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CORRECTIONS FOR COMPONENT IMPERFECTIONS AND AZIMUTH ERRORS IN AN AUTOMATIC SELF-COMPENSATING ELLIPSOMETER

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ABSTRACT

An analysis of the first-order effects of component imperfections and azimuth errors in an automatic, self-compensating ellipsometer is presented. Twenty-three parameters in a linearized theory are used to compute the ellipsometer parameters Δ and ψ from polarizer, quarter wave compensator, and analyzer azimuths. These parameters are evaluated using 56 experimental measurements. The effectiveness of the corrections is recognized by a substantial decrease of the differences between 4-zone measurements. The theoretical dependence of the magnitude of errors on the orientation of the polarizer, analyzer, and quarter wave plate was experimentally verified.

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NOTATION

A,a	Analyzer azimuth (degree). As a subscript, refers to the component.
C,c	Quarter wave plate azimuth (degree). As a subscript, refers to the component.
C^*	FCI-C
E_x	Electrical field vector perpendicular to plane of specimen surface.
E_y	Electric field vector parallel to plane of specimen surface.
FC	Refers to measurements made with Faraday cells.
FCI	Glass core in the polarizer Faraday cell azimuth. Refers to component as a subscript.
FCR	Glass core in the analyzer Faraday cell azimuth. Refers to component as a subscript.
NFC	Refers to measurements made without the Faraday cells.
NW	Refers to measurements made without windows.
P,p	Polarizer azimuth (degree). As a subscript, refers to component.
P^*	P-FCI
$R(\omega)$	Rotator matrix describing orientation of principle frame of reference for a component.
r_s	Reflection coefficient in direction perpendicular to specimen surface.
r_p	Reflection coefficient in direction parallel to specimen surface.

T_j	Transmission matrix for component j.
T_j^*	$R(j)T_j\ R(-j)$
t_{1j}	Describes non-ideal effect of component j on the electric field intensity (degree).
T_{2j}	Describes non-ideal effect of component j on phase of the electric field.
WI,WR	Entrance and exit window azimuths. As a subscript, refers to components.
Z_j	Refers to azimuths of component j.
Δ	Relative phase shift due to reflection from specimen surface (degree).
$\delta\beta$	Specimen mispositioning error (degree).
$\delta\Delta$	Deviation in Δ due to component imperfections.
$\delta\rho_j$	Deviation in ρ_j due to imperfections in component j.
$\delta\psi$	Deviation in ψ due to component imperfections.
δZ_j	Error in azimuth of component j.
ε_x	Phase of electric field vector in x-direction.
ε_y	Phase of electric field vector in y-direction.
γ_{Z_j}	Coupling constant for Z_j error δZ_j.
γ_{ρ_j}	Coupling constant for ρ_j error $\delta\rho_j$.
λ	Wavelength of light, angstroms.
ϕ	Angle of incidence (degree).

ρ_j	Relative transmittance for component j.
ρ_s	Specimen reflectance ratio, $= r_p/r_s$
ψ	Relative intensity parameter due to reflection from specimen surface (degree).
ν	Frequency of light.

I. BACKGROUND AND THEORY

Ellipsometry is used to study the surface properties of materials immersed in an optically transparent medium. The two quantities of interest in ellipsometry studies are the changes, due to reflection from the surface, in the relative amplitude ψ and the relative phase Δ of two orthogonal components of the electric field vector which describe the state of polarization of the probing light beam.

When the components of the ellipsometer contain imperfections, their effects on the phase and amplitude of the light must be included in the interpretation of ellipsometer measurements. Imperfections in the ellipsometer are assumed to be either calibration errors in the graduated azimuth circles of the polarizer, analyzer, and quarter wave plate, deviation from quarter wave retardation, or flaws in the optical components due to strain-generated birefringence and polarization-dependent absorption.

The purpose of this analysis is to obtain a set of equations for ψ and Δ which compensate for the component imperfections. This analysis extends the word of Azzam and Bashara[1] on error corrections for a manual compensating ellipsometer to the use of an automatic self-compensating ellipsometer. The self-compensating ellipsometer uses Faraday cells to rotate the plane of polarization, thus enabling the ellipsometer to follow rapid changes in the state of the surface. The Faraday cells add two more imperfect components to the analysis, which results in more error terms in the equations for ψ and Δ. The Faraday cells are assumed to be both birefringent and dicroic, thus producing both relative phase and relative attenuation effects.

A. Components of the Ellipsometer

The components of the ellipsometer are arranged in the following sequence (Figure 1): polarizer P, polarizer Faraday cell FCI, quarter wave plate C, entrance cell window WI, specimen S, exit cell window WR, analyzer Faraday cell FCR, and analyzer A. The abbreviations used to represent each component (P, FCI, etc.) will also be used in other contexts: as subscripts to designate to which component a parameter refers; and to represent angles of rotation. These latter usages will be introduced in the following sections.

B. Rotated and Standard Coordinate Systems for the Measurement of the Nulling Angles

There are two possible coordinate systems for the measurement of the nulling azimuth angles of the analyzer and polarizer. In one system, the nulling angles are called standard azimuth angles and are denoted by a and p (for analyzer and polarizer). In the other, they are called rotated azimuth angles and are denoted by A and P.

The two coordinate systems are defined by the experimental set-up, specifically by the positioning of the specimen, which in turn depends on the nature of the experiment. Figures 2 and 3 illustrate the two different experimental set-ups. If the specimen plane is horizontal (Figure 2), then the coordinate system is standard; if the specimen plane is vertical, the coordinate system is rotated (Figure 3).

The orientation of the orthogonal components, E_x and E_y, of the electric field vector is defined relative to the plane of the specimen. The E_y-axis is parallel to the plane of the specimen, and the positive E_x-axis is in the direction of a vector in the plane defined by the

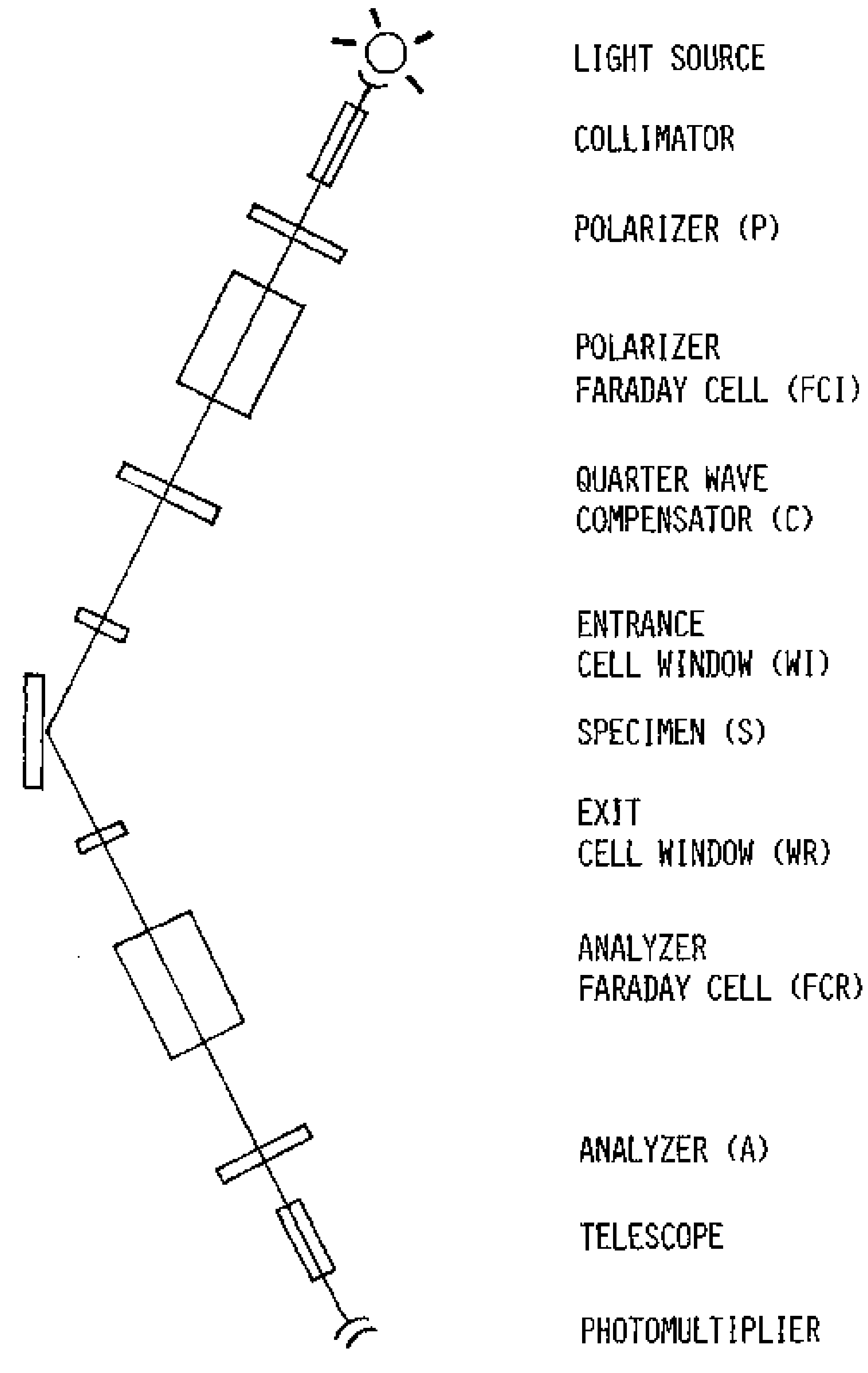

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Fig. 1. Components of the automatic self-compensating ellipsometer.

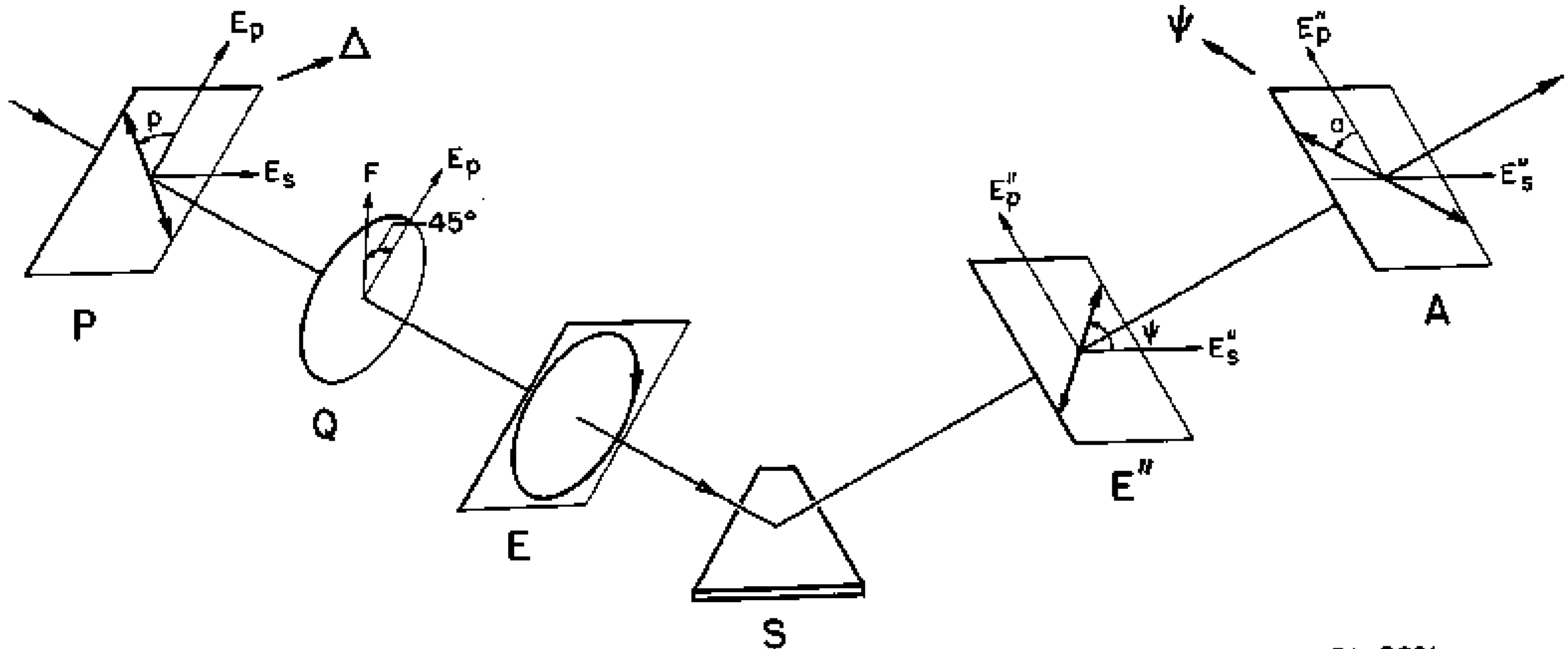

Fig. 2. Horizontal specimen orientation defining standard coordinate system.

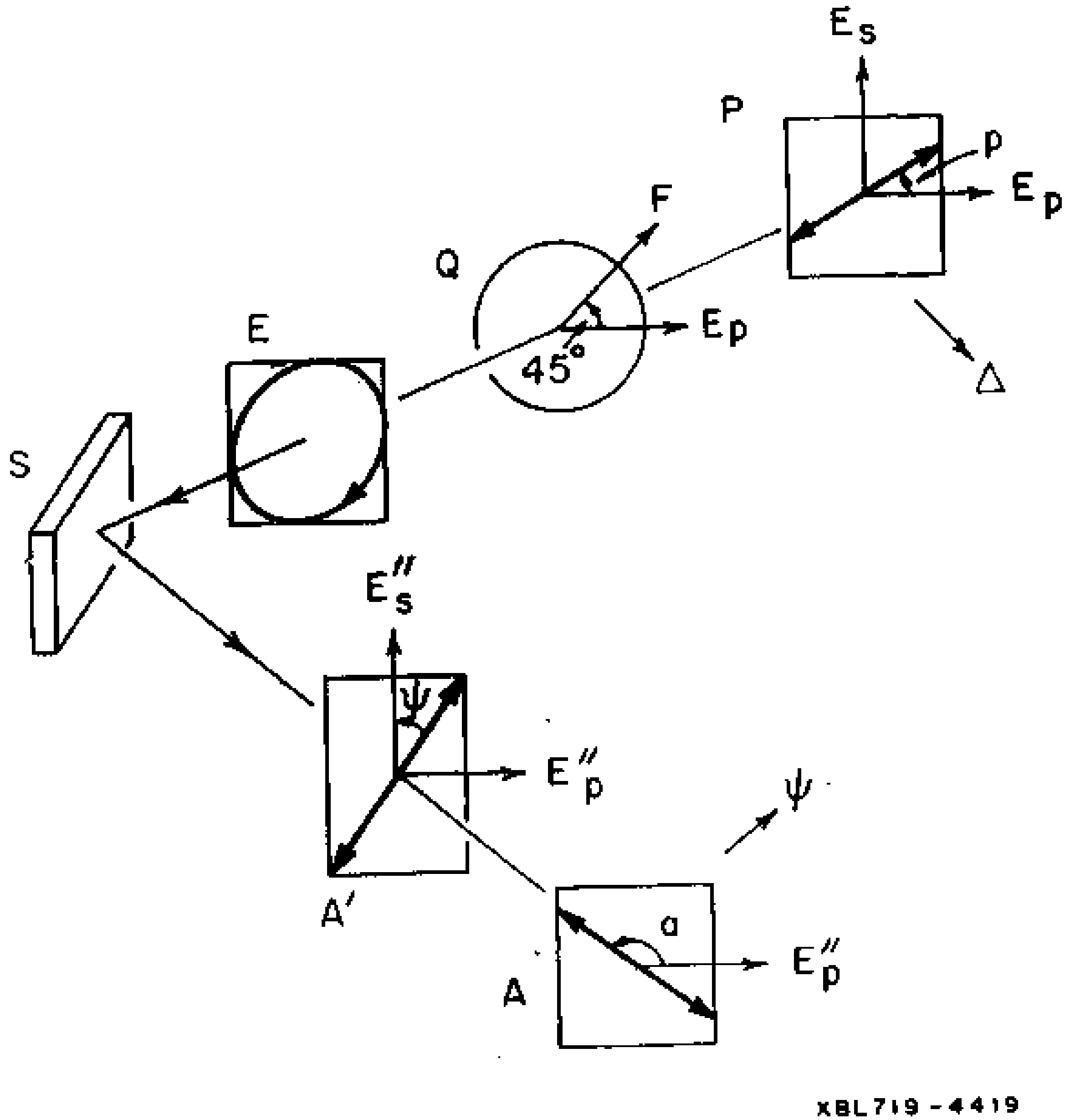

Fig. 3. Vertical specimen orientation defining rotated coordinate system. Azimuths p, a, and q referred to as (capital) P, A, and Q in text.

incident and reflected beams. Rotating the specimen plane counterclockwise from a vertical position (rotated system) to a horizontal position (standard system) therefore rotates the E_x-E_y coordinate system by 90° (in the counterclockwise direction).

Though the azimuth circle of the polarizer can be moved, the transmission axis of the polarizer remains fixed with respect to the azimuth circle. The same is true in regard to the transmission axis of the analyzer. The azimuth angle of the polarizer is measured counterclockwise from the positive E_x axis to the transmission axis of the polarizer (indicated in the figures by a heavy double pointed arrow), and the azimuth angle of the analyzer is measured counterclockwise from the positive E_x'' axis to the analyzer transmission axis (see Figures 2 and 3).

Because the two coordinate systems are related by a 90° rotation, the polarizer and analyzer azimuths in each system are related by:

$$P = p \pm 90° \ ,$$

$$A = a \pm 90° \ .$$

In keeping with the rotation for the analyzer and polarizer azimuth angles, the setting of the quarter wave plate is denoted by c if the nulling angles have been measured in the standard system or by C if the nulling angles have been measured in the rotated system, and

$$C = c \pm 90° \ .$$

Some derivations and results given in this paper do not depend on the coordinate system used to measure the nulling angles. For that reason, *unless otherwise noted,* A, P and C will be used throughout this paper to refer in general to the analyzer and polarizer azimuth angles and the setting of the quarter wave plate in *either* coordinate system. If the results depend on the coordinate system used, both sets of results will be given.

C. <u>Zones and Groups</u>

The values of ψ, the relative amplitude, and Δ, the relative phase, are not measured directly, but are calculated from the nulling azimuth angles of the analyzer and polarizer respectively. The ranges of ψ and Δ and the nulling azimuth angles are :

ψ:	$0° - 90°$	a,A:	$0° - 180°$
Δ:	$0° - 360°$	p,P:	$0° - 180°$

Though the value of Δ depends explicitly only on P, the equation defining the ideal value of Δ in terms of P is not the same for all values of Δ. In fact, the defining equation for Δ depends also on the range of A and on C, though neither A nor C appear in the equation. For this reason, it is convenient to distinguish 16 categories called *zones* to which a measurement, consisting of a value of P, C and A can belong. The 16 zones are defined as follows: The range of A is divided into two subranges, 0°-90° and 90°-180°; the range of P is divided into four subranges, 0°-45°, 45°-90°, 90°-135° and 135°-180°, and the value of C is limited to two possibilities, 45° and -45° (135°). Taking all possible combinations of the subranges of A and P and the value of C yields 16 zones. Moreover, the zones are grouped into four larger categories called groups, which depend on the range of Δ. It is a convention that the groups which correspond to a certain range of Δ are lettered and that the four zones, which comprise one of the lettered groups, are numbered according to the range of the polarizer azimuth.[2] However, the numbering of zones is not the same for rotated polarizer azimuths as for standard polarizer aximuths. Table E1 gives the definition of groups and Table E2 the definition of zones.

The ideal (uncorrected) value of ψ and Δ for each zone, as well as the corresponding ranges of the polarizer and analyzer azimuths for both the standard and rotated azimuth system, are given in Appendix A.

D. One-Zone and Four-Zone Measurements

A *one-zone measurement* consists of a value of A, P and C. The value of C may be either 45° or 135° and is set before the nulling angles A and P are found. A one-zone measurement may be taken on either a bare, unchanging surface or on a surface that is rapidly changing. After the nulling angles have been measured, the zone to which the triplet (C, A, P) belongs can be found from Table I which contains the zone and group divisions. The triplet will belong to one and only one of these zones, hence the name one-zone measurement. Once the correct zone is determined, the ideal values of ψ and Δ may be calculated using the appropriate formulas.

TABLE E1. Definition of group lettering

Group	Range of Δ
A	0°-90°
B	90°-180°
C	180°-270°
D	270°-360°

TABLE E2. Definition of zone numbering

Zone	Range of standard azimuth p	Range of rotated azimuth P
1	0°-45°	90°-135°
2	45°-90°	135°-180°
3	90°-135°	0°-45°
4	135°-180°	45°-90°

A *four-zone measurement* is actually four one-zone measurements that all belong to the same group, but fall into the four different zones within that group. Since each set (C, A, P) defines a value of ψ and a value of Δ, a four-zone measurement is only meaningful if taken on an unchanging surface so that the four one-zone measurements all define the same values of ψ and Δ. Ideally the values of ψ and Δ calculated from the four one-zone measurements should be the same. This is rarely the case, however, and the conclusion drawn is that the components of the ellipsometer contain imperfections which affect the measurements. The assumption that disagreement in the four-zone measurements is a result of imperfect components is central to this analysis, because the aim of using the corrected equations (to be derived in following sections) is to improve the agreement of the four one-zone measurements.

Given below is a four-zone measurement and values of ψ and Δ calculated from the ideal equations in Table I. For comparison, the corrected values of ψ and Δ are given.

E. Theory

The analysis of Azzam and Bashara uses the Jones Calculus to describe the optical effect of each component on the state of polarization of the light as it passes through the ellipsometer. The Jones Calculus, invented in 1940 and 1941 by R. Clark Jones,[3] uses a matrix to represent an optical device such as a polarizer or retarder and a vector to describe the state of polarization of a beam of light. The passage of the light through a component corresponds to multiplication of the

Four-zone measurement made on bare zinc

Zone	c	a	p
B1	135°	37.68°	5.23°
B2	45°	40.06°	84.03°
B3	135°	142.70°	95.35°
B4	45°	145.09°	174.42°

Ideal values of ψ and Δ calculated from four-zone measurement

Zone	ψ°	Δ°
B1	37.68	100.46
B2	40.06	101.94
B3	37.30	100.70
B4	34.91	101.16

Corrected values of ψ and Δ

Zone	ψ	Δ
B1	37.473	100.422
B2	37.509	100.632
B3	37.472	100.392
B4	37.452	100.742

vector representing the light by the component's matrix. The effect of imperfections in a component is treated as a first-order Taylor series expansion about the ideal component value.

F. The Jones Calculus

To define the spatial arrangement of the ellipsometer, we use a right-handed rectangular coordinate system with x, y and z axes. The components of the ellipsometer lie along the z axis which is perpendicular to the planes containing the components. When the axes of a component are parallel to the x and y axes, the component is said to be in its principal frame of reference. In this case, the Jones transmission matrix of either a partial polarizer or a retardation plate is given by $e^{i\phi}N$, where N equals

$$N = \begin{pmatrix} N_x & 0 \\ 0 & N_y \end{pmatrix}$$

where

$$N_x = \exp\left[-i\ (2\pi d/\lambda)\ (n_x - ik_x)\right]$$

$$N_y = \exp\left[-i\ (2\pi d/\lambda)\ (n_y - ik_y)\right]$$

and d is the thickness of the plate; λ is the wavelength of the light in a vacuum; n_x and n_y are the principal indices of refraction in the x and y directions; and k_x and k_y are the principal extinction coefficients in the x and y directions.

For a partial polarizer the principal indices of refraction are equal, $n_x = n_y = n$, and the Jones matrix has the form $e^{i\phi} N_p$ where

$$N_p = \begin{pmatrix} P_x & 0 \\ 0 & P_y \end{pmatrix} ,$$

$e^{i\phi}$ is the absolute phase factor and P_x and P_y are the principal transmittances in the x and y directions, respectively. If the absolute phase of the two components of light is of no interest, then N_p may be used alone as the matrix representation of the partial polarizer. This form of the Jones matrix is called the standard matrix.

For a retardation plate, the principal extinction coefficients, k_x and k_y, are both zero and the Jones matrix has the form $e^{i\phi} N_R$,

$$N_R = \begin{pmatrix} e^{i\gamma} & 0 \\ 0 & e^{-i\gamma} \end{pmatrix} \quad \text{and} \quad \gamma = \delta/2$$

where δ is the retardance. As in the previous case, the absolute phase factor $e^{i\phi}$ may be omitted so that the standard matrix of a retardation plate is N_R. Appendix 2 of Reference (4) lists the standard Jones matrices for commonly used optical devices.

For this analysis the standard Jones matrix is further simplified by dividing each entry in the matrix by the entry in the upper left-hand corner of the matrix, and then dropping the divisor. The representation of the partial polarizer is then

$$T_P = \begin{pmatrix} 1 & 0 \\ 0 & \rho_P \end{pmatrix} ,$$

where ρ_P is the relative transmittance for the polarizer, P_y/P_x.
For the retardation plate,

$$T_R = \begin{pmatrix} 1 & 0 \\ 0 & \rho_R \end{pmatrix} ,$$

where $\rho_R = e^{-i\delta}$ is also called the relative transmittance.

Using this simplification, each component of the ellipsometer has the same diagonal form when in its principal frame of reference. Thus, for the jth component,

$$T_j = \begin{pmatrix} 1 & 0 \\ 0 & \rho_j \end{pmatrix}$$

where ρ_j is the relative transmittance. However, if the component is not in its principal frame of reference, then its Jones matrix T_j^* is related to the Jones matrix T_j by

$$T_j^* = R(\omega)\, T_j\, R(-\omega)$$

where $R(\omega)$ and $R(-\omega)$ are the rotator matrices

$$R(\omega) = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix} \quad \text{and} \quad R(-\omega) = \begin{pmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{pmatrix} .$$

The angle ω measures the amount by which the coordinate system associated with the component has been rotated with respect to the fixed x-y coordinate system.

The form of the Jones vector that will be used in this analysis is just

$$\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

where E_x and E_y are the scalar components of the electric field vector along the x and y axes. E_x and E_y are further defined by

$$E_x = A_x \exp[i(\varepsilon_x + 2\pi\nu t)]$$

$$E_y = A_y \exp[i(\varepsilon_y + 2\pi\nu t)]$$

where A_x and A_y are the maximum values of E_x and E_y, and ϵ_x and ϵ_y are the phase components of E_x and E_y at time $t=0$. In problems not concerned with time, the time factor $e^{i2\pi\nu t}$ can be dropped so that

$$E_x = A_x \exp(i\epsilon_x)$$

$$E_y = A_y \exp(i\epsilon_y) \quad .$$

Subscripts on $\vec{E}$, E_x and E_y will be used to designate through which component the light has just passed.

With the major elements of the Jones calculus defined, we can now express the optical effect of the sequence of ellipsometer components as a chain of matrix multiplication:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}_{final} = T_A^* \, T_{FCR}^* \, T_{WR}^* \, T_S \, T_{WI}^* \, T_C^* \, T_{FCI}^* \;\; R(-P) \begin{pmatrix} E_x \\ E_y \end{pmatrix}_{initial}$$

Two points should be noted in connection with the matrix multiplication given above. First, because the Jones vector representation of light is limited to polarized light, the first use of the Jones vector is to describe the state of polarization of the light after it has passed through the polarizer. Therefore, the Jones matrix for the polarizer

T_P^* does not appear in the matrix chain through the counter rotator matrix R(-P) does. Secondly, the Jones matrix for the specimen is

$$T_s = \begin{pmatrix} \rho_s & 0 \\ 0 & 1 \end{pmatrix}$$

where ρ_s is the specimen reflectance ratio.

G. Component Imperfections Treated as First-Order Series Expansions

For the purpose of this analysis, the relative transmittance of an optical component contains the measure of the imperfection within that component. For a small imperfection in the jth component, the relative transmittance ρ_j is given by

$$\rho_j = \rho_j^{o} + \delta\rho_j \quad , \tag{1}$$

where ρ_j^{o} is the ideal relative transmittance and $\delta\rho_j$ is a measure of the imperfection. Since the imperfection may affect both the intensity and the relative phase of the light,

$$\delta\rho_j = t_{1j} + i\, t_{2j} \quad , \tag{2}$$

where t_{1j} measures of the effect of the imperfection on the intensity and t_{2j} measures the effect on the relative phase .

Due to calibration errors the true azimuth angle Z_j (this refers to C, A or P) may vary from the measured azimuth Z_j° by a small deviation δZ_j:

$$Z_j = Z_j^{\circ} + \delta Z_j \tag{3}$$

When the light flux transmitted by the analyzer is a minimum, the value of ρ_s, the ratio of the complex reflection coefficients (r_p/r_s) for the specimen, is given by equation (4):

$$\rho_s = f(Z_j, \rho_j) \tag{4}$$

The first order Taylor series expansion of (4) about ρ_j and Z_j is

$$\rho_s = f(Z_j, \rho_j) + \sum_j \gamma_j \, \delta Z_j + \sum_j \gamma_{\rho_j} \, \delta_{\rho_j} \tag{5}$$

The coupling constants γ_{Zj} and $\gamma_{\rho j}$ determine the way in which the azimuth error δZ_j and a component imperfection $\delta\rho_j$ couple onto an error in ρ_s.

The zeroth-order approximation to ρ_s, given by

$$\rho_s^{\circ} = f(Z_j^{\circ}, \rho_j^{\circ}) \quad , \tag{6}$$

is the value of ρ_s when the measured azimuths are substituted into the error-free ellipsometry equations in Table I. The corresponding corrections to ψ and Δ, $\delta\psi$ and $\delta\Delta$, are found by taking the logarithmic differential of $\rho_s = \tan\psi\, e^{i\Delta}$,

$$\delta\rho_s/\rho_s = 2\delta\psi/\sin 2\psi + i\delta\Delta \qquad (7)$$

The expressions for $\delta\psi$ and $\delta\Delta$ are therefore,

$$\begin{aligned} \delta\psi &= \tfrac{1}{2}\sin 2\psi \;\; \mathrm{Re}(\delta\rho_s/\rho_s) \\ \delta\Delta &= \mathrm{Im}\,(\delta\rho_s/\rho_s) \end{aligned} \qquad (8)$$

H. Birefringence in the Polarizer Faraday Cell

The coupling constants for the polarizer, analyzer, quarter wave plate, and entrance and exit cell windows, as well as the corresponding corrections to ψ and Δ are derived in Reference 1. It can be shown that $\gamma_{\rho_{FCR}}$ has the same form as $\gamma_{\rho_{WR}}$. The derivation of the constant $\gamma_{\rho_{FCI}}$ is presented here; the procedure will first be outlined and the algebraic steps and simplifications will follow.

For the purpose of determining $\gamma_{\rho_{FCI}}$, the other components are assumed to be ideal; therefore

$$\rho_s = \rho_s^\circ + \gamma_{\rho_{FCI}}\, \delta_{\rho_{FCI}} \qquad (A)$$

The imperfection in the glass core of the Faraday cell is assumed to affect both the intensity and the relative phase of the light beam so that

$$\delta\rho_{FCI} = t_{1FCI} + i\, t_{2FCI}$$

The measure of the imperfection $\delta\rho_{FCI}$ is embedded in the Jones matrix for the Faraday cell:

$$T^{*}_{FCI}\ (FCI) = R(FCI) \begin{pmatrix} 1 & 0 \\ 0 & \rho_{FCI} \end{pmatrix} R(-FCI) \qquad (B)$$

where

$$\rho_{FCI} = \overset{\circ}{\rho}_{FCI} + \delta\rho_{FCI} \quad . \qquad (C)$$

The result of the chain of matrix multiplications (from right to left), $T^{*}_{S}\ T^{*}_{C}\ T^{*}_{FCI}\ T^{*}_{\rho}$, acting on the incident beam, is the vector $(E_{x,s},\ E_{y,s})$. The ratio of the components of this vector defines the specimen reflectance ratio:

$$\rho_{s} = \frac{E_{x,s}}{E_{y,s}} \qquad (D)$$

This ratio contains the term $\delta\rho_{FCI}$ which has been carried through the matrix multiplication like a constant. Using algebraic simplifications and trigonometric identities, the single-term expression for ρ_s given in equation (D), can be separated into two terms, one of which is multiplied by $\delta\rho_{FCI}$. Comparison of this to equation (A) yields expressions for ρ_s° and $\gamma\rho_{FCI}$. These expressions are then substituted into the equation for the logarithmic differential of ρ_s, which in this case is,

$$\frac{\delta\rho_s}{\rho_s} = \frac{\gamma_{\rho_{FCI}}\ \delta\rho_{FCI}}{\rho_s^\circ} \tag{E}$$

where ρ_s in the denominator of the term on the right hand side has been approximated by its ideal value ρ_s°, and

$$\delta\rho_{FCI} = t_{1FCI} + i\ t_{2FCI}$$

When the resulting expression of equation (E) is separated into its real and imaginary parts, it yields the corrections to ψ and Δ that must be made to compensate for the imperfections in the polarizer Faraday cell.

The details of the matrix multiplication are given next.

The light beam leaving the polarizer is described by a Jones vector as,

$$\underline{E}_P = (1, \rho_P)$$

For the purpose of determining γ_{FCI}, $\rho_P = 0$, because in the linear analysis, component imperfections in the polarizer do not affect the analysis for the Faraday cell. On passing through the polarizer Faraday cell, the Jones vector becomes

$$\underline{E}_{FCI} = T_{FCI}R(-P^*)\,\underline{E}_P$$

$$= (\cos P^*,\ \rho_{FCI}\sin P^*)$$

where $P^* = P - FCI$

After passing through the compensator,

$$\underline{E}_C = T_C\,R(-C^*)\,\underline{E}_{FCI} \tag{10}$$

$$= (F_1,\ F_2)$$

where,

$$F_1 = \cos C^*\cos P^* - \rho_{FCI}\sin C^*\sin P^* \tag{11a}$$

$$F_2 = \rho_C(\sin C^*\cos P^* + \rho_{FCI}\cos C^*\sin P^*) \tag{11b}$$

and $C^* = FCI - C$.

Upon reflection from the specimen, the light is linearly polarized so that $\Delta = \pi$ and

$$\rho_s = -\tan\psi. \tag{12}$$

The Jones vector is given by

$$\begin{aligned} \underline{E}_s &= T_s R(-C)\, \underline{E}_c \\ &= (E_{x,s}, E_{y,s}) \end{aligned} \tag{13}$$

where

$$T_s = \begin{bmatrix} \rho_s & 0 \\ 0 & 1 \end{bmatrix},$$

and

$$E_{x,s} = \rho_s(F_1\cos C - F_2\sin C), \tag{14a}$$

$$E_{y,s} = (F_1\sin C + F_2\cos C) \quad . \tag{14b}$$

For extinction (or flux minimum), the value of ψ is given by A ± 180°; substituting this into equation (12) gives

$$\rho_s = -\tan A.$$

Combining this fact with the following definition of ρ_s,

$$\rho_s = \frac{E_{x,s}}{E_{y,s}} ,$$

where equations (14a) and (14b) have been substituted for $E_{x,s}$ and $E_{y,s}$ yields the following expression for ρ_s:

$$\rho_s = \frac{-\tan A[\sin C \cos C^* \cos P^* - \rho_{FCI} \sin C \sin C^* \sin P^* + \rho_c \cos C \sin C^* \cos P^* + \rho_c \rho_{FCI} \cos C \cos C^* \sin P^*]}{[\cos C \cos C^* \cos P^* - \rho_{FCI} \cos C \sin C^* \sin P^* - \rho_c \sin C \sin C^* \cos P^* - \rho_c \rho_{FCI} \sin C \cos C^* \sin P^*]}$$

$$= \frac{-\tan A[\tan C - \rho_{FCI} \tan C \tan C^* \tan P^* + \rho_c \tan C^* + \rho_c \rho_{FCI} \tan P^*]}{[1 - \rho_{FCI} \tan P^* \tan C^* - \rho_c \tan C \tan C^* - \rho_c \rho_{FCI} \tan C \tan P^*]}$$

$$= \frac{-\tan A[\tan C(1-\rho_{FCI} \tan C^* \tan P^*) + \rho_c(\tan C^* + \rho_{FCI} \tan P^*)]}{[1 - \rho_{FCI} \tan C^* \tan P^*) - \rho_c \tan C (\tan C^* + \rho_{FCI} \tan P^*)]}$$

$$= \frac{\tan A [\tan C + \rho_c (\tan C^* + \rho_{FCI} \tan P^*)/(1 - \rho_{FCI} \tan C^* \tan P^*)]}{\rho_c \tan C(\tan C^* + \rho_{FCI} \tan P^*)/(1-\rho_{FCI} \tan C^* \tan P^*) -1} \qquad (16)$$

Assuming that the ideal relative transmittance for the Faraday cells is one, we can make the substitution $\rho_{FCI} = 1 + \delta\rho_{FCI}$ in equation (16) to obtain,

$$\rho_S^* = \frac{\tan A \left[\dfrac{\tan C + \rho_c(\tan C^* + \tan P^* + \delta\rho_{FCI} \tan P^*)}{1 - \tan C^* \tan P^* + \delta\rho_{FCI} \tan C^* \tan P^*} \right]}{\rho_c \tan C \dfrac{(\tan C^* + \tan P^* + \delta\rho_{FCI} \tan P^*)}{(1 - \tan C^* \tan P^* - \delta\rho_{FCI} \tan C^* \tan P^*)} - 1} \tag{17}$$

The terms,

$$\frac{\rho_c(\tan C^* + \tan P^*) + \rho_c \delta\rho_{FCI} \tan P^*}{(1 - \tan C^* \tan P^*) - \delta\rho_{FCI} \tan C^* \tan P^*}$$ in the numerator, and

$$\frac{(\tan C^* + \tan P^*) + \delta\rho_{FCI} \tan P^*}{(1 - \tan C^* \tan P^*) - \delta\rho_{FCI} \tan C^* \tan P^*}$$ in the denominator

are both of the form $(x + \varepsilon_1)/(y + \varepsilon_2)$, which may be expanded as,

$$\frac{(X + \varepsilon_1)}{(Y + \varepsilon_2)} = \frac{X}{Y} + \frac{\varepsilon_1}{Y} - \frac{\varepsilon_2 X}{Y^2} \tag{18}$$

This expansion is valid for $\varepsilon_2 \ll 1$, which is the case for small values of $\delta\rho_{FCI}$.

The numerator term is expanded as follows:

$$\frac{\rho_c(\tan C^* + \tan P^*) + \rho_c\ \delta\rho_{FCI}\ \tan P^*}{(1 - \tan C^*\ \tan P^*) - \delta\rho_{FCI}\ \tan C^*\ \tan P^*} = \frac{\rho_c(\tan C^* + \tan P^*)}{(1 - \tan C^*\ \tan P^*)} + \frac{\rho_c\ \delta\rho_{FCI}\ \tan P^*}{(1 - \tan C^*\ \tan P^*}$$

$$+ \frac{\rho_c(\tan C^* + \tan P^*)\ \delta\rho_{FCI}\ \tan C^*\ \tan P^*}{(1 - \tan C^*\ \tan P^*)^2}$$

Using the trigonometric identity $\tan(\alpha + \beta) = (\tan\alpha + \tan\beta)/(1 - \tan\alpha\tan\beta)$, and combining the second and third terms, the expression becomes

$$\frac{\rho_c(\tan C^*\ \tan P^*) + \rho_c\ \delta\rho_{FCI}\tan P^*}{(1 - \tan C^*\tan P^*) - \delta\rho_{FCI}\ \tan C^*\ \tan P^*}$$

$$= \rho_c\ \tan(C^* + P^*) + \frac{\rho_c\ \delta\rho_{FCI}[\tan P^* + \tan C^*\ \tan P^*\ \tan(C^* + P^*)]}{(1 - \tan C^*\ \tan P^*)} \qquad (19)$$

The expansion of the term in the denominator is:

$$\frac{(\tan C^* + \tan P^*) + \delta\rho_{FCI} \tan P^*}{(1 - \tan C^* \tan P^*) - \delta\rho_{FCI} \tan C^* \tan P^*} = \frac{\tan C^* + \tan P^*}{(1 - \tan C^* \tan P^*)}$$

$$+ \frac{\delta\rho_{FCI} \tan P^*}{(1 - \tan C^* \tan P^*)} + \frac{(\tan C^* - \tan P^*)\ \delta\rho_{FCI} \tan C^* \tan P^*}{(1 - \tan C^* \tan P^*)^2}$$

$$= \tan(C^* + P^*) + \frac{\delta\rho_{FCI}[\tan P^* + \tan C^* \tan P^* \tan(C^* + P^*)}{(1 - \tan C^* \tan P^*)} \qquad (20)$$

The result of putting the expansions of the numerator and denominator terms into equation (17) is another expression of the form of equation (18).

$$\frac{\rho_s}{\tan A} = \frac{[\tan C + \rho_c \tan(C^* + P^*)] + \rho_c \delta\rho_{FCI} \tan P^* \dfrac{[1 + \tan C^* \tan(C^* + P^*)]}{(1 - \tan C^* \tan P^*)}}{[\rho_c \tan C \tan(C^* + P^*) - 1] + \rho_c \delta\rho_{FCI} \tan C \tan P^* \dfrac{[1 + \tan C^* \tan(C^* + P^*)]}{(1 - \tan C^* \tan P^*)}}$$

$$= \frac{\tan C + \rho_c \tan(C^* + P^*)}{\rho_c \tan C \tan(C^* + P^*) - 1} + \frac{\rho_c \delta\rho_{FCI} \tan P^* [1 + \tan C^* \tan(C^* + P^*)]}{(1 - \tan C^* \tan P^*)[\rho_c \tan C \tan(C^* + P^*) - 1}$$

$$- \frac{[\tan C + \rho_c \tan(C^* + P^*)]\ [\rho_c \delta\rho_{FCI} \tan C \tan P^* (1 + \tan C^* \tan(C^* + P^*))]}{(1 - \tan C^* \tan P^*)\ [\rho_c \tan C \tan(C^* + P^*) - 1]^2}$$

Combining the second and third terms and replacing $(C^* + P^*)$ by $(P - C)$ gives,

$$\frac{\rho_s}{\tan A} = \frac{\tan C + \rho_c \tan(P - C)}{\rho_c \tan C \tan(P - C) - 1}$$

$$+ \frac{\rho_c \delta\rho_{FCI} \tan P^* [1 + \tan C^* \tan(P-C)] \left[\rho_c \tan C \tan(P-C) - 1 - \tan C\left(\tan C + \rho_c \tan(P-C)\right)\right]}{(1 - \tan C^* \tan P^*) [\rho_c \tan C \tan(C^* + P^*) - 1]^2}$$

$$= \frac{\tan C + \rho_c \tan(P-C)}{\rho_c \tan C \tan(P - C) - 1} - \frac{\rho_c \delta\rho_{FCI} \tan P^* \sec^2 C[1 + \tan C^* \tan(P-C)]}{(1 - \tan C^* \tan P^*) [\rho_c \tan C \tan(P-C) - 1]^2} \tag{21}$$

Comparing the last expression to $\rho_s = \rho_s^\circ + \gamma_{FCI} \delta\rho_{FCI}$ shows that

$$\rho_s^\circ = \frac{\tan A[\tan C + \rho_c \tan(P-C)]}{\rho_c \tan C \tan(P-C) - 1} \tag{22}$$

$$\gamma_{FCI} = \frac{-\tan A \, \rho_c \tan P^* \sec^2 C[1 + \tan C^* \tan(P-C)]}{(1 - \tan C^* \tan P^*) [\rho_c \tan C \tan(P-C) - 1]^2} \tag{23}$$

The expression for γ_{FCI} can be simplified as follows:

$$\gamma_{FCI} = \frac{-\tan A\,\rho_c \sec^2 C[1 + \tan C^* \tan(P-C)]}{(1/\tan P^* - \tan C^*)[\rho_c \tan C \tan(P-C) - 1]^2}$$

$$= \frac{-\tan A\,\rho_c \sec^2 C[1 + \sin C^* \sin(P-C)/\cos C^* \cos(P-C)]}{(\cos P^*/\sin P^* - \sin C^*/\cos C^*)[\rho_c \tan C \tan(P-C) - 1]^2}$$

$$= \frac{-\tan A\,\rho_c \sec^2 C\,\sin P^*[\cos C^* \cos(P-C) + \sin(P-C)]}{\cos(P-C)(\cos P^* \cos C^* - \sin P^* \sin C^*)[\rho_c \tan C \tan(P-C) - 1]^2}$$

$$= \frac{-\tan A\,\rho_c \sec^2 C \sin P^* \cos(C^* - (P-C))}{\cos(P-C)\cos(C^* + P^*)[\rho_c \tan C \tan(P-C) - 1]^2} \qquad (24)$$

$$= \frac{-\tan A\,\rho_c \sec^2 C \sin P^* \cos P^*}{\cos(P-C)[\rho_c \tan C \tan(P-C) - 1]^2}$$

$$\frac{-\tan A\,\rho_c \sec^2 C \sec^2(P-C) \sin 2P^*}{2[\rho_c \tan C \tan(P-C) - 1]^2}$$

The fractional error of ρ due to birefringence in the Faraday cell core is

$$\left(\frac{\partial\rho}{\rho}\right)_{FCI} = \left(\frac{\gamma_{FCI}}{\rho}\right)\delta\rho_{FCI} \qquad (25)$$

where $\delta\rho_{FCI} = t_{1FCI} + it_{2FCI}$. Ideally, the quarterwave plate affects only the phase shift measured by Δ and not the intensity of the beam and under this condition $\psi = 45°$, $\Delta = 90°$ and $\rho_c^{\circ} = -i$. When, in addition, $C = \pi/4$, equation (22) gives,

$$\rho_o^+ = \frac{-\tan A[1 - i\tan(P - \pi/4)]}{[i\tan(P - \pi/4) + 1]}$$

$$= \frac{-\tan A(1 + \tan P - i\tan P + i)}{(i\tan P - i + 1 + \tan P)}$$

$$= \frac{-\tan A(\cos P + \sin P - i\sin P + i\cos P)}{(i\sin P - i\cos P + \cos P + \sin P)}$$

$$= -\tan A\,\frac{(e^{-iP} + ie^{-iP})}{(e^{iP} - ie^{iP})}$$

$$= -\tan A\, e^{-i2P}\left(\frac{1 + i}{1 - i}\right)$$

$$= -i\tan A\, e^{-i2P} \qquad (26a)$$

Similarly, for $c = -\pi/4$, equation (22) reduces to

$$\rho_\circ^- = i \tan A \, e^{-i2P} \quad . \tag{26b}$$

We can now use equations (24), (25), and (26) to solve for $\delta\psi_{FCI}$ and $\delta\Delta_{FCI}$. First, for $C = \pi/4$ and $\rho_c = -i$,

$$\gamma_{FCI} = \frac{i \tan A \sec\left(P - \frac{\pi}{4}\right) \sin 2P^*}{\left[i \tan\left(P - \frac{\pi}{4}\right) + 1\right]^2}$$

$$= \frac{i \tan A \sin 2P^*}{\cos^2\left(P - \frac{\pi}{4}\right)\left[\frac{i(\tan P - 1)}{(1 + \tan P) + 1}\right]^2}$$

$$= \frac{i \tan A \sin 2P^*(1 + \tan P)^2}{\left(\frac{1}{\sqrt{2}} \cos P + \frac{1}{\sqrt{2}} \sin P\right)^2 [\tan P(i + 1) + (-i + 1)]^2}$$

$$= \frac{i \tan A \sin 2P^*\left[\frac{(\cos P + \sin P)}{\cos P}\right]^2}{\frac{1}{2}(\cos P + \sin P)^2\left[\frac{(1 - i)(\cos P + i \sin P)}{\cos P}\right]^2}$$

$$= \frac{2i \tan A \sin 2P^*}{(1 - i)^2 \, e^{i2P}}$$

$$= -\tan A \sin 2P^* \, e^{-i2P}$$

$$= -\tan A \; e^{-i2P} (\sin 2P \cos 2FCI - \cos 2P \sin 2FCI) \tag{27}$$

For $\rho = \rho_o^+$,

$$\frac{\gamma_{FCI}}{\rho} = -i(\sin 2P \cos 2FCI - \cos 2P \sin 2FCI) \qquad (28a)$$

and for $C = -\pi/4$, $\rho = \rho_o^-$,

$$\frac{\gamma_{FCI}}{\rho} = i(\sin 2P \cos 2FCI - \cos 2P \sin 2FCI) \qquad (28b)$$

Equation (25) becomes

$$\left(\frac{\partial\rho}{\rho}\right)_{FCI} = \mp i(\sin 2P \cos 2FCI - \cos 2P \sin 2FCI)(t_{1FCI} + it_{2FCI}) \qquad (29)$$

for $C = \pm\pi/4$, and the corrections to Δ and ψ are given by

$$\delta\Delta = \text{Im}\left(\frac{\partial\rho}{\rho}\right)_{FCI} = \mp i(\sin 2P \cos 2FCI - \cos 2P \sin 2FCI)\ t_{1FCI} \qquad (30)$$

and .

$$\delta\psi = \tfrac{1}{2} \sin 2\psi \ \text{Re}\ \frac{\partial\rho}{\rho}_{FCI}$$

$$= \pm\tfrac{1}{2} \sin 2A(\sin 2P \cos 2FCI - 2P \sin 2FCI)\ t_{2FCI} \qquad (31a)$$

for those zones in which $\sin 2\psi = \sin 2A$.

For those zones in which $\sin 2\psi = -\sin 2A$,

$$\delta\psi = \mp \tfrac{1}{2} \sin 2A(\sin 2P \cos 2FCI - \cos 2P \sin 2FCI)\, t_{2FCI} \;. \qquad (31b)$$

When the error terms have all been derived, they are added together to give the total correction to ψ and to Δ. Care must be taken that the parameters appear with the correct signs; the signs depend on the setting of C and the range of A and the range of P. The result is that 32 equations are needed to characterize Δ and 32 more are needed to characterize ψ. Sixteen equations, one for each zone, are used when the nulling angles have been measured in standard azimuth angles, and the other sixteen are used when the nulling angles have been measured in rotated azimuth angles. The equations for ψ and Δ are written out in full in Tables IIa and IIb (Appendix A). Tables IIc, IId, IIIa, and IIIb contain the signs of the parameters in the equations for ψ and Δ for all groups.

If the ideal-value equations for Δ (Table I) are solved for 2p and the resulting expression substituted for 2p into the corrected equations for Δ, the final result is a set of equations for p. Replacing 2a in the equations for ψ with expressions in terms of the ideal value of ψ yields a set of equations for a. If the same sort of substitution is made for 2P in the equations for Δ (for rotated azimuths), the resulting set of equations for P are identical to those for p. The same is true of the equations for A; they are identical to those for a. The equations for A and P are used in linear combinations to solve for the parameters. The procedure will be outlined in the next section.

Tables IVa and IVb show the complete equations for A and P for group B. The analysis also includes intensity effects in the analyzer Faraday cell. This involves use of the coupling constant γ_{FCR}, having the same form as γ_{WR}, derived in Ref. (1), and $\delta\rho_{FCR} = t_{1FCR} + i\,t_{2FCR}$. $(\delta\Delta)_{FCR}$ and $(\delta\psi)_{FCR}$ are found as in equations (30) and (31).

II. EVALUATION OF THE PARAMETERS

A. Introduction

Nineteen parameters have been used to characterize the component imperfections, window birefringence, and azimuth angle errors. Eight of the parameters appear only in the equations for ψ and are solved using the equations for A_2, A_2, A_3 and A_4. The other parameters, with one exception, appear only in the equations for Δ and are evaluated using the P_i equations. The exception is δC which is in all of the equations.

The expressions contained in this section are those that were used to evaluate the parameters. Some alternate expressions have also been indicated if there is a choice between determining a parameter from measurements made on either a dielectric or a metal. In general, the expressions presented are the simplest. A discrepancy of nearly half a degree in the value of t_{1FCR} sin2FCR did not occur between dielectric and metal measurements; the value for the metal measurements was chosen.

The equations for Δ and ψ in terms of the azimuth angles are different for each group of zones. As a result, the definition of a parameter depends on the group in which the measurements occur. Once defined, however, the parameter does apply to other zones. The parameter definition does not depend on the use of standard or rotated azimuths for the measurement.

B. Experimental Procedure

Inspection of the complete equations for ψ and Δ in Table IVa and IVb show that the magnitude of the corrections depends upon ϕ and Δ. By the appropriate choice of the specimen and the arrangement of the ellipsometer, this functional dependence on ψ and Δ may be used to simplify the expressions. For reflection from a dielectric, $\Delta_o = 0°$ and $\psi_o = 45°$.

Measurements were taken in each of the four zones for the group determined by the experimental value of Δ. It will be shown that the appropriate averaging of the azimuths A and P for the zones isolates certain component parameters. Isolation of parameters is also achieved by making four-zone measurements with and without various components. It should be noted that when the Faraday cells are removed, manual ellipsometer measurements are necessary. These were performed by monitoring the output voltage from the photo-multiplier tube, and determining the minimum intensity by averaging azimuths for A and P which give equal voltages on each side of the minimum.

The ellipsometer was first placed in the straight-through position. With Faraday cells in place, the calibration circles of A and P were adjusted to give minimums for A being perpendicular to P. The calibration circle for C was adjusted to give the smallest average deviation when set parallel to A and P. The Faraday cells were removed, and a four-zone measurement was made.

The ellipsometer was then aligned to reflect at 75° from a surface facing up. Alignment was performed by auto-collimation from mirrors on a prism accurate to 16 seconds of arc. Without the Faraday cells, multiple four-zone measurements were made from a clean, uncoated glass prism. The specimen was realigned for each four-zone measurement, to allow the azimuth angle $\delta\beta$ to be averaged to zero in the analysis of the data. With the specimen fixed, the Faraday cells were inserted at a fixed orientation. One four-zone measurement was performed. The procedure for the dielectric was repeated for a metal surface. Finally, four zone measurements with and without the cell windows were conducted for reflection from the metal specimen.

C. Expressions for the Parameters

The following subscripts have been used to identify the set of measurements used in the calculation of each parameter:

S — measurements made while the ellipsometer is in the straight-through position,

D — measurements made using a dielectric as a specimen,

FC,NFC — measurements made with and without the Faraday cells in place, respectively,

W,NW — measurements made with and without the windows in place, respectively,

The windows are always used in combination with the Faraday cells (though the Faraday cells are sometimes used alone), and in the context of this paper, FC, NFC, and NW will all refer to measurements made on either a dielectric or metal specimen. In addition, the notation $\langle\ \rangle$ represents the average of multiple measurements of the quantity inside the brackets.

Whenever Δ and ψ appear in the expressions for the parameters, they refer to the four-zone average calculated from the set of measurements that is being used to evaluate the parameter. If the measurements are taken with the windows and/or Faraday cells in place, correction terms must be added to the four-zone average for Δ. For measurements made with only the Faraday cells the correction term is $t_{2FCR} \cos 2FCR$; for measurements made with both the windows and Faraday cells in place, the correction term is the sum of $t_{2FCR} \cos 2FCR$, $t_{2WI} \cos 2WI$, and $t_{2WR} \cos 2WR$.

The correction terms are the same for all groups. Table V contains the formulas for the four-zone averages for Δ and ψ for each group and the correction terms for the four-zone average of Δ.

Because most of the parameters do not depend directly on other parameters, the order in which the parameters are calculated is not important except in a few instances. Some of the terms for the Faraday cells or windows involve a four-zone average of Δ. Therefore they indirectly depend on $t_{2WI}\cos 2WI$, $t_{2WR}\cos 2WR$, and/or $t_{2FCR}\cos 2FCR$, terms which appear explicitly only in the equations for Δ. The expressions for these parameters are given first.

From measurements made on a dielectric:

$$t_{2FCR}\cos 2FCR = \tfrac{1}{2}[(P_1-P_2+P_3-P_4)_{FC} - (P_1-P_2+P_3-P_4)_{NFC}] \quad (32a)$$

for groups A and C

$$= \tfrac{1}{2}[(-P_1+P_2-P_3+P_4)_{FC} - (-P_1+P_2-P_3+P_4)_{NFC} \quad (32b)$$

for groups B and D .

The terms $t_{2WI}\cos 2WI$ and $t_{2WR}\cos 2WR$ always appear as a sum in the equations for Δ and P_i; therefore it is not necessary to solve for each parameter individually. From measurements made on a metal:

$$t_{2WI}\cos 2WI + t_{2WR}\cos 2WR = \tfrac{1}{2}[(P_1-P_2+P_3-P_4)_W - (P_1-P_2+P_3-P_4)_{NW}] \quad (33a)$$

for groups A and C,

$$= \tfrac{1}{2}[(-P_1+P_2-P_3+P_4)_W - (-P_1+P_2-P_3+P_4)_{NW}] \quad (33b)$$

for groups B and D .

The next ten expressions are for the parameters that appear in the equations for ψ, (from measurements made while the ellipsometer is in straight-through position):

$$\delta C_c + \delta A_c = 90° - \tfrac{1}{4}(A_1+A_2+A_3+A_4)_s \qquad \text{for groups A and D,} \qquad (34a)$$

$$\delta C_c - \delta A_c = 90° - \tfrac{1}{4}(A_1+A_2+A_3+A_4)_s \qquad \text{for groups B and C,} \qquad (34b)$$

From multiple measurements made on a dielectric with neither the Faraday cells nor windows in place:

$$\sin 2\psi_D \cos\Delta_D \delta C_c + \delta A_c = 90° - \tfrac{1}{4}(A_1+A_2+A_3+A_4)_D$$

for groups A and D,

$$\sin 2\psi_d \cos\Delta_D \delta C_c - \delta A_c = 90° - \tfrac{1}{4}(A_1+A_2+A_3+A_4)_D \qquad (35b)$$

for groups B and C.

Combining the appropriate form of equations (34) and (35) and solving for δC_c gives

$$\delta C_c = \frac{(A_1+A_2+A_3+A_4)_s - (A_1+A_2+A_3+A_4)_D}{4(\cos\Delta_D \sin 2\psi_D - 1)} \quad \text{for all groups.} \qquad (36)$$

The value of δC_c may now be substituted into equation (34) to solve for δA_c.

$$\delta A_c = 90° - \tfrac{1}{4}(A_1+A_2+A_3+A_4)_s - \delta C_c \qquad \text{for groups A and D,} \qquad (37a)$$

$$\delta A_c = -90° + \tfrac{1}{4}(A_1+A_2+A_3+A_4)_s + \delta C_c \qquad \text{for groups B and C.} \qquad (37b)$$

From measurements made without Faraday cells on a dielectric:

$$t_{2P} = \frac{(A_1 - A_2 - A_3 + A_4)}{4\sin 2\psi} \qquad \text{for groups A and B ,} \qquad (38a)$$

$$= \frac{(-A_1 + A_2 + A_3 - A_4)}{4\sin 2\psi} \qquad \text{for groups C and D .} \qquad (38b)$$

From multiple measurements, without Faraday cells, on a metal:

$$t_{1C(1,3)} = \frac{(A_1 + A_3) - 180° - 2(\sin 2\psi \cos\Delta\, \delta C_c - \delta A_c)}{\sin 2\psi \sin\Delta} \qquad (39a)$$

$$t_{1C(2,4)} = \frac{-(A_2 + A_4) + 180° + 2(\sin 2\psi \cos\Delta\, \delta C_c - \delta A_c)}{\sin 2\psi \sin\Delta} \qquad (39b)$$

for groups B and C. For groups A and D,

$$t_{1C(1,3)} = \frac{-(A_1 + A_3) + 180° - 2(\sin 2\psi \cos\Delta\, \delta C_c + \delta A_c)}{\sin 2\psi \sin\Delta} \qquad (39c)$$

$$t_{1C(2,4)} = \frac{(A_2 + A_4) - 180° + 2(\sin 2\psi \cos\Delta\, \delta C_c + \delta A_c)}{\sin 2\psi \sin\Delta} \qquad (39d)$$

If the measure of the quarter waveplate in zones 1 and 3 is $q = 45°$ or $Q = 135°$, then $t^+_{1C} = t_{1C(1,3)}$ and $t^-_{1C} = t_{1C(2,4)}$. Otherwise, $t^+_{1C} = t_{1C(2,4)}$ and $t^-_{1C} = t_{1C(1,3)}$.

From measurements made with the Faraday cells on a dielectric:

$$t_{2FCI}\cos 2FCI = \frac{[(A_1+A_2+A_3+A_4)_{FC} - (A_1+A_2+A_3+A_4)_{NFC}]}{2\sin\ 2\psi\cos\Delta} \qquad (40a)$$

for groups A and D,

$$= \frac{-[(A_1+A_2+A_3+A_4)_{FC} - (A_1+A_2+A_3+A_4)_{FCI}]}{2\sin\ 2\psi\cos\Delta} \qquad (40b)$$

for groups B and C.

From measurements made using the Faraday cells on a metal:

$$t_{2FCI}\sin 2FCI = \frac{-[(A_1+A_2+A_3+A_4)_{FC} - (A_1+A_2+A_3+A_4)_{NFC}] + 2t_{2FCI}\cos 2FCI(\sin 2\psi\cos\Delta)}{2\sin\ 2\psi\ \sin\Delta} \qquad (41a)$$

for group A,

$$= \frac{-[(A_1+A_2+A_3+A_4)_{FC} - (A_1+A_2+A_3+A_4)_{NFC}] - 2t_{2FCI}\cos 2FCI(\sin 2\psi\cos\Delta)}{2\sin\ 2\psi\ \sin\Delta} \qquad (41b)$$

for group B,

$$= \frac{[(A_1+A_2+A_3+A_4)_{FC} - (A_1+A_2+A_3+A_4)_{NFC}] - 2t_{2FCI}\cos 2FCI(\sin 2\psi\cos\Delta)}{2\sin\ 2\psi\ \sin\Delta} \qquad (41c)$$

for group C,

$$= \frac{[(A_1+A_2+A_3+A_4)_{FC} - (A_1+A_2+A_3+A_4)_{NFC}] + 2t_{2FCI}\cos FCI(\sin 2\psi\cos\Delta)}{2\sin\ 2\psi\ \sin\Delta} \qquad (41d)$$

for group D.

From measurements made with the Faraday cells on either a dielectric or metal specimen:

$$t_{1FCR}\cos 2FCR = \frac{(A_1-A_2-A_3+A_4)_{FC} - (A_1-A_2-A_3+A_4)_{NFC}}{2\sin 2\psi} \quad (42a)$$

for groups A and B ,

$$= \frac{(-A_1+A_2+A_3-A_4)_{FC} - (-A_1+A_2+A_3-A_4)_{NFC}}{2\sin 2\psi} \quad (42b)$$

for groups C and D ,

and

$$t_{1FCR}\sin 2FCR = \frac{(A_1-A_2+A_3-A_4)_{FC} - (A_1-A_2+A_3-A_4)_{NFC}}{2\cos 2\psi} \quad (43a)$$

for groups A and C ,

$$= \frac{(-A_1+A_2-A_3+A_4)_{FC} - (A_1+A_2-A_3+A_4)_{NFC}}{2\cos 2\psi} \quad (43b)$$

for groups B and D .

From measurements made with both Faraday cells and windows in place on a metal:

$$t_{2WI}\sin 2WI = \frac{-[(A_1+A_2+A_3+A_4)_{W} - (A_1+A_2+A_3+A_4)_{NW}]}{2\sin 2\psi\ \sin\Delta} \quad (44a)$$

for groups A and B ,

$$= \frac{(A_1+A_2+A_3+A_4)_{W} - (A_1+A_2+A_3+A_4)_{NW}}{2\sin 2\psi\ \sin\Delta} \quad (44b)$$

for groups C and D .

The next six expressions are for the remaining parameters that appear only in the equations for Δ. Because the terms t_{1P} and δP_C always appear as a sum in the equations for Δ and P, they are evaluated as a single term. From measurements made without the Faraday cells in place, on either a dielectric or metal specimen:

$$t_{1P} + \delta P_C = \delta C_C + 90° - \tfrac{1}{4}(P_1 + P_2 + P_3 + P_4) \quad \text{for all groups .} \tag{45}$$

From measurements made without the Faraday cells, on a metal:

$$t_{2C(1,3)} = \frac{(-P_1 + P_3)}{\sin\Delta} \quad \text{and} \quad t_{2C(2,4)} = \frac{(-P_2 + P_4)}{\sin\Delta} \tag{46a}$$

for groups A and C,

$$t_{2C(1,3)} = \frac{(P_1 - P_3)}{\sin\Delta} \quad \text{and} \quad t_{2C(2,4)} = \frac{(P_2 - P_4)}{\sin\Delta} \tag{46b}$$

for groups B and D.

From measurements made with the Faraday cells on a dielectric:

$$t_{2FCR} \sin 2FCR = \tfrac{1}{2} \tan 2\psi [(P_1 - P_2 - P_3 + P_4)_{FC} - (P_1 - P_2 - P_3 + P_4)_{NRC}] \tag{47a}$$

for groups A and B,

$$= \tfrac{1}{2} \tan 2\psi [(-P_1 + P_2 + P_3 - P_4)_{FC} - (-P_1 + P_2 + P_3 - P_4)_{NFC}] \tag{47b}$$

for groups C and D.

From measurements made with the Faraday cells, on either a dielectric or metal specimen,

$$t_{2FCR}\sin 2FCR = \tfrac{1}{2}\tan 2\psi[(P_1-P_2-P_3+P_4)_{FC} - (P_1-P_2-P_3+P_4)_{NFC} \qquad (48a)$$

for groups A and B,

$$= \tfrac{1}{2}\tan 2\psi[(-P_1+P_2+P_3-P_4)_{FC} - (-P_1+P_2+P_3-P_4)_{NFC} \qquad (48b)$$

for groups C and D.

From measurements made with the Faraday cells, on either dielectric or metal specimen:

$$t_{1FCI}\cos 2FCI = \frac{(P_1+P_2-P_3-P_4)_{FC} - (P_1+P_2-P_3-P_4)_{NFC}}{2\cos\Delta} \quad \text{for all groups.} \qquad (49)$$

From measurements made with the Faraday cells on a metal:

$$t_{1FCI}\sin 2FCI = \frac{[P_1-P_2-P_3+P_4)_{FC} - (P_1-P_2-P_3+P_4)_{NFC}] + 2t_{2FCR}\sin 2FCR(\cot 2\psi)}{2\sin\Delta} \qquad (50a)$$

for groups A and B,

$$= \frac{[(P_1-P_2-P_3+P_4) - (P_1-P_2-P_3+P_4)_{NFC}] - 2t_{2FCR}\sin 2FCR(\cot 2\psi)}{2\sin\Delta} \qquad (50b)$$

for groups C and D.

$$t_{2WR}\sin 2WR = \tfrac{1}{2}\left[(P_1-P_2-P_3+P_4)_W - (P_1-P_2-P_3+P_4)_{NW}\right] \quad (51a)$$

for groups A and B,

$$= \tfrac{1}{2}\left[(-P_1+P_2+P_3-P_4)_W - (-P_1+P_2+P_3-P_4)_{NW}\right] \quad (51b)$$

for groups C and D.

Now that all of the parameters have been defined, the specimen mis-positioning error can be determined. For groups A and D,

$$\delta\beta^{\pm} = \frac{\tfrac{1}{2}(R^{\pm} - \mathrm{Res}A^{\pm}) - \sin 2\psi \cos\Delta\ \delta C_c - \delta A_c}{\cos 2\phi + \sin 2\psi \cos\Delta} \quad (52a)$$

and for groups B and C,

$$\delta\beta^{\pm} = \frac{\tfrac{1}{2}(R^{\pm} - \mathrm{Res}A^{\pm}) + \sin 2\psi \cos\Delta\ \delta C_c - \delta A_c}{\cos 2\phi - \sin 2\psi \cos\Delta} . \quad (52b)$$

The sign refers to the setting of the quarter wave plate. If $q = 45°$ or $Q = 135°$ for zones 1 and 3, then $\mathrm{Res}A^{+} = A_1 + A_3$ and $\mathrm{Res}A^{-} = A_2 + A_4$. Otherwise, $\mathrm{Res}A^{+} = A_2 + A_4$ and $\mathrm{Res}A^{-} = A_1 + A_3$. Table VI contains the corresponding expressions for R which are different for each group.

Finally, we have for the azimuth angle errors,

$$\delta C^{\pm} = \delta C_c + \delta\beta^{\pm} \quad (53)$$

$$\delta P^{\pm} = \delta P_c + \delta\beta^{\pm} \quad (54)$$

$$\delta A^{\pm} = \delta A_c + (\cos 2\phi)\delta\beta^{\pm} \quad (55)$$

where the errors δA_c, δP_c, and δC_c in the calibration circle azimuths are given in equations (37), (45), and (36). Note that, for the purpose of calculation, the combinations $t_{1P} + \delta P^{\pm} - \delta C^{\pm}$ which appear in the equations for Δ and P are equal to $t_{1P} + \delta P_c - \delta C_c$.

Table VII contains the numerical values of the parameters calculated for our experiments.

D. Index of Parameter Expressions

Parameters that appear in the equations for ψ and A:

δA_c . . . . equation (37)
δC_c . . . . equation (36)
δB . . . . equation (52)
δA . . . . equation (55)
δC . . . . equation (53)
t_{1C} . . . . equation (39)
t_{2P} . . . . equation (38)
$t_{2FCI}\cos 2FCI$ . . . . equation (40)
$t_{2FCI}\sin 2FCI$ . . . . equation (41)
$t_{1FCR}\cos 2FCR$ . . . . equation (42)
$t_{1FCR}\sin 2FCR$ . . . . equation (43)
$t_{2WI}\sin 2WI$ . . . . . equation (44)

Parameters that appear in the equations for Δ and P:

δC_c . . . . equation (36)
δP_c . . . . equation (45)
t_{1P} . . . . equation (45)

$\delta\beta$	equation (52)
δC	equation (53)
δP	equation (54)
t_{2C}	equation (46)
$t_{1FCI}\cos 2FCI$ . .	equation (49)
$t_{1FCI}\sin 2FCI$ . .	equation (50)
$t_{2FCR}\cos 2FCR$ . .	equation (32)
$t_{2FCR}\sin 2FCR$ . .	equation (48)
$t_{2WI}\cos 2WR$ . . .	equation (33)
$t_{2WR}\cos 2WR$ . . .	equation (33)
$t_{2WR}\sin 2WR$ . . .	equation (51)

III. RESULTS AND DISCUSSION

A. Parameter Values

The magnitudes of the twenty parameters (Table VII, Appendix A) indicate that the quarter wave plate has the largest imperfections. Intensity effects give a maximum error of 1.72 degrees ($t_{1c}^{+} = -3.45°$) in the determination of the relative amplitude ψ, while shifts in the relative phase give a maximum error of 0.43 degrees in the determination of Δ ($t_{2c}^{+} = -0.43°$). Errors in the divided circle readings are next in importance, with δA_c and δC_c combining to give a maximum error of 1.1 degrees in ψ. The holders for the retarder and polarizing prisms would allow correction for these effects by the rotation of the components.

Both relative phase and relative attenuation effects are present in the Faraday cells. The largest parameters are for the analyzer Faraday cell ($t_{1FCR} \sin 2FCR = 0.242$, $t_{2FCR} \cos 2FCR = -0.253$). The windows of the stagnant cell exhibit minor birefringence, with only $t_{2WI} \sin 2WI = -0.228$ being significant. However, the windows of the ultra-high vacuum chamber have a much larger effect ($t_{2WI} \cos 2WI + t_{2WR} \cos 2WR = -1.54$, $t_{2WI} \sin 2WI = 0.632$).

B. The Variation of Imperfection Effects with ψ and Δ

The complete equations (Table I, Appendix A) show that the corrections for component imperfections have a trigonometric dependence on ψ and Δ. This results from the use of the rotator matrix (p. 15) to orient the principal reference frame of each component. Figures 4 and 5 indicate this dependence. The computer program discussed in Appendix C was used to correct experimental measurements on the anodic dissolution of zinc

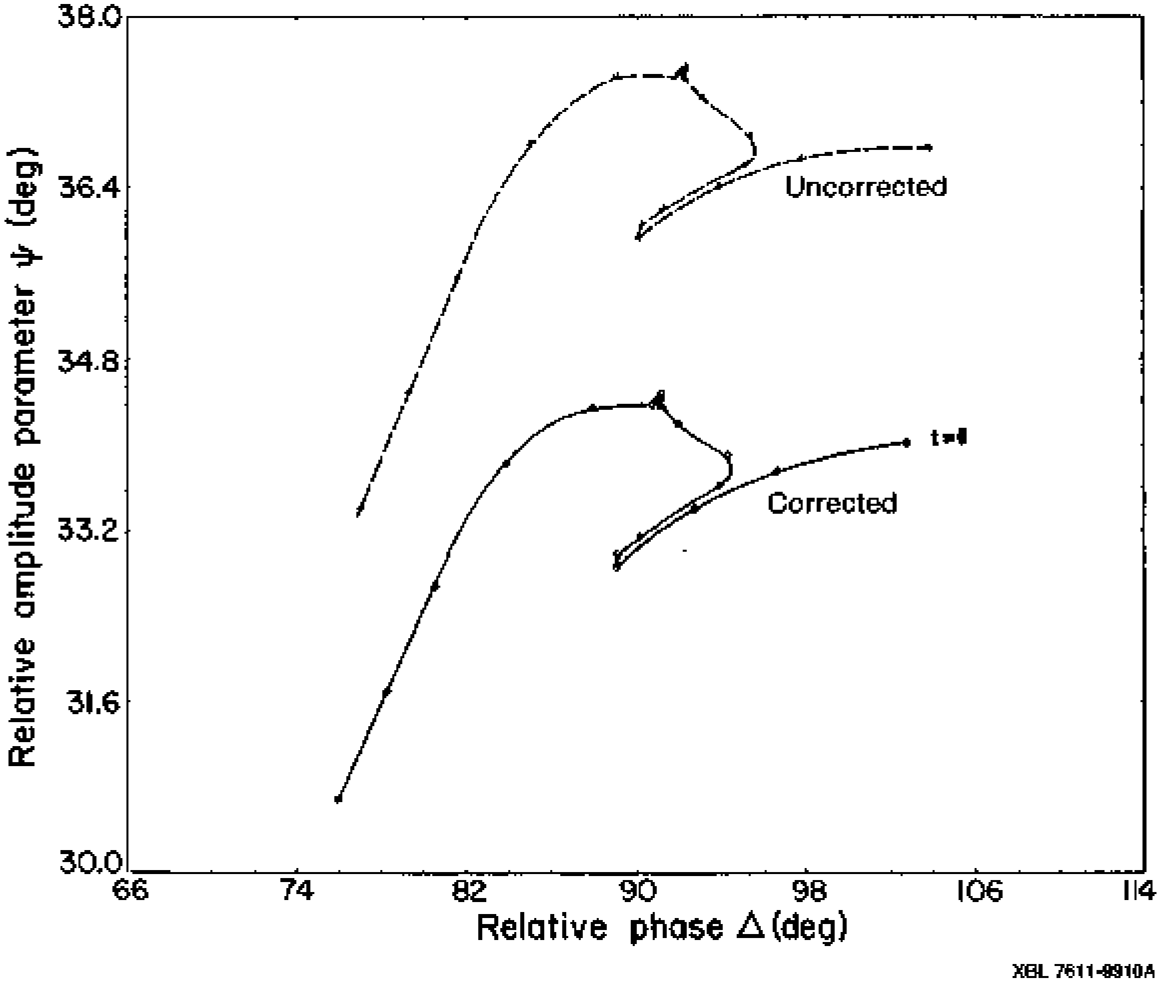

Fig. 4. Correction for ellipsometer imperfections
Zn, 0.5M KOH, -1.0V vs. Hg/HgO.

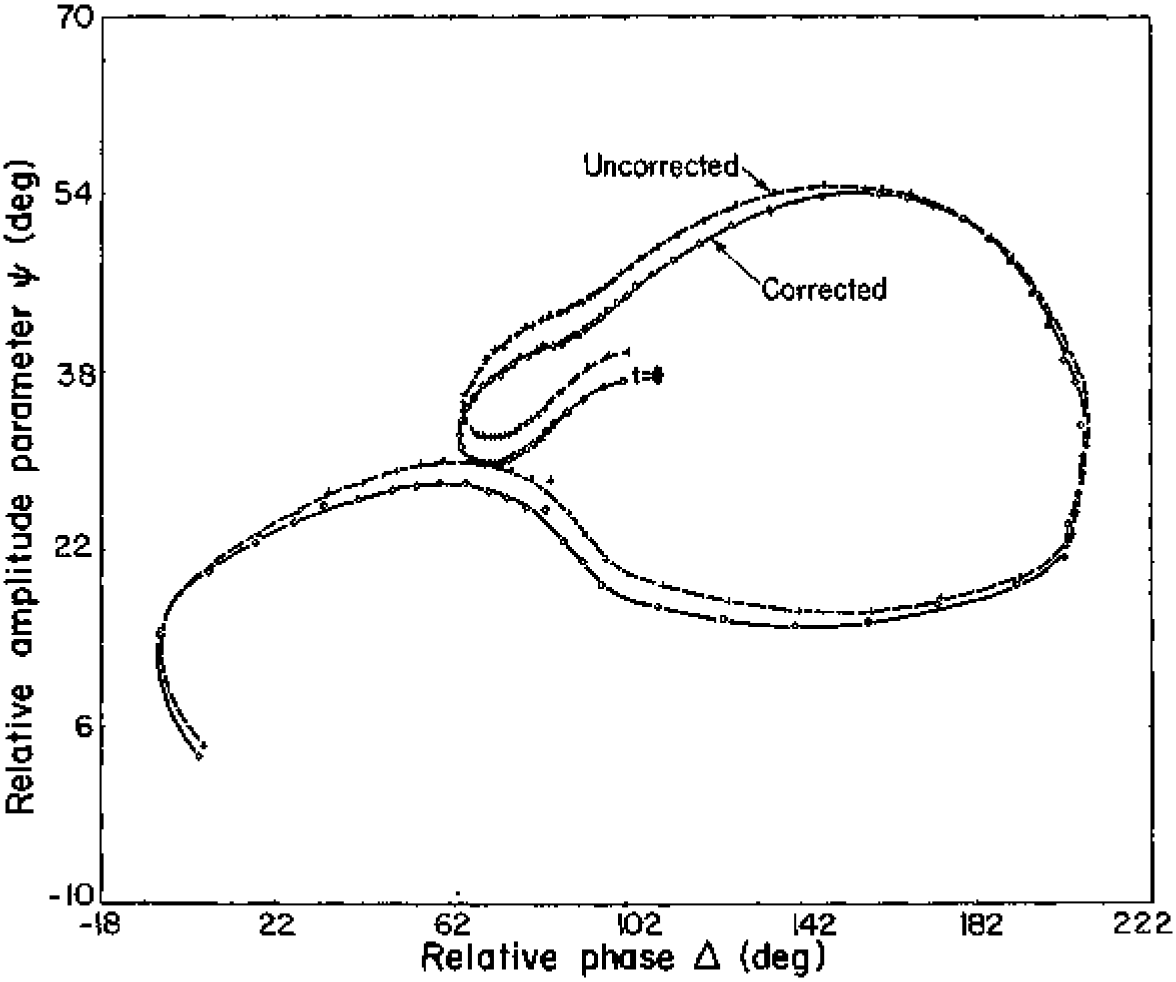

Fig. 5. Correction for ellipsometer imperfections
Zn, 0.5M KOH, -1.2V vs. Hg/HgO.

in alkaline solution. Figure 4 shows that over a small range of Δ and ψ, the corrections result in a parallel shift of the curve calculated by assuming ideal components. Figure 5 shows that over large ranges of Δ and ψ, the magnitude of the corrections vary and may even change sign. An approximate correction procedure discussed in Appendix B shows explicitly the variation of the corrections for component imperfections with the orientation of the polarizer, analyzer, and quarter wave plate.

C. Verification of the Theory

Attempts were made to verify the theory used to derive the parametric equations by correcting 4-zone measurements on materials covering a range of Δ and ψ values. The measurements on Cd, Ag, Cu, and Zn were randomly chosen from a large number of experiments conducted over a period of five months following the calibration of the ellipsometer. The measurements on Ag_2O and CdO samples prepared from compressed powders were made fourteen months after the calibration. For comparison, Δ_o and ψ_o, calculated assuming ideal components, and the range of the corrected Δ and ψ values are also presented.

The corrected 4-zone measurements indicate that the calibration procedure gives an excellent account of the effects of component imperfections in the determination of the relative amplitude parameter ψ. Over ranges of Δ values of 0.34 to 101.82, and ψ values of 22.84 to 43.05, the maximum spread in corrected ψ values is ±0.06 degrees for the Ag_2O sample. For the metal specimens, even better results were obtained, with the maximum spread being ±0.025 degrees. The corrections to the relative phase Δ at first appear less accurate, with the largest spread

Corrected 4-zone measurements.

Material	p	a	q	Δ_o	Δ	ψ_o	ψ
Cd 0001	86.20	40.72	45	97.60	96.48	40.72	38.13
Single crystal	176.44	144.47	45	97.12	96.86	35.53	38.10
	93.33	142.05	135	96.66	96.74	37.96	38.14
	3.06	38.36	135	96.12	96.45	38.36	38.13
			Average:	96.88 ±0.74	96.63 ±0.20	38.14 ±2.60	38.13 ±0.02
Ag 100	93.22	45.75	45	83.56	82.45	45.75	43.05
Single crystal	3.68	139.69	45	82.64	82.37	40.31	43.06
	86.20	137.34	135	82.40	82.46	42.66	43.05
	175.96	43.49	135	81.92	82.27	43.49	43.07
			Average:	82.63 ±0.82	82.39 ±0.10	43.05 ±2.72	43.06 ±0.01
Cu, off axis	101.59	39.16	45	66.82	65.75	39.16	37.39
Single crystal	11.71	144.37	45	66.58	66.26	35.63	37.39
	78.05	142.27	135	66.1	66.22	37.73	37.40
	167.65	37.11	135	65.3	65.58	37.11	37.40
			Average:	66.20 ±0.76	65.95 ±0.34	37.40 ±1.77	37.395 ±0.005
Zn 1010	83.64	39.52	45	102.72	101.61	39.52	36.82
Single crystal	173.99	145.92	45	102.02	101.75	34.08	36.77
	95.72	143.59	135	101.44	101.53	36.41	36.79
	5.54	37.19	135	101.08	101.40	37.19	36.77
			Average:	101.82 ±0.82	101.57 ±0.18	36.80 ±2.27	36.80 ±0.025
Ag polycrystal	93.52	44.46	45	82.96	81.85	44.46	41.75
	3.24	141.05	45	83.52	83.25	38.95	41.69
	86.49	138.72	135	82.98	83.05	41.28	41.73
	175.72	42.20	135	81.44	81.78	42.20	41.75
			Average:	82.73 ±1.04	82.48 ±0.735	41.72 ±2.76	41.72 ±0.03

Corrected 4-zone measurements (continued)

Material	P	A	Q	Δ_o	Δ	ψ_o	ψ
Ag_2O powder	140.65	67.80	135	11.30	11.95	22.20	22.87
	50.69	113.45	135	11.38	12.13	23.45	22.83
	40.18	113.65	45	9.64	9.59	23.65	22.94
	129.91	67.95	45	10.18	9.83	22.05	22.94
			Average:	10.62 ±0.98	10.87 ±1.26	22.84 ±0.79	22.89 ±0.06
CdO powder	134.56	59.87	135	0.88	0.12	30.13	30.35
	44.62	120.48	135	0.76	0.12	30.48	30.33
	45.16	120.33	45	0.32	0.37	30.33	30.31
	135.20	59.67	45	0.40	0.75	30.33	30.32
			Average:	0.59 ±0.29	0.34 ±0.41	30.32 ±0.19	30.33 ±0.02
PbO_2 powder	129.09	62.49	135	11.82	11.15	27.51	28.24
	38.70	118.87	135	12.60	11.87	28.87	28.22
	50.96	118.31	45	11.92	11.99	28.31	28.22
	140.70	61.86	45	11.40	11.72	28.14	28.22
			Average:	11.94 ±0.66	11.70 ±0.55	28.21 ±0.70	28.22 ±0.02

for the metal samples being ±0.74 for the polycrystalline Ag surface, while the spread in corrected 4-zone measurements increases over the range of uncorrected values for the two compressed-powder oxide samples.

An explanation for the range of the corrected 4-zone values of Δ is anisotropy of the specimen surface. For the idealized planar, isotropic surface, Δ and ψ would not depend upon the zone of measurement. However, structural irregularities such as surface roughness, strain-induced birefringence, or crystallographic orientation would lead to directionally-dependent optical properties of the surface. Evidence supporting anisotropy is that the range of corrected values of Δ is greatest among the single crystal specimens for the off-axis Cu surface. Also, it is certain that the oxide samples were strained by the elastic contraction of a protective brass ring following compression at 50,000 psia.

Another result which should be emphasized is that averaging of 4-zone measurements does not eliminate the effects of imperfections in the determination of the relative phase Δ (Table V, Appendix A). The residual error $t_{2FCR} \cos 2FCR + t_{2WI} \cos 2WI + t_{2WR} \cos 2WR$ remains. This suggests that the glass rod with fewer imperfections is used for the analyzer Faraday cell.

IV. CONCLUSIONS

The application of parametric equations to the calibration of our automatic ellipsometer indicates that the quarter wave plate contains the largest optical imperfections. The Faraday cells used for self-compensation contain both dicroism and birefringence. The windows of the ultra-high vacuum system exhibit significant birefringence.

The effectiveness of the calibration procedure remains uncertain due to the possibility of surface anisotropies in the specimen being measured. The results of corrections to the determination of the relative amplitude parameter ψ suggest an exceptional accuracy of ±0.02 degrees. The consideration of surface anisotropies will be necessary to obtain a more definitive verification of the calibration procedure.

The calibration was performed for the monochromatic Hg 5461 Å line. It is expected that the 20 parameter values determined by the analysis will be functions of the wavelength of light.

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APPENDIX A

TABLE I

TABLE I

ZONE ASSIGNMENTS AND IDEAL VALUES OF Ψ AND Δ

ZONE	STANDARD AZIMUTH ANGLES					ROTATED AZIMUTH ANGLES				
	p	a	c	Δ°	ψ°	P	A	C	Δ°	ψ°
A1	0° - 45°	90° - 180°	45°	90° - 2p	180° - a	9. - 135°	0° - 90°	135°	270° - 2P	90° - A
A2	45° - 50°	90° - 180°	135°	2p - 90°	180° - a	135° - 180°	0° - 90°	45°	2P - 270°	90° - A
A3	90° - 135°	0° - 90°	45°	270° - 2p	a	0° - 45°	90° - 180°	135°	90° - 2P	A - 90°
A4	135° - 180°	0° - 90°	135°	2p - 270°	a	45° - 90°	90° - 180°	45°	2P - 90°	A - 90°
B1	0° - 45°	0° - 90°	135°	90° + 2p	a	90° - 135°	90° - 180°	45°	2P - 90°	A - 90°
B2	45° - 90°	0° - 90°	45°	270° - 2p	a	135° - 180°	90° - 180°	135°	450° - 2P	A - 90°
B3	90° - 135°	90° - 180°	135°	2p - 90°	180° - a	0° - 45°	0° - 90°	45°	2P + 90°	90° - A
B4	135° - 180°	90° - 180°	45°	450° - 2p	180° - a	45° - 90°	0° - 90°	135°	270° - 2P	90° - A
C1	0° - 45°	0° - 90°	45°	270° - 2p	a	90° - 135°	90° - 180°	135°	450° - 2P	A - 90°
C2	45° - 90°	0° - 90°	135°	2p + 90°	a	135° - 180°	90° - 180°	45°	2P - 90°	A - 90°
C3	90° - 135°	90° - 180°	45°	450° - 2p	180° - a	0° - 45°	0° - 90°	135°	270° - 2P	90° - A
C4	135° - 180°	90° - 180°	135°	2p - 90°	180° - a	45° - 90°	0° - 90°	45°	2P + 90°	90° - A
D1	0° - 45°	90° - 180°	135°	2p + 270°	180° - a	90° - 135°	0° - 90°	45°	2P + 90°	90° - A
D2	45° - 90°	90° - 180°	45°	450° - 2p	180° - a	135° - 180°	0° - 90°	135°	630° - 2P	90° - A
D3	90° - 135°	0° - 90°	135°	2p - 90°	a	0° - 45°	90° - 180°	45°	2P + 270°	A - 90°
D4	135° - 180°	0° - 90°	45°	630° - 2p	a	45° - 90°	90° - 180°	135°	450° - 2P	A - 90°

TABLE IIA

COMPLETE CORRECTED EQUATIONS FOR Ψ FOR GROUP B

(NULLING ANGLES MEASURED IN STANDARD AZIMUTHS)

TABLE IIA

ZONE	
B1	$\psi = a + \delta A + \delta C \sin 2a \sin 2p - \tfrac{1}{2} t_{1c}^{-} \sin 2a \cos 2p - t_{2p} \sin 2a - \tfrac{1}{2} t_{2fci} \cos 2FCI \sin 2a \sin 2p + \tfrac{1}{2} t_{2fci} \sin 2FCI \sin 2a \cos 2p - \tfrac{1}{2} t_{1fcr} \cos 2FCR \sin 2a + \tfrac{1}{2} t_{1fcr} \sin 2FCR \cos 2a + \tfrac{1}{2} t_{2wi} \sin 2WI \sin 2a \cos 2p$
B2	$\psi = a + \delta A + \delta C \sin 2a \sin 2p - \tfrac{1}{2} t_{1c}^{+} \sin 2a \cos 2p + t_{2p} \sin 2a - \tfrac{1}{2} t_{2fci} \cos 2FCI \sin 2a \sin 2p - \tfrac{1}{2} t_{2fci} \sin 2FCI \sin 2a \cos 2p + \tfrac{1}{2} t_{1fcr} \cos 2FCR \sin 2a - \tfrac{1}{2} t_{1fcr} \sin 2FCR \cos 2a - \tfrac{1}{2} t_{2wi} \sin 2WI \sin 2a \cos 2p$
B3	$\psi = 180° - a - \delta A - \delta C \sin 2a \sin 2p + \tfrac{1}{2} t_{1c}^{-} \sin 2a \cos 2p + t_{2p} \sin 2a + \tfrac{1}{2} t_{2fci} \cos 2FCI \sin 2a \sin 2p - \tfrac{1}{2} t_{2fci} \sin 2FCI \sin 2a \cos 2p + \tfrac{1}{2} t_{1fcr} \cos 2FCR \sin 2a - \tfrac{1}{2} t_{1fcr} \sin 2FCR - \tfrac{1}{2} t_{2wi} \sin 2WI \sin 2a \cos 2p$
B4	$\psi = 180° - a - \delta A - \delta C \sin 2a \sin 2p + \tfrac{1}{2} t_{1c}^{+} \sin 2a \cos 2p - t_{2p} \sin 2a + \tfrac{1}{2} t_{2fci} \cos 2FCI \sin 2a \sin 2p + \tfrac{1}{2} t_{2fci} \sin 2FCI \sin 2a \cos 2p - \tfrac{1}{2} t_{1fcr} \cos 2FCR \sin 2a + \tfrac{1}{2} t_{1fcr} \sin 2FCR + \tfrac{1}{2} t_{2wi} \sin 2WI \sin 2a \cos 2p$

TABLE IIB

COMPLETE CORRECTED EQUATIONS FOR Δ FOR GROUP B

(NULLING ANGLES MEASURED IN STANDARD AZIMUTHS)

TABLE IIB

ZONE	
B1	$\Delta = 90° + 2p - 2\delta C + 2\delta P + 2t_{1p} - t_{2c}^{-} \cos 2p + t_{1fci} \cos 2FCI \sin 2p - t_{1fci} \sin 2FCI \cos 2p$ $+ t_{2fcr} \cos 2FCR - t_{2fcr} \sin 2FCR \cot 2a + t_{2wi} \cos 2WI + t_{2wr} \cos 2WR - t_{2wr} \sin 2WR \cot 2a$
B2	$\Delta = 270° - 2p + 2\delta C - 2\delta P - 2t_{1p} - t_{2c}^{+} \cos 2p - t_{1fci} \cos 2FCI \sin 2p + t_{1fci} \sin 2FCI \cos 2p$ $+ t_{2fcr} \cos 2FCR - t_{2fcr} \sin 2FCR \cot 2a + t_{2wi} \cos 2WI + t_{2wr} \cos 2WR - t_{2wr} \sin 2WR \cot 2a$
B3	$\Delta = 2p - 90° - 2\delta C + 2\delta P + 2t_{1p} - t_{2c}^{-} \cos 2p + t_{1fci} \cos 2FCI \sin 2p - t_{1fci} \sin 2FCI \cos 2p$ $+ t_{2fcr} \cos 2FCR - t_{2fcr} \sin 2FCR \cot 2a + t_{2wi} \cos 2WI + t_{2wr} \cos 2WR - t_{2wr} \sin 2WR \cot 2a$
B4	$\Delta = 450° - 2p + 2\delta C - 2\delta P - 2t_{1p} - t_{2c}^{+} \cos 2p - t_{1fci} \cos 2FCI \sin 2p + t_{1fci} \sin 2FCI \cos 2p$ $+ t_{2fcr} \cos 2FCR - t_{2fcr} \sin 2FCR \cot 2a + t_{2wi} \cos 2WI + t_{2wr} \cos 2WR - t_{2wr} \sin 2WR \cot 2a$

Note: $\delta C - \delta P = (\delta C_c + \delta B) - (\delta P_c + \delta B)$

$= \delta C_c - \delta P_c$

TABLE IIc

SIGNS OF THE PARAMETERS IN THE CORRECTED EQUATIONS FOR Ψ FOR ALL GROUPS

(NULLING ANGLES MEASURED IN STANDARD AZIMUTH ANGLES)

TABLE IIc

Zone	δA	δC sin2a X sin2p	$\frac{1}{2} t_{1c}$ X sin2a cos2p	t_{2p} X sin2a	$\frac{1}{2} t_{2fci}$ cos2FCI X sin2a sin2p	$\frac{1}{2} t_{2fci}$ sin2FCI X sin2a cos2p	$\frac{1}{2} t_{1fcr}$ X cos2FCR sin2a	$\frac{1}{2} t_{1fcr}$ X sin2FCR cos2a	$\frac{1}{2} t_{2wi}$ sin2WI X sin2a cos2p
A1	-	+	+	-	-	+	-	+	+
A2	-	+	+	+	-	-	+	-	-
A3	+	-	-	+	+	-	+	-	-
A4	+	-	-	-	+	+	-	+	+
B1	+	+	-	-	-	+	-	+	+
B2	+	+	-	+	-	-	+	-	-
B3	-	-	+	+	+	-	+	-	-
B4	-	-	+	-	+	+	-	+	+
C1	+	+	+	+	-	+	+	-	+
C2	+	+	+	-	-	-	-	+	-
C3	-	-	-	-	+	-	-	+	-
C4	-	-	-	+	+	+	+	-	+
D1	-	+	-	+	-	+	+	-	+
D2	-	+	-	-	-	-	-	+	-
D3	+	-	+	-	+	-	-	+	-
D4	+	-	+	+	+	+	+	-	+

TABLE IID

SIGNS OF THE PARAMETERS IN THE CORRECTED EQUATIONS FOR △ FOR ALL GROUPS

(NULLING ANGLES MEASURED IN STANDARD AZIMUTH ANGLES)

TABLE IID

Zone	2δC	2δP	$2t_{1p}$	t_{2c} X cos2p	t_{1fci} X cos2FCI sin2p	t_{1fci} X sin2FCI cos2p	t_{2fcr} X cos2FCR	t_{2fcr} X sin2FCR cot2a	t_{2wi} X cos2WI	t_{2wr} X cos2WR	t_{2wr} X sin2WR cot2a
A1	+	-	-	-	+	+	+	-	+	+	-
A2	-	+	+	-	-	-	+	-	+	+	-
A3	+	-	-	-	+	+	+	-	+	+	-
A4	-	+	+	-	-	-	+	-	+	+	-
B1	-	+	+	-	+	-	+	-	+	+	-
B2	+	-	-	-	-	+	+	-	+	+	-
B3	-	+	+	-	+	-	+	-	+	+	-
B4	+	-	-	-	-	+	+	-	+	+	-
C1	+	-	-	+	-	-	+	-	+	+	-
C2	-	+	+	+	+	+	+	-	+	+	-
C3	+	-	-	+	-	-	+	-	+	+	-
C4	-	+	+	+	+	+	+	-	+	+	-
D1	-	+	+	+	-	+	+	-	+	+	-
D2	+	-	-	+	+	-	+	-	+	+	-
D3	-	+	+	+	-	+	+	-	+	+	-
D4	+	-	-	+	+	-	+	-	+	+	-

TABLE IIIA

SIGNS OF THE PARAMETERS IN THE CORRECTED EQUATIONS FOR Ψ FOR ALL GROUPS

(NULLING ANGLES MEASURED IN ROTATED AZIMUTH ANGLES)

TABLE IIIA

Zone	δA	δC X sin2A sin2P	$\frac{1}{2}$ t_{1c} X sin2A cos2P	t_{2P} X sin2A	$\frac{1}{2}$ t_{2fci} X cos2FCI sin2A sin2P	$\frac{1}{2}$ t_{2fci} X sin2FCI sin2A cos2P	$\frac{1}{2}$ t_{1fcr} X cos2FCR sin2A	$\frac{1}{2}$ t_{1fcr} X sin2FCR cos2A	$\frac{1}{2}$ t_{2wi} X sin2WI sin2A cos2P
A1	-	+	-	+	-	+	+	-	+
A2	-	+	-	-	-	-	-	+	-
A3	+	-	+	-	+	-	-	+	-
A4	+	-	+	+	+	+	+	-	+
B1	+	+	-	+	-	+	+	-	+
B2	+	+	-	-	-	-	-	+	-
B3	-	-	+	-	+	-	-	+	-
B4	-	-	+	+	+	+	+	-	+
C1	+	+	-	-	-	+	-	+	+
C2	+	+	-	+	-	-	+	-	-
C3	-	-	+	+	+	-	+	-	-
C4	-	-	+	-	+	+	-	+	+
D1	-	+	-	-	-	+	-	+	+
D2	-	+	-	+	-	-	+	-	-
D3	+	-	+	+	+	-	+	-	-
D4	+	-	+	-	+	+	-	+	+

TABLE IIIB

SIGNS OF THE PARAMETERS IN THE CORRECTED EQUATIONS FOR Δ FOR ALL GROUPS

(NULLING ANGLES MEASURED IN ROTATED AZIMUTH ANGLES)

TABLE IIIB

Zone	2δC	2δP	$2t_{1P}$	t_{2c} X cos2P	t_{1fci} X cos2PCI sin2P	t_{1fci} X sin2PCI cos2P	t_{2fcr} X cos2PCR	t_{2fcr} X sin2PCR cot2A	t_{2wi} X cos2WI	t_{2wr} X cos2WR	t_{2wr} X sin2WR cot2A
A1	+	-	-	+	-	-	+	-	+	+	-
A2	-	+	+	+	+	+	+	-	+	+	-
A3	+	-	-	+	-	-	+	-	+	+	-
A4	-	+	+	+	+	+	+	-	+	+	-
B1	-	+	+	+	-	+	+	-	+	+	-
B2	+	-	-	+	+	-	+	-	+	+	-
B3	-	+	+	+	-	+	+	-	+	+	-
B4	+	-	-	+	+	-	+	-	+	+	-
C1	+	-	-	-	+	+	+	-	+	+	-
C2	-	+	+	-	-	-	+	-	+	+	-
C3	+	-	-	-	+	+	+	-	+	+	-
C4	-	+	+	-	-	-	+	-	+	+	-
D1	-	+	+	-	+	-	+	-	+	+	-
D2	+	-	-	-	-	+	+	-	+	+	-
D3	-	+	+	-	+	-	+	-	+	+	-
D4	+	-	-	-	-	+	+	-	+	+	-

TABLE IVA

TABLE IVA

COMPLETE CORRECTED EQUATIONS FOR THE ANALYZER AZIMUTH

(NULLING ANGLES MEASURED IN STANDARD AZIMUTHS)

ZONE	
B1	$A = \psi - \delta A + \delta C \sin 2\psi \cos\Delta + \frac{1}{2} t_{1c}^{-} \sin 2\psi \sin\Delta + t_{2p} \sin 2\psi - \frac{1}{2} t_{2fci} \cos 2FCI \sin 2\psi \cos\Delta - \frac{1}{2} t_{2fci} \sin 2FCI \sin 2\psi \sin\Delta + \frac{1}{2} t_{1fcr} \cos 2FCR \sin 2\psi - \frac{1}{2} t_{1fcr} \sin 2FCR \cos 2\psi - \frac{1}{2} t_{2wi} \sin 2WI \sin 2\psi \sin\Delta$
B2	$A = \psi - \delta A + \delta C \sin 2\psi \cos\Delta - \frac{1}{2} t_{1c}^{+} \sin 2\psi \sin\Delta - t_{2p} \sin 2\psi - \frac{1}{2} t_{2fci} \cos 2FCI \sin 2\psi \cos\Delta - \frac{1}{2} t_{2fci} \sin 2FCI \sin 2\psi \sin\Delta - \frac{1}{2} t_{1fcr} \cos 2FCR \sin 2\psi + \frac{1}{2} t_{1fcr} \sin 2FCR \cos 2\psi - \frac{1}{2} t_{2wi} \sin 2WI \sin 2\psi \sin\Delta$
B3	$A = 180^\circ - \psi - \delta A + \delta C \sin 2\psi \cos\Delta + \frac{1}{2} t_{1c}^{-} \sin 2\psi \sin\Delta - t_{2p} \sin 2\psi - \frac{1}{2} t_{2fci} \cos 2FCI \sin 2\psi \cos\Delta - \frac{1}{2} t_{2fci} \sin 2FCI \sin 2\psi \sin\Delta - \frac{1}{2} t_{1fcr} \cos 2FCR \sin 2\psi - \frac{1}{2} t_{1fcr} \sin 2FCR \cos 2\psi - \frac{1}{2} t_{2wi} \sin 2WI \sin 2\psi \sin\Delta$
B4	$A = 180^\circ - \psi - \delta A + \delta C \sin 2\psi \cos\Delta - \frac{1}{2} t_{1c}^{+} \sin 2\psi \sin\Delta + t_{2p} \sin 2\psi - \frac{1}{2} t_{2fci} \cos 2FCI \sin 2\psi \cos\Delta - \frac{1}{2} t_{2fci} \sin 2FCI \sin 2\psi \sin\Delta + \frac{1}{2} t_{1fcr} \cos 2FCR \sin 2\psi + \frac{1}{2} t_{1fcr} \sin 2FCR \cos 2\psi - \frac{1}{2} t_{2wi} \sin 2WI \sin 2\psi \sin\Delta$

TABLE IVB

TABLE IVB

COMPLETE CORRECTED EQUATIONS FOR THE POLARIZER AZIMUTH

(NULLING ANGLES MEASURED IN STANDARD AZIMUTHS)

$$P = \tfrac{1}{2}(\Delta - 90°) + \delta C - \delta P - t_{1p} + \tfrac{1}{2} t_{2c}^{+} \sin\Delta + \tfrac{1}{2} t_{1fci} \cos 2FCI \cos\Delta + \tfrac{1}{2} t_{1fci} \sin 2FCI \sin\Delta$$
$$- \tfrac{1}{2} t_{2fcr} \cos 2FCR + \tfrac{1}{2} t_{2fcr} \sin 2FCR \cot 2\psi - \tfrac{1}{2} t_{2wi} \cos 2WI - \tfrac{1}{2} t_{2wr} \cos 2WR + \tfrac{1}{2} t_{2wr} \sin 2WR \cot 2\psi$$

$$P = \tfrac{1}{2}(270° - \Delta) + \delta C - \delta P - t_{1p} + \tfrac{1}{2} t_{2c}^{-} \sin\Delta + \tfrac{1}{2} t_{1fci} \cos 2FCI \cos\Delta - \tfrac{1}{2} t_{1fci} \sin 2FCI \sin\Delta$$
$$+ \tfrac{1}{2} t_{2fcr} \cos 2FCR - \tfrac{1}{2} t_{2fcr} \sin 2FCR \cot 2\psi + \tfrac{1}{2} t_{2wi} \cos 2WI + \tfrac{1}{2} t_{2wr} \cos 2WR - \tfrac{1}{2} t_{2wr} \sin 2WR \cot 2\psi$$

$$P = \tfrac{1}{2}(90° + \Delta) + \delta C - \delta P - t_{1p} - \tfrac{1}{2} t_{2c}^{+} \sin\Delta - \tfrac{1}{2} t_{1fci} \cos 2FCI \cos\Delta - \tfrac{1}{2} t_{1fci} \sin 2FCI \sin\Delta$$
$$- \tfrac{1}{2} t_{2fcr} \cos 2FCR - \tfrac{1}{2} t_{2fcr} \sin 2FCR \cot 2\psi - \tfrac{1}{2} t_{2wi} \cos 2WI + \tfrac{1}{2} t_{2wr} \cos 2WR - \tfrac{1}{2} t_{2wr} \sin 2WR \cot 2\psi$$

$$P = \tfrac{1}{2}(450° - \Delta) + \delta C - \delta P - t_{1p} - \tfrac{1}{2} t_{2c}^{-} \sin\Delta - \tfrac{1}{2} t_{1fci} \cos 2FCI \cos\Delta + \tfrac{1}{2} t_{1fci} \sin 2FCI \sin\Delta$$
$$+ \tfrac{1}{2} t_{2fcr} \cos 2FCR + \tfrac{1}{2} t_{2fcr} \sin 2FCR \cot 2\psi + \tfrac{1}{2} t_{2wi} \cos 2WI + \tfrac{1}{2} t_{2wr} \cos 2WR + \tfrac{1}{2} t_{2wr} \sin 2WR \cot 2\psi$$

Note: $\delta C - \delta P = (\delta C_c + \delta\beta) - (\delta P_c + \delta\beta)$; Definition is for Group B

$= \delta C_c - \delta A$

TABLE IVc

SIGNS OF THE PARAMETERS IN THE CORRECTED EQUATIONS FOR THE ANALYZER AZIMUTH FOR ALL GROUPS

(NULLING ANGLES MEASURED IN EITHER STANDARD OR ROTATED AZIMUTH ANGLES)

TABLE IVc

Zone	δA	$\delta C \times \sin 2\psi \cos\Delta$	$\frac{1}{2} t_{1c} \times \sin 2\psi \sin\Delta$	$t_{2p} \times \sin 2\psi$	$\frac{1}{2} t_{2fci} \times \cos 2FCI \sin 2\psi \cos\Delta$	$\frac{1}{2} t_{2fci} \times \sin 2FCI \sin 2\psi \sin\Delta$	$\frac{1}{2} t_{2fcr} \times \cos 2FCR \sin 2\psi$	$\frac{1}{2} t_{1fcr} \times \sin 2FCR \cos 2\psi$	$\frac{1}{2} t_{2wi} \times \sin 2WI \sin 2\psi \sin\Delta$
A1	-	-	-	+	+	-	+	+	-
A2	-	-	+	-	+	-	-	-	-
A3	-	-	-	-	+	-	-	+	-
A4	-	-	+	+	+	-	+	-	-
B1	-	+	+	+	-	-	+	-	-
B2	-	+	-	-	-	-	-	+	-
B3	-	+	+	-	-	-	-	-	-
B4	-	+	-	+	-	-	+	+	-
C1	-	+	+	-	-	+	-	+	+
C2	-	+	-	+	-	+	+	-	+
C3	-	+	+	+	-	+	+	+	+
C4	-	+	-	-	-	+	-	-	+
D1	-	-	-	-	+	+	-	-	+
D2	-	-	+	+	+	+	+	+	+
D3	-	-	-	+	+	+	+	-	+
D4	-	-	+	-	+	+	-	+	+

TABLE IVD

SIGNS OF THE PARAMETERS IN THE CORRECTED EQUATIONS FOR THE POLARIZER AZIMUTH FOR ALL GROUPS

(NULLING ANGLES MEASURED IN EITHER STANDARD OR ROTATED AZIMUTH ANGLES)

TABLE IVD

Zone	δC	δP	t_{1p}	$\frac{1}{2} t_{2c} \times \sin\Delta$	$\frac{1}{2} t_{1fcr} \times \cos 2PCI \cos\Delta$	$\frac{1}{2} t_{1fcr} \times \sin 2PCI \sin\Delta$	$\frac{1}{2} t_{2fcr} \times \cos 2PCR$	$\frac{1}{2} t_{2fcr} \times \sin 2PCR \cos 2\psi$	$\frac{1}{2} t_{2wi} \times \cos 2WI$	$\frac{1}{2} t_{2wr} \times \cos 2WR$	$\frac{1}{2} t_{2wr} \times \sin 2WR \cot 2\psi$
A1	+	-	-	-	+	+	+	+	+	+	+
A2	+	-	-	-	+	-	-	-	-	-	-
A3	+	-	-	+	-	-	+	-	+	+	-
A4	+	-	-	+	-	+	-	+	-	-	+
B1	+	-	-	+	+	+	-	+	-	-	+
B2	+	-	-	+	+	-	+	-	+	+	-
B3	+	-	-	-	-	-	-	-	-	-	-
B4	+	-	-	-	-	+	+	+	+	+	+
C1	+	-	-	-	+	+	+	-	+	+	-
C2	+	-	-	-	+	-	-	+	-	-	+
C3	+	-	-	+	-	-	+	+	+	+	+
C4	+	-	-	+	-	+	-	-	-	-	-
D1	+	-	-	+	+	+	-	-	-	-	-
D2	+	-	-	+	+	-	+	+	+	+	+
D3	+	-	-	-	-	-	-	+	-	-	+
D4	+	-	-	-	-	+	+	-	+	+	-

TABLE V

FOUR-ZONE AVERAGES FOR Ψ AND Δ

GROUP	STANDARD AZIMUTH ANGLES		ROTATED AZIMUTH ANGLES	
	ψ	Δ	ψ	Δ
A	$-\frac{1}{2}(a_1+a_2-a_3-a_4) + 90°$	$-\frac{1}{2}(p_1-p_2+p_3-p_4) + \Delta c$	$-\frac{1}{2}(A_1+A_2-A_3-A_4)$	$-\frac{1}{2}(P_1-P_2+P_3-P_4) + \Delta c$
B	$+\frac{1}{2}(a_1+a_2-a_3-a_4) + 90°$	$+\frac{1}{2}(p_1-p_2+p_3-p_4) + 180° + \Delta c$	$+\frac{1}{2}(A_1+A_2-A_3-A_4)$	$+\frac{1}{2}(P_1-P_2+P_3-P_4) + 180° + \Delta c$
C	$+\frac{1}{2}(a_1+a_2-a_3-a_4) + 90°$	$-\frac{1}{2}(p_1-p_2+p_3-p_4) + 180° + \Delta c$	$+\frac{1}{2}(A_1+A_2-A_3-A_4)$	$-\frac{1}{2}(P_1-P_2+P_3-P_4) + 180° + \Delta c$
D	$-\frac{1}{2}(a_1+a_2-a_3-a_4) + 90°$	$+\frac{1}{2}(p_1-p_2+p_3-p_4) + 360° + \Delta c$	$-\frac{1}{2}(A_1+A_2-A_3-A_4)$	$+\frac{1}{2}(P_1-P_2+P_3-P_4) + 360° + \Delta c$

For measurements made without Faraday cells and windows, $\Delta c = 0$

For measurements made with Faraday cells, without windows, $\Delta c = t_{2fcr}\cos 2FCR$.

For measurements made with Faraday cells and windows, $\Delta c = t_{2fcr}\cos 2FCR + t_{2wi}\cos 2WI + t_{2wr}\cos 2WR$.

TABLE VI

TABLE VI

EXPRESSIONS FOR "R"

ZONE	
A1, A3	$R^{+} = + t_{1fcr} \sin 2FCR \cos 2\psi + \sin 2\psi \, [t_{2fci} \cos 2FCI \cos\Delta - \sin\Delta (t_{1c}^{+} + t_{2fci} \sin 2FCI + t_{2wi} \sin 2WI)]$
A2, A4	$R^{-} = - t_{1fcr} \sin 2FCR \cos 2\psi + \sin 2\psi \, [t_{2fci} \cos 2FCI \cos\Delta - \sin\Delta (-t_{1c}^{-} + t_{2fci} \sin 2FCI + t_{2wi} \sin 2WI)]$
B1, B3	$R^{-} = - t_{1fcr} \sin 2FCR \cos 2\psi + \sin 2\psi \, [-t_{2fci} \cos 2FCI \cos\Delta - \sin\Delta \, (-t_{1c}^{-} + t_{2fci} \sin 2FCI + t_{2wi} \sin 2WI)]$
B2, B4	$R^{+} = + t_{1fcr} \sin 2FCR \cos 2\psi + \sin 2\psi \, [-t_{2fci} \cos 2FCI \cos\Delta - \sin\Delta \, (t_{1c}^{+} + t_{2fci} \sin 2FCI + t_{2wi} \sin 2WI)]$
C1, C3	$R^{+} = + t_{1fcr} \sin 2FCR \cos 2\psi + \sin 2\psi \, [-t_{2fci} \cos 2FCI \cos\Delta - \sin\Delta \, (-t_{1c}^{+} - t_{2fci} \sin 2FCI - t_{2wi} \sin 2WI)]$
C2, C4	$R^{-} = - t_{1fcr} \sin 2FCR \cos 2\psi + \sin 2\psi \, [-t_{2fci} \cos 2FCI \cos\Delta - \sin\Delta \, (+t_{1c}^{-} - t_{2fci} \sin 2FCI - t_{2wi} \sin 2WI)]$
D1, D3	$R^{-} = - t_{1fcr} \sin 2FCR \cos 2\psi + \sin 2\psi \, [t_{2fci} \cos 2FCI \cos\Delta - \sin\Delta \, (+t_{1c}^{-} - t_{2fci} \sin 2FCI - t_{2wi} \sin 2WI)]$
D2, D4	$R^{+} = + t_{1fcr} \sin 2FCR \cos 2\psi + \sin 2\psi \, [t_{2fci} \cos 2FCI \cos\Delta - \sin\Delta \, (-t_{1c}^{+} - t_{2fci} \sin 2FCI - t_{2wi} \sin 2WI)]$

TABLE VII

TABLE VII

VALUES OF THE PARAMETERS
(IN DEGREES)

$\delta Ac = 1.088$

$\delta Cc = 1.128$

$\delta Pc + t_{1p} = 1.353$

$t_{2p} = 0.021$

$t_{1c}^{+} = -3.4527$
$t_{1c}^{-} = -1.653$
$t_{2c}^{+} = -0.4286$
$t_{2c}^{-} = -0.1633$

+ Corresponds to Q = 45°
− Corresponds to Q = 135°

$t_{1fci}\ \cos 2FCI = -0.1102$
$t_{1fci}\ \sin 2FCI = -0.0014$
$t_{1fcr}\ \cos 2FCR = -0.0078$
$t_{1fcr}\ \sin 2FCR = +0.2415$
$t_{2fci}\ \sin 2FCI = 0.0443$
$t_{2fci}\ \cos 2FCI = -0.1424$
$t_{2fcr}\ \cos 2FCR = -0.2525$
$t_{2fcr}\ \sin 2FCR = 0.0742$

Stagnant Cell Windows	Vacuum Chamber Windows
$t_{2wi}\ \cos 2WI + t_{2wr}\ \cos 2WR = 0.0050$	− 1.54
$t_{2wr}\ \sin 2WR = 0.0172$	− .03
$t_{2wi}\ \sin 2WI = -0.2281$	+ .6322

TABLE VIII

TABLE VIII

COMPLETE EQUATION FOR STANDARD ANALYZER AZIMUTH FOR ZONE B1

$$A = \psi - (\delta Ac + \delta B \cos 2\psi) + (\delta Cc + \delta B) \sin 2\psi \cos\Delta + \tfrac{1}{2} t_{1c}^{-} \sin 2\psi \sin\Delta + t_{2p} \sin 2\psi - \tfrac{1}{2} t_{2fci} \cos 2FCI \sin 2\psi \cos\Delta$$
$$- \tfrac{1}{2} t_{2fci} \sin 2FCI \sin 2\psi \sin\Delta + \tfrac{1}{2} t_{1fcr} \cos 2FCR \sin 2\psi - \tfrac{1}{2} t_{1fcr} \sin 2FCR \cos 2\psi - \tfrac{1}{2} t_{2wi} \sin 2WI \sin 2\psi \sin\Delta$$

<u>From measurements made while the ellipsometer is in the straight-through position</u> (no specimen)

- Without the Faraday cells or the cell windows.

<u>Simplifications</u>

1. No terms for the Faraday cells or cell windows.
2. $\delta B = 0$
3. $\Delta° = 0° \Rightarrow \cos\Delta° = 1,\ \sin\Delta° = 0$
4. $\psi° = 45° \Rightarrow \sin 2\psi° = 1$

<u>Reduced Equation</u>

$A = \psi - \delta Ac + \delta Cc + t_{2p}$

<u>From measurements made using a dielectric as a specimen</u>

A. Without the Faraday cells or cell windows.
B. With the Faraday cells, without the cell windows.

A. <u>Simplifications</u>

1. No terms for the Faraday cells or cell windows.
2. $\Delta° = 0° \Rightarrow \cos\Delta° = 1,\ \sin\Delta° = 0$

A. <u>Reduced Equation</u>

$A = \psi - \delta A + \delta C \sin 2\psi + t_{2p} \sin 2\psi$

B. <u>Simplifications</u>

1. No terms for the cell windows.
2. $\Delta° = 0° \Rightarrow \cos\Delta° = 1,\ \sin\Delta° = 0$

B. <u>Reduced Equation</u>

$$A = \psi - \delta A + \delta C \sin 2\psi + t_{2p} \sin 2\psi - \tfrac{1}{2} t_{2fci} \cos 2FCI \sin 2\psi + \tfrac{1}{2} t_{1fcr} \cos 2FCR \sin 2\psi - \tfrac{1}{2} t_{1fcr} \sin 2FCR \cos 2\psi$$

<u>From measurements made using a polished metal as a specimen</u>

A. Without the Faraday cells or cell windows.
B. With the Faraday cells, without the cell windows.
C. With both the Faraday cells and the cell windows (complete equation).

A. <u>Simplifications</u>

- No terms for the Faraday cells or cell windows.

A. <u>Reduced Equation</u>

$$A = \psi - \delta A + \delta C \sin 2\psi \cos\Delta + t_{2p} \sin 2\psi + \tfrac{1}{2} t_{1c} \sin 2\psi \sin\Delta$$

B. <u>Simplifications</u>

- No terms for the cell windows

B. <u>Reduced Equation</u>

$$A = \psi - \delta A + \delta C \sin 2\psi \cos\Delta + t_{2p} \sin 2\psi + \tfrac{1}{2} t_{1c}^{-} \sin 2\psi \sin\Delta - \tfrac{1}{2} t_{2fci} \cos 2FCI \sin 2\psi \cos\Delta - \tfrac{1}{2} t_{2fci} \sin 2FCI \sin 2\psi \sin\Delta - \tfrac{1}{2} t_{1fcr} \cos 2FCR \sin 2\psi - \tfrac{1}{2} t_{1fcr} \sin 2FCR \cos 2\psi$$

APPENDIX B. AN APPROXIMATE DETERMINATION OF ψ AND Δ

Once the parameter values describing component imperfections have been determined, experimental azimuths may be transformed to the relative phase Δ and relative intensity parameter ψ. The specimen mispositioning error $\delta\beta$ (equation 52, p. 45) is the only correction term which varies between experiments. This term affects only ψ, and has the magnitude

$$|\delta\beta| = |\delta\beta[\cos 2\phi + \sin 2a \sin 2p]|$$

where ϕ is the angle of incidence. For the corrected 4-zone measurements on Cd (p. 52), $|\delta\psi|$ = 0.213, 0.236, 0.108, and 0.109 degrees.

A graphical correction procedure may be used if reduced computation time justifies the uncertainty in ψ introduced by neglecting $\delta\beta$. Computer-generated plots are constructed for each set of the 20 parameter values, with $\delta\beta = 0$. The corrections to Δ and ψ due to component imperfections $\delta\Delta$ and $\delta\psi$ are presented as functions of the polarizer and analyzer azimuths p and a. Figures B1 and B3 show the corrections for $q = 45°$; Figures B2 and B4 show the corrections for $q = 135°$. The parameter values in Table VII (Appendix A, p. 71) with the stagnant cell windows have been used. The correction terms $\delta\Delta$ and $\delta\psi$ are added to the values of Δ and ψ calculated by assuming ideal components.

It should be noted that graphical procedures introduce human error in the reading of the plots. For data acquisition systems with computational capabilities, a form of the computer program in Appendix C should be used.

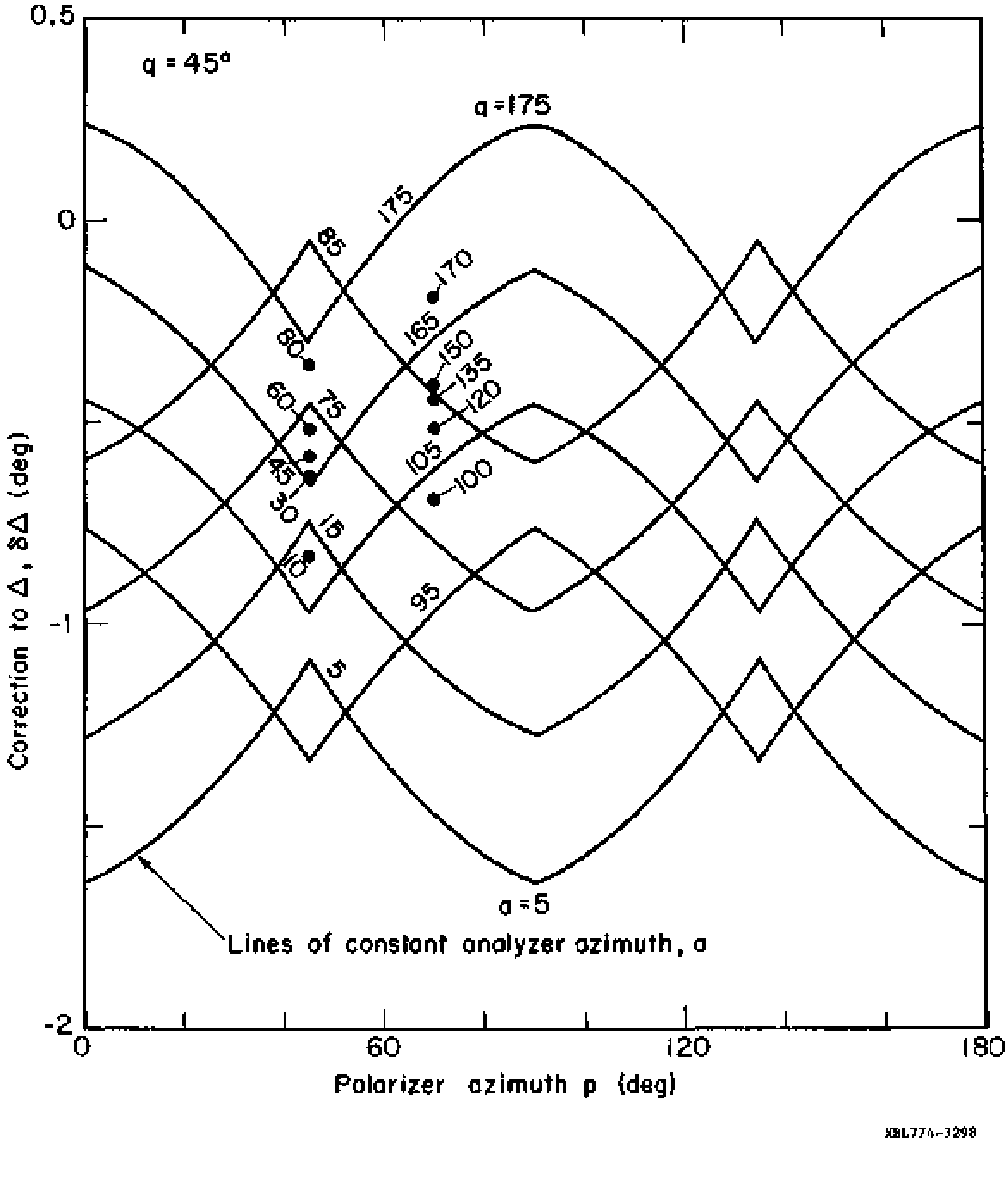


Fig. B1. Correction to Δ due to component imperfections. Quarter wave compensator at 45°, specimen mispositioning error $\delta\beta = 0$.

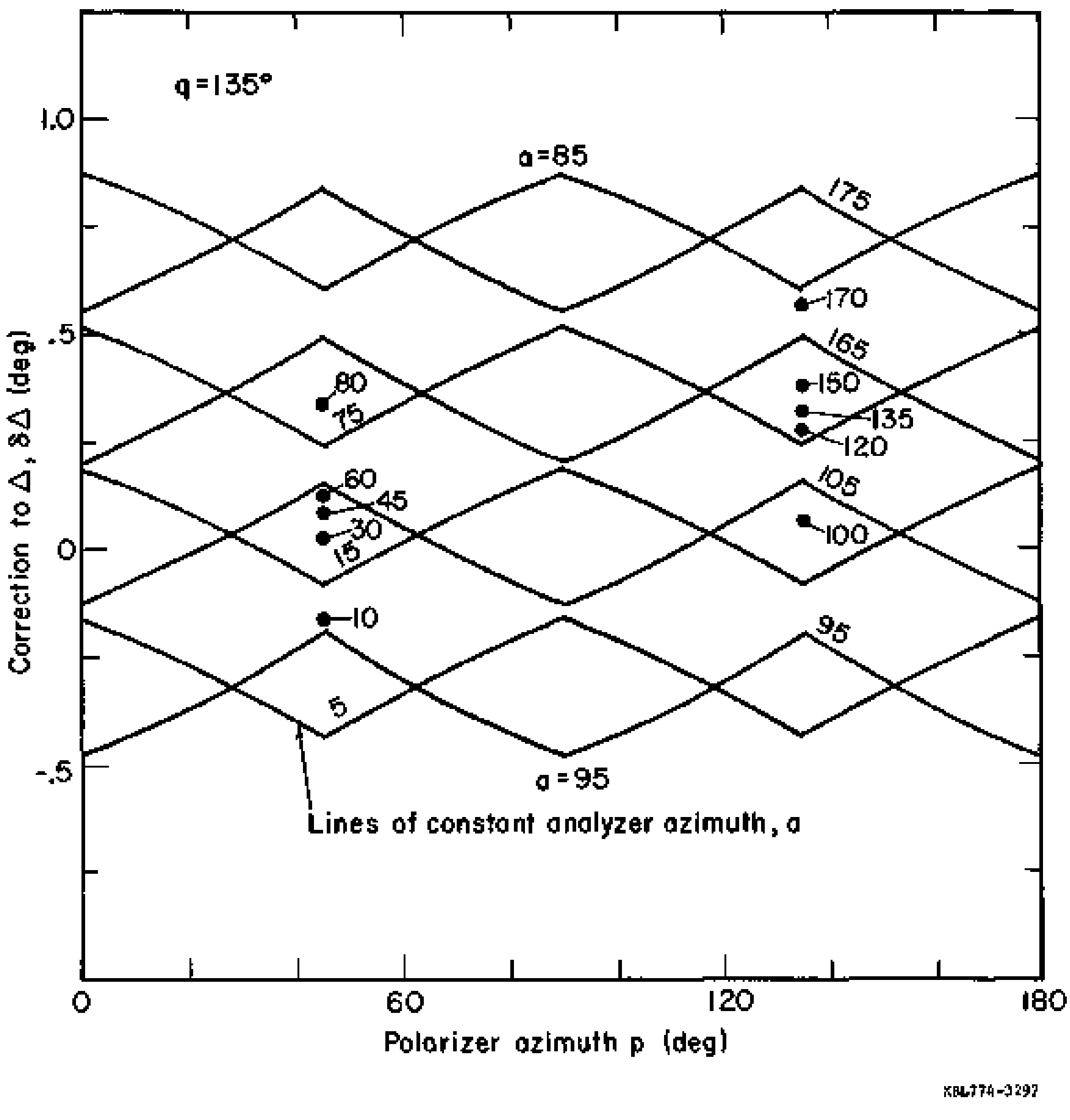
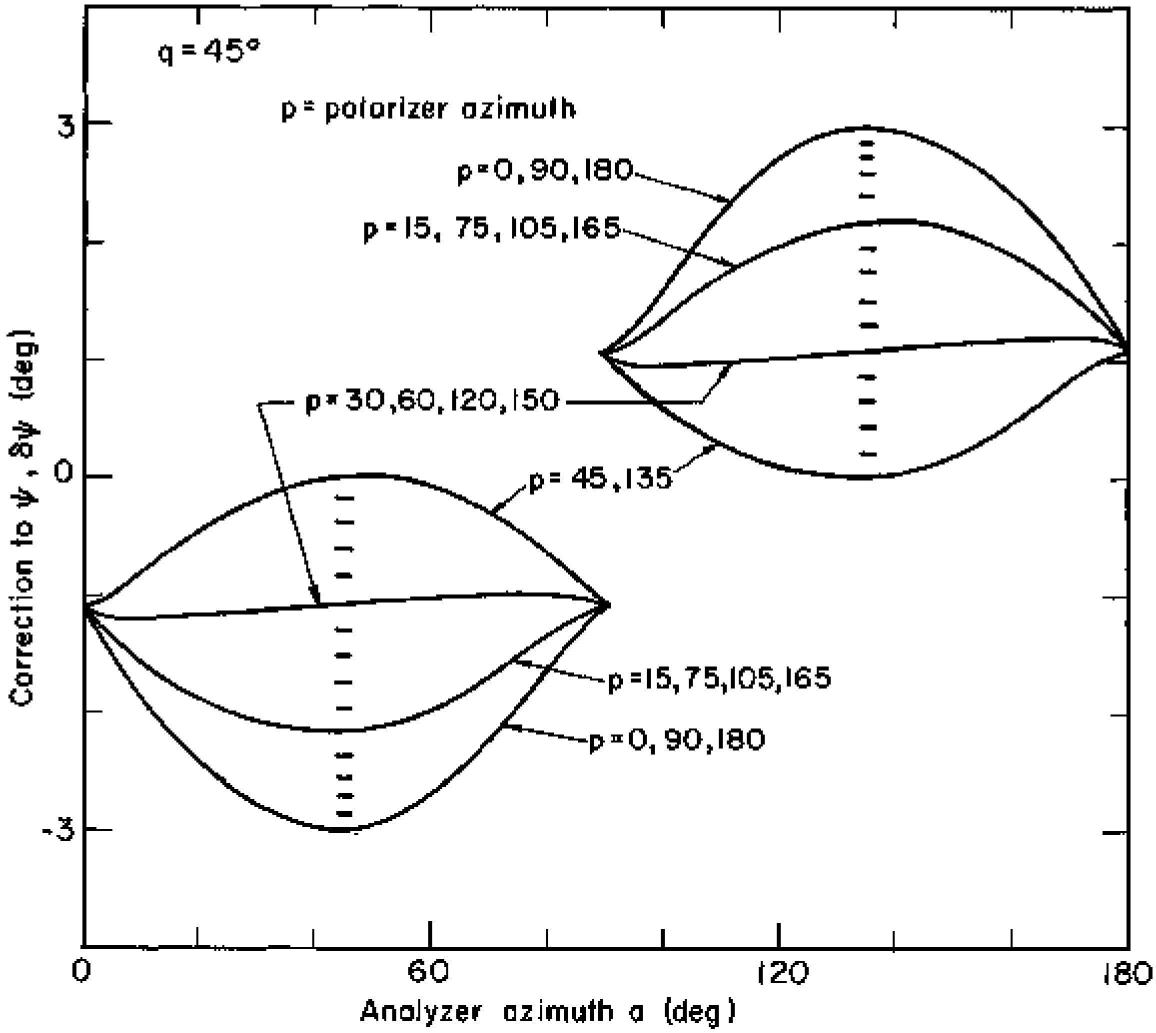
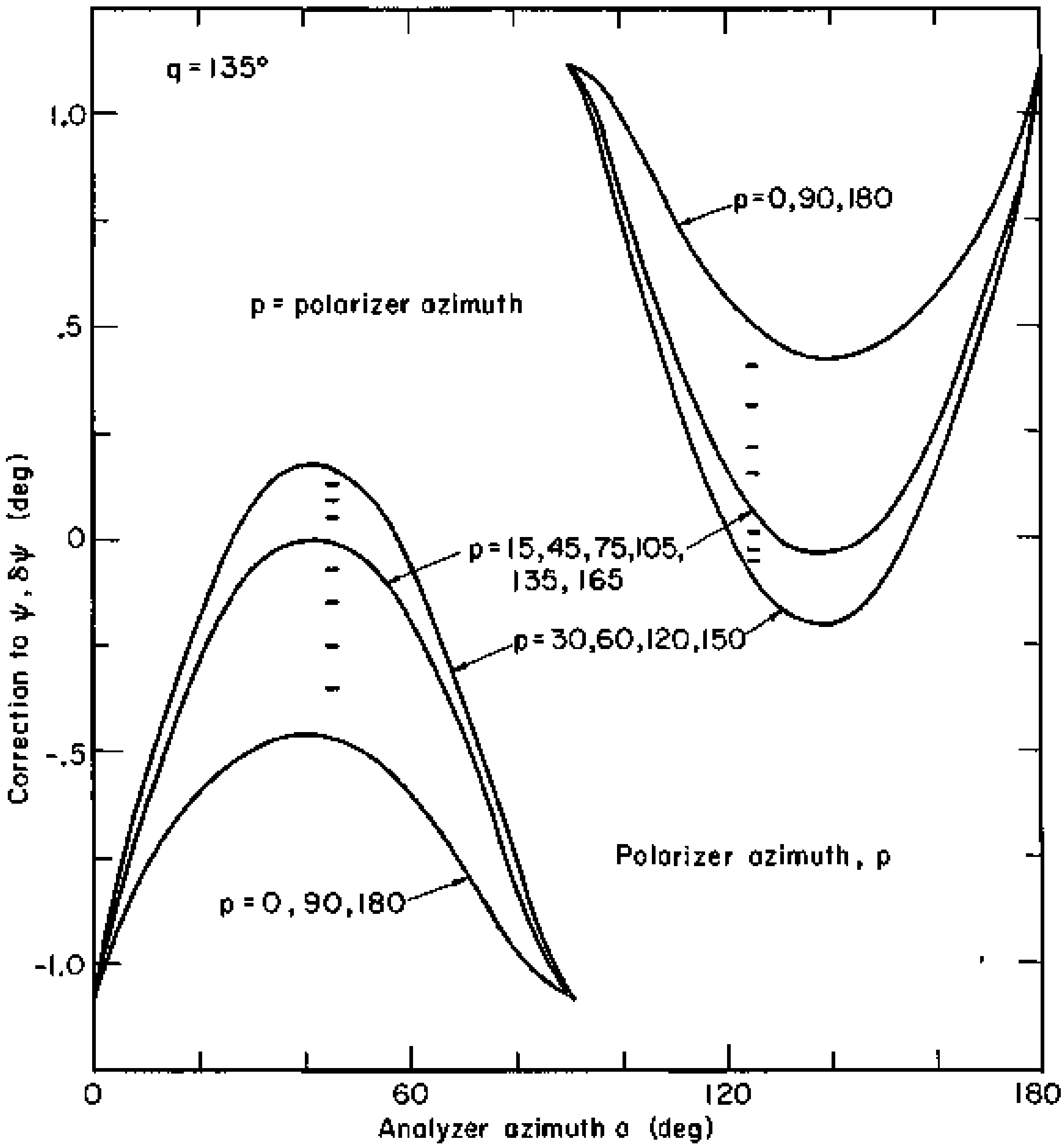


Fig. B2. Correction to Δ due to component imperfections. Quarter wave compensator at 135°, specimen mispositioning error $\delta\beta = 0$.



XBL774-3296

Fig. B3. Correction to ψ due to component imperfections. Quarter wave compensator at 45°, specimen mispositioning error $\delta\beta = 0$.



XBL774-3295

Fig. B4. Correction to ψ due to component imperfections. Quarter wave compensator at 135°, specimen mispositioning error $\delta\beta = 0$.

APPENDIX C. PROGRAM COMPER

The computer program COMPER calculates values of the relative phase Δ and the relative amplitude parameter ψ using the error analysis outlined in this report. In addition, the corrected values of Δ and ψ may be displayed graphically by the CALCOMP plotter.

The program is structured to interpret sets of one-zone measurements from experiments on changing surfaces. Before initiating the surface changes, a 4-zone measurement should be taken on a reference state of the surface, to allow calculation of the specimen mispositioning error δβ (equation 52 p.45). From this 4-zone measurement, the Res A (p. 45) and the 4-zone averages of Δ and ψ (Table V, Appendix A) must be calculated and entered as input data to the program.

Input Variables

INUMBER	is the number of data sets in the input file.
WPLOT	controls plotting on the CALCOMP plot. If WPLOT = 1., a graphical display of ψ vs. Δ is given. If WPLOT = 0., plotting statements are ignored.
TITLE,RANGE	are 80 character alphanumeric labels for the output.
INDCAT	indicates the specimen orientation. For INDCAT = 1, the specimen is vertical and the measured azimuths are in rotated form. For INDCAT = 0, the specimen is horizontal and the measured azimuths are in standard form.
NO	is the number of data points in the set.
C	is the quarter-wave plate setting (either 45° or 135°).

DELTAV is the 4-zone average of Δ.

PSIAV is the 4-zone average of ψ.

RESA $= A_2 + A_4$ for $q = 45°$ or $Q = 135°$;
$= A_1 + A_3$ for $q = 135°$ or $Q = 45°$.

PHI is the angle of incidence (degree).

A(I),P(I) are the analyzer and polarizer azimuths for data point I (degree).

A Sample Data Set

Column:	0 - 9	10-19	20-29	30-39	40-49	50-59	60-69	70-79
	INUMBER	WPLOT						
one set	TITLE →							
	RANGE →							
	INDCAT	NO	C	DELTAV	PSIAV	RESA	PHI	
	A(I)	P(I)	A(I+1)	P(I+1)	A(I+2)	P(I+2)	A(I+3)	P(I+3)
	A(J+1)	P(J+1)	etc.,	through NO number of points				

Note: Fixed-point variables (first letter is I-N) must be right-justified in its column.

The computer program with sample output follows.

```
      PROGRAM COMPER(INPUT,OUTPUT,TAPE98,PLOT,TAPE99=PLOT)
      COMMON/CCPOOL/XMIN,XMAX,YMIN,YMAX,CCXMIN,CCXMAX,CCYMIN,CCYMAX
      COMMON/CCFACT/FACTOR
      DIMENSION TITLE(8), RANGE(8), PSIC(1000), DELTAC(1000)
      DIMENSION PSI(1000), DELTA(1000), A(1000), P(1000)
      INTEGER HG
C
      READ 100, INUMBR,NPLOT
  100 FORMAT (I9,F10.0)
      ITRACK = 0
C
C     INUMBR IS THE NUMBER OF DATA SETS TO BE READ IN.  ITRACK IS USED
C        TO KEEP TRACK OF THE NUMBER OF DATA SETS THAT HAVE BEEN RUN
C        THROUGH THE PROGRAM.
C
C      IF NPLOT=1.,A PLOT OF THE CORRECTED VALUES IS GIVEN BY
C      THE CALCOMP PLOTTER.  IF NPLOT=0., NO PLOT.
C
C     THE VALUES OF THE CONSTANT PARAMETERS THAT APPEAR IN THE EQUATIONS
C        FOR PSI AND DELTA ARE SET.
C
      DELAC = -1.098
      DELCC = 1.128
      DELPC = 1.333
      T1P = 0.
      T2P = .021
      T1CP = -3.4327
      T1CN = -1.653
      T2CP = -.4286
      T2CN = -.1633
C
C     IN THE FOLLOWING VARIABLE NAMES, I AND R ARE ABBREVIATIONS FOR
C        INCIDENCE AND REFLECTION, RESPECTIVELY. SN AND CS ARE ABBREVIA-
C        TIONS FOR SINE AND COSINE.  F REFERS TO THE FARADAY CELLS, AND
C        W REFERS TO THE GLASS WINDOWS OF THE FLOW CELL.
C     THE VARIABLE NAMES CAN BE INTERPRETED AS FOLLOWS ----
C        T2SNWI STANDS FOR T2WISIN2WI ---- T2SNWR FOR T2WRSIN2WR
C        T1SNFI FOR T1FISIN2FI ---- T1SNFR FOR T1FRSIN2FR
C        T1CSFI FOR T1FICOS2FI ---- T1CSFR FOR T1FRCOS2FR
C        T2SNFI FOR T2FISIN2FI ---- T2SNFR FOR T2FRSIN2FR
C        T2CSFI FOR T2FICOS2FI ---- T2CSFR FOR T2FRCOS2FR
C        SCSWIR FOR THE SUM OF T2WICOS2WI AND T2WRCOS2WR.
C
      T1CSFI = -.1162
      T1SNFI = -.2014
      T1CSFR = -.0078
      T1SNFR = .2415
      T2CSFI = .0443
      T2SNFI = -.1424
      T2CSFR = -.2525
      T2SNFR = .0742
      T2SNWI = -.2281
      T2SNWR = .0172
```

```
      SCSWIR = .005
C
C     THE NEXT FIVE VARIABLES, WHICH ARE PART OF THE CALCOMP SUBROUTINE,
C       PROVIDE FOR A 12 BY 10 INCH GRID FOR PLOTTING DELTA AGAINST PSI.
C
      FACTOR = 100.
      CCXMIN = 2.
      CCXMAX = 14.
      CCYMIN = 0.0
      CCYMAX = 10.0
C
C     THE DATA FOR THE EXPERIMENT IS READ IN.
C
  111 READ 200, TITLE, RANGE
  200 FORMAT (8A10/8A10)
      READ 300, INOCAT, NO, C, DELTAV, PSIAV, RESA, PHI
  300 FORMAT (I9,I10,5F10.0)
C
C     INOCAT IS SET TO 1 IF THE INPUT A AND P VALUES REFER TO THE
C       ROTATED AZIMUTH ANGLES.  IT IS SET TO 0 IF THE A AND P VALUES
C       REFER TO STANDARD AZIMUTH ANGLES.
C     NO IS THE NUMBER OF DATA POINTS.
C     C IS THE QUARTER WAVE PLATE ANGLE (EITHER 45 OR 135 DEGREES).
C
      DO 20 I = 1,NO,4
      READ 400, A(I),P(I),A(I+1),P(I+1),A(I+2),P(I+2),A(I+3),P(I+3)
   20 CONTINUE
  400 FORMAT (F9.0,7F10.0)
C
C     **      **      **      **      **      **      **      **      **      **
C
C     THE FOLLOWING VARIABLES, T1C, T2C, I, HAVE TWO POSSIBLE VALUES
C        WHICH DEPEND ON THE VALUE OF C.  (I CONTROLS THE + AND - SIGNS
C        IN THE EXPRESSION FOR R.)
C
      IF ((INOCAT.EQ.0).AND.(C.EQ.45.)) GO TO 25
      IF ((INOCAT.EQ.1).AND.(C.EQ.135.)) GO TO 25
      T1C = T1CM
      T2C = T2CM
      I = 1
      GO TO 27
   25 T1C = T1CP
      T2C = T2CP
      I = 2
C
C     THE VARIABLES J, K, AND L CONTROL THE + AND - SIGNS IN THE EXPRES-
C        SIONS FOR R AND DELB.  THEIR VALUES DEPEND ON THE GROUP THAT
C        DELTAV BELONGS TO.
C
   27 IF ((DELTAV.GE.0.).AND.(DELTAV.LE.90.)) GO TO 32
      IF ((DELTAV.GE.90.).AND.(DELTAV.LE.180.)) GO TO 34
      IF ((DELTAV.GE.180.).AND.(DELTAV.LE.270.)) GO TO 36
      IF ((DELTAV.GE.270.).AND.(DELTAV.LE.360.)) GO TO 38
   32 J = 2
      K = 1
      L = 2
      GO TO 39
   34 J = 1
      K = 1
      L = 1
      GO TO 39
   36 J = 1
```

```
      K = 2
      L = 2
      GO TO 39
   38 J = 2
      K = 2
      L = 1
C
C     THE SUBROUTINE DEGRAD CHANGES DEGREE MEASUREMENTS TO RADIANS.
C
   39 CALL DEGRAD(PHI)
      CALL DEGRAD(PSIAV)
      CALL DEGRAD(DELTAV)
C
C     SINE AND COSINE FUNCTIONS WHICH APPEAR MORE THAN ONCE IN THE
C       EXPRESSIONS FOR R AND DELE ARE CALCULATED.
C
      CS2PSI = COS(2.*PSIAV)
      SN2PSI = SIN(2.*PSIAV)
      COSDEL = COS(DELTAV)
      SINDEL = SIN(DELTAV)
C
C     **      **      **      **      **      **      **      **      **      **
C
      R = SN2PSI*(((-1)**J)*COSDEL*T2CSFI + SINDEL*(((-1)**K)*(T2SNFI+T2
     CSNMI) + ((-1)**(K+I))*T1C)) + ((-1)**I)*CS2PSI*T1SNFR
C
C     DELB IS THE AZIMUTH CORRECTION ANGLE.
C
      DELB = ((R - RISA)/2. + ((-1)**(J+1))*SN2PSI*COSDEL*DELCC - DELAC)
     C/(((-1)**J)*SN2PSI*COSDEL + COS(2.*PHI))
C
      DELC = DELCC + DELB
      DELP = DELPC + DELB
      DELA = DELAC + DELB*COS(2.*PHI)
C
C     **      **      **      **      **      **      **      **      **      **
C
C     THE FOLLOWING LOOP CALCULATES DELTAO, PSIO, DELTA, AND PSI.
C
      DO 30 I = 1,NO
C
C     THE EQUATIONS FOR DELTA, PSI, DELTAO, AND PSIO ARE WRITTEN IN
C         TERMS OF THE STANDARD AZIMUTH ANGLES.  IF THE INPUT A AND P
C         VALUES ARE ROTATED AZIMUTH ANGLES --- IF INDCAT = 1 --- THEY
C         MUST BE EXPRESSED IN AN EQUIVALENT STANDARD AZIMUTH ANGLE FORM.
C         THE SUBROUTINE CONVRT MAKES THE TRANSFORMATION FROM ROTATED
C         ANGLES TO STANDARD ANGLES.
C
      IF (INDCAT.EQ.1) CALL CONVRT(A,P,I)
C
C     THE FOLLOWING IF STATEMENTS DETERMINE THE ZONE AND GROUP OF THE
C       INPUT SET OF A AND P ANGLES.  M AND N ARE ASSIGNED CERTAIN
C       INTEGER VALUES DEPENDING ON THE RANGE OF P AND A.  L, A LINEAR
C       COMBINATION OF M AND N GIVES THE LINE NUMBER OF THE CORRECT SET
C       OF EQUATIONS FOR DELTAO AND PSIO.  THE FIRST EIGHT SETS OF
C       EQUATIONS ARE FOR C = 45, INDCAT = 0 OR C = 135, INDCAT = 1.
C       THE SECOND EIGHT SETS ARE FOR C = 135, INDCAT = 0 OR C = 45,
C       INDCAT = 1.
C
      IF ((P(I).GE.0.).AND.(P(I).LE.45.)) M = 1
      IF ((P(I).GE.45.).AND.(P(I).LE.90.)) M = 2
      IF ((P(I).GE.90.).AND.(P(I).LE.135.)) M = 3
```

```
      IF ((P(I).GE.135.).AND.(P(I).LE.180.)) M = 4
      IF ((A(I).GE.0.).AND.(A(I).LE.90.)) N = 1
      IF ((A(I).GE.90.).AND.(A(I).LE.180.)) N = 2
      L = 2*M + N - 2
      IF ((INDCAT.EQ.1).AND.(C.EQ.45.)) L = L + 8
      IF ((INDCAT.EQ.6).AND.(C.EQ.135.)) L = L + 8
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16) L
C
C     **     **     **     **     **     **     **     **     **     **
C
C     IGROUP IS ASSIGNED THE VALUES 1, 2, 3, OR 4 IF A AND P FALL INTO
C        GROUP A, B, C, OR D, RESPECTIVELY.
C
C     ***** GROUP C1
    1 PSIO(I) = A(I)
      DELTAO(I) = 270. - 2.*P(I)
      IGROUP = 3
      GO TO 50
C
C     ***** GROUP A1
    2 PSIO(I) = 180. - A(I)
      DELTAO(I) = 90. - 2.*P(I)
      IGROUP = 1
      GO TO 50
C
C     ***** GROUP B2
    3 PSIO(I) = A(I)
      DELTAO(I) = 270. - 2.*P(I)
      IGROUP = 2
      GO TO 50
C
C     ***** GROUP D2
    4 PSIO(I) = 180. - A(I)
      DELTAO(I) = 450. - 2.*P(I)
      IGROUP = 4
      GO TO 50
C
C     ***** GROUP A3
    5 PSIO(I) = A(I)
      DELTAO(I) = 270. - 2.*P(I)
      IGROUP = 1
      GO TO 50
C
C     ***** GROUP C3
    6 PSIO(I) = 180. - A(I)
      DELTAO(I) = 450. - 2.*P(I)
      IGROUP = 3
      GO TO 50
C
C     ***** GROUP D4
    7 PSIO(I) = A(I)
      DELTAO(I) = 630. - 2.*P(I)
      IGROUP = 4
      GO TO 50
C
C     ***** GROUP B4
    8 PSIO(I) = 180. - A(I)
      DELTAO(I) = 450. - 2.*P(I)
      IGROUP = 2
      GO TO 50
C
C     ***** GROUP B1
```

```
    9 PSIO(I) = A(I)
      DELTAO(I) = 90. + 2.*P(I)
      IGROUP = 2
      GO TO 50
C
C     ***** GROUP D1
   10 PSIO(I) = 180. - A(I)
      DELTAO(I) = 2.*P(I) + 270.
      IGROUP = 4
      GO TO 50
C
C     ***** GROUP C2
   11 PSIO(I) = A(I)
      DELTAO(I) = 2.*P(I) + 90.
      IGROUP = 3
      GO TO 50
C
C     ***** GROUP A2
   12 PSIO(I) = 180. - A(I)
      DELTAO(I) = 2.*P(I) - 90.
      IGROUP = 1
      GO TO 50
C
C     ***** GROUP D3
   13 PSIO(I) = A(I)
      DELTAO(I) = 2.*P(I) + 90.
      IGROUP = 4
      GO TO 50
C
C     ***** GROUP B3
   14 PSIO(I) = 180. - A(I)
      DELTAO(I) = 2.*P(I) - 90.
      IGROUP = 2
      GO TO 50
C
C     ***** GROUP A4
   15 PSIO(I) = A(I)
      DELTAO(I) = 2.*P(I) - 270.
      IGROUP = 1
      GO TO 50
C
C     ***** GROUP C4
   16 PSIO(I) = 180. - A(I)
      DELTAO(I) = 2.*P(I) - 90.
      IGROUP = 3
C
C     **     **     **     **     **     **     **     **     **     **
C
C     A(I) AND P(I) ARE CONVERTED TO RADIAN MEASUREMENT.  NOTE, HOWEVER,
C       THAT XA AND XP ARE ACTUALLY TWICE THE RADIAN EQUIVALENTS OF A(I)
C       AND P(I).
C
   50 XA = (ACOS(-1.)*A(I))/90.
      XP = (ACOS(-1.)*P(I))/90.
C
C     SINE, COSINE, AND COTANGENT TERMS WHICH APPEAR SEVERAL TIMES IN
C       THE EQUATIONS FOR PSI AND DELTA ARE COMPUTED.
C
      COS2A = COS(XA)
      SIN2A = SIN(XA)
      COS2P = COS(XP)
      SIN2P = SIN(XP)
```

```
      COT2A = COS(XA)/SIN(XA)
C
C     **      **      **      **      **      **      **      **      **      **
C
C     DELTA(I) AND PSI(I) ARE THE CORRECTED (FOR COMPONENT IMPERFEC-
C       TIONS) VALUES OF DELTAO(I) AND PSIO(I).
C
C     THE VARIABLES JZ, KZ, JG, KG, LG, HG, NG, AND M CONTROL THE + AND
C        - SIGNS IN THE EQUATIONS FOR PSI(I) AND DELTA(I). JZ, KZ, AND M
C        DEPEND ONLY ON THE ZONE, NG DEPENDS ONLY ON THE GROUP, AND THE
C        OTHER VARIABLES DEPEND ON BOTH THE GROUP AND THE ZONE.
C
      IF ((IGROUP.EQ.1).OR.(IGROUP.EQ.3)) GO TO 52
        IG = 2
        KG = 1
      GO TO 54
   52   IG = 1
        KG = 2
   54 IF ((IGROUP.EQ.1).OR.(IGROUP.EQ.2)) GO TO 56
        JG = 2
        LG = 1
        NG = 2
      GO TO 58
   56   JG = 1
        LG = 2
        NG = 1
   58 IF ((IGROUP.EQ.1).OR.(IGROUP.EQ.4)) GO TO 60
        HG = 1
        MG = 2
      GO TO 61
   60   HG = 2
        MG = 1
   61 IF ((M.EQ.1).OR.(M.EQ.4)) GO TO 62
        JZ = 2
        JG = JG + 1
      GO TO 64
   62   JZ = 1
   64 IF ((M.EQ.1).OR.(M.EQ.2)) GO TO 66
        KZ = 2
        MG = MG + 1
        KG = KG + 1
      GO TO 68
   66   KZ = 1
   68 IF ((M.EQ.1).OR.(M.EQ.3)) GO TO 70
        HG = HG + 1
        IG = IG + 1
        LG = LG + 1
C
   70 DELTA(I) = DELTAO(I) + ((-1)**IG)*2.*(T1P+DELP) + ((-1)**NG)*T2C*C
     COS2P + ((-1)**(IG+1))*2.*DELC + SCSWIR + T2CSFR - COT2A*(T2SNWR+T2
     CSNFR) + ((-1)**LG)*T1CSFI*SIN2P + ((-1)**HG)*T1SNFI*COS2P
C
      PSI(I) = PSIO(I) + SIN2A*(((-1)**JG)*T2P + SIN2P*(((-1)**KZ)*T2CSF
     CI/2. + ((-1)**(KZ+1))*DELC) + COS2P*(((-1)**(JZ+1))*(T2SNFI+T2SNWI
     C) + ((-1)**KG)*T1O)/2.) + ((-1)**NG)*DELA + ((-1)**JG)*(T1CSFR*SIN
     C2A)/2. + ((-1)**(JG+1))*(T1SNFR*COS2A)/2.
C
   30 CONTINUE
      ITRACK = ITRACK + 1
      IF(HPLOT.EQ.0.) GO TO 650
C
C     **      **      **      **      **      **      **      **      **      **
```

```
C
C     THE FOLLOWING SECTION CONTAINS THE CAL-COMP PLOTTER COMMANDS.
C
      XMIN = 360.
      XMAX = 0.
      YMIN = 180.
      YMAX = 0.
      DO 90 J = 1,NO
         IF (DELTA(J).LT.XMIN) XMIN = DELTA(J)
         IF (DELTA(J).GT.XMAX) XMAX = DELTA(J)
         IF (PSI(J).LT.YMIN) YMIN = PSI(J)
         IF (PSI(J).GT.YMAX) YMAX = PSI(J)
         IF (DELTAO(J).LT.XMIN) XMIN = DELTAO(J)
         IF (DELTAO(J).GT.XMAX) XMAX = DELTAO(J)
         IF (PSIO(J).LT.YMIN) YMIN = PSIO(J)
         IF (PSIO(J).GT.YMAX) YMAX = PSIO(J)
   90 CONTINUE
      SXMIN = XMIN
      SXMAX = XMAX
      SYMIN = YMIN
      SYMAX = YMAX
C
C     THE X (DELTA) SCALE IS 12 INCHES LONG, WITH 20 SMALL SQUARES TO
C        THE INCH.  THE Y (PSI) SCALE IS 10 INCHES LONG, ALSO WITH 20
C        SMALL SQUARES TO THE INCH.
C
      XFACTR = 1.2
      YFACTR = 1.0
      XSCALE = (XMAX - XMIN)/240.
      YSCALE = (YMAX - YMIN)/200.
      SCALEF = 0.
      IF (XSCALE.EQ.FLOAT(IFIX(XSCALE))) XSCALE = XSCALE + 0.005
      IF (YSCALE.EQ.FLOAT(IFIX(YSCALE))) YSCALE = YSCALE + 0.005
C
C     THE SUBROUTINE GSCALE CALCULATES A CONVENIENT UNIT OF MEASURE FOR
C        THE SMALL SQUARES ON THE GRAPH.
C
      XMID = (XMIN + XMAX)/2.
      YMID = (YMIN + YMAX)/2.
      IXMID = IFIX(XMID+0.5)
      IF (MOD(IXMID,2).NE.0) IXMID = IXMID + 1
      IYMID = IFIX(YMID+0.5)
      IF (MOD(IYMID,2).NE.0) IYMID = IYMID + 1
   72 CALL GSCALE(XSCALE,SCALEF,XFACTR)
      XMIN = FLOAT(IXMID) - SCALEF
      XMAX = FLOAT(IXMID) + SCALEF
      IF ((SXMIN.GE.XMIN).AND.(SXMAX.LE.XMAX)) GO TO 74
      CALL FXSCAL(XSCALE)
      GO TO 72
   74 CALL GSCALE(YSCALE,SCALEF,YFACTR)
      YMIN = FLOAT(IYMID) - SCALEF
      YMAX = FLOAT(IYMID) + SCALEF
      IF ((SYMIN.GE.YMIN).AND.(SYMAX.LE.YMAX)) GO TO 76
      CALL FXSCAL(YSCALE)
      GO TO 74
C
C     XMIN, XMAX, YMIN, YMAX ARE THE UPPER AND LOWER LIMITS ON THE
C        VALUES OF DELTA AND PSI WHICH WILL BE PLOTTED BY CAL-COMP.
C
   76 WRITE (99,40)
   40 FORMAT (1H=)
      CALL CCGRID(1,6,4,6HNOLBLS,1,5,4)
```

```
      CALL FIXLBL(6,5,2,-1,-1)
      CALL CCLTR(6.65,-0.3,0.2,5HDELTA)
      CALL CCLTR(1.12,5.0,0,2,3HPSI)
      CALL CCLTR(2.25,0.5,0.1,49HTHE IDEAL VALUES OF DELTA AND PSI ARE R
     CEPRESENTED)
      CALL CCLTR(2.25,0.25,0.1,47H  BY A PLUS, THE CORRECTED VALUES BY A
     C DIAMOND.)
      CALL CCLTR(2.25,9.75,0.2,TITLE,80)
      CALL CCLTR(2.25,9.5,0,2,RANGE,80)
      CALL CCPLCT(DELTA,PSI,NO,6HNOJOIN,6,1)
      CALL CCPLCT(DELTAC,PSIO,NO,6HNOJOIN,23,1)
      CALL CCLTR(CCXMAX,0.,0,1,1H )
      CALL CCNEXT
C
C     **      **      **      **      **      **      **      **      **      **
C
C     THE LAST SECTION CONTAINS THE OUTPUT COMMANDS AND FORMATS.
C
  650 CONTINUE
      PRINT 101, TITLE, RANGE
  101 FORMAT(*1*,8A10/8A10)
      PRINT 201,C
  201 FORMAT (*C*,*THE QUARTER WAVE PLATE AZIMUTH IS *,F4.0,* DEGREES.*)
      PHI = (PHI*180.)/ACOS(-1.)
      PRINT 301, PHI
  301 FORMAT (*0*,*THE ANGLE OF INCIDENCE IS *,F8.3,* DEGREES.*)
      PRINT 401, DELB
  401 FORMAT (*0*,*THE AZIMUTH CORRECTION ERROR IS *,F8.3,* DEGREES.*)
      IF (INDCAT.EQ.0) PRINT 501
      IF (INDCAT.EQ.1) PRINT 503
  501 FORMAT(*0*,*A AND P REFER TO THE STANDARD AZIMUTH ANGLES OF THE AN
     CALYZER AND POLARIZER, RESPECTIVELY.*)
  503 FORMAT(*0*,*A AND P REFER TO THE ROTATED AZIMUTH ANGLES OF THE ANA
     CLYZER AND POLARIZER, RESPECTIVELY.*)
      PRINT 601, NO
  601 FORMAT (*0*,*NUMBER OF DATA POINTS -- *,I4)
      PRINT 701
  701 FORMAT (*C*,10X,*A*,11X,*P*,9X,*DELTA*,6X,*DELTAO*,8X,*PSI*,8X,*PS
     CIC*)
C
C     IF INDCAT = 1, A AND P ARE CONVERTED BACK TO ROTATED AZIMUTH
C        ANGLES.
C
      DO 80 I=1,NO
      IF (INDCAT.EQ.1) CALL CONVRT(A,P,I)
      PRINT 801, A(I), P(I), DELTA(I), DELTAO(I), PSI(I), PSIO(I)
   80 CONTINUE
  801 FORMAT (*0*,6X,5(F8.3,4X),F8.3)
      IF (ITRACK.NE.INUMBR) GO TO 111
      IF(NPLOT.EQ.1.) CALL CCEND
      STOP
      END
      SUBROUTINE GSCALE(SCALEI,SCALEF,FACTR)
      IF (SCALEI.GE.0.9) GO TO 65
      IF (SCALEI.LT.0.09) SCALE = SCALEI*100.
      IF ((SCALEI.GE.0.09).AND.(SCALEI.LT.0.9)) SCALE = SCALEI*10.
      ISCALE = IFIX(SCALE) + 1
      IF (SCALE.EQ.FLOAT(IFIX(SCALE))) ISCALE = IFIX(SCALE)
      IF ((ISCALE.EQ.1).OR.(ISCALE.EQ.5)) GO TO 75
      IF (MOD(ISCALE,2).NE.0) ISCALE = ISCALE + 1
      IF (ISCALE.EQ.6) ISCALE = 8
   75 IF ((SCALE.GT.2.).AND.(SCALE.LE.2.5)) GO TO 85
```

```
      IF (SCALEI.LT.0.09) SCALEF = (FLOAT(ISCALE))*FACTR
      IF ((SCALEI.GE.0.09).AND.(SCALEI.LT.0.9)) SCALEF = (FLOAT(ISCALE))
     C*10.*FACTR
   65 IF ((SCALEI.GE.0.9).AND.(SCALEI.LE.1.0)) SCALEF = (FLOAT(ISCALE))*
     C100.*FACTR
      IF (SCALEI.GT.1.0) SCALEF = 150.*FACTR
      GO TO 95
   85 IF (SCALEI.LT.0.09) SCALEF = 2.5*FACTR
      IF ((SCALEI.GE.0.09).AND.(SCALEI.LT.0.9)) SCALEF = 25.*FACTR
   95 RETURN
      END
      SUBROUTINE CONVRT(A,P,IN)
      DIMENSION A(1000), P(1000)
      IF (P(IN).GT.180.) P(IN) = P(IN) - 180.
      IF ((A(IN).GE.90.).AND.(A(IN).LE.180.)) GO TO 24
      A(IN) = A(IN) + 90.
      GO TO 26
   24 A(IN) = A(IN) - 90.
   26 IF ((P(IN).GE.90.).AND.(P(IN).LE.180.)) GO TO 28
      P(IN) = P(IN) + 90.
      GO TO 22
   28 P(IN) = P(IN) - 90.
   22 RETURN
      END
      SUBROUTINE DEGRAD(X)
      X = (ACOS(-1.)*X)/180.
      RETURN
      END
      SUBROUTINE FXSCAL(SCALEI)
      IF (SCALEI.LE.0.1) SCALEI = SCALEI + .01
      IF (SCALEI.GT.0.1) SCALEI = SCALEI + 0.1
      RETURN
      END
      SUBROUTINE FIXLBL(NX1,NY1,KSIZE,NXX,NYY)
      COMMON/CCPOOL/XMIN,XMAX,YMIN,YMAX,CCXMIN,CCXMAX,CCYMIN,CCYMAX
      COMMON/CCFACT/FACTOR
      NXP=NXX $ NYP=NYY
      NXP=MIN0(NXP,7) $ NYP=MIN0(NYP,7) $ FACT=KSIZE/FACTOR
      XFCTR=1.0 $ YFCTR=1.0
      CCXINT=(CCXMAX-CCXMIN)/NX1 $ CCYINT=(CCYMAX-CCYMIN)/NY1
      XINT=ABS(XMAX-XMIN)/NX1 $ YINT=ABS(YMAX-YMIN)/NY1
      TF=3.0*FACT $ SF=7.0*FACT $ NDPRX=1 $ NDPRY=1

C     X-AXIS NORMALIZATION.........

      VMV=AMAX1(ABS(XMIN),ABS(XMAX))+1.0E-10 $ XM=10.0**NXP
      IF((NXP.EQ.0).OR.((NXP.GT.0).AND.(VMV.GE.XM.OR.
    2 (XINT.LT.(0.1-1.0E-10)) )).OR.((NXP.LT.0).AND.
    3 (XINT.LT.(XM-1.0E-10))   ))GO TO 46
      IF(NXP.LT.0)NDPRX=-NXP
      NXP=0 $ GO TO 50
   46 Z=ALOG10(VMV) $ Z=SIGN((ABS(Z)+1.0E-8),Z) $ NXP=Z
      IF(Z.LT.0.0)NXP=NXP-1 $ XFCTR=10.0**NXP

C     Y-AXIS NORMALIZATION..........

   50 VMV=AMAX1(ABS(YMIN),ABS(YMAX)) +1.0E-10 $ YM=10.0**NYP
      IF((NYP.EQ.0).OR.((NYP.GT.0).AND.(VMV.GE.YM.OR.
    2 (YINT.LT.(0.1-1.0E-10)) )).OR.((NYP.LT.0).AND.
    3 (YINT.LT.(YM-1.0E-10))   ))GO TO 53
      IF(NYP.LT.0)NDPRY=-NYP $ NYP=0 $ GO TO 60
   53 Z=ALOG10(VMV) $ Z=SIGN((ABS(Z)+1.0E-8),Z) $ NYP=Z
```

```
      IF(Z.LT.0.0)NYP=NYP-1 $ YFCTR=10.0**NYP
   60 XINT=XINT/XFCTR $ YINT=YINT/YFCTR $ YMF=YMIN/YFCTR

C     TEST THE X-LABELS FOR NON-INTEGER VALUES..........

      X=XMIN/XFCTR-XINT $ XM=XMAX/XFCTR+1.0E-8
   64 X=X+XINT $ IF(X.GT.XM) GO TO 65 $ AX=ABS(X)
      TMP=ABS(FLOAT(INT(AX+0.5))-AX) $ IF(TMP.LE.1.0E-8) GO TO 64
      GO TO 66
   65 NOPRX=0

C     TEST THE Y-LABELS FOR NON-INTEGER VALUES..........

   66 YM=YMAX/YFCTR+1.0E-8 $ Y=YMIN/YFCTR-YINT
   68 Y=Y+YINT $ IF(Y.GT.YM) GO TO 69 $ AY=ABS(Y)
      TMP=ABS(FLOAT(INT(AY+0.5))-AY) $ IF(TMP.LE.1.0E-8) GO TO 68
      GO TO 70
   69 NOPRY=0

C     LABEL THE X-AXIS..........

   70 CCX=CCXMIN $ CCY=CCYMIN-10.0*FACT $ X=XMIN/XFCTR
   75 IF(X.GT.XM)GO TO 90
      CALL FIXX1(NTEMP,NC,NP,NOPRX,X)
      NMD=MOD(NC,2)
      IF(NMD.EQ.0)XT=CCX-FACT*(3.5*NC+5.5)
      IF(NMD.NE.0)XT=CCX-FACT*(3.5*(NC-1)+9.0)
      CALL COLTR(XT,CCY,0,KSIZE,NTEMP,NP)
      CCX=CCX+CCXINT $ X=X+XINT $ GO TO 75

C     LABEL THE Y-AXIS..........

   90 CCY=CCYMIN-TF
      Y=YMF
   95 IF(Y.GT.YM)RETURN
      CALL FIXX1(NTEMP,NC,NP,NOPRY,Y)
      IF(Y.EQ.YMF)CCY=CCY+TF
      CCX=CCXMIN-SF*NP
      CALL COLTR(CCX,CCY,0,KSIZE,NTEMP,NP)
      IF(Y.EQ.YMF)CCY=CCY-TF
      CCY=CCY+CCYINT $ Y=Y+YINT $ GO TO 95
      END
      SUBROUTINE FIXX1(NTEMP,NC,NP,NOPRX,XX)
      DIMENSION NFMT(1)
      DATA NHOLD/5555550543357605523/,TMP/0.999999/
      DATA NHOLD2/77777777777777778  /,NHOLD3/55330000000000000008/
      DATA  NHOLD4/46330000000000000008/ ,NHOLD5/778/,NZERO/4H 0   /
      X=XX $ AX=ABS(X) $ IF(AX.LT.1.E-7) GO TO 79 $ NOPL=1
      IF(AX.GE.1.0) GO TO 76 $ NN=7 $ GO TO 761
   76 IF(AX.GE.10.0)NOPL=NOPL+(ALOG10(AX)+1.0E-10)
      NN=8-NOPL
  761 ENCODE(10,77,NTP)NN
   77 FORMAT(I10)
      JEM=64*(NTP.AND.NHOLD5)
      NFMT=NHOLD.OR.JEM
      ENCODE(10,NFMT,NTEMP)X
      IF(AX.GE.TMP.OR.X.EQ.0.0)GO TO 78
      NTEMP=NTEMP.AND.NHOLD2
      IF(X.GT.0.0)NTEMP=NTEMP.OR.NHOLD3
      IF(X.LT.0.0)NTEMP=NTEMP.OR.NHOLD4
   78 NC=NOPL $ NP=NOPL+1
      IF(NOPRX.EQ.0.OR.X.EQ.0.0)RETURN $ N1=NOPRX+1 $ NC=NC+N1
```

```
      NP=NP+N1
      RETURN
   79 NTLMP=NZERO $ NP=4 $ NC=1
      RETURN
      END
```

ZN IN 0.5 M KOH
CONSTANT POTENTIAL E= -1.2 VOLTS VS HG/HGO

THE QUARTER WAVE PLATE AZIMUTH IS 45. DEGREES.

THE ANGLE OF INCIDENCE IS 75.000 DEGREES.

THE AZIMUTH CORRECTION ERROR IS -.029 DEGREES.

A AND P REFER TO THE STANDARD AZIMUTH ANGLES OF THE ANALYZER AND POLARIZER

NUMBER OF DATA POINTS -- 10

A	P	DELTA	DELTAO	PSI	PSIO
39.980	83.630	101.634	102.740	37.311	39.980
39.630	85.830	97.218	98.340	36.857	39.630
38.580	88.130	92.603	93.740	35.716	38.580
37.410	89.930	88.991	90.140	34.490	37.410
36.380	91.450	85.953	87.100	33.527	36.380
35.680	92.370	84.115	85.260	32.874	35.680
34.900	93.090	82.676	83.820	32.137	34.900
34.360	93.920	81.059	82.200	31.641	34.360
33.920	94.690	79.482	80.620	31.243	33.920
33.530	95.280	78.304	79.440	30.887	33.530

APPENDIX D. THE GROUP DEPENDENCE OF THE SIGNS OF THE PARAMETER VALUES.

The parametric equations describing component imperfections were derived for Group B (Tables IIa and IIb, Appendix A). Many of the terms in the equations are multiplied by trigonometric functions of the polarizer and analyzer azimuths. Allowance for the sign changes of sin2P and cos2P (vectors b and a in Figure D1) must be made in applying the derivation to Groups A, C, and D.

The signs of the parameters for all groups (Tables IIc and IId, Appendix A) were obtained by using Group B as a standard. As an example (Figure D1), the terms multiplied by sin2P change sign in zones C1 and D4 from the equation describing B2. Similarly, the terms multiplied by cos2P for zones A1 and A3 have signs opposite those in the expression for B2. The same allowance was made in obtaining expressions for calculating Δ and ψ from rotated polarizer and analyzer azimuths (vertical specimen).

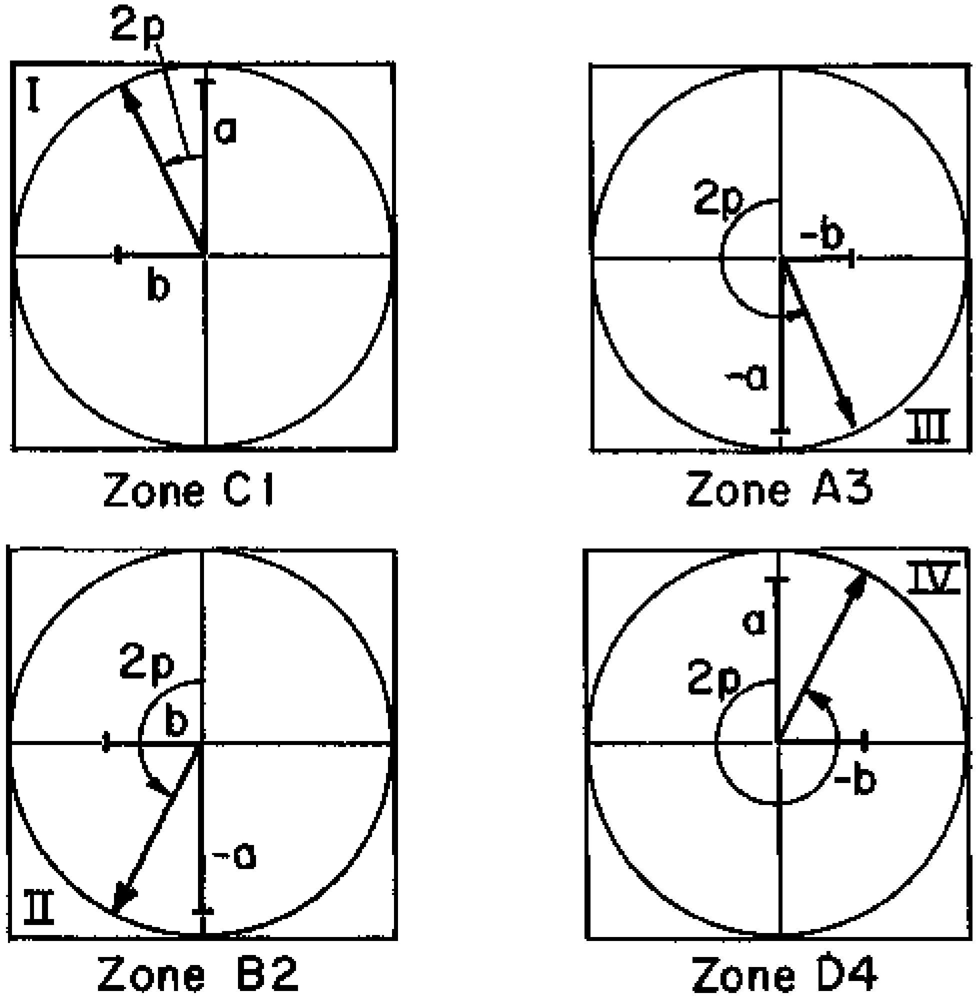


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Fig. D1. Group dependence of the signs of the parameter values. Vector components a and b represent sin2P and cos2P.

APPENDIX E. MEASUREMENTS USED TO DETERMINE PARAMETER VALUES.

For the following experimental measurements, the notation NW and NFC refers to the absence of windows and Faraday cells. Five sets of 4-zone measurements, with re-alignment of the specimen between each set, were made on the dielectric and metal surfaces to allow averaging out the specimen mispositioning error $\delta\beta$. The horizontal surfaces faced up and the angle of incidence was 75°.

I. Straight-through Position; NW, NFC

q	a	p	Zone
45.00	134.97	45.01	A4
45.00	44.94	134.92	A2
135.00	44.97	135.98	A1
135.00	134.96	44.93	A3

IIa. Reflection from a dielectric surface; NW, NFC, $\phi = 75°$.

q	a	p	Zone
45.0	116.26	45.78	A4
45.0	64.02	135.78	A2
135.0	63.98	133.92	A1
135.0	116.26	43.94	A3
135	116.21	44.03	A3
135	64.07	134.05	A1
45	64.03	135.65	A2
45	116.47	45.47	A4

IIa. (continued)

q	a	p	Zone
45	116.21	45.76	A4
45	63.94	135.79	A2
135	64.09	134.06	A1
135	116.37	43.99	A3
135	116.42	43.96	A3
135	64.12	133.98	A1
45	64.08	135.97	A2
45	116.40	46.02	A4
45	116.40	45.85	A4
45	64.03	136.00	A2
135	64.12	133.85	A1
135	116.42	44.00	A3

IIb. Reflection from a dielectric surface; NW, $\phi = 75°$.

q	a	p	Zone
135	116.60	44.13	A3
135	64.35	133.98	A1
45	64.26	135.99	A2
45	116.58	45.84	A4

IIIa. Reflection from a metal surface; NW, NFC, $\phi = 75°$.

q	a	p	Zone
135	62.70	86.84	B4
135	122.53	176.46	B2
45	120.28	92.93	B1
45	60.46	3.52	B3

IIIa. (continued)

q	a	p	Zone
45	60.53	3.57	B3
45	120.15	93.26	B1
135	122.29	176.85	B2
135	62.67	86.38	B4
135	62.56	86.77	B4
135	122.40	176.73	B2
45	120.27	93.00	B1
45	60.43	3.29	B3
135	62.56	86.98	B4
135	122.44	176.67	B2
45	120.31	93.21	B1
45	60.38	2.83	B3
45	60.20	2.44	B3
45	120.50	92.25	B1
135	122.64	177.24	B2
135	62.46	86.92	B4

IIIb. Reflection from a metal surface; NW, $\phi = 75°$

q	a	p	Zone
135	62.36	87.45	B4
135	122.47	177.29	B2
45	120.34	92.10	B1
45	60.14	2.38	B3

IIIc. Reflection from a metal surface; $\phi = 75°$

135	62.11	87.75	B4
135	122.06	177.85	B2
45	119.94	91.47	B1
45	59.92	2.06	B3

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