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APERTURE STUDIES ON RHIC FOR 30 AND 100 GeV/NUCLEON OPERATION WITH RANDOM AND SYSTEMATIC MAGNET ERRORS*

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Abstract

Tracking studies on RHIC for the emittances and momentum spreads necessary for operation at 30 and 100 GeV/nucleon have been made when the effects of random and systematic magnet errors are included. All test particles are stable for the emittances required. Tune shifts from systematic b_2 are compensated by the chromaticity correcting sextupoles of the arcs. Tune shifts from systematic b_3 and b_4 are compensated by correction coils placed adjacent to the arc quadrupoles. The emittances required for operation are significantly smaller than the emittance corresponding to the dynamic aperture.

Introduction

The aperture required for RHIC operation at 30 and 100 GeV/nucleon in the presence of random and systematic multipoles and with correction elements has been determined by particle tracking with PATRICIA.[1] In addition to determining the dynamic aperture corresponding to the largest initial betatron amplitude for which motion is stable throughout the duration of the tracking run (400 turns), the requirement that the betatron tune of the test particle must stay within the region selected for RHIC operation, i.e. $28.000 \leq \nu \leq 28.833$ for both the horizontal and vertical motion has been imposed.

The range over which RHIC must operate includes: 1) injection at 11 GeV/nucleon where systematic multipoles resulting from magnetization effects are important, 2) operation as a collider at 30 GeV/nucleon where systematic multipoles are small but where beam size due to intrabeam scattering is large, and 3) operation at 100 GeV/nucleon where the beam size is comparatively small but where systematic multipoles due to saturation effects are important. The present study is limited to consideration of 30 and 100 GeV/nucleon operation at the design intensity of $N_B = 1.1 \times 10^9$ Au ions/bunch as well as possible high intensity operation at 100 GeV/nucleon with $N_b = 5.5 \times 10^9$ Au ions/bunch.

The requirements for betatron aperture and the momentum spread needed for the rf bucket are listed in Table 1. The entries for the betatron amplitude $A_\beta = 6\sigma$(mm) correspond to the total emittance $\varepsilon_t = \pi A_\beta^2/\beta$ where $\varepsilon_t = \varepsilon_x + \varepsilon_y$ with ε_x and ε_y being the emittances in the horizontal and vertical planes, respectively. Hence at 30 GeV/nucleon, the total emittance ε_t is $\varepsilon_t = \pi A_\beta^2/\beta = 6.48\pi$ mm mradians ($\beta = 50$m).

Table 1. Parameters for RHIC operation.

Ion	Au	Au	Au	Au
β^* (m)	6	6	3	3
E_{kin} (GeV/amu)	11	30	100	100
N_B	5.5×10^9 5xDesign	1.1×10^9 1xDesign	1.1×10^9 1xDesign	5.5×10^9 5xDesign
$A_\beta = 6\sigma_\beta$(mm)	16	18	9.6	12.2
$\Delta p/p$ (%)	±0.36	±0.5	±0.26	±0.36

Magnet Multipoles

Magnet multipoles for standard bore dipoles and quadrupoles are listed in Table 2. Random multipoles are attributed to construction tolerances and are assumed to be independent of magnet excitation.

The systematic multipoles do depend on magnet excitation with magnetization effects dominating at 11 GeV/nucleon and saturation dominating at 100 GeV/nucleon. The systematic multipoles at 30 GeV/nucleon should be quite small, however based on FNAL experience,[2] the random multipoles may not cancel completely and may produce an effective systematic multipole whose strength can be as large as ±30% of the random multipoles. These are the values listed in Table 2 for 30 GeV/nucleon operation. The tabulation applies to dipoles and quadrupoles having the standard coil ID of 8.8 cm. Multipoles for special, large bore dipoles and quadrupoles have been scaled from Table 2 by a factor of $(4.4/r)^m$ with r being the coil radius of the large bore magnet and $m = n + 1$ for dipoles and $m = n$ for quadrupoles.

Estimates of the strength of systematic multipoles that produce a specified tune shift have been determined using formulae for one dimensional uncoupled motion.[3] For RHIC the multipole strength that produces a tune shift $\Delta\nu = 0.003$ (one tenth of the operating region) has been determined for each order of multipole $2 \leq n \leq 10$ for the maximum momentum spread and betatron amplitudes listed in Table 1.[4] These strengths are tabulated in Table 2 under the heading "Tol". These tolerances are used as a means of estimating the importance of specific multipoles and may be used to impact on magnet design. As would be expected, the b_2 and b_4 systematic multipoles in the dipoles dominate all other multipoles. The tolerances for dipoles were obtained using average values of beta functions in a ring filled with dipoles; the values for quadrupoles were scaled by the ratio of the dipole to quadrupole length (9.46/1.24 = 7.6).

Lattice

The RHIC lattice has been described elsewhere.[5] Briefly, there are two independent storage rings that intersect at six points where β^* can be varied from 3 to 6 meters. Each ring has three superperiods consisting of an inner arc of twelve cells, an insertion, an outer arc of twelve cells, and a second insertion. The cells have a nominal phase advance of 90°. For the present study, small tune changes have been made by changing the phase advance per cell without rematching the insertions. The lattice is antisymmetric with respect to the crossing points; the radius of curvature is 243.88 m.

The placement of elements in the cells of the inner and outer arcs is shown in Figure 1. Effects of higher order fields are simulated by kicks at elements MF and MD located at the center of quadrupoles and elements MB located at the ends of the dipoles. Chromaticity is corrected with sextupoles SF1, SF2, and SD_o in the inner arcs and SF_o, SD3, and SD4 in the outer arcs. Finally, two families of correction elements (MC1 and MC2) 0.5m long are located adjacent to the defocusing and focusing quadrupoles respectively. These elements will have several correction coils, however in the present study only octupole and decapole corrections have been used.

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Table 2. Systematic and random multipoles for standard bore RHIC dipoles and quadrupoles.

	DIPOLES									QUADRUPOLES							
	11 GeV Systematic		30 GEV Systematic		100 GeV Systematic			Random		11 GeV Systematic		30 GeV Systematic		100 GeV Systematic			Random
	Tol[a]	Exp[b]	Tol	Exp	Tol (1x)	Tol (5x)	Exp	σb_n	σa_n	Tol	Exp	Tol	Exp	Tol (1x)	Tol (5x)	Exp	$\sigma b_n = \sigma a_n$
b'_2	0.17	-3	0.13	±1.4	0.25	0.18	6.5	4.6	1.3								3.7
b'_3	0.14		0.10	±0.4	0.35	0.21		1.3	2.2								2.3
b'_4	0.29	<1	0.15	±0.7	1.0	0.44	-4.7	2.2	0.57								1.7
b'_5	0.24		0.16	±0.2	1.9	0.67		0.53	0.91	2.2	-0.2	1.2	±0.3	15	5.1	0.6	1.2
b'_6	0.46		0.19	±0.2	4.5	1.20	-0.5	0.83	0.23								0.85
b'_7	0.57		0.21	±0.05	9.4	2.00		0.18	0.34								0.59
b'_8	0.81		0.25	±0.08		3.30		0.28	0.08								0.41
b'_9	1.00		0.28	±0.02				0.06	0.11	8.0	<0.1	2.1	±0.1	340	42	<0.1	0.27
b'_{10}	1.4		0.32	±0.03				0.09	0.03								0.18

a Tol indicates strength of systematic multipole that produces $\Delta\nu = 0.003$.
b Exp indicates expected strength of systematic multipoles in units of 10^{-4} at 2.5 cm (0.6 x coil radius)

Sextupoles

The lattice with two families of sextupoles for chromaticity correction displays a strong dependence of tune on momentum that is unacceptable. Improved tune dependence on momentum is achieved when pairs of focusing sextupoles of the inner arcs alternate in strength by ΔSF and pairs of defocusing sextupoles of the outer arcs alternate by ΔSD.[6] The dependence of tune on momentum for the two sextupole schemes is shown in Figure 2. For operation of RHIC in the $\beta^* = 6$m configuration, $\Delta SF/SF_o = \Delta SD/SD_o = 0.31$ gives optimum correction of tune, however for $\beta^* = 3$m, $\Delta SF/SF_o = 0.24$ and $\Delta SD/SD_o = 0.175$ is required. When systematic sextupoles are included, the values of SF_o and SD_o change, but to a good approximation ΔSF and ΔSD are unchanged—see Table 3.

Operation at 100 GeV/nucleon

Higher Order Systematic Multipoles

The tune dependence on momentum introduced by b_4 and b_6 multipoles in the dipoles and b_5 and b_9 multipoles in the quadrupoles is shown in Figure 3. Contributions from all but the b_4 multipoles are negligible.

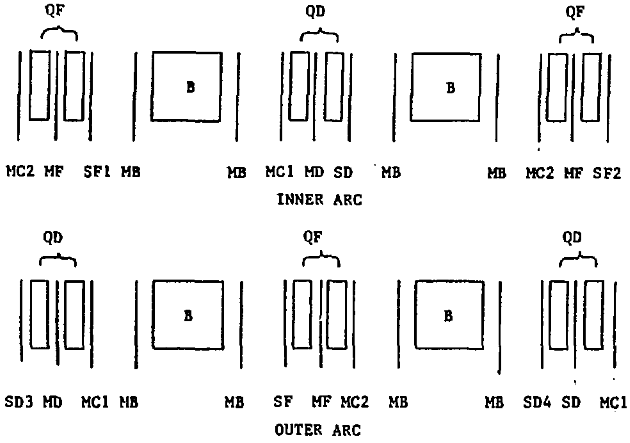

Figure 1. Layout of cells of the inner and outer arcs indicating placement of elements. Kicks (MF and MD) representing the effects of multipole fields are placed at the center of quadrupoles QF and QD. Half of the multipole fields of the dipoles are attributed to elements MB at each end of the dipoles (B). Elements SF_o, SF1, SF2, SD_o, SD3, and SD4 are sextupoles, and elements MC1 and MC2 are correction elements.

Complete correction of the dependence of linear tune on momentum from the systematic b_4 plus the residual contribution from the sextupoles seen in Figure 2 is possible with decapole correctors in the arcs, however this correction produces a strong dependence of particle tune on both momentum and emittance. Instead, the decapole correctors were adjusted to minimize tune dependence on emittance. This correction produced an acceptable dependence of tune on amplitude and momentum in the momentum interval needed for RHIC operation at 100 GeV/nucleon. The tune dependence from both b_4 corrections for one dimensional motion ($\varepsilon_x = 0$ and $\varepsilon_y = \varepsilon_t$) is shown in Figure 4.

Systematic and Random Multipoles

With the systematic b_2 and b_4 corrected, the contribution from random sextupoles (σa_2 and σb_2) as well as from random decapoles (σa_4 and σb_4) was determined separately when all other random multipoles were set to zero strength. For the random sextupoles, the values of SF_o and SD_o required to achieve zero chromaticity changed by as much as ±5%. The values of ΔSF and ΔSD were also adjusted slightly, but this adjustment was probably unnecessary for operation confined to $-0.36 \leq \Delta P/P \leq 0.36\%$.

The contribution to the linear tune from random b_4 is less than $\Delta\nu = \pm 0.01$ at $\Delta P/P = \pm 1\%$ and is less than $\Delta\nu = \pm 0.001$ for $-0.5 \leq \Delta P/P \leq 0.5\%$. In addition,

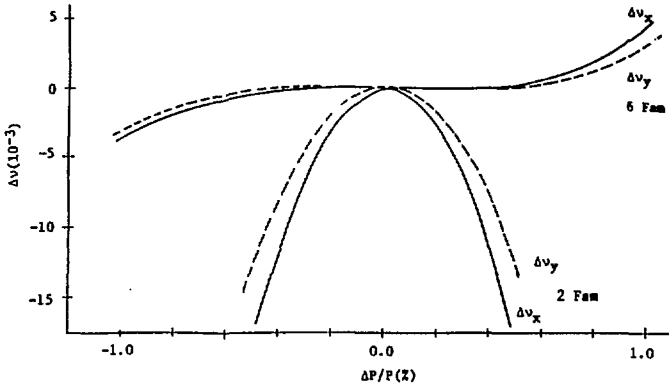

Figure 2. Tune dependence on momentum for two sextupole configurations: 1) two families with sextupoles SF_o and SD_o in both the inner and outer arcs, and 2) six family scheme (with four power supplies) having $SF1 = SF_o + \Delta SF$, $SF2 = SF_o - \Delta SF$, and SD_o in the inner arcs and SF_o, $SD3 = SD_o - \Delta SD$, and $SD4 = SD_o + \Delta SD$ in the outer arcs.

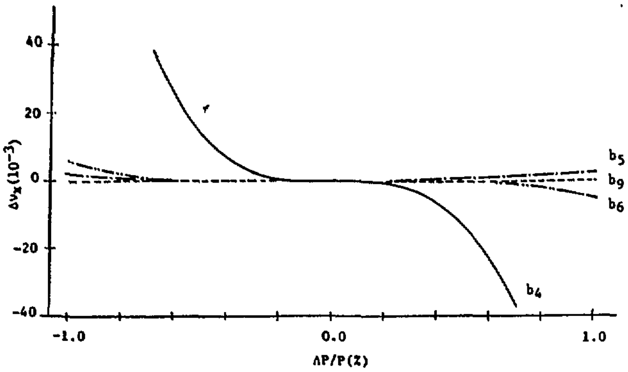

Figure 3. Contribution to the linear tune from the systematic b_4 and b_6 of all dipoles and b_5 and b_9 of all quadrupoles (arcs and insertions). In the momentum interval $-0.36 \leq P/P \leq +0.36\%$ required for 100 GeV/nucleon operation, only b_4 needs correction.

tracking showed that the tune shift at maximum emittance was not increased significantly ($\Delta\nu = 0.002$), so no additional adjustment of the decapole corrector was made.

Tracking at 100 GeV/nucleon

Next all systematic and random multipoles were included, and tracking runs of 400 turns were made using several different initial conditions and six different machines (six different sets of random multipoles). With $\varepsilon_t=\varepsilon_x+\varepsilon_y$, particles were tracked with $\varepsilon_x=\varepsilon_y$ for two launching conditions; $x\neq0$, $y\neq0$ and $x\neq0$, $y'\neq0$ (traditionally these conditions have represented the results for the four combinations $(x,y)\neq0$, $(x,y')\neq0$, $(x',y)\neq0$, and $(x',y')\neq0$).

Four particles having $\varepsilon_t=0$, 1.0, 1.84, and 2.98π mm mradians were tracked at nine different momenta: $\Delta P/P = \pm0.36\%$, $\pm0.26\%$, $\pm0.20\%$, $\pm0.10\%$, and 0.0%. The combination of $\varepsilon_t=1.84\pi$ mm mradians and $\Delta P/P=\pm0.26\%$ represents the requirement for 100 GeV/nucleon operation at the normal intensity $N_B=1.1 \times 10^9$ Au ions/bunch, and the combination $\varepsilon_t=2.98\pi$ mm mradians and $\Delta P/P = \pm0.36\%$ is the emittance and momentum spread needed for operation at five times the nominal RHIC intensity. At these modest emittances and momentum spreads, all particles were stable. The average tune of each particle was determined from its total phase advance around a normalized phase ellipse during the tracking run. A plot of the region of tune occupied by particles with the two launching conditions $x\neq0$, $y\neq0$ and $x\neq0$, $y'\neq0$ is shown in Figure 5. The plot includes the tunes at each of the nine momenta and four emittances mentioned above. At large emittances and ΔP/P, the horizontal and vertical tunes become equal, and the particle travels up the diagonal (ΔP/P>0) or down the diagonal (ΔP/P<0) as its emittance increases. The shaded area of Figure 5 indicates the region of tune occupied by particles having emittances and momenta corresponding to operation with normal intensity; the enclosed, unshaded area indicates the additional tune region necessary for operation at high intensity. The particle at $\varepsilon_t = 2.98\pi$ mm mradians and $\Delta P/P = +0.36\%$ goes beyond the region of tune bounded by $\nu = 28.833$.

Table 3. Sextupoles for chromaticity correction.

	β^*=6m	β^*=3m	β^*=3m + SYSTEMATICS
SF_o[a]	-0.1423	-0.1662	-0.1038
SD_o	0.2888	0.3374	0.4233
ΔSF	-0.0441	-0.0399	-0.0399
ΔSD	0.0895	0.0590	0.0550
CHX[b]	-48.88	-57.11	-13.90
CHY	-48.84	-57.14	-39.78

a Integrated strengths.
b Chromaticities before correction.

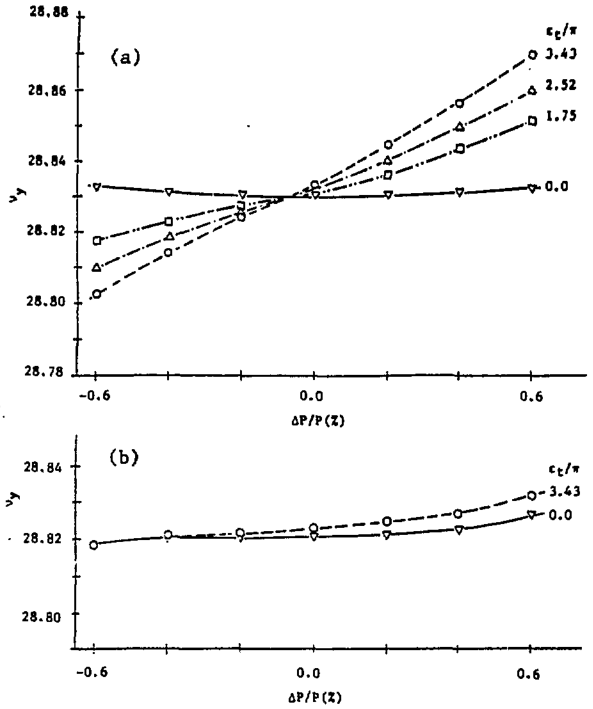

Figure 4. Dependence of tune ν_y on momentum and emittance for two degrees of decapole correction when $\varepsilon_x=0$ and $\varepsilon_y=\varepsilon_t$: a) decapoles at MC1 and MC2 adjusted to flatten the linear tune ($\Delta\nu<0.0005$) in the range $-1.0<\Delta P/P<1.0\%$. $MC1=4.11\times10^4\ m^{-4}$ and $MC2=1.27\times10^4\ m^{-4}$. and b) decapoles in MC1 and MC2 adjusted to minimize tune shift with emittance. $MC1=1.81\times10^4\ m^{-4}$ and $MC2=1.37\times10^4\ m^{-4}$.

In addition to determining the tune dependence in the interval of emittance and ΔP/P necessary for operation, the dynamic aperture has been determined over this interval of ΔP/P to determine how near to its stability limit RHIC will be operating. The dynamic aperture as well as the aperture required for normal and high intensity operation are compared on Figure 6. The dynamic aperture is significantly larger than the aperture required for normal operation.

Operation at 30 GeV/nucleon

As mentioned earlier, effective systematic multipoles equal to ±30% of the normal random multipoles have been assumed for 30 GeV/nucleon operation. Little correction is needed--the nominal strengths of SF_o and SD_o must be adjusted--see Table 3, and the major contributions to the tune dependence on momentum have been corrected by octupoles and decapoles in the correction elements of the arcs (The tune shift at ±0.5% is ∿ 0.007 for each of these multipoles). With both random and systematic multipoles present, the dynamic aperture has been determined over the interval $-0.5 \leq \Delta P/P \leq 0.5\%$ and is compared in Figure 7 with the aperture required for operation at normal intensity. In all cases the tunes couple at large amplitudes but remain within the region bounded by $28.800 \leq \nu \leq 28.833$.

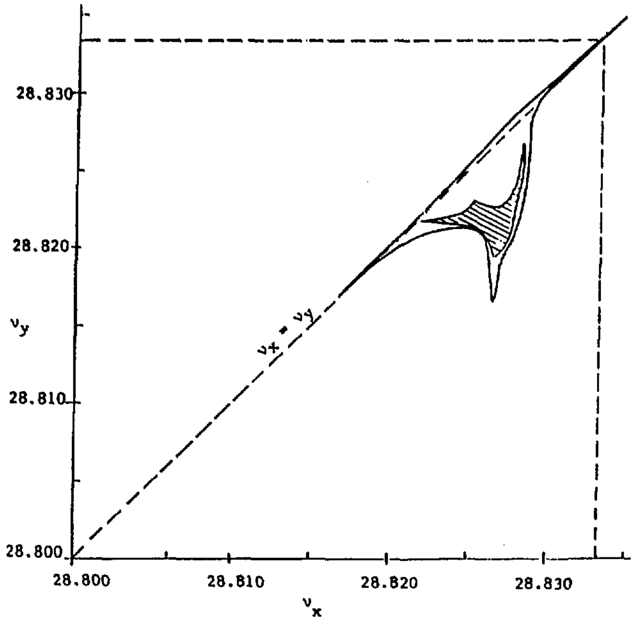

Figure 5. Region of tune space occupied by test particles having launching conditions ($x\neq0$, $y\neq0$) and ($x\neq0$, $y'\neq0$). The shaded area indicates the region for operation at normal intensity ($-0.26 \le \Delta P/P \le 0.26\%$ and $\epsilon_t \le 1.84\pi$ mm mradian), while the unshaded area indicates the additional tune region needed for high intensity operation ($-0.36 \le \Delta P/P \le 0.36\%$ and $\epsilon_t = 2.98\pi$ mm mradians). For all $\Delta P/P$, the particles migrate towards and then up or down the diagonal depending upon whether $\Delta P/P$ is greater or less than zero.

Discussion

Tracking runs made at $\Delta P/P=0$ with six family sextupole correction (four power supplies) and no systematic or random multipoles indicate a dynamic aperture of $\epsilon_t=82.5\pi$ mm mradians (with $\epsilon_x=\epsilon_t/2$, $\hat{x}=41.3$mm) for $\beta^*=6$m and $\epsilon_t=65.9\pi$ mm mradians ($\hat{x}=33.0$mm) for $\beta^*=3$m. When random multipoles only are included, these apertures are reduced to $\hat{x}=20$mm for $\beta^*=6$m and $\hat{x}=14.6$mm for $\beta^*=3$m. When systematic multipoles and correction coils are added, the apertures are only slightly affected: $\hat{x}=19$mm for $\beta^*=6$m and $\hat{x}=14.6$mm for $\beta^*=3$m.

These results agree with a previous study[7] made at $\beta^*=6$m with only random multipoles present when averages of the ten cases considered, rather than the worst case values, are used, and they also agree at $\Delta P/P=0\%$ with results from a study with random and systematic multipoles but no correction coils.[8] From these three studies it is concluded that the major limitation to the aperture results from random multipoles and that systematic multipoles have little effect on the aperture at $\Delta P/P\sim0$. For $\Delta P/P\neq0$, lumped correction coils compensating the tune shifts from the low order multipoles maintain the tune within the desired region and keep the dynamic aperture near its value at $\Delta P/P=0\%$.

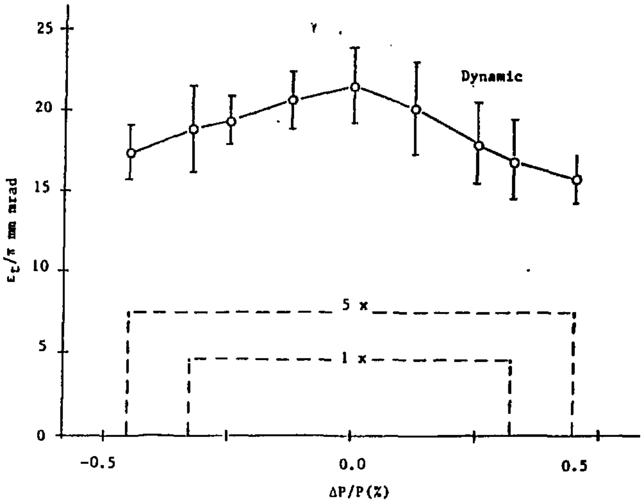

Figure 6. Comparison of the dynamic aperture with the aperture required for 100 GeV/nucleon operation with $\beta^* = 3$m at either 1 x design intensity or 5 x design intensity.

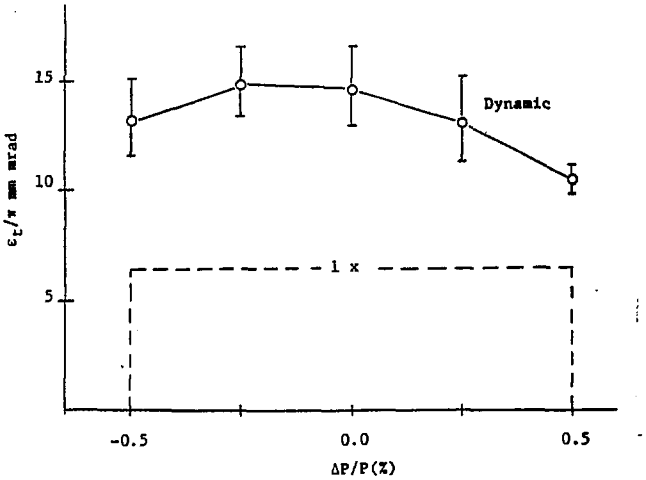

Figure 7. Comparison of the dynamic aperture with the aperture required for 30 GeV/nucleon operation with $\beta^* = 6$m and 1 x the design intensity.

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