1260	0.1327	0.1397	0.1469	0.1543	0.1618

σ(c*)

n\c	2.1	2.2	2.3	2.4	2.5	2.6	2.8	3.0	3.2	3.5	3.8	4.0
13	0.5781	0.6072	0.6367	0.6664	0.6964	0.7266	0.7880	0.8496	0.9121	1.0076	1.1022	1.1559
14	0.5463	0.5737	0.6014	0.6294	0.6576	0.6861	0.7437	0.8020	0.8609	0.9518	1.0405	1.0999
15	0.5192	0.5451	0.5713	0.5977	0.6245	0.6514	0.7060	0.7613	0.8172	0.9013	0.9878	1.0450
16	0.4956	0.5202	0.5452	0.5703	0.5958	0.6214	0.6734	0.7260	0.7793	0.8601	0.9421	0.9969
17	0.4750	0.4985	0.5223	0.5463	0.5706	0.5951	0.6448	0.6952	0.7462	0.8235	0.9020	0.9546
18	0.4567	0.4792	0.5020	0.5250	0.5483	0.5718	0.6195	0.6678	0.7168	0.7911	0.8665	0.9171
19	0.4403	0.4620	0.4839	0.5060	0.5284	0.5510	0.5969	0.6434	0.6905	0.7622	0.8348	0.8836
20	0.4256	0.4464	0.4676	0.4889	0.5105	0.5323	0.5765	0.6214	0.6669	0.7361	0.8062	0.8534
21	0.4122	0.4324	0.4528	0.4734	0.4943	0.5154	0.5581	0.6015	0.6455	0.7125	0.7804	0.8260
22	0.4001	0.4195	0.4393	0.4593	0.4795	0.4999	0.5413	0.5834	0.6261	0.6910	0.7568	0.8011
23	0.3889	0.4078	0.4269	0.4463	0.4659	0.4857	0.5259	0.5668	0.6082	0.6712	0.7352	0.7783
24	0.3786	0.3969	0.4156	0.4344	0.4535	0.4727	0.5118	0.5515	0.5918	0.6531	0.7153	0.7572
25	0.3691	0.3869	0.4050	0.4234	0.4419	0.4607	0.4987	0.5374	0.5766	0.6363	0.6970	0.7378
26	0.3602	0.3776	0.3953	0.4131	0.4312	0.4495	0.4865	0.5242	0.5625	0.6208	0.6799	0.7197
27	0.3520	0.3690	0.3862	0.4036	0.4212	0.4391	0.4752	0.5120	0.5494	0.6063	0.6640	0.7029
28	0.3443	0.3609	0.3777	0.3947	0.4119	0.4293	0.4647	0.5006	0.5371	0.5927	0.6492	0.6873
29	0.3371	0.3533	0.3697	0.3863	0.4032	0.4202	0.4548	0.4900	0.5257	0.5801	0.6353	0.6726
30	0.3303	0.3461	0.3622	0.3785	0.3950	0.4117	0.4455	0.4799	0.5149	0.5682	0.6223	0.6588
31	0.3239	0.3394	0.3552	0.3711	0.3873	0.4036	0.4367	0.4705	0.5047	0.5570	0.6100	0.6458
32	0.3179	0.3331	0.3485	0.3642	0.3800	0.3960	0.4285	0.4616	0.4952	0.5464	0.5984	0.6335
33	0.3122	0.3271	0.3422	0.3576	0.3731	0.3888	0.4207	0.4532	0.4861	0.5364	0.5875	0.6219
34	0.3068	0.3214	0.3362	0.3513	0.3665	0.3820	0.4133	0.4452	0.4775	0.5269	0.5771	0.6109
35	0.3016	0.3160	0.3306	0.3454	0.3603	0.3755	0.4063	0.4376	0.4694	0.5179	0.5672	0.6005
36	0.2967	0.3109	0.3252	0.3397	0.3544	0.3693	0.3996	0.4304	0.4617	0.5094	0.5579	0.5906
37	0.2921	0.3060	0.3201	0.3343	0.3488	0.3635	0.3932	0.4235	0.4543	0.5012	0.5489	0.5811
38	0.2876	0.3013	0.3151	0.3292	0.3435	0.3579	0.3871	0.4170	0.4473	0.4935	0.5404	0.5721
39	0.2834	0.2968	0.3105	0.3243	0.3383	0.3525	0.3813	0.4107	0.4405	0.4860	0.5323	0.5635
40	0.2793	0.2925	0.3060	0.3196	0.3334	0.3474	0.3758	0.4047	0.4341	0.4789	0.5245	0.5553
41	0.2754	0.2884	0.3017	0.3151	0.3287	0.3425	0.3705	0.3990	0.4280	0.4721	0.5171	0.5474
42	0.2716	0.2845	0.2976	0.3108	0.3242	0.3378	0.3654	0.3935	0.4221	0.4656	0.5099	0.5398
43	0.2681	0.2807	0.2936	0.3067	0.3199	0.3333	0.3605	0.3882	0.4164	0.4594	0.5031	0.5326
44	0.2646	0.2771	0.2898	0.3027	0.3158	0.3290	0.3558	0.3832	0.4109	0.4534	0.4965	0.5256
45	0.2613	0.2736	0.2862	0.2989	0.3118	0.3248	0.3513	0.3783	0.4057	0.4476	0.4902	0.5189
46	0.2581	0.2703	0.2826	0.2952	0.3079	0.3208	0.3469	0.3736	0.4007	0.4420	0.4841	0.5124
47	0.2550	0.2670	0.2792	0.2916	0.3042	0.3169	0.3427	0.3691	0.3958	0.4366	0.4782	0.5062
48	0.2520	0.2639	0.2760	0.2882	0.3006	0.3132	0.3387	0.3647	0.3911	0.4315	0.4725	0.5002
49	0.2491	0.2609	0.2728	0.2849	0.2971	0.3096	0.3348	0.3605	0.3866	0.4265	0.4670	0.4944
50	0.2463	0.2579	0.2697	0.2817	0.2938	0.3061	0.3310	0.3564	0.3822	0.4216	0.4617	0.4888
55	0.2337	0.2447	0.2558	0.2672	0.2786	0.2902	0.3139	0.3379	0.3624	0.3997	0.4377	0.4634
60	0.2228	0.2333	0.2439	0.2547	0.2656	0.2766	0.2991	0.3220	0.3453	0.3809	0.4171	0.4416
65	0.2134	0.2233	0.2335	0.2438	0.2542	0.2648	0.2863	0.3082	0.3305	0.3645	0.3992	0.4225
70	0.2050	0.2146	0.2243	0.2342	0.2442	0.2543	0.2750	0.2960	0.3174	0.3501	0.3833	0.4058
75	0.1975	0.2067	0.2161	0.2256	0.2353	0.2450	0.2649	0.2851	0.3057	0.3372	0.3692	0.3909
80	0.1908	0.1997	0.2088	0.2179	0.2272	0.2367	0.2558	0.2754	0.2953	0.3257	0.3566	0.3775
85	0.1848	0.1934	0.2021	0.2110	0.2200	0.2291	0.2477	0.2665	0.2858	0.3152	0.3452	0.3654
90	0.1793	0.1876	0.1961	0.2047	0.2134	0.2222	0.2402	0.2586	0.2772	0.3057	0.3348	0.3544
95	0.1742	0.1823	0.1905	0.1989	0.2074	0.2159	0.2334	0.2512	0.2694	0.2970	0.3252	0.3443
100	0.1696	0.1774	0.1854	0.1936	0.2018	0.2102	0.2271	0.2445	0.2621	0.2891	0.3165	0.3350

$\sqrt{\beta_1}(c^*)$

n\c	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
13	0.8235	0.9126	0.9599	1.0085	1.0546	1.0969	1.1341	1.1765	1.2011	1.2226	1.2586	1.2638
14	0.7867	0.8671	0.9110	0.9557	0.9980	1.0365	1.0705	1.1068	1.1302	1.1508	1.1792	1.1885
15	0.7524	0.8255	0.8667	0.9084	0.9476	0.9832	1.0147	1.0467	1.0692	1.0889	1.1126	1.1237
16	0.7205	0.7875	0.8266	0.8658	0.9026	0.9359	0.9655	0.9945	1.0161	1.0350	1.0557	1.0677
17	0.6909	0.7527	0.7901	0.8273	0.8622	0.8938	0.9219	0.9486	0.9695	0.9878	1.0065	1.0189
18	0.6634	0.7208	0.7568	0.7925	0.8259	0.8561	0.8830	0.9081	0.9283	0.9461	0.9634	0.9759
19	0.6377	0.6915	0.7264	0.7609	0.7930	0.8221	0.8481	0.8720	0.8916	0.9089	0.9252	0.9377
20	0.6138	0.6645	0.6985	0.7320	0.7632	0.7914	0.8166	0.8395	0.8587	0.8756	0.8912	0.9035
21	0.5914	0.6396	0.6729	0.7056	0.7360	0.7635	0.7880	0.8102	0.8290	0.8456	0.8606	0.8728
22	0.5704	0.6164	0.6493	0.6814	0.7111	0.7381	0.7620	0.7836	0.80[illegible]	0.8183	0.8328	0.8449
23	0.5508	0.5949	0.6274	0.6590	0.6883	0.7147	0.7382	0.7593	0.7775	0.7934	0.8075	0.8194
24	0.5323	0.5749	0.6071	0.6383	0.6672	0.6932	0.7164	0.7370	0.7549	0.7705	0.7844	0.7961
25	0.5149	0.5563	0.5882	0.6191	0.6477	0.6734	0.6962	0.7165	0.7341	0.7495	0.7631	0.7747
26	0.4985	0.5388	0.5706	0.6013	0.6296	0.6550	0.6775	0.6975	0.7149	0.7301	0.7434	0.7548
27	0.4830	0.5224	0.5541	0.5847	0.6127	0.6379	0.6602	0.6799	0.6971	0.7120	0.7251	0.7364
28	0.4683	0.5070	0.5387	0.5691	0.5969	0.6219	0.6440	0.6635	0.6805	0.6952	0.7082	0.7193
29	0.4544	0.4925	0.5242	0.5545	0.5822	0.6070	0.6289	0.6482	0.6650	0.6796	0.6923	0.7033
30	0.4412	0.4788	0.5106	0.5408	0.5684	0.5930	0.6148	0.6339	0.6504	0.6649	0.6774	0.6883
31	0.4287	0.4659	0.4977	0.5279	0.5554	0.5799	0.6015	0.6204	0.6368	0.6511	0.6635	0.6742
32	0.4168	0.4536	0.4855	0.5157	0.5431	0.5675	0.5890	0.6077	0.6240	0.6381	0.6503	0.6609
33	0.4054	0.4420	0.4740	0.5042	0.5315	0.5558	0.5772	0.5957	0.6119	0.6258	0.6379	0.6484
34	0.3946	0.4310	0.4631	0.4933	0.5206	0.5448	0.5660	0.5844	0.6004	0.6142	0.6262	0.6365
35	0.3842	0.4205	0.4527	0.4830	0.5102	0.5343	0.5554	0.5737	0.5896	0.6033	0.6151	0.6253
36	0.3743	0.4106	0.4429	0.4731	0.5004	0.5244	0.5454	0.5636	0.5793	0.5928	0.6045	0.6146
37	0.3649	0.4010	0.4335	0.4638	0.4910	0.5150	0.5358	0.5539	0.5695	0.5829	0.5945	0.6045
38	0.3558	0.3920	0.4246	0.4549	0.4821	0.5060	0.5268	0.5447	0.5602	0.5735	0.5850	0.5949
39	0.3471	0.3833	0.4160	0.4464	0.4736	0.4974	0.5181	0.5359	0.5513	0.5645	0.5759	0.5857
40	0.3388	0.3750	0.4079	0.4383	0.4655	0.4892	0.5098	0.5276	0.5428	0.5559	0.5672	0.5769
41	0.3307	0.3670	0.4001	0.4305	0.4577	0.4814	0.5019	0.5195	0.5347	0.5477	0.5589	0.5685
42	0.3230	0.3594	0.3926	0.4231	0.4503	0.4739	0.4944	0.5119	0.5269	0.5398	0.5509	0.5604
43	0.3156	0.3520	0.3854	0.4160	0.4432	0.4668	0.4871	0.5046	0.5195	0.5323	0.5433	0.5527
44	0.3085	0.3450	0.3785	0.4092	0.4363	0.4599	0.4802	0.4975	0.5124	0.5250	0.5359	0.5453
45	0.3016	0.3382	0.3719	0.4026	0.4298	0.4533	0.4735	0.4908	0.5055	0.5181	0.5289	0.5381
46	0.2949	0.3317	0.3655	0.3963	0.4235	0.4469	0.4671	0.4842	0.4989	0.5114	0.5221	0.5313
47	0.2885	0.3254	0.3594	0.3903	0.4174	0.4408	0.4609	0.4780	0.4925	0.5050	0.5156	0.5247
48	0.2823	0.3194	0.3535	0.3844	0.4116	0.4350	0.4549	0.4720	0.4864	0.4988	0.5093	0.5183
49	0.2763	0.3136	0.3478	0.3788	0.4059	0.4293	0.4492	0.4661	0.4805	0.4928	0.5032	0.5122
50	0.2705	0.3079	0.3424	0.3734	0.4005	0.4238	0.4437	0.4605	0.4748	0.4870	0.4974	0.5062
55	0.2442	0.2825	0.3176	0.3489	0.3760	0.3991	0.4187	0.4352	0.4491	0.4609	0.4709	0.4794
60	0.2216	0.2607	0.2965	0.3281	0.3552	0.3781	0.3973	0.4135	0.4270	0.4385	0.4482	0.4564
65	0.2019	0.2420	0.2783	0.3101	0.3372	0.3599	0.3789	0.3947	0.4079	0.4191	0.4285	0.4364
70	0.1846	0.2255	0.2624	0.2944	0.3214	0.3440	0.3627	0.3782	0.3911	0.4020	0.4111	0.4188
75	0.1693	0.2111	0.2484	0.2806	0.3075	0.3299	0.3483	0.3636	0.3763	0.3868	0.3957	0.4032
80	0.1556	0.1982	0.2360	0.2683	0.2951	0.3173	0.3355	0.3505	0.3629	0.3733	0.3819	0.3892
85	0.1434	0.1867	0.2249	0.2572	0.2840	0.3060	0.3240	0.3387	0.3509	0.3610	0.3695	0.3766
90	0.1323	0.1764	0.2149	0.2473	0.2740	0.2958	0.3135	0.3281	0.3400	0.3499	0.3582	0.3651
95	0.1223	0.1670	0.2058	0.2383	0.2649	0.2865	0.3040	0.3183	0.3300	0.3397	0.3478	0.3546
100	0.1132	0.1585	0.1976	0.2301	0.2566	0.2780	0.2953	0.3094	0.3209	0.3304	0.3383	0.3450

$\sqrt{\beta_1}(c^*)$

n\c	2.1	2.2	2.3	2.4	2.5	2.6	2.8	3.0	3.2	3.5	3.8	4.0
13	1.2829	1.2908	1.2979	1.2613	1.3065	1.3090	1.3168	1.3192	1.3177	1.3092	1.3074	1.3370
14	1.2039	1.2127	1.2203	1.2295	1.2304	1.2336	1.2410	1.2434	1.2429	1.2330	1.2345	1.2335
15	1.1374	1.1466	1.1544	1.1618	1.1654	1.1691	1.1764	1.1791	1.1793	1.1796	1.1726	1.1696
16	1.0804	1.0898	1.0978	1.1048	1.1094	1.1134	1.1205	1.1237	1.1245	1.1233	1.1192	1.1159
17	1.0309	1.0404	1.0484	1.0553	1.0603	1.0646	1.0718	1.0753	1.0765	1.0757	1.0724	1.0694
18	0.9875	0.9970	1.0050	1.0118	1.0171	1.0214	1.0287	1.0325	1.0341	1.0337	1.0311	1.0283
19	0.9490	0.9584	0.9664	0.9732	0.9785	0.9830	0.9903	0.9944	0.9963	0.9964	0.9941	0.9916
20	0.9146	0.9241	0.9318	0.9386	0.9440	0.9483	0.9558	0.9601	0.9623	0.9627	0.9609	0.9586
21	0.8836	0.8928	0.9006	0.9073	0.9127	0.9166	0.9246	0.9291	0.9315	0.9322	0.9308	0.9287
22	0.8555	0.8645	0.8723	0.8789	0.8843	0.8900	0.8963	0.9008	0.9034	0.9044	0.9033	0.9014
23	0.8299	0.8388	0.8465	0.8530	0.8559	0.8634	0.8703	0.8750	0.8777	0.8790	0.8781	0.8764
24	0.8064	0.8152	0.8228	0.8292	0.8349	0.8395	0.8465	0.8512	0.8540	0.8555	0.8550	0.8534
25	0.7848	0.7934	0.8009	0.8073	0.8128	0.8174	0.8245	0.8292	0.8321	0.8338	0.8342	0.8321
26	0.7648	0.7733	0.7807	0.7870	0.7925	0.7970	0.8041	0.8088	0.8118	0.8137	0.8130	0.8124
27	0.7462	0.7547	0.7619	0.7682	0.7735	0.7781	0.7851	0.7899	0.7929	0.7949	0.7946	0.7909
28	0.7289	0.7373	0.7444	0.7506	0.7559	0.7604	0.7674	0.7722	0.7752	0.7774	0.7773	0.7762
29	0.7128	0.7210	0.7281	0.7342	0.7394	0.7439	0.7508	0.7556	0.7587	0.7609	0.7610	0.7601
30	0.6976	0.7058	0.7128	0.7188	0.7239	0.7284	0.7352	0.7400	0.7432	0.7455	0.7457	0.7449
31	0.6834	0.6914	0.6983	0.7043	0.7094	0.7138	0.7206	0.7254	0.7285	0.7309	0.7312	0.7306
32	0.6700	0.6780	0.6848	0.6907	0.6957	0.7000	0.7068	0.7116	0.7147	0.7172	0.7176	0.7170
33	0.6574	0.6652	0.6720	0.6778	0.6828	0.6870	0.6938	0.6985	0.7017	0.7042	0.7047	0.7042
34	0.6455	0.6532	0.6598	0.6656	0.6705	0.6747	0.6814	0.6861	0.6893	0.6919	0.6925	0.6920
35	0.6341	0.6418	0.6483	0.6540	0.6589	0.6631	0.6697	0.6744	0.6776	0.6802	0.6808	0.6805
36	0.6234	0.6309	0.6374	0.6430	0.6479	0.6520	0.6586	0.6632	0.6664	0.6691	0.6698	0.6695
37	0.6131	0.6206	0.6270	0.6326	0.6374	0.6415	0.6480	0.6526	0.6558	0.6585	0.6592	0.6590
38	0.6034	0.6108	0.6171	0.6226	0.6273	0.6314	0.6378	0.6425	0.6456	0.6483	0.6492	0.6490
39	0.5941	0.6014	0.6077	0.6131	0.6178	0.6218	0.6282	0.6328	0.6360	0.6387	0.6396	0.6394
40	0.5852	0.5924	0.5987	0.6040	0.6086	0.6126	0.6190	0.6235	0.6267	0.6294	0.6303	0.6302
41	0.5767	0.5839	0.5900	0.5953	0.5999	0.6038	0.6101	0.6147	0.6178	0.6206	0.6215	0.6214
42	0.5686	0.5756	0.5817	0.5870	0.5915	0.5954	0.6017	0.6062	0.6093	0.6121	0.6131	0.6130
43	0.5608	0.5678	0.5738	0.5790	0.5835	0.5874	0.5935	0.5980	0.6011	0.6039	0.6049	0.6049
44	0.5533	0.5602	0.5662	0.5713	0.5758	0.5796	0.5857	0.5902	0.5933	0.5961	0.5971	0.5971
45	0.5461	0.5529	0.5589	0.5640	0.5684	0.5721	0.5782	0.5826	0.5857	0.5885	0.5896	0.5896
46	0.5392	0.5459	0.5518	0.5569	0.5612	0.5650	0.5710	0.5754	0.5784	0.5812	0.5824	0.5824
47	0.5325	0.5392	0.5450	0.5500	0.5543	0.5581	0.5640	0.5684	0.5714	0.5742	0.5754	0.5754
48	0.5261	0.5327	0.5385	0.5434	0.5477	0.5514	0.5573	0.5616	0.5647	0.5675	0.5686	0.5687
49	0.5198	0.5264	0.5321	0.5371	0.5413	0.5450	0.5508	0.5551	0.5581	0.5609	0.5621	0.5622
50	0.5139	0.5204	0.5260	0.5309	0.5351	0.5387	0.5445	0.5488	0.5518	0.5546	0.5558	0.5560
55	0.4867	0.4930	0.4984	0.5031	0.5071	0.5105	0.5161	0.5203	0.5232	0.5260	0.5272	0.5275
60	0.4634	0.4695	0.4747	0.4792	0.4830	0.4864	0.4918	0.4957	0.4986	0.5013	0.5027	0.5029
65	0.4432	0.4491	0.4541	0.4584	0.4621	0.4653	0.4705	0.4744	0.4772	0.4799	0.4812	0.4815
70	0.4254	0.4311	0.4359	0.4401	0.4437	0.4468	0.4518	0.4556	0.4583	0.4609	0.4623	0.4626
75	0.4096	0.4151	0.4198	0.4238	0.4273	0.4303	0.4352	0.4388	0.4415	0.4441	0.4454	0.4458
80	0.3955	0.4008	0.4053	0.4092	0.4126	0.4155	0.4203	0.4238	0.4264	0.4289	0.4303	0.4307
85	0.3827	0.3878	0.3922	0.3960	0.3993	0.4022	0.4068	0.4102	0.4127	0.4152	0.4166	0.4170
90	0.3710	0.3760	0.3803	0.3841	0.3873	0.3900	0.3945	0.3978	0.4003	0.4028	0.4041	0.4045
95	0.3604	0.3653	0.3695	0.3731	0.3762	0.3789	0.3833	0.3865	0.3890	0.3914	0.3927	0.3931
100	0.3506	0.3554	0.3595	0.3630	0.3661	0.3687	0.3729	0.3761	0.3785	0.3809	0.3822	0.3826

$\beta_2(c^*)$

n\c	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	2.1921	3.6828	4.0491	3.8993	5.7323	5.1346	5.3151	5.5177	5.0595	6.1864	6.2573	6.5253
14	2.3617	3.6733	4.0135	3.9201	5.3807	4.9723	5.1338	5.3068	4.9211	5.8236	5.8901	6.0817
15	2.4929	3.6610	3.9764	3.9254	5.1053	4.8250	4.9696	5.1187	4.7919	5.5268	5.5888	5.7306
16	2.5977	3.6466	3.9388	3.9197	4.8839	4.6922	4.8220	4.9520	4.6727	5.2811	5.3388	5.4477
17	2.6832	3.6307	3.9015	3.9062	4.7022	4.5730	4.6900	4.8044	4.5635	5.0753	5.1292	5.2160
18	2.7539	3.6141	3.8651	3.8874	4.5507	4.4661	4.5722	4.6739	4.4637	4.9012	4.9516	5.0230
19	2.8129	3.5969	3.8298	3.8651	4.4225	4.3702	4.4669	4.5582	4.3722	4.7523	4.7997	4.8602
20	2.8627	3.5796	3.7960	3.8407	4.3128	4.2841	4.3727	4.4554	4.2882	4.6239	4.6685	4.7211
21	2.9048	3.5624	3.7637	3.8151	4.2179	4.2065	4.2882	4.3637	4.2105	4.5122	4.5543	4.6009
22	2.9406	3.5455	3.7329	3.7890	4.1351	4.1366	4.2122	4.2817	4.1380	4.4142	4.4541	4.4962
23	2.9712	3.5289	3.7038	3.7629	4.0624	4.0734	4.1438	4.2080	4.0693	4.3278	4.3657	4.4041
24	2.9975	3.5128	3.6762	3.7371	3.9980	4.0160	4.0819	4.1416	4.0027	4.2509	4.2870	4.3225
25	3.0200	3.4972	3.6501	3.7120	3.9406	3.9639	4.0257	4.0816	3.9357	4.1823	4.2167	4.2498
26	3.0394	3.4821	3.6255	3.6876	3.8893	3.9163	3.9746	4.0272	3.8631	4.1206	4.1535	4.1846
27	3.0561	3.4676	3.6023	3.6642	3.8430	3.8728	3.9279	3.9776	3.7728	4.0648	4.0964	4.1258
28	3.0705	3.4537	3.5803	3.6416	3.8011	3.8329	3.8852	3.9323	3.6202	4.0143	4.0446	4.0725
29	3.0830	3.4403	3.5596	3.6200	3.7631	3.7963	3.8461	3.8908	2.9650	3.9682	3.9974	4.0240
30	3.0937	3.4275	3.5401	3.5994	3.7284	3.7625	3.8100	3.8526	4.5541	3.9261	3.9542	3.9796
31	3.1030	3.4153	3.5217	3.5797	3.6967	3.7313	3.7767	3.8175	4.0543	3.8874	3.9145	3.9389
32	3.1111	3.4036	3.5043	3.5609	3.6675	3.7024	3.7459	3.7850	3.9237	3.8518	3.8780	3.9014
33	3.1180	3.3924	3.4879	3.5431	3.6406	3.6755	3.7174	3.7549	3.8521	3.8189	3.8442	3.8668
34	3.1239	3.3817	3.4723	3.5261	3.6158	3.6506	3.6908	3.7270	3.8018	3.7885	3.8129	3.8347
35	3.1291	3.3715	3.4576	3.5100	3.5928	3.6273	3.6661	3.7010	3.7620	3.7602	3.7838	3.8049
36	3.1335	3.3618	3.4437	3.4947	3.5714	3.6056	3.6431	3.6767	3.7286	3.7338	3.7568	3.7772
37	3.1373	3.3525	3.4306	3.4801	3.5515	3.5852	3.6215	3.6540	3.6995	3.7092	3.7315	3.7512
38	3.1405	3.3436	3.4181	3.4662	3.5329	3.5662	3.6013	3.6328	3.6734	3.6861	3.7078	3.7270
39	3.1432	3.3351	3.4062	3.4530	3.5155	3.5483	3.5823	3.6128	3.6498	3.6645	3.6856	3.7043
40	3.1455	3.3270	3.3950	3.4404	3.4992	3.5315	3.5644	3.5941	3.6282	3.6443	3.6648	3.6829
41	3.1474	3.3192	3.3843	3.4285	3.4839	3.5156	3.5476	3.5764	3.6083	3.6252	3.6451	3.6628
42	3.1490	3.3118	3.3742	3.4171	3.4695	3.5006	3.5318	3.5598	3.5897	3.6072	3.6266	3.6438
43	3.1503	3.3047	3.3645	3.4062	3.4559	3.4865	3.5168	3.5440	3.5724	3.5902	3.6091	3.6259
44	3.1514	3.2979	3.3553	3.3958	3.4431	3.4731	3.5026	3.5291	3.5561	3.5741	3.5926	3.6089
45	3.1522	3.2915	3.3466	3.3860	3.4310	3.4604	3.4892	3.5150	3.5409	3.5589	3.5769	3.5929
46	3.1528	3.2852	3.3382	3.3765	3.4195	3.4484	3.4764	3.5017	3.5265	3.5444	3.5621	3.5777
47	3.1533	3.2793	3.3303	3.3675	3.4086	3.4370	3.4643	3.4889	3.5128	3.5307	3.5480	3.5632
48	3.1536	3.2736	3.3227	3.3589	3.3983	3.4261	3.4528	3.4769	3.4999	3.5176	3.5345	3.5494
49	3.1538	3.2681	3.3154	3.3507	3.3885	3.4158	3.4418	3.4653	3.4877	3.5052	3.5217	3.5363
50	3.1539	3.2629	3.3085	3.3428	3.3792	3.4059	3.4314	3.4544	3.4761	3.4934	3.5095	3.5238
55	3.1530	3.2397	3.2780	3.3080	3.3386	3.3628	3.3857	3.4064	3.4255	3.4416	3.4562	3.4691
60	3.1508	3.2207	3.2531	3.2795	3.3059	3.3280	3.3487	3.3675	3.3847	3.3996	3.4129	3.4247
65	3.1481	3.2050	3.2326	3.2559	3.2790	3.2992	3.3182	3.3354	3.3511	3.3649	3.3772	3.3880
70	3.1453	3.1918	3.2154	3.2361	3.2566	3.2751	3.2925	3.3085	3.3229	3.3357	3.3471	3.3571
75	3.1425	3.1806	3.2008	3.2192	3.2376	3.2546	3.2707	3.2855	3.2989	3.3109	3.3214	3.3308
80	3.1399	3.1711	3.1883	3.2047	3.2213	3.2370	3.2520	3.2657	3.2783	3.2894	3.2993	3.3081
85	3.1375	3.1629	3.1775	3.1920	3.2071	3.2217	3.2356	3.2485	3.2603	3.2707	3.2801	3.2883
90	3.1353	3.1557	3.1681	3.1809	3.1947	3.2082	3.2213	3.2334	3.2444	3.2543	3.2631	3.2708
95	3.1333	3.1495	3.1598	3.1712	3.1837	3.1963	3.2086	3.2200	3.2304	3.2398	3.2481	3.2554
100	3.1315	3.1440	3.1524	3.1625	3.1739	3.1857	3.1973	3.2081	3.2180	3.2268	3.2347	3.2417

$\beta_2(c^*)$

n\c	2.1	2.2	2.3	2.4	2.5	2.6	2.8	3.0	3.2	3.5	3.8	4.0
13	6.5396	6.5864	6.6175	6.5545	6.5973	6.6087	6.6360	6.6504	6.6108	6.5366	6.4995	6.6866
14	6.1045	6.1451	6.1723	6.1269	6.1748	6.1862	6.2070	6.2121	6.1867	6.1064	6.0993	6.0873
15	5.7598	5.7955	5.8211	5.7663	5.8361	5.8478	5.8650	5.8676	5.8504	5.8381	5.7797	5.7572
16	5.4787	5.5127	5.5375	5.8796	5.5595	5.5715	5.5871	5.5893	5.5775	5.5559	5.5192	5.4969
17	5.2480	5.2799	5.3040	5.3422	5.3300	5.3421	5.3571	5.3599	5.3517	5.3323	5.3030	5.2825
18	5.0551	5.0853	5.1087	5.1335	5.1368	5.1490	5.1638	5.1674	5.1618	5.1454	5.1207	5.1024
19	4.8917	4.9205	4.9431	4.9640	4.9723	4.9844	4.9992	5.0037	5.0001	4.9863	4.9650	4.9487
20	4.7517	4.7803	4.8011	4.8202	4.8307	4.8427	4.8574	4.8627	4.8606	4.8491	4.8306	4.8159
21	4.6306	4.6565	4.6780	4.6961	4.7075	4.7193	4.7341	4.7400	4.7391	4.7296	4.7133	4.7001
22	4.5248	4.5497	4.5703	4.5876	4.5995	4.6111	4.6259	4.6323	4.6323	4.6245	4.6101	4.5981
23	4.4316	4.4555	4.4754	4.4921	4.5041	4.5155	4.5302	4.5370	4.5378	4.5313	4.5185	4.5077
24	4.3490	4.3720	4.3912	4.4073	4.4193	4.4304	4.4449	4.4521	4.4535	4.4482	4.4368	4.4269
25	4.2753	4.2974	4.3159	4.3315	4.3434	4.3542	4.3686	4.3760	4.3779	4.3736	4.3634	4.3543
26	4.2092	4.2304	4.2483	4.2634	4.2751	4.2856	4.2998	4.3074	4.3097	4.3063	4.2971	4.2888
27	4.1494	4.1699	4.1872	4.2018	4.2133	4.2236	4.2376	4.2452	4.2479	4.2452	4.2369	4.2292
28	4.0953	4.1150	4.1317	4.1459	4.1572	4.1671	4.1809	4.1885	4.1916	4.1896	4.1821	4.1750
29	4.0460	4.0650	4.0812	4.0949	4.1059	4.1156	4.1292	4.1370	4.1402	4.1386	4.1318	4.1253
30	4.0008	4.0192	4.0349	4.0482	4.0590	4.0685	4.0818	4.0895	4.0929	4.0919	4.0857	4.0796
31	3.9594	3.9772	3.9924	4.0053	4.0159	4.0251	4.0381	4.0459	4.0494	4.0488	4.0432	4.0375
32	3.9213	3.9385	3.9533	3.9658	3.9762	3.9850	3.9979	4.0056	4.0092	4.0089	4.0038	3.9985
33	3.8860	3.9027	3.9170	3.9292	3.9394	3.9480	3.9606	3.9682	3.9719	3.9720	3.9673	3.9624
34	3.8534	3.8696	3.8835	3.8953	3.9052	3.9136	3.9250	3.9335	3.9373	3.9376	3.9334	3.9287
35	3.8230	3.8387	3.8522	3.8638	3.8735	3.8816	3.8937	3.9012	3.9051	3.9056	3.9017	3.8974
36	3.7947	3.8100	3.8231	3.8343	3.8438	3.8518	3.8635	3.8711	3.8749	3.8757	3.8721	3.8681
37	3.7683	3.7832	3.7959	3.8069	3.8161	3.8239	3.8355	3.8428	3.8467	3.8477	3.8444	3.8406
38	3.7436	3.7580	3.7705	3.7811	3.7901	3.7977	3.8091	3.8164	3.8203	3.8214	3.8184	3.8148
39	3.7204	3.7345	3.7466	3.7570	3.7658	3.7732	3.7844	3.7915	3.7955	3.7967	3.7940	3.7905
40	3.6986	3.7123	3.7241	3.7343	3.7429	3.7501	3.7611	3.7681	3.7721	3.7734	3.7709	3.7677
41	3.6781	3.6915	3.7030	3.7129	3.7213	3.7284	3.7391	3.7461	3.7500	3.7515	3.7492	3.7461
42	3.6588	3.6718	3.6830	3.6927	3.7009	3.7078	3.7184	3.7253	3.7291	3.7307	3.7286	3.7257
43	3.6405	3.6532	3.6642	3.6736	3.6816	3.6884	3.6988	3.7055	3.7094	3.7111	3.7091	3.7064
44	3.6232	3.6356	3.6463	3.6555	3.6634	3.6701	3.6802	3.6869	3.6907	3.6925	3.6907	3.6881
45	3.6068	3.6189	3.6294	3.6384	3.6461	3.6526	3.6626	3.6692	3.6730	3.6748	3.6732	3.6707
46	3.5913	3.6031	3.6133	3.6222	3.6297	3.6361	3.6459	3.6524	3.6562	3.6580	3.6565	3.6541
47	3.5765	3.5881	3.5981	3.6067	3.6141	3.6204	3.6300	3.6364	3.6401	3.6421	3.6407	3.6384
48	3.5624	3.5737	3.5836	3.5920	3.5993	3.6054	3.6148	3.6211	3.6249	3.6268	3.6256	3.6234
49	3.5490	3.5601	3.5697	3.5780	3.5851	3.5911	3.6004	3.6056	3.6103	3.6123	3.6111	3.6091
50	3.5363	3.5471	3.5565	3.5647	3.5716	3.5775	3.5866	3.5927	3.5964	3.5984	3.5974	3.5954
55	3.4804	3.4902	3.4988	3.5061	3.5125	3.5179	3.5262	3.5319	3.5354	3.5376	3.5369	3.5353
60	3.4350	3.4440	3.4518	3.4586	3.4644	3.4694	3.4771	3.4824	3.4857	3.4879	3.4875	3.4862
65	3.3975	3.4058	3.4130	3.4192	3.4246	3.4292	3.4363	3.4413	3.4445	3.4466	3.4464	3.4453
70	3.3659	3.3736	3.3803	3.3860	3.3910	3.3953	3.4020	3.4066	3.4096	3.4118	3.4117	3.4108
75	3.3390	3.3461	3.3523	3.3577	3.3624	3.3664	3.3726	3.3770	3.3799	3.3820	3.3820	3.3812
80	3.3157	3.3224	3.3282	3.3333	3.3377	3.3414	3.3473	3.3514	3.3541	3.3562	3.3563	3.3556
85	3.2955	3.3017	3.3072	3.3120	3.3161	3.3196	3.3252	3.3291	3.3317	3.3336	3.3338	3.3332
90	3.2776	3.2835	3.2887	3.2932	3.2971	3.3004	3.3057	3.3094	3.3119	3.3138	3.3140	3.3135
95	3.2618	3.2675	3.2723	3.2766	3.2803	3.2834	3.2884	3.2919	3.2943	3.2961	3.2964	3.2960
100	3.2477	3.2531	3.2577	3.2617	3.2652	3.2682	3.2730	3.2763	3.2786	3.2804	3.2807	3.2803

TABLE 15a. Percentage Points of V* from V* Moments (Direct) and C* Moments (Indirect)

	N=15		N=20			N=25	
%	DIRECT	I	DIRECT	I	M C	DIRECT	I
C=1.0							
1	0.5454	0.5475	0.5956	0.5964	0.595	0.6309	0.6315
5	0.6312	0.6249	0.6757	0.6726	0.677	0.7067	0.7048
10	0.6813	0.6743	0.7220	0.7186	0.723	0.7500	0.7481
90	1.1747	1.1736	1.1705	1.1711	1.170	1.1645	1.1649
95	1.2744	1.2479	1.2613	1.2525	1.261	1.2481	1.2441
99	1.4921	1.3651	1.4606	1.4107	1.465	1.4319	1.4099
C=1.5							
1	0.3864	0.3869	0.4243	0.4245	0.425	0.4502	0.4504
5	0.4532	0.4515	0.4840	0.4834	0.484	0.5049	0.5046
10	0.4902	0.4886	0.5171	0.5165	0.518	0.5352	0.5349
90	0.8046	0.8042	0.7942	0.7940	0.794	0.7858	0.7856
95	0.8605	0.8565	0.8427	0.8416	0.842	0.8291	0.8286
99	0.9769	0.9591	0.9429	0.9393	0.943	0.9179	0.9170
C=2.0							
1	0.2985	0.2986	0.3290	0.3290	0.330	0.3496	0.3496
5	0.3523	0.3520	0.3766	0.3765	0.376	0.3929	0.3928
10	0.3821	0.3819	0.4029	0.4028	0.403	0.4168	0.4167
90	0.6212	0.6209	0.6113	0.6110	0.611	0.6039	0.6037
95	0.6606	0.6604	0.6450	0.6448	0.645	0.6338	0.6337
99	0.7394	0.7411	0.7121	0.7127	0.714	0.6930	0.6932
C=2.5							
1	0.2424	0.2425	0.2679	0.2679	0.269	0.2852	0.2852
5	0.2875	0.2873	0.3079	0.3077	0.308	0.3215	0.3214
10	0.3125	0.3123	0.3299	0.3298	0.330	0.3414	0.3413
90	0.5102	0.5100	0.5014	0.5012	0.501	0.4949	0.4948
95	0.5416	0.5412	0.5282	0.5281	0.528	0.5187	0.5186
99	0.6032	0.6026	0.5806	0.5806	0.582	0.5649	0.5650
C=3.0							
1	0.2037	0.2038	0.2256	0.2257	0.226	0.2405	0.2405
5	0.2425	0.2423	0.2601	0.2600	0.260	0.2718	0.2718
10	0.2641	0.2640	0.2792	0.2791	0.279	0.2891	0.2890
90	0.4350	0.4349	0.4271	0.4270	0.427	0.4214	0.4213
95	0.4617	0.4615	0.4499	0.4498	0.450	0.4416	0.4416
99	0.5136	0.5133	0.4941	0.4940	0.495	0.4807	0.4806

M. C. is 10**5 simulation.

TABLE 15b. Percentage Points of C* from C* Moments (Direct) and V* Moments (Indirect)

	N=15		N=20			N=25	
%	DIRECT	I	DIRECT	I	M.C.	DIRECT	I
C=1.0							
1	0.7439	0.6880	0.7225	0.7008	0.699	0.7229	0.7131
5	0.8075	0.7919	0.8047	0.7996	0.800	0.8098	0.8074
10	0.8554	0.8546	0.8571	0.8576	0.858	0.8615	0.8618
90	1.5112	1.4945	1.4108	1.4037	1.401	1.3512	1.3475
95	1.6420	1.6242	1.5156	1.5078	1.505	1.4406	1.4365
99	1.9000	1.9078	1.7285	1.7311	1.696	1.6233	1.6250
C=1.5							
1	1.0429	1.0237	1.0652	1.0611	1.061	1.0917	1.0906
5	1.1713	1.1657	1.1930	1.1911	1.192	1.2125	1.2118
10	1.2513	1.2507	1.2684	1.2680	1.268	1.2826	1.2823
90	2.1558	2.1480	2.0267	2.0242	2.022	1.9497	1.9484
95	2.3543	2.3449	2.1819	2.1788	2.181	2.0802	2.0786
99	2.7980	2.8015	2.5221	2.5238	2.519	2.3610	2.3621
C=2.0							
1	1.3648	1.3682	1.4234	1.4247	1.420	1.4666	1.4672
5	1.5459	1.5455	1.5867	1.5861	1.587	1.6172	1.6168
10	1.6538	1.6529	1.6830	1.6823	1.684	1.7055	1.7050
90	2.8389	2.8372	2.6747	2.6739	2.673	2.5754	2.5748
95	3.1086	3.1056	2.8844	2.8829	2.885	2.7507	2.7498
99	3.7336	3.7341	3.3524	3.3524	3.344	3.1327	3.1326
C=2.5							
1	1.7089	1.7072	1.7804	1.7805	1.766	1.8349	1.8351
5	1.9242	1.9227	1.9776	1.9769	1.976	2.0177	2.0172
10	2.0555	2.0547	2.0956	2.0950	2.096	2.1259	2.1255
90	3.5518	3.5487	3.3430	3.3417	3.343	3.2175	3.2168
95	3.8968	3.8925	3.6103	3.6085	3.606	3.4405	3.4394
99	4.6964	4.6992	4.2086	4.2092	4.197	3.9281	3.9282
C=3.0							
1	2.0409	2.0395	2.1299	2.1293	2.125	2.1960	2.1958
5	2.2976	2.2964	2.3644	2.3637	2.363	2.4138	2.4134
10	2.4552	2.4547	2.5058	2.5054	2.507	2.5437	2.5434
90	4.2785	4.2753	4.0235	4.0220	4.025	3.8703	3.8695
95	4.7006	4.6967	4.3505	4.3486	4.346	4.1428	4.1417
99	5.6789	5.6835	5.0821	5.0837	5.072	4.7391	4.7398

M. C. is 10**5 simulation.

3.7 Comments on the asymptotic variances In Table 16 comparisons are given for $n = 20$,

TABLE 16 Asymptotic Variance compared to best Assessment for Samples of 20

	c = 1.0	1.5	2.0	2.5	3.0
Asymptotic Value	0.05	0.0769	0.1246	0.1926	0.2819
Preferred Assessment	0.0478	0.0943	0.1640	0.2606	0.3861
% Error	4.6	18.5	24.0	26.1	27.0

and it is rather suprising to notice that the error increases with c.

Another aspect of this is to consider percentage points based on asymptotic normality $(\mu_1'(c^*) = c$, $\mu_2(c^*)$ taken to be the n^{-1} term, $\sqrt{\beta_1} = 0$, $\beta_2 = 3)$ and the best 4-moment distributional approximation (Table 17). The largest error occurs for c=1 at the 1% level but for lower levels decreases as c increases; however the upper level errors, small for c small, increase with c. As might be expected sample size plays an important role, but for $n > 75$ or so, asymptotic normality appears to be acceptable for $1 < c < 2$. For smaller samples discrepancies can be marked (Figures 3a & 3b).

It is pertinent here to mention that in the $(\sqrt{\beta_1}, \beta_2)$ plane, the skewness and kurtosis of c^* for fixed c respond rather oddly to changes in sample size (Figure 4). The locus of $(\sqrt{\beta_1}, \beta_2)$ points moves away from the normal point as n increases from 10 or so, reaching its worst abnormality for samples of about 25. This phenomenon may explain why we have found least summation problem difficulties for $n > 25$.

4. APPROXIMATE JOINT ACCEPTANCE REGION FOR c* AND b*

The four moments for c*, and b* could be used to set up an acceptance region. For Johnson (1949) has given a system of curves which are based on transformations of a standard normal variate z $\left(z \in N(0,1)\right)$. Briefly the systems are:

$$S_B: \quad z = \gamma + \delta \ \ell n\{y/(1-y)\}, \qquad 0 < y < 1, \qquad (4.1)$$
$$S_U: \quad z = \gamma + \delta \sinh^{-1} y, \qquad -\infty < y < \infty$$

where in the (β_1, β_2) plane S_B applies for points above (smaller β_2) the lognormal line (S_L) and S_U to points below (larger β_2) S_L. The parameters γ, δ are determined from the skewness and kurtosis, the approximation being completed by adjusting the variate y to have the correct location and scale. As an illustration consider the case c=2, b=1, and n=20. Since we have not developed moment series for the moment estimator b* of b, we use a simulation run of 100,000 samples, leading to $\mu_1'(\bar{c}) = 2.0700$, $\sigma(\bar{c})=0.3952$, $\sqrt{\beta_1}(c)=0.8963$, and $\beta_2(\bar{c})=4.7229$, the corresponding moment parameters for $\bar{b}$ being 0.9965, 0.1182, 0.1302, and 3.0176 respectively (the parameters for $\bar{c}$ compare quite favorably with those in Table 5). For $\bar{b}$ in the simulation the approximation $m_1'/\Gamma(1+1/\bar{c})$ was used, with $\bar{c}$ derived from v* using (2.3). The Johnson transformations are

$$z_1 = \gamma_1 + \delta_1 \sinh^{-1}\{(\bar{c} - \xi_1)/\lambda_1\}, \quad z_2 = \gamma_2 + \delta_2 \ \ell n\{(\bar{b} - \xi_2)/(\lambda_2 + \xi_2 - \bar{b})\}$$

where $\gamma_1 = -4.650150$, $\delta_1 = 3.082742$, $\xi_1 = 0.989711$, $\lambda_1 = 0.476904$;

$\gamma_2 = 8.223113$, $\delta_2 = 9.164447$, $\xi_2 = -0.532078$, $\lambda_2 = 5.268673$.

Now z_1 and z_2 are approximate normal variates (D'Agostino and Pearson (1973) refer to these as equivalent normal variates; we think it should be kept in mind that they are such but only to the extent that four moments (of $\bar{c}$ or $\bar{b}$) are used in the process); however they are correlated, and the correlation ρ can only be approximated from simulation studies, the mathematical approach presenting formidable problems.

An approximation then is that

$$\chi^2 = (z_1^2 - 2\rho z_1 z_2 + z_2^2)/(1 - \rho) \qquad (4.2)$$

is distributed as χ^2 with ν=2 degrees of freedom. In Figure 5 we give for c=2, b=1, n=20, and 1000 samples,

(i) A dot diagram in the (c, b) plane for samples of 20.

(ii) The 90, 95 and 99% contour derived from the χ^2 statistic

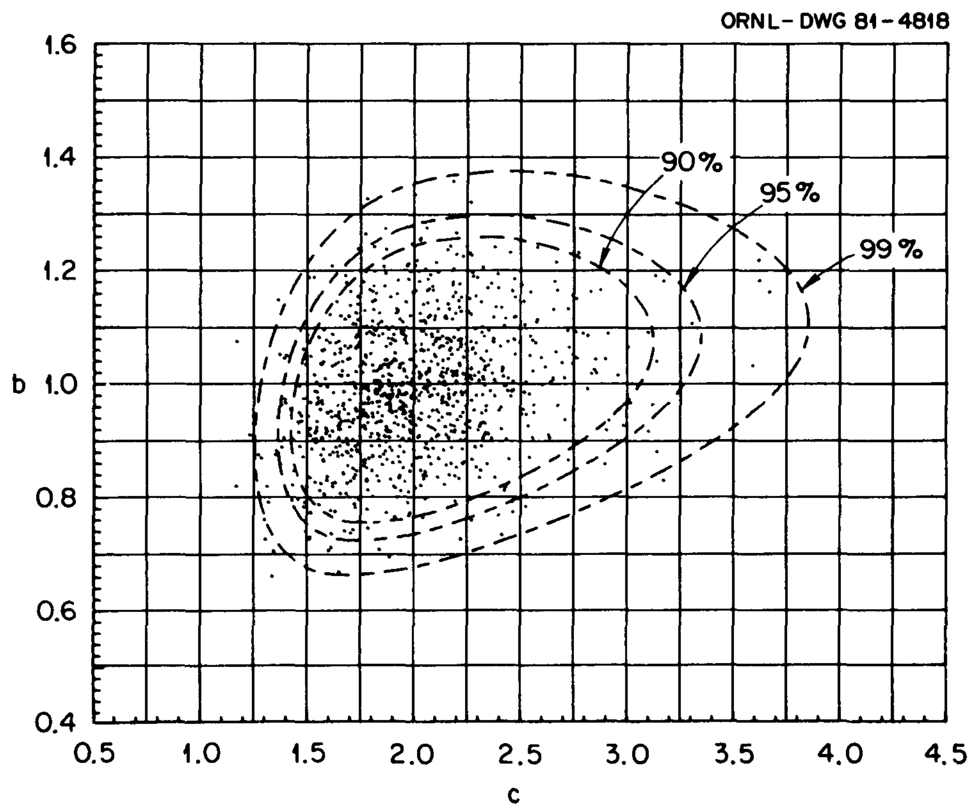

Figure 5. Acceptance contours for (c, b) in Sample of 20 from Weibull distribution with c = 2, b = 1

TABLE 17. Approximate Percentage Error in Assuming C* Asymptotically Distributed

		Percentiles					
		1	5	10	90	95	99
C=1.0	N=20	34	21	17	9	10	12
	30	21	14	12	6	6	8
	50	12	8	7	4	4	5
	75	7	5	5	2	3	3
C=1.5	N=20	20	13	10	9	10	15
	30	13	8	7	6	7	10
	50	7	5	4	3	4	6
	75	5	3	3	2	3	4
C=2.0	N=20	17	11	8	9	11	16
	30	11	7	5	6	7	11
	50	7	4	3	3	4	6
	75	4	3	2	2	3	4
C=2.5	N=20	17	10	8	9	11	16
	30	11	7	5	6	7	11
	50	7	4	3	3	4	7
	75	4	3	2	2	3	4
C=3.0	N=20	17	10	7	9	11	17
	30	11	7	5	6	7	11
	50	7	4	3	3	4	7
	75	5	3	2	2	3	5

(Percentage error = $100/P\alpha - N\alpha|/P\alpha$, where $P\alpha$ is the 4-moment Pearson percentage points, N the corresponding normal).

It will be seen that the χ^2 contour is responsive to the overall pattern of dots and not obviously symmetric. An actual count of dots shows 98, 52 and 10 outside the 90, 95 and 99% χ^2-contours respectively, the correct numbers being 100, 50, and 10.

5. REMARKS ON BASIC ASYMPTOTICS FOR c*, b*, AND v*

Defining h_{11}, h_{12}, and h_{22} to be the coefficients of the n^{-1} covariances in c*, b* ($h_{11} = n\ \mathrm{var}_1 c^*$, $h_{12}\ \mathrm{cov}_1(c^*,b^*)$, $h_{22} = n\ \mathrm{var}_1 b^*$) we have the approximation (see Newby, 1980) that the quadratic form

$$\chi^2 = \{(c^*-c)^2 b^2 h_{22} - 2(c^*-c)(b^*-b)bh_{12} + (b^*-b)^2 h_{11}\}/(n\Delta), \quad (5.1)$$

where $\Delta = b^2(h_{11}\ h_{22} - h_{12}^2)/n^2$, associated with asymptotic normality, is distributed X^2 with two degrees of freedom.

5.1 $\underline{h_{11}}$

From (1.1) by traditional first order principles, using the subscript 1 to denote first order terms in n^{-1},

$$\text{where } h_{11} = \mathrm{Var}_1\ c^* = (v+v^{-1})^{-2}(\mathrm{Var}_1\ v^*)\ H(c) \quad (5.2)$$
$$H(c) = c^4\{\psi(1+2/c) - \psi(1+1/c)\}^{-2}.$$
$$(\psi(x) = (d/dx)\ln\Gamma(x)).$$

5.2 $\underline{h_{22}}$

From (1.2) $b^* = m_1'/\Gamma(1+1/c^*)$ where c* is a function of v* which in turn is $(\sqrt{m_2})/m_1'$. Thus we need only consider first order incremental terms ∂b^*, $\partial m_1'$, and ∂c^*. We have

$$\partial b^*/b = \partial m_1'/\mu_1' + (\partial c^*/c^2)\ \psi(1+1/c) \quad (5.3)$$

where from (1.1),

$$\Gamma(1+2/c^*)/\Gamma^2(1+1/c^*) = 1 + v^{*2}$$

so that taking logarithmic derivatives

$$\partial c^*/c = \partial v^*/[(v + v^{-1})\{\psi(1+1/c) - \psi(1+2/c)\}] \qquad (5.4)$$

with

$$\partial v^*/v = \partial m_2/(2\mu_2) - \partial m_1'/\mu_1'.$$

Substituting in (5.3), squaring and using the expectations (Kendall and Stuart, 1969)

$$E(\partial m_2^2) = n^{-1}\mathrm{Var}_1 m_2 = (\mu_4 - \mu_2^2)/n$$

$$E(\partial m_2 \partial m_1') = n^{-1}\mathrm{Cov}_1(m_1', m_2) = \mu_3/n \qquad (5.5)$$

$$E(\partial m_1'^2) = n^{-1}\mathrm{Var}_1 m_1' = \mu_2/n$$

we find

$$h_{22} = n\ \mathrm{Var}_1 b^* = b^2\{P^2\mu_2 + 2PQ\mu_3 + Q^2(\mu_4 - \mu_2^2)\} \qquad (5.6)$$

where

$$P = 1/\mu_1' - \sqrt{\mu_2}\,\psi(1+1/c)/[\mu_1'^2(v+v^{-1})\{\psi(1+1/c) - \psi(1+2/c)\}]$$

$$Q = \psi(1+1/c)\ /[2\sqrt{\mu_2}\,\mu_1'(v+v^{-1})\{\psi(1+1/c) - \psi(1+2/c)\}].$$

5.3 $\underline{h_{12}}$

From (5.3) we have

$$\partial m_1'/\mu_1' = \partial b^*/b - (\partial c^*/c^2)\ \psi(1+1/c) \qquad (5.7)$$

so that

$$\mu_2/\mu_1'^2 = v^2$$

$$= \{[\mathrm{Var}_1\ b^*]/b^2 - 2\psi(1+1/c)[\mathrm{Cov}_1(b^*,c^*)]/(bc^2) \qquad (5.8)$$

$$+ \psi^2(1+1/c)[\mathrm{Var}_1\ c^*]/c^4\}n.$$

Computing the three asymptotic covariances, our h_{11} agrees with Newby's, but there is disagreement for h_{12} and h_{22}. Newby (1980) has a strange equation given below, on his (2.1.2) to the effect that

$$q = \partial b^*/\partial h^* = db^*/dm_1' \, dm_1'/dh^* \quad \text{(with } h^* = h\text{)}$$

which we are unable to justify. A few correct values are (with b = 1):

c	h_{11}	h_{12}	h_{22}
1.0	1.0	0.422784	1.178747
2.0	2.491787	0.263372	0.277837
2.5	3.851253	0.259234	0.177641
3.0	5.637150	0.257120	0.123336

6. ILLUSTRATIONS

Barlow et al (1979) give data on the stress-rupture life of Kevlar/epoxy spherical pressure vessels. They discuss a Bayesian analysis of the Weibull model used, the two parameter case with shape parameter c and scale parameter b. Their data for the 70% stress level is given in Table 18 for which n=49, and c is estimated to be 2.023. The value of v* from the data is 0.511851 which used in (2.3) gives the unbiased c to be 2.018. We use c=2 and n=49 in our tabulations to compute the standard deviation, etc., of c*, finding $\sigma = 0.2379$, $\sqrt{\beta_1} = 0.5122$, and $\beta_2 = 3.5363$. Now using the unbiased c, along with the other moment parameters in a 4-moment Pearson approximating distribution, we find for the 0.025 and 0.975 probability levels of c, the values 1.61 and 2.54. These differ only slightly from the asymptotic normal values 1.56 and 2.44. As far as one can judge, we have more information on the distribution of the moment estimator c* than is available on the maximum likelihood estimator of c. Even so the joint distribution of

TABLE 18

ORDERED LIST OF KEVLAR 49/EPOXY STRANDS TESTED AT 70% STRESS LEVEL

Rank	T_f	Rank	T_f	Rank	T_f	Rank	T_f
1	1051	14	5817	27	9711	40	12044
2	1337	15	5905	28	9806	41	13520
3	1389	16	5956	29	10205	42	13670
4	1921	17	6068	30	10396	43	14110
5	1942	18	6121	31	10861	44	14496
6	2322	19	6473	32	11026	45	15395
7	3629	20	7501	33	11214	46	16179
8	4006	21	7886	34	11362	47	17092
9	4012	22	8108	35	11604	48	17568
10	4063	23	8546	36	11608	49	17568
11	4921	24	8666	37	11745		
12	5445	25	8831	38	11762		
13	5620	26	9106	39	11895		

(Time to failure (T_f) in h).

the moment estimators of c and b is undoubtedly complicated and out of reach at the present.

Cain and Knight (1981) describe an application of the two parameter Weibull (in our notation m=c, σ_0=b) to failure prediction in composites. It is interesting to note that they find values of c, the shape parameter, in samples of 40 to 50 specimens in the region of 20. This from a statistical point of view, seems unusually large, and corresponds to a density which one would expect a beta curve (Pearson Type I) to fit. In any event, we ourselves have met few situations for which the shape parameter exceeds 4. However, the approach to the moment estimators described in our report would almost certainly apply to cases for which c exceeds 4. It should be noted, in addition, that Cain and Knight do not seem to be concerned with distributional properties of estimators, and leave unsettled the main reasons for preferring one method of estimation over another.

7. ACCURACY

The majority of the computations were carried out on IBM Model 370/3033 in double-double precision. It is possible there is a certain loss of accuracy especially with the n^{-20} - n^{-24} coefficients for the v* moments, and the n^{-9} - n^{-12} coefficients for the c* moments.

In particular 48 central moments of the Weibull variate itself are needed and these have to be evaluated from the non-central moments μ_r' given in (2.1). Since central moments involve products of alternating signs, accuracy may easily be lost. Keep in mind also that these moments are used as elements (not linear) in the series for the moments of v*, followed by further usage in the series for c*.

An Illustration of loss of accuracy is given for the case of the mean value of the logarithm of the mean in sampling from an exponential density (Weibull, c=b=1). We have in samples of n

$$\mathrm{Var}\{\ln(m_1')\} = \Psi^{(1)}(n) \qquad (7.1)$$

in terms of a polygamma function. Thus in terms of Bernoulli numbers

$$\mathrm{Var}\{\ln(m_1')\} \sim \frac{1}{n} + \frac{1}{2n^2} + \sum_{k=1}^{\infty} \frac{B_{2k}}{n^{2k+1}} \qquad (7.2)$$

where $B_2 = 1/6$, $B_4 = -1/30$, $B_6 = 1/42$, $B_8 = -1/30$, $B_{10} = 5/66$, $B_{12} = -691/2730$.

Using a Taylor expansion and expressions for the moments of $(m_1' - \mu_1')$, we have derived (Table 19) the first few terms in $\mathrm{Var}\{\ln(m_1')\}$ using double and double-double precision arithmetic. Loss of accuracy soon shows itself for the former, and is in clear evidence for the higher order even powers of n.

8. CONCLUDING REMARKS

If there is interest in the distribution of the coefficient of variation then Table 7 gives the first four moments from which, using a four moment Pearson approximant, the non-extreme percentage points can be found, assuming c is known. If c is not known, then the unbiased formula (2.14) uses the sample moment estimator v* to provide an almost unbiased estimate of c; if v* turns out to be greater than one, then a root transformation is possibly the best strategy, and if $\bar{c}$ exceeds 2.5 or so the normal approximant should be appropriate.

TABLE 19

Var $\{\ell n(m_1')\}$ in Sampling from Exponential: Loss of Accuracy

s	Exact	RQ	RD
7	0.0238095238095	0.0238095238095	0.02380952378
8	0.0	-0.41 Q-26	0.19 D-9
9	-0.033333333333	-0.033333333333	-0.033333305
10	0.0	0.62 Q-24	0.11 D-05
11	0.075757575758	0.075757575758	0.07578
12	0.0	-0.14 Q-19	0.19 D-03
13	-0.253113553136	-0.253113553136	-0.229
14	0.0	0.26 Q-17	0.17 D+01
15	1.16666666667	1.16666666667	0.145 D+03
16	0.0	-0.58 Q-13	0.63 D+04
17	-7.0921568627	-7.0921568627	0.77 D+05
18	0.0	-0.23 Q-09	0.54 D+07
19	54.9711779336	54.9711779336	0.50 D+09
20	0.0	-0.20 Q-06	0.77 D+11
21	-529.1242424	-529.1242487	0.45 D+13
22	0.0	0.49 Q-03	0.21 D+15
23	6192.123188	6192.134205	0.42 D+16
24	0.0	-0.24 Q+01	0.22 D+18

RQ ~ recursive formula using double-double arithmetic

RD ~ recursive formula using double arithmetic

It is risky to use the asymptotic normal approximation to the distribution of c* (shape), for in small samples (less than 50 or so according to the value of c) there is likely to be significant bias, some departure of the variance from the asymptotic value, and in addition skewness and kurtosis which cannot be ignored; even if one or two of these features can be ignored, the compounding of the four can lead to serious error. For example it is quite possible to come up with accurate assessments of mean and variance of c*, but a skewness exceeding a half and kurtosis exceeding 4, can throw the standard percentiles out by as much as 30-50%. Our guess would be that if the shape parameter really lies between 1 and 3, then a safe sample size would be 75 or more.

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APPENDIX A

First Order Terms in the Mean and Variance of the Coefficient of Variation

Let $Ev^* \doteq v + v_1/n$. $\quad (v = \sqrt{\mu_2}/\mu_1')$

Then if $m_1' - \mu_1' = x$

$m_2' - \mu_2' = y \quad (Ex = Ey = 0)$

$$E\left(\frac{\sqrt{m_2}}{m_1'}\right) = E\sqrt{\left(\frac{\mu_2' + y}{(\mu_1' + x)^2} - 1\right)}$$

$$= v\,E\sqrt{\left(1 + \frac{y}{\mu_2} - \frac{2x\mu_2'}{\mu_1'\mu_2} + \frac{3x^2\mu_2'}{\mu_1'^2\mu_2} - \frac{2xy}{\mu_1'\mu_2} + \cdots\right)}$$

$$= v\,E\left(1 + Ax^2 + Hxy + By^2 + \cdots\right), \qquad \text{(A1)}$$

$$A = \frac{3\mu_2'}{2\mu_1'^2\mu_2} - \frac{\mu_2'^2}{2\mu_1'^2\mu_2^2},$$

where $$H = -\frac{1}{\mu_1'\mu_2} + \frac{\mu_2'}{2\mu_1'\mu_2^2},$$

$$B = -\frac{1}{8\mu_2^2}.$$

Moreover, $Ex^2 = \mu_2/n$,

$Exy = (\mu_3' - \mu_1'\mu_2')/n$

$Ey^2 = (\mu_4' - \mu_2'^2)/n$

Hence v_1 is found from (A1).

Similarly,

$$E\frac{m_2}{m_1'^2} = \frac{\mu_2}{\mu_1'^2} + \left(\frac{3\mu_2'\mu_2}{\mu_1'^4} - \frac{2\mu_3'}{\mu_1'^3} + \frac{2\mu_1'\mu_2'}{\mu_1'^3}\right)/n + \cdots,$$

from which the first order term in the variance of v* is

$$\left(\frac{3\mu_2'\mu_2}{\mu_1'^4} - \frac{2\mu_3'}{\mu_1'^3} + \frac{2\mu_2'}{\mu_1'^2} - \frac{2\sqrt{\mu_2}\,v_1}{\mu_1'} \right) / n \quad . \qquad \text{(A2)}$$

APPENDIX B

Exact Distribution of v* when n = 4 and c= 1

Lam (1980) has derived the density of the variance (actually the second central sample moment) and non-central t in exponential sampling when n= 4. He has communicated to us the density of v* and other details for n = 2, 3, 4.

n = 2 $\mu_1'(v^*) = 1/2$, $\mu_2'(v^*) = 1/3$, $\mu_3'(v^*) = 1/4$, $\mu_4'(v^*) = 1/5$

$$Pr(v^* < v) = \begin{cases} 0, & v \leq 0 \\ v, & 0 < v < 1 \\ 1, & v > 1 . \end{cases}$$

n = 3 $\mu_1'(v^*) = 0.650620$

$\mu_2(v^*) = 0.0766939$

$\sqrt{\beta_1}(v^*) = 0.194222$

$\beta_2(v^*) = 2.598861$

$$Pr(v^* < v) = \begin{cases} 0, & v < 0 \\ 2\pi v^2/(3\sqrt{3}), & 0 < v < 1/\sqrt{2} \\ 2\pi v^2/(3\sqrt{3}) - 2(v^2/\sqrt{3})\tan^{-1}(2v^2 - 1)^{1/2} & \\ \qquad + \{(2v^2 - 1)/3\}^{1/2}, & 1/\sqrt{2} < v < \sqrt{2} \\ 0, & v > \sqrt{2}. \end{cases}$$

$n = 4 \quad \mu_1'(v^*) = 0.727087$

$\mu_2(v^*) = 0.0713451$

$\sqrt{\beta_1}(v^*) = 0.390596$

$\beta_2(v^*) = 3.031612$

$$\Pr(v^* < \nu) = \begin{cases} 0, & v < 0 \\ \pi v^3/2, & 0 < v < 1/\sqrt{3} \\ (\pi/2)(\sqrt{3}v^2 - v^3 - 1/(3\sqrt{3})), & 1/\sqrt{3} < v < 1 \\ \sum_{s=1}^{4} A_s(v), & 1 < v < \sqrt{3} \\ 1, & v > \sqrt{3} \end{cases}$$

Here,

$$A_1(v) = (\pi/2)(\sqrt{3}v^2 - v^3 - 1/(3\sqrt{3}))$$

$$A_2(v) = (1/(2\sqrt{3}))(1 - 9v^2)\tan^{-1}\{(3/2)(v^2 - 1)\}^{1/2}$$

$$A_3(v) = 3v^3\tan^{-1}((v^2 - 1)/(2v^2))^{1/2}$$

$$A_4(v) = \{(v^2 - 1)/2\}^{1/2}.$$

Some percentiles are given in Table B1.

TABLE B1

Percentiles of v^* in exponential sampling.

Percentile	0.1	1	5	10	90	95	99	99.9
n=3	0.0288	0.0909	0.2033	0.2876	1.0354	1.1441	1.2924	1.3755
n=4	0.0860	0.1853	0.3169	0.3993	1.0862	1.2120	1.4231	1.5874

APPENDIX C

A Series with one-signed coefficients.

The series for the fourth moment of c* for small n and c > 1.5 or so highlights typical summation problems. Consider sign patterns:

Signs of Successive Coefficients

c = 1.5	++++-+-+-+-+
2.0	+++++++++-+-
2.1	+++++++++++-
2.2	++++++++++++
2.5	++++++++++++
3.0	++++++++++++

This excess of positive terms in itself is a problem. But in addition, although magnitudes are moderate (last coefficient in the vicinty of 10^{18} for c=1.5, reducing to 10^{10} at c=3.0), there is magnitude irregularity. At c=2, successive ratios of coefficients are: 17.5, 11.5, 9.2, 8.1, 7.8, 4.0, 18.2, 17.9, 38.4, and 36.9. At c=3, the ratios are: 20, 12, 9, 8, 6, 5.6, 6.2, 9.5, 9.7, and 2.0. Clearly patterns of magnitude are obscure for the higher order coefficients. We conclude that the only answer is to derive more terms, with a careful analysis of loss of precision.

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