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AUTHOR(S): B. R. Suydam

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University of California

LOS ALAMOS SCIENTIFIC LABORATORY
Post Office Box 1663 Los Alamos, New Mexico 87545
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CAN OPTICAL PHASE CONJUGATORS PRODUCE A VERY SHORT CONJUGATE PULSE?

B. R. Suydam
University of California
Los Alamos National Laboratory
Los Alamos, NM 87545

Abstract

In studying the transient behavior of phase conjugation via four wave mixing, it has been noted that very short probe pulses invariably produce considerably broadened conjugate pulses unless the conjugator is inordinately short. We have therefore undertaken the present investigation to determine whether a short conjugate pulse might be produced by judicious programming of the probe. We find that the probe can indeed be tailored to produce an arbitrarily short conjugate signal and, if the conjugating medium exhibits linear absorption, then the total energy in the probe signal is finite. The required probe programming can be determined by numerical evaluation of a Fourier transform.

Introduction

In this paper we shall be considering an optical phase conjugator which operates by four wave mixing as first described by Hellwarth[1]. The conjugating medium will be a dielectric exhibiting the Kerr effect with essentially instantaneous response, pumped far from resonance by two exactly counterpropagating, unmodulated plane waves of frequency ω_o. It is assumed that the probe is so weak that the pump is not depleted.

Among many possible uses[9], optical phase conjugators may prove to be valuable devices in the field of laser fusion. Here, and perhaps elsewhere, one deals with quite short pulses, so the transient behavior becomes important. The classical paper on this subject is that of Bobroff and Haus[2], who give the conjugate response to a δ-function probe input. Fisher et al[3] have carried the analysis further, have developed a convenient scheme to compute the conjugate response given the probe envelope, and have presented results of some typical probe pulse shapes. Generally speaking, if the probe pulse was as short as a nanosecond the conjugate signal was considerably smeared unless the conjugator was vary short. Upon seeing these results it was natural to ask whether a short conjugate pulse might be produced by properly programming the probe rather than by arbitrarily shortening the conjugator. In this paper we therefore address the question. Can the probe be so tailored that the conjugate output pulse is shorter than the round trip transit time of light in the conjugator, i.e. shorter than the δ-function response? The answer turns out to be, indeed it can. Naturally we shall accept only such programming as results in finite total input energy. It is therefore convenient to use mathematical terminology which I shall now define. If the complex function F(t) is such that

$$\int_{-\infty}^{\infty} |F(t)|^2 dt \quad \text{converges} \quad .$$

then we say that F(t) belongs to L_2. We require that the probe envelope function belong to L_2.

General Result

The electric fields of the probe and conjugate waves are represented, as usual, by

$$E_p = \tfrac{1}{2}\mathcal{E}_p \exp\{-i\omega_o(t - z/v)\} + cc \quad .$$
$$E_c = \tfrac{1}{2}\mathcal{E}_c \exp\{-i\omega_o(t + z/v)\} + cc \qquad (1)$$

and the envelopes $\mathcal{E}_p$, $\mathcal{E}_c$ are presumed to vary slowly in t and in z. Clearly v is the phase velocity in the medium. It is convenient to define

$$F(t) = \mathcal{E}_p^* (o,t) \quad .$$
$$G(t) = (i/\kappa)\, \mathcal{E}_c(o,t) \qquad (2)$$

where κ is the coupling coefficient which, in our case, is a real constant. Thus F(t) is the complex conjugate of the probe envelope and G(t) is the normalized envelope of the conjugate signal, both evaluated at z = 0, where the probe enters and the conjugate exits. The coupled wave equations are solved by Laplace transforming them and obtaining the result[2,3]

$$g(s) = h(\sigma)f(s) \quad ,$$

$$h(\sigma) \equiv \frac{\sin\beta\ell}{\beta\cos\beta\ell + (\sigma/v)\sin\beta\ell} \quad , \tag{3}$$

$$\sigma \equiv s + \tfrac{1}{2}\alpha v \quad , \quad \beta \equiv \sqrt{\kappa^2 - (\sigma/v)^2} \quad .$$

Here α is the linear absorption coefficient of the conjugating medium, ℓ is its length and $f(s)$, $g(s)$ are respectively the Laplace transforms of $F(t)$, $G(t)$; s is of course the complex frequency variable $s = \gamma - i\Omega$. Laplace transforming the first of Eqs. (3) and using the Faltung theorum, we have

$$G(t) = \int_{-\infty}^{\infty} H(t - \tau)\, F(\tau)\, d\tau \quad , \tag{4}$$

where $H(t)$ is the Laplace transform of $h(\sigma)$.

Our problem is to solve the integral equation, Eq. (4); that is, given the desired response $G(t)$, find the required input $F(t)$. Formally the problem is trivial, for Eq. (3) immediately gives us

$$f(s) = [h(\sigma)]^{-1}\, g(s) \tag{5}$$

whence, Laplace transforming

$$F(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} [h(\sigma)]^{-1}\, g(s)\, e^{st}\, ds \quad . \tag{6}$$

The difficulty with this procedure is that, while $h(\sigma)$ is a well behaved transfer function, its reciprocal is not. Thus we have no a priori assurance that the integral converges, let alone that $F(t)$ should belong to L_2. The major result here reported is that the integral of Eq. (6) does indeed converge and defines a function $F(t)$ which belongs to L_2 if and only if the following conditions are all satisfied:

(a) $\alpha > 0$;

(b) $\kappa^2 < (\alpha/2)^2 + (\pi/\ell)^2$;

(c) Both $G(t)$ and its derivative $G'(t)$ belong to L_2.

When these conditions are satisfied we can choose $\gamma = 0$ in Eq. (6) which then expresses the required input function $F(t)$ as an ordinary Fourier transform. Thus when conditions (a), (b) and (c) are all satisfied the method developed in Ref. 3 can be used to compute $F(t)$ numerically.

A Simplified Example

The proof of the above stated result, which is a bit long, will be given in a fuller paper on this subject. In this fuller paper I shall also discuss what goes wrong when $\alpha = 0$ and what can be done about it. For this paper it seems more instructive to illustrate our general result by examining the simplified case of a weak conjugator, as first discussed by Marburger[4]. In the limit $\kappa \to 0$, Eq. (3) becomes, after renormalization,

$$h_o(\sigma) = (1/\sigma T)\,[1 - e^{-\sigma T}] \quad , \tag{7}$$

$$T \equiv 2\ell/v \quad .$$

Note that T is the round trip light transit time. The Laplace transform of this is

$$H_o(t) = \begin{cases} 0 \text{ for } t < 0 \\ (1/T)\exp[-\tfrac{1}{2}\alpha vt] \text{ for } 0 < t < T \\ 0 \text{ for } t > T \end{cases} \quad , \tag{8}$$

and Eq. (4) takes the form

$$G(t) = \int_{t-T}^{t} \exp[-\tfrac{1}{2}\alpha v(t-\tau)] \quad F(\tau)d\tau \quad . \tag{9}$$

By direct differentiation we readily see that

$$G'(t) + \tfrac{1}{2}\alpha v\, G(t) = (1/T)\,\{F(t) - e^{-\frac{1}{2}\alpha vT}\, F(t-T)\} \quad , \tag{10}$$

which can readily be solved to yield

$$F(t) = T \sum_{n=0}^{\infty} e^{-\frac{1}{2}n\alpha vT}\,\{G'(t-nT) + \tfrac{1}{2}\alpha v\, G(t-nT)\} \quad . \tag{11}$$

This result is perfectly general for the response function given by Eq. (8), but we shall limit our attention to conjugate signals G(t) of the form

$$G(t) = \begin{cases} 0 \text{ for } t < 0 \quad , \\ \text{arbitrary except that } G'(t) \text{ is} \\ \text{continuous for } 0 \le t \le T_1 \quad , \\ 0 \text{ for } t > T_1 \quad , \end{cases} \tag{12}$$

$$T_1 < T \quad ,$$

as we are specifically interested in producing arbitrarily short output pulses. We see that Eq. (11) expresses F(t) as an infinite string of similar nonoverlapping pulses of ever diminishing amplitude and that

$$\int_{-\infty}^{\infty} |F(t)|^2\, dt = T^2\,[1 - e^{-\alpha vT}]^{-1} \int_{-\infty}^{\infty} |G'(t) + \tfrac{1}{2}\alpha v G(t)|^2\, dt \quad . \tag{13}$$

With G(t) defined by Eq. (12), G(t) and G'(t) both belong to L_2 and F(t) belongs to L_2 if and only if $\alpha > 0$.

The exact solution to our problem is an infinite string of pulses, and this may be inconveniently many. Let us therefore see what happens if we mutilate the input by taking only a finite string. Specifically we choose

$$\hat{F}(t) = T \sum_{n=0}^{N-1} e^{-\frac{1}{2}n\alpha vT}\,\{G'(t-nT) + \tfrac{1}{2}\alpha v G(t-nT)\} \tag{14}$$

and from this mutilated input we calculate the mutilated output

$$\hat{G}(t) = G(t) - e^{-\frac{1}{2}N\alpha vT}\, G(t-NT) \quad . \tag{15}$$

Clearly if $\frac{1}{2}N\alpha vT$ (which = $N\alpha\ell$) is sufficiently large, $\hat{G}(t)$ differs insignificantly from the desired shape G(t), and this is true even if N = 1. Thus from a practical point of view, we have the curious result that the longer the conjugator the better. In fact when N = 1 and $\ell \to \infty$, $\hat{G}(t) = G(t)$ exactly and the required input is

$$F(t) = T[G'(t) + \tfrac{1}{2}\alpha v G(t)] \quad \text{for } \ell = \infty \quad . \tag{16}$$

Let us summarize what our simple model has shown. Set N = 1 into Eqs. (14) and (15). The probe signal then has the form of Eq. (16) which, if α is small, describes what is very nearly a zero pi pulse. As this probe enters the conjugator the desired conjugate signal, G(t), is generated. Once all of the probe pulse is within the conjugator, contributions from the positive and from the negative portions of the probe envelope exactly cancel and lead to a zero conjugate output. When the probe leaves the conjugator this balance is upset so that an undesired "echo" is generated and reaches the conjugator input as the term $-\exp[-\frac{1}{2}\alpha vT]\, G(t-T)$. We can handle this echo in three different ways:

1) We can introduce a second probe pulse which exactly cancels the echo of the first, then a third probe pulse to cancel the new echo and so on. The result is the series of Eq. (11).

2) We can make the attenuation so high that the echo is of insignificant magnitude.

3) By making the conjugator long we can so delay the echo that it is no longer of consequence. Or, of course, we can combine these three strategies in any way we choose.

When he conjugator is not weak we have no neat analytic solution to the basic integral equation, such as Eq. (11) above. Rather we must, in general, choose the response function G(t) desired, calculate its Fourier transform $g(-i\Omega)$ and then evaluate numerically the Fourier transform, Eq. (6) with γ set equal to zero.

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