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Comments on "ECH Current Drive By Asymmetric Heating Around the Median Plane" (GA Technologies Report # GA-A18656, October 1986) by Tihiro Ohkawa

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Comments on
"ECH Current Drive By Asymmetric Heating Around the Median Plane"
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In a recent GA report[1] Ohkawa describes a novel electron cyclotron current-drive (ECCD) scheme. Ohkawa finds a net toroidal plasma current driven by "up-down" asymmetries in the electron pressure anisotropy, i.e., differences in $p_\perp - p_\parallel$ evaluated at the same B between the top and bottom half of the poloidal cross-section. Such up-down asymmetries are associated with the s-dependent, first order in ν/ω_b part of the electron distribution function (s is distance along a magnetic field line). The current-drive efficiency estimated by Ohkawa is competitive with other r.f. current-drive schemes[2]. Ohkawa's scheme is attractive because it appears that this scheme can produce an r.f.-driven current even when the electron-cyclotron power is dissipated on trapped electrons (Ohkawa explicitly makes this claim in his GA report[1]); and the scheme requires no selectivity in the sign of the parallel velocity of the electrons heated by the electron cyclotron wave. These are both significant advantages over the ECCD mechanism of Fisch and Boozer[2].

In sections I through III of this memo we analyze the current-drive calculation presented by Ohkawa in Ref. 1. We conclude that, in the limit of long mean-free-path appropriate to current tokamak experiments and tokamak reactors, the current Ohkawa attempts to compute in Ref. 1 [see, e.g., Eq. (12) of Ref. 1] from a fluid model is a neoclassical correction to the Fisch-Boozer current of Ref. 2. Neoclassical effects that give rise to corrections of this sort were first pointed out by Ohkawa in an earlier paper on ECH current drive[3]. These neoclassical effects (which are associated with magnetic field variations and trapped particles) have been treated more accurately by using a kinetic rather than fluid theory in several recent papers[4]. They are important because they can reduce the ECCD efficiency below the level computed by Fisch and Boozer[2].

In addition to presenting a fluid calculation of an ECH driven current, Ohkawa's approach raises the possibility that currents can be driven by localizing the ECH heating zone in order to create spatial variations in the plasma pressure that will in turn drive plasma currents. Such effects can be analyzed within kinetic theory by solving for the electron distribution function through first order in ν/ω_b, where ν is a characteristic rate of evolution of the electron distribution function due to both Coulomb collisions and ECH heating, while ω_b is the bounce frequency for trapped particles or the transit frequency for passing particles. Such schemes are analyzed in section IV, where we find that even at first order in ν/ω_b the trapped region of phase space cannot contribute to the parallel current. In addition, we find that the current-drive efficiency is reduced by a factor of ν/ω_b relative to the current-drive efficiency calculated by Fisch and Boozer[2] and others[4]. Hence, ECCD schemes that depend on localized heating cannot yield current-drive efficiencies that are competitive with current-drive schemes like that of Fisch and Boozer[2] which work by producing asymmetries in the electron distribution function at zero order in ν/ω_b.

Finally, in section V we comment on the effect of making $\nu_{rf} \gg \nu_{..}$, while keeping $\nu_{..}/\omega_b \ll 1$. We find that if, on the one hand, $\nu_{..}/\omega_b \ll \nu_{rf}/\omega_b \ll 1$, the ECCD efficiency is still reduced by a factor of $\nu_{..}/\omega_b$ relative to the current-drive efficiency calculated by Fisch and Boozer; $\nu_{rf}/\omega_b \sim 1$, on the other hand, requires very large ECH power levels, and, while the complete analysis remains to be done, it seems unlikely to result in a great increase in the ECCD efficiency.

I. Ohkawa's Fluid Equation

Ohkawa begins his analysis in Ref. 1 by taking the parallel component of the fluid momentum balance equation, having assumed only that the material pressure tensor can be written in the form

$$\bar{\bar{\mathbf{P}}} = p_\perp \bar{\bar{\mathbf{I}}} + (p_\| - p_\perp)\mathbf{BB}/B^2$$

and having written "$q_e n\eta j_\|$" for the net parallel component of the Coulomb collision and r.f. diffusion terms. (We will discuss this identification further in section II). This results in Ohkawa's Eq. (1), which we choose to write in the form

$$-\text{"}q_e n\eta j_\|\text{"} = m_e n\frac{\partial u}{\partial t} + m_e n u\frac{\partial u}{\partial s} + \frac{\partial p_\|}{\partial s} + (p_\perp - p_\|)\frac{1}{B}\frac{\partial B}{\partial s} - q_e n E_\| \ .$$

We follow Ohkawa in considering only steady-states ($\partial/\partial t = 0$), and focusing on non-Ohmic current drive by assuming that $E_\| = -\partial\phi/\partial s$. Ohkawa takes a field line average of this equation, and proceeds to discuss and model the various terms. We believe that it is instructive to analyze the parallel momentum balance equation before taking a field line average since there are important cancellations which Ohkawa has missed.

We note first that the axial variations in the perpendicular and parallel pressures are related. For any $f(\epsilon,\mu,\sigma,s)$,

$$\begin{aligned}\frac{\partial p_\|/B}{\partial s} &= \frac{\partial}{\partial s}\frac{1}{B}\frac{2\pi B}{m_e^2}\int\frac{d\epsilon d\mu}{|v_\||}\sum_\sigma f m_e(v_\| - u)^2 \\ &= \frac{\partial}{\partial s}\left[\frac{2\pi}{m_e^2}\sqrt{\frac{m_e}{2}}\int d\epsilon d\mu\sum_\sigma f 2\sqrt{\epsilon - \mu B - q_e\phi} - \frac{m_e n u^2}{B}\right] \\ &= -\frac{p_\perp}{B^2}\frac{\partial B}{\partial s} - \frac{q_e n}{B}\frac{\partial\phi}{\partial s} - m_e\frac{\partial}{\partial s}\frac{nu^2}{B} + \frac{2\pi}{m_e^2}\sqrt{\frac{m_e}{2}}\int d\epsilon d\mu\sum_\sigma\frac{\partial f}{\partial s}2\sqrt{\epsilon - \mu B - q_e\phi}\ .\end{aligned}$$

Using the above expression to eliminate $\partial p_\|/\partial s$ from the parallel momentum equation, we obtain

$$\begin{aligned}-\text{"}q_e n\eta j_\|\text{"} &= m_e n u\frac{\partial u}{\partial s} + q_e n\frac{\partial\phi}{\partial s} + \frac{\partial p_\|}{\partial s} + \frac{p_\perp - p_\|}{B}\frac{\partial B}{\partial s} \\ &= -m_e u B\frac{\partial nu/B}{\partial s} + \mathfrak{S} = \mathfrak{S}\end{aligned}$$

where the last equality follows from particle conservation, i.e.,

$$\frac{\partial n/B}{\partial t} + \frac{\partial (nu/B)}{\partial s} = 0 .$$

We have defined

$$\Im(s) = \frac{2\pi B}{m_e^2}\sqrt{\frac{m_e}{2}}\int d\epsilon d\mu \sum_\sigma \frac{\partial f}{\partial s} 2(\epsilon - \mu B - q_e\phi)^{1/2}$$
$$= \int d^3v\, m_e v_\parallel^2 \frac{\partial f}{\partial s} .$$

Note the cancellation point by point of the large terms on the right-hand side of our parallel momentum balance equation. This is important because these terms will become the integrand in Ohkawa's Eq. (2). The remaining term arises only from the explicit s-dependence of the distribution function (note that $\partial f/\partial s$ is to be taken at constant ϵ and μ). In the long mean-free-path ordering ($\nu/\omega_b \ll 1$) the explicit dependence of f on s is weak (see section III below). Hence, this term is small point by point. This cancellation is lost in Ohkawa's analysis because he, in effect, ignores the term in $\partial p_\parallel/\partial s$, retaining only the term in $(p_\perp - p_\parallel)\partial B/\partial s$. However, Okawa still finds that the integral on the right-hand side of Eq. 2 in Ref. 1 vanishes at leading order in ν/ω_b; therefore, he has not made a qualitative error by dropping the term.

It is instructive, again making use of particle conservation, to write the parallel momentum balance equation in the form

$$-\text{“}q_e n\eta j_\parallel\text{”} = B\frac{\partial}{\partial s}\frac{1}{B}(p_\parallel + m_e n u^2) + \frac{p_\perp}{B}\frac{\partial B}{\partial s} + q_e n\frac{\partial \phi}{\partial s} .$$

The final two terms describe the transfer of momentum from the electrons to the magnetic field due to the μ-grad B force, and from the electrons to the ions due to the electrostatic potential. Note that $\Im(s)$, written in this way, is again the sum of large terms which cancel at leading order.

We are now ready to take the field line average of the parallel momentum balance equation. The most natural field line average to take is the flux tube average, $\int ds/B$. This operator neatly annihilates the parallel momentum flux term, leaving

$$\oint \frac{ds}{B}\,\text{“}q_e n\eta j_\parallel\text{”} = -\oint\frac{ds}{B}\left(\frac{p_\perp}{B}\frac{\partial B}{\partial s} - q_e n\frac{\partial\phi}{\partial s}\right) ,$$

which has the straight-foward interpretation that the collisional transfer of momentum to the electrons is equal to the sum of μ-grad B and electrostatic forces on the electron distribution.

II. The Collisional Momentum Transfer

It is important to note that the only meaning of the collisional momentum transfer term, "$q_e n\eta j_{||}$", is through its definition

$$\text{“}q_e n\eta j_{||}\text{”} \equiv -\int d^3v\, m_e v_{||} C(f)\ ;$$

where the operator $C(f)$ includes both the Coulomb collisions and the quasilinear diffusion associated with the electron cyclotron heating. In non-Ohmic current-drive schemes this term bears little direct relation to the plasma current.

To see this, consider first the ECCD mechanism described by Fisch and Boozer[2]. In this mechanism the ECH is assumed to input zero net momentum into the electrons. Nevertheless a current is driven by creating an "asymmetric resistivity". That is, the left-going electrons transfer momentum to the ions at a greater rate (i.e., they are more "resistive") than the right-going electrons. This is possible if the right-going electrons have a larger mean energy than the left-going electrons, as then the right-going electrons will be less collisional than the left-going electrons. A steady-state in which there is no net transfer of momentum to the ions is then achieved with more right-going than left-going electrons, and hence, a net current. In this scenario we have $\int d^3v\, m_e v_{||} C(f) \equiv 0$ since both the collisional and r.f. momentum transfer rates vanish. However, there is a net current.

Now consider a lower-hybrid current-drive scheme. In this case the r.f. diffusion puts momentum into the electrons and this momentum is removed by collisions with ions. The net source of momentum into the electrons from both r.f. and Coulomb collisions vanishes, so again we have $\int d^3v\, m_e v_{||} C(f) \equiv 0$. One might try to gain some information about the current by separating the collision operator into two pieces: C_{rf} to describe the r.f. diffusion, and C_c to describe the Coulomb collisions; and making the identification "$q_e n\eta j_{||}$" $= -\int d^3v\, m_e v_{||} C_c(f)$. Unfortunately, even after allowing for the fact that the current-carrying electrons may have energies large compared to the thermal energy one finds that this approach underestimates the current-drive efficiency by a factor of four[5] precisely because it ignores the effect of an asymmetric resistivity described in the previous paragraph.

We have shown that the identification

$$-\int d^3v\, m_e v_{||} C(f) \equiv \text{“}q_e n\eta j_{||}\text{”}$$

can lead to serious qualitative errors. Is this identification ever of any use? The flux-tube average of the parallel momentum balance equation derived in section II suggests that this identification may be of some qualitative use in understanding the reduction in the current-drive efficiency due to magnetic field variations and trapped particles. To see this, consider a plasma in which there are no appreciable variations in the potential ϕ along a field line, and assume that we are driving a current with ECH as described by Fisch and

Boozer[2]. Our system differs from that considered in Ref. 2 in that we will allow for periodic variations in B. In this current-drive scheme there is little net momentum input from the ECH. Hence, $\int d^3v\, m_e v_\parallel C(f)$ gives the net transfer of momentum from the electrons to the ions. The parallel momentum balance equation tells us that this collisional momentum transfer is balanced by the net μ-grad B force on the electrons.

If we are using the ECH to drive a net current of electrons to the right, there will be a tendency for energetic right-going electrons to be scattered from the passing region of phase space into the trapped region by the ECH. This results in both a reduction in the ECH current-drive efficiency as calculated in Ref. 4, and a net transfer of momentum between the magnetic field and the electron distribution. It is the balance between this net force on the electrons from the magnetic field and the collisional drag on the ions that is being described by the flux-tube average of the parallel momentum equation. Hence, a qualitative estimate of the current lost due to this neoclassical effect might be obtained from the parallel momentum balance equation. This is what we believe Ohkawa has qualitatively calculated in Ref. 1. Further investigation of this hypothesis requires a short excursion into "bounce-averaged" kinetic theory, which we present in the next section.

III. Expanding the Kinetic Equation in ν/ω_b

A hierarchy of equations can be generated by ordering the steady-state kinetic equation in ν/ω_b, where ν is a characteristic rate of change in the distribution function due to r.f. heating, Coulomb collisions, etc.; and $\omega_b \sim v_\parallel/L$ is the axial bounce frequency of a trapped particle or the transit frequency of a passing particle. The electron distribution function may then be written as

$$f_e \approx f_0(\epsilon,\mu,\sigma) + f_1(\epsilon,\mu,\sigma,s) + \dots$$

where ϵ is the energy, μ is the magnetic moment, and σ is the sign of the parallel velocity. In writing the arguments of f_0 we have anticipated the result from the zero-order equation in this hierarchy,

$$\sigma |v_\parallel| \frac{\partial f_0}{\partial s} = 0 \ ,$$

that f_0 must be independent of s. The effect of r.f. absorption on f_0 is described by the bounce-averaged kinetic equation,

$$\overline{C(f_0)} = 0 \ ,$$

which arises as a solubility constraint on the first-order equation in the hierarchy. Here again the "collision" operator $C(f_0)$ includes r.f. diffusion as well as Coulomb collisions. $\overline{C(f_0)}$ is the bounce average of the collision operator:

$$\overline{C(f_0)} \equiv \frac{1}{\tau_b} \int_{-s_t}^{s_t} \frac{ds}{|v_\parallel|} C(f_0) \ ,$$

where

$$\tau_b \equiv \int_{-s_t}^{s_t} \frac{ds}{|v_\parallel|}$$

and s_t is the turning point for trapped electrons or the magnetic field maximum for passing electrons. This bounce-averaged kinetic equation has been used extensively in the previously mentioned studies of ECCD (see, e.g., Ref. 5). It has been shown that the absorption of ECH power can produce a parallel current at leading order. The mechanism is essentially that first described by Fisch and Boozer[2], although trapped particle effects reduce the current-drive efficiency through the mechanism pointed out by Ohkawa in his earlier report[3].

The mechanism described in Ohkawa's most recent report involves the explicit s-dependence of the electron distribution function. Hence, it must appear through f_1. The variation in f_1 is described by the first order kinetic equation:

$$\sigma |v_\parallel| \frac{\partial f_1}{\partial s} = C^{(\sigma)}(f_0) \; .$$

The bounce-average of this equation yields $\overline{C(f_0)} = 0$ as a solubility constraint on f_1 as mentioned above. Here we integrate the equation once in s to find

$$f_1^{(\sigma)} = \sigma \int_{-s_t}^{s} \frac{ds}{|v_\parallel|} C^{(\sigma)}(f_0) + \alpha^{(\sigma)}(\epsilon, \mu) \; .$$

We are now ready to return to the parallel component of the momentum equation. Assume that we are driving current j_0 at zero order in ν/ω_b (i.e., $j_0 = q_e \int d^3v \, v_\parallel f_0$), and that the magnetic field has axial variations. What will the lowest significant order momentum balance equation look like? At zero order in ν/ω_b we simply get $0 = 0$. At first order we obtain a nonzero contribution to $\Im(s)$ from f_1. Upon inserting our expression for f_1 into the definition of $\Im(s)$, we obtain

$$\Im(s) = \int d^3v \, m_e v_\parallel C(f_0) \; .$$

Thus we see that $\Im(s)$, which (apart from a perfect derivative flow term) represents the sum of the μ-grad B and electrostatic forces on the electron distribution, balances the net collisional drag on the electrons, a first-order quantity in ν/ω_b. Hence, the hypothesis presented at the end of section II, that the parallel component of the momentum equation describes the balance between neoclassical terms associated with axial variations in B, trapped particles, etc., and electron-ion drag, is born out. Unfortunately, this same analysis shows the futility of attempting to compute the contribution to j_0 from these neoclassical effects using the parallel component of the momentum equation since it is necessary to have solved the bounce-averaged kinetic equation for f_0 in order to compute the relevant part of f_1, [i.e., $\sigma \int_{-s_t}^{s} \frac{ds}{|v_\parallel|} C^{(\sigma)}(f_0)$]; if we have already computed f_0 it is easier and more accurate to compute j_0 by simply taking the appropriate moment of f_0.

We now understand why Ohkawa obtained a current of the same order as the Fisch-Boozer current by examining the parallel component of the momentum balance equation—he has simply estimated the neoclassical correction to the Fisch-Boozer current, and this correction is of the same order of magnitude as the Fisch-Boozer current. We have also shown that in the long mean-free-path limit (i.e., when $\nu/\omega_b \ll 1$) the parallel component of the momentum balance equation is inappropriate for calculating these neoclassical corrections to j_0 since it is necessary to know f_0 and hence j_0 in order to compute the terms in the momentum balance equation to lowest significant order. The fundamental reason for this is that the net momentum transfer from the μ-grad B force, etc. is a consequence of the neoclassical correction to j_0 that Ohkawa is trying to compute.

IV. Currents at First Order in ν/ω_b.

It is nevertheless possible to drive a current in the manner that Ohkawa suggests—although we will show now that the efficiency is smaller than that estimated in Ref. 1 by a factor of ν/ω_b. To see why there should be *some* current driven by a $v_{\|}$-symmetric heating mechanism, consider the following simple model. We take a periodic (e.g., sinusoidal) magnetic well and introduce a heating zone part way up one side of each well. When an electron crosses a heating zone it receives a fixed increment in magnetic moment independent of $v_{\|}$, and with no change in the parallel energy at the heating zone $\epsilon - \mu B_h$, where B_h is the field at the heating zone. We model collisions by introducing a spatially uniform drag toward some thermal value μ_t of magnetic moment, again with no change in $\epsilon - \mu B_h$. If we follow two electrons with equal but opposite parallel velocities leaving the heating zone with magnetic moments such that they are very close to the velocity-space separatrix dividing passing and trapped electrons, then the electron travelling toward the closer magnetic mirror can take considerably longer to get over the field maximum than the one traveling toward the more distant mirror, as the latter has more time to drag away from the separatrix before it gets to the mirror. Hence, for an exponentially small number of electrons, the time to transit once through the periodic structure can differ by order unity for particles with opposite signs of parallel velocity. For the bulk of electrons, there is an asymmetry as well, but only of order ν/ω_b; the net current is smaller than the Fisch-Boozer current by a factor of order ν/ω_b.

We analyze these schemes by considering the contribution of the first order distribution function to the parallel current,

$$j_1 = q_e \int d^3v \, v_{\|} f_1 \ ,$$

which may be written as

$$
\begin{aligned}
j_1 &= q_e \int d^3v\, v_{\|} f_1 \\
&= q_e \frac{2\pi B}{m_e^2} \int \frac{d\epsilon d\mu}{|v_{\|}|} |v_{\|}| \left[\sum_\sigma \int_{-s_t}^{s} \frac{ds}{|v_{\|}|} C^{(\sigma)}(f_0) + \alpha^{(+)} - \alpha^{(-)} \right] \\
&= q_e B \int_{-s_t}^{s} \frac{ds}{B} \int d^3v C(f_0) + q_e \frac{2\pi B}{m_e^2} \int \frac{d\epsilon d\mu}{|v_{\|}|} |v_{\|}| \left[\alpha^{(+)} - \alpha^{(-)} \right] \\
&= q_e \frac{2\pi B}{m_e^2} \int \frac{d\epsilon d\mu}{|v_{\|}|} |v_{\|}| \left[\alpha^{(+)} - \alpha^{(-)} \right]
\end{aligned}
$$

where we have used the fact that the "collision" operator $C(f_0)$ conserves particles, i.e.,

$$\int d^3v C(f_0) = 0 \; ,$$

and the relation

$$\int d^3v \cdots = \sum_\sigma \frac{2\pi B}{m_e^2} \int \frac{d\epsilon d\mu}{v_{\|}} \cdots \; .$$

For the trapped electrons the constants of integration, $\alpha^{(-)}$ and $\alpha^{(-)}$ are related by the boundary condition

$$f_1^{(+)}(\epsilon, \mu, -s_t) = f_1^{(-)}(\epsilon, \mu, -s_t) \; .$$

Hence

$$\alpha^{(+)} = \alpha^{(-)} \equiv \alpha(\epsilon, \mu)$$

for the trapped particles. We see that only the passing region of phase space contributes to the first order parallel current. Hence, mechanisms that attempt to drive a current at first order in ν/ω_b cannot drive a current when acting on the trapped electrons.

Evidently, the first order current is determined by

$$\Delta\alpha(\epsilon, \mu) \equiv \alpha^{(+)}(\epsilon, \mu) - \alpha^{(-)}(\epsilon, \mu) \; .$$

An equation for $\Delta\alpha(\epsilon, \mu)$ is obtained from the solubility constraint for the second order distribution function (i.e., $\overline{C(f_1)} = 0$):

$$\overline{C(\Delta\alpha)} = -\frac{1}{\tau_b} \int_{-s_t}^{s_t} \frac{ds}{|v_{\|}|} C \left[\int_{-s_t}^{s} \frac{ds'}{|v'_{\|}|} \left(C^{(+)}(f_0) - C^{(-)}(f_0) \right) \right] \; .$$

This equation together with the boundary condition that $\Delta\alpha$ vanish on the separatrix between passing and trapped particles determines the first order current carried by passing electrons.

Defining superscript e and o to be even and odd parts in $v_\parallel$ respectively, we have for any function

$$g^{(e)} = \frac{g^{(+)} + g^{(-)}}{2}$$

and

$$g^{(o)} = \sigma \frac{g^{(+)} - g^{(-)}}{2} \ .$$

The integrand in the source term in the equation for the s-independent part of the first order distribution function, $\Delta\alpha$, can now be written as

$$C^{(+)}(f_0) + C^{(-)}(f_0) = 2\left(C^{(e)}(f_0^{(e)}) + C^{(o)}(f_0^{(o)})\right) \ .$$

Note that if there is a 0-order current, there will also be an odd part of the 0-order Coulomb collision operator. Hence both the even and odd parts of f_0 contribute to the first-order current, and so efforts to preferentially heat electrons with a particular sign of parallel velocity, which are crucial to the Fisch-Boozer current-drive mechanism, are less important to schemes which attempt to drive a current at first order in ν/ω_b by localizing the ECH heating. Unfortunately, the current-drive efficiency of a scheme that relies on the ν/ω_b correction to the electron distribution function must be smaller than the Fisch-Boozer efficiency by a factor of order ν/ω_b since power is dissipated at zero order in ν/ω_b in both these schemes and the Fisch-Boozer scheme, while the resulting current is smaller by a factor of order ν/ω_b in schemes which generate current only at first order in ν/ω_b.

V. Estimate of "First-order current" for arbitrary ν_{rf}/ω_b

One might hope to overcome the low efficiency of schemes that generate a current only at first order in ν/ω_b by operating in a regime in which $\nu/\omega_b \sim 1$. Tokamak reactors presumably will operate with $\nu_c/\omega_b \ll 1$, but perhaps the possiblilty of $\nu_{rf}/\omega_b \sim 1$ should be considered. Here ν_c and ν_{rf} are the Coulomb collision frequency and r.f. scattering rates. While it is difficult to analyze the kinetic equation when $\nu_{rf}/\omega_b \sim 1$*, we can analyze the case where ν_c and ν_{rf} are both small compared to ω_b but ν_{rf}/ν_c is arbitrary; we find that the first-order current j_1 is smaller than the Fisch-Boozer current j_{FB} by a factor of order ν_c/ω_b. Hence it is unlikely that much will be gained by letting ν_{rf} approach ω_b. The argument proceeds as follows: By construction Ohkawa's resistivity term is

$$\text{"}q_e n\eta j_\parallel\text{"} = -m\int d^3v\, v_\parallel C(f)$$

where C is the sum of the collisional and r.f. diffusion operators. We assume for the moment that there are no Ohmic or Fisch-Boozer currents, so that the collision and r.f. operators

* indeed, in this limit, the r.m.s. change in perpendicular velocity on a single passage through resonance is of order of the mean speed v_t of the distribution, in which case not only our expansion of the kinetic equation, but also the treatment of the r.f. operator as diffusive breaks down. This limit would also require a tremendous expenditure of power.

are symmetric in $v_\parallel$. Then $q_e n\eta j_\parallel$ is nonzero only to the extent that f is not symmetric in $v_\parallel$. If we order the kinetic equation in powers of ν/ω_b, then there is no current at zero order, since the zero-order distribution function is symmetric in $v_\parallel$ unless driven assymetric by external means such as ohmic or Fisch-Boozer drive, which we have excluded. Now, what is the order of magnitude of f_1? f_1 is driven by the local $C(f_0)$; we recall that, by construction, $\overline{C(f_0)} = 0$. In the limits where either ν_{rf} or ν_{c} approaches zero, the local $C(f_0)$ also approaches zero, in the former case because f_0 approaches a Maxwellian which is locally annihilated by the collision operator, and in the latter case because the r.f. operator is localized along a field line, so that, in the limit, the C operator is proportional to a delta function in position times the bounce-averaged operator. Hence f_1 must be down from f_0 by factors of both ν_{c}/ν and ν_{rf}/ν as well as ν/ω_b. Then we see that the right-hand side of the above definition of the resistivity term is of order $mnv_t\nu(\nu/\omega_b)(\nu_{c}/\nu)(\nu_{rf}/\nu)$. Since the absorbed power is $P_d = amn\overline{D}$, where a is a constant of order unity (which depends on harmonic number) and $\overline{D}$ is the bounce-averaged, velocity-space-averaged diffusion coefficient, which is in turn approximately $\nu_{rf}v_t^2$, the current density is of order $eP_d/mv_t\omega_b$, which gives J/P_d of order ν_{c}/ω_b smaller than the Fisch-Boozer current. Here we have taken η to be the Spitzer resistivity; the estimate would be still smaller if we took an r.f.-enhanced resistivity for η.

In summary, it appears that schemes that heat symetrically in $v_\parallel$ cannot drive current in tokamak reactors as efficiently as schemes, like the Fisch-Boozer ECH current-drive mechanism, which do selectively heat electrons with a particular sign of the parallel velocity.

References

[1]T. Ohkawa, "ECH Current Drive by Asymmetric Heating Around the Median Plane", GA Technologies Report # GA–A18656, (October, 1986).

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[3]T. Ohkawa, GA Technologies Report # GA–A13847 (February, 1976).

[4] see, e.g., T.M. Antonsen, Phys. Fluids **29**, 2235 (1986), and references therein.

[5]see, e.g., N.J. Fisch, "Theory of Current Drive in Plasmas", to be published in Reviews of Modern Physics.