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GLOBAL ICRF MODELING IN LARGE NON-CIRCULAR TOKAMAK PLASMAS WITH FINITE TEMPERATURE*

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ABSTRACT

Full wave ICRF coupling calculations[1] in two and three dimensions have been extended to treat tokamaks with non-circular flux surfaces and conducting boundaries. The magnetic field configuration is derived from a Solovév equilibrium[2] with finite poloidal magnetic fields. The conducting boundary may be of arbitrary shape. The mode conversion model is that of Colestock et al.[3] in which the fourth order finite temperature wave equation is reduced to a second order equation which describes the effects of mode conversion on the fast wave but neglects the detailed structure of the ion Bernstein wave. Results show the effect of non-circular cross section on excitation, wave propagation, and absorption in Doublet III-D and JET. Also, in the limit of circular cross section, toroidal phasing of the resonant double loop antenna design for TFTR is studied.

INTRODUCTION

High power ICRF experiments are already underway at JET and are planned in the near future for TFTR and Doublet III-D. To provide reliable and accurate estimates of wave fields and power deposition in these experiments, we extend the full-wave solution of Maxwell's equations in Ref 1 to include approximate warm plasma effects and non-circular magnetic cross-section with conducting boundaries of arbitrary shape.

MODEL EQUATIONS

An approximate magnetic equilibrium of the Solovév type is assumed where

$$B^0(r,\theta) = \frac{\partial\psi}{\partial\theta}\frac{\hat{r}}{r} - \frac{\partial\psi}{\partial r}\hat{\theta} + \chi\,\hat{z} \tag{1}$$

and the poloidal and axial flux functions are

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$$
\begin{aligned}
\psi(r,\theta) &= \frac{-\iota_0 B_0}{2R_T}\left[\frac{(1+x/R_T)^2}{\kappa^2}y^2 + (1+\frac{x}{2R_T})^2 x^2\right] \\
\chi(r,\theta) &= \frac{B_0}{1+x/R_T} \quad .
\end{aligned} \tag{2}
$$

Since this equilibrium is independent of z, the wave electric field can be Fourier decomposed as

$$
\vec{E}(r,\theta,z) = \sum_{k_z} \vec{E}_{k_z}(r,\theta)e^{ik_z z} \tag{3}
$$

to allow a two-dimensional solution for $E_{k_z}(r,\theta)$ from the vector wave equation

$$
-\nabla\times\nabla\times\vec{E}_{k_z} + \frac{\omega^2}{c^2}\vec{E}_{k_z} + i\omega\mu_0\sum_s \vec{J}_s = -i\omega\mu_0\vec{J}_{\text{ext}} \tag{4}
$$

In the mode conversion model of P. L. Colestock et al,[3] the plasma current $\sum_s \vec{J}_s$ is represented with the warm plasma dielectric tensor

$$
\overleftrightarrow{K} = \overleftrightarrow{\epsilon}^{(0)} + ik_{\perp,\text{fast}}\overleftrightarrow{\epsilon}^{(1)} - k^2_{\perp,\text{fast}}\overleftrightarrow{\epsilon}^{(2)} \tag{5}
$$

where $k_{\perp,\text{fast}}$ is the fast wave root of the local dispersion relation for an infinite uniform plasma with $\vec{B}$ = constant and $k_{\|} \to k_z$:

$$
Det\left[\vec{n}\vec{n} - |\vec{n}|^2\vec{I} + \overleftrightarrow{\epsilon}^{(0)} + ik_\perp\overleftrightarrow{\epsilon}^{(1)} - k_\perp^2\overleftrightarrow{\epsilon}^{(2)}\right] = 0 \tag{6}
$$

Assuming $E_z = 0$ for $\omega \sim O(\Omega_{ci})$ (zero electron mass limit), this is 4th order in $k_\perp$ and quadratic in $k_\perp^2$. Equation (6) is solved locally at each point on the (r,θ) finite difference mesh. Then discarding the Bernstein wave root and keeping $k_{\perp,\text{fast}}$ only, the wave equation is formally reduced to a 2nd order partial differential equation describing the fast wave only:

$$
-\frac{1}{k_0^2}(\nabla\times\nabla\times\vec{E}) + \underbrace{\left(\overleftrightarrow{\epsilon}^{(0)} + ik_\perp\overleftrightarrow{\epsilon}^{(1)} - k_\perp^2\overleftrightarrow{\epsilon}^{(2)}\right)}_{\overleftrightarrow{K}}\cdot\vec{E} = -\frac{i}{\omega\epsilon_0}\vec{J}_{\text{ext}} \tag{7}
$$

Dotting this vector equation with the unit vectors $\hat{e}_1 = \nabla\psi/|\nabla\psi|$ and $\hat{e}_2 = \hat{b}\times\hat{e}_1$, we obtain two equations for the two components of $\vec{E}$ perpendicular to $\vec{B}$. Local energy deposition is computed from

$$
\dot{W}_{k_z}(r,\theta) = \frac{\omega\epsilon_0}{2}Im\left\{\vec{E}^*\cdot(\overleftrightarrow{K} - \overleftrightarrow{I})\cdot\vec{E}\right\} \tag{8}
$$

Because of the approximate model used, both mode converted and dissipated energy appear in Eq (8) as power dissipation.

NUMERICAL RESULTS

Figure 1a shows density contours and the conducting wall for the non-circular tokamak geometry of Doublet III-D. Elongation is 1.8 and the antenna is 40 cm high and located on the low field side of the plasma. The dashed line shows the second harmonic resonance in a pure hydrogen plasma with $f = 55\,\mathrm{MHz}$. Keeping 80 toroidal modes ($k_z R_T = n_{\mathrm{toroidal}} = -40 \rightarrow 40$) to represent an antenna half-width of 17.5 cm in the toroidal (z) direction, we find contours of $Re(E_x)$ and power absorbed at the antenna midplane as shown in Fig (1b) and (1c) respectively. The temperature assumed was $T_e = T_i = 3\,\mathrm{keV}$. Figure 2 shows the toroidal spectrum for this result. The solid curve shows the unweighted two-dimensional results for power absorbed versus toroidal wave number k_z. The dashed curve shows the weighted contribution of each toroidal mode in the Fourier sum which represents the antenna assumed in Fig 2. Similar calculations have been done for JET with an elongation of 1.66 and a 6.8% He^3 minority in D. The JET results show Alfven resonance[4] ($\omega^2 = k_{\parallel}^2 v_A^2$) near the high field edge and the associated rapid spatial increase in $k_{\perp}$.

Resonant double loop antennas have been studied for the circular geometry of TFTR with a 5% He^3-D mixture and $n_e = 10^{14}\mathrm{cm}^{-3}$, $T_e = T_i = 10\,\mathrm{keV}$. Figure 3 compares contours of $Re(E_x)$ in the equatorial (r, z) plane for a double loop antenna where the loops are in phase (Fig 3a) and out of phase (Fig 3b). Note the more complete absorption and hence lower transmission in the out of phase case. Higher transmission as in Fig 3a can lead to greater susceptibility to cavity modes at low density due to reflections from the opposite wall.

REFERENCES

1. E. F. Jaeger et al., Comput. Phys. Commun. **40**, 33 (1986).
2. L. S. Solovév, Soviet Phys. JETP **26**, 400 (1968).
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FIGURE CAPTIONS

Fig. 1. (a) Non-circular tokamak geometry:
Density contours (solid) and second harmonic resonance surface (dashed) for hydrogen in Doublet III-D.
(b) Contours of constant $Re(E_x)$ at antenna midplane.
(c) Contours of constant absorbed power at antenna midplane.

Fig. 2. Toroidal spectrum for antenna half-width of 17.5 cm in z:
Solid lines show unweighted two-dimensional power absorption versus k_z and dashed curves show weighted spectrum for the assumed antenna.

Fig. 3. Contours of constant $Re(E_x)$ in equatorial plane for TFTR resonant double loop antenna with loops in phase (a) and out of phase (b).

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