8-9-88

QUENCH PRESSURE, THERMAL EXPULSION, AND NORMAL ZONE PROPAGATION IN INTERNALLY COOLED SUPERCONDUCTORS*

Lawrence Dresner
Oak Ridge National Laboratory, P.O. Box 2009
Oak Ridge, TN 37831-8054

CONF-880812--34

DE89 005668

Abstract

When a nonrecovering normal zone appears in an internally cooled superconductor, the pressure in the conductor rises, helium is expelled from its ends, and the normal zone grows in size. This paper presents a model of these processes that allows calculation of the pressure, the expulsion velocity, and the propagation velocity with simple formulas. The model is intended to apply to conductors such as the cable-in-conduit conductor of the Westinghouse LCT (WH-LCT) coil, the helium volumes of which have very large length-to-diameter ratios (3×10^5). The predictions of the model agree with the rather limited data available from propagation experiments carried out on the WH-LCT coil.

Introduction

When a nonrecovering normal zone appears in an internally cooled superconductor, the pressure in the conductor rises, helium is expelled from its ends, and the normal zone grows in size. A variety of coupled physical processes, each simple in itself, underlies these three manifestations of the nonrecovering normal zone. Thus, heating of the helium by the normal conductor causes its pressure to rise, but the rise in pressure is limited by the expansion of the helium. The expansion of the helium is restrained by inertia and by turbulent friction with the walls and the wires of the conductor. This partially restrained expansion determines the rates of pressure rise and thermal expulsion. These rates are also determined by the power input to the helium, which in turn depends on the size of the normal zone. The normal zone grows with time, spreading because of heat transfer from the normal part of the conductor to the part that is still superconducting. Such heat transfer takes place by conduction through the copper matrix and by convection by the expanding warm helium.

As these events unfold, two kinds of fronts propagate outward from the initial site of the normal zone: namely, normal-superconducting fronts in the superconductor and hot-cold fronts in the helium. So the problem we face is a two-component, four-region problem. It is further complicated by the fact that the helium and the metal are not in thermal equilibrium, so that the interfacial heat transfer between these phases must be taken into account. Furthermore, the helium, being in a state close to its critical state, may exhibit rapid changes in its density and specific heat as its pressure and temperature change. Finally, the specific heat, thermal conductivity, and normal-state resistivity of the metal are strong functions of the temperature.

This brief summary shows the reader that a complete calculation of quench pressure, thermal expulsion rate, and propagation velocity can only be carried out on a large computer and even then is a rather daunting task. A useful preliminary, having considerable value in its own right, would be the discovery of simple, easily solved problems that describe roughly but reliably what happens. One such problem, to which this paper is devoted, is to calculate the pressure rise in and expansion of a heated slug of helium centrally located in a long, slender pipe. On either side of the heated slug resides cold helium, whose resistance to motion restrains the expansion of the heated slug. Heat transfer between the warm and cold helium is neglected so that the warm helium always consists of the same atoms as originally present in the heated slug. The warm helium is treated as a perfect gas having a uniform pressure and temperature; its temperature is presumed known as a function of time. The cold helium is treated as a dense fluid.

The computations outlined below are intended to apply to internally cooled superconductors like that of the WH-LCT coil. The length-to-diameter ratio of the helium volume inside this conductor is of the order of 3×10^5. Because of this extremely large length-to-diameter ratio, any pressure gradient in the cold helium is almost entirely expended in overcoming friction, and accordingly we neglect the inertia of the cold helium. This simplification enables us to derive an explicit formula for the pressure rise at the hot-cold interface as a function of the trajectory of that interface.[1] This formula occupies a central role in the work of this paper.

Heated Slug in a Long Pipe

In the long, slender, helium-filled pipe, the origin of the distance coordinate is placed at the center of the heated slug, whose initial length is $2L$. After a time t, the warm helium expands to a total length $2X$, displacing outward the cold helium on either side of it. Early enough, the disturbance created in the cold helium by the expansion of the heated slug has not yet reached the open ends of the pipe. The pipe then may be treated as though it were of infinite length. If the displacement of the hot-cold interface is proportional to the nth power of the time ($2/3 < n < 2$), then the pressure rise $P - P_0$ at the interface is given by[1]

$$P - P_0 = A\rho_0 c^2 \left[\frac{4f(X-L)}{D}\right]^{3/2} \left(\frac{D}{4fct}\right) \quad . \qquad (1)$$

[A list of symbols and their definitions can be found at the end of the paper. The dimensionless coefficient A depends on n and is given approximately by $A(n) = 0.204 + 0.473n$.] As mentioned above, the warm helium is treated as a perfect gas of uniform temperature T and uniform pressure P. Thus its equation of state is

$$PX = \rho_0 LRT \quad . \qquad (2)$$

The pressures P in (1) and (2) are the same, so if T is known as a function of the time t, the pressure P and the position X of the interface can be calculated. The method is to eliminate P between (1) and (2), write the result as

$$y\left[A\rho_0 c^2 \left(\frac{4fL}{D}\right)^{3/2} \left(\frac{D}{4fct}\right)(y-1)^{3/2} + P_0\right] - \rho_0 RT = 0;$$
$$y = \frac{X}{L} \quad , \qquad (3)$$

and solve for y by the Newton-Raphson method.

When enough time has elapsed and the disturbance created in the cold helium by the expansion of the heated slug has long since reached the open ends of the pipe, the cold helium is in uniform slug flow. Then the pressure rise at the hot-cold interface is given by

*Research sponsored by the Office of Fusion Energy, U.S. Department of Energy, under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

Manuscript received August 22, 1988.

DISCLAIMER

$$P - P_0 = 2f\rho_0\dot{X}^2 l/D \ . \tag{4}$$

If we assume that the velocity of expansion is roughly uniform, we can set $\dot{X} = (X - L)/t$. Now if we eliminate P between (2) and (4), we obtain

$$y\left[\frac{2f\rho_0 l}{D}\left(\frac{L}{t}\right)^2(y-1)^2 + P_0\right] - \rho_0 RT = 0 \ , \tag{5}$$

which can again be solved by the Newton-Raphson method.

To continue our calculations, we need the temperature as a function of time. A simple estimate that is fairly reasonable, as we shall see below, is the hot-spot temperature calculated with the conventional formula

$$\int_0^t J_{\mathrm{Cu}}^2 dt = \int_{T_b}^{T} \frac{S_{\mathrm{eff\,Cu}}}{\rho_{\mathrm{Cu}}} dT \ . \tag{6}$$

The effective heat capacity of the copper includes contributions from the Nb_3Sn superconductor and from the steel jacket. The contribution from the steel jacket is included because the time scales we are considering (many seconds) are long compared with the thermal equilibration time of the 2-mm-thick steel jacket of the Westinghouse conductor. Figures 1(a), 1(b), and 1(c) show the temperature, pressure, and expansion velocity of heated slugs of helium of various initial lengths as functions of time for a current density J_{Cu} of 15 kA/cm^2 and a field of 8.0 T, which are the standard operating conditions for the WH-LCT coil. The temperature rises slowly at first and then more rapidly, this behavior being caused by the interplay of the temperature dependences of the effective heat capacity and the resistivity of the copper. The pressure remains relatively constant during the interval from 0.1 to 10 s. The velocity at first decreases, then increases. The larger the initial length of the normal zone, the larger both the pressure and the velocity.

The temperature of the heated slug calculated as described above is an overestimate for several reasons. In the first place, different elements of the normal zone are resistive for different lengths of time. The temperature used above is that for the central element, which is resistive longest and is therefore hottest. Second, the heat capacity of the helium has been ignored. And finally, the work done by the warm helium on the cold has been ignored.

Because of the growth of the normal zone, the mass of resistive metal becomes large compared with the mass of the contiguous, warm helium, which always consists of the same atoms that were in the initial normal zone. Moreover, the volumetric heat capacity of the metal grows rapidly with temperature, outstripping that of the helium at roughly 20–30 K. So the contribution of the helium to the total heat capacity of the normal zone is small. As an illustration, let us consider a normal zone initially 2 m long (L = 1 m) 3 s after its creation. According to Eqs. (2), (5), and (6), the temperature is 60 K, the expansion ratio y = 6.5, and the pressure is 31 atm. Raising the metal to 60 K requires a heat addition of 49 J/cm^3 of copper. The change in internal energy of the helium in going from 4.0 K and 15 atm to 60 K and 31 atm is 192 J/g. There are 28 g of helium in the initial normal zone and there are 1560 cm^3 of copper in the expanded normal zone, so the increase in internal energy of the helium divided by the volume of the copper is 3.5 J/cm^3, or about 6% of the heat addition to the metal.

The initial volume of the helium in the normal zone is 176 cm^3. At t = 3 s, the volume of helium is 1140 cm^3. Since the expansion takes place at a relatively constant pressure of 31 atm, the work done by the expanding helium is 3000 J. When divided by the volume of copper in the expanded normal zone, this gives 1.9 J/cm^3, or about 4% of the heat addition to the metal. So the lion's share of the Joule heat produced in the normal zone goes to the metal.

About different elements of the normal zone being resistive for different lengths of time, we can say the following. When the expansion ratio is large, the average time that elements have been resistive is about half the elapsed time. When the expansion ratio is small, the average time is greater than half the elapsed time, but then the helium makes a larger contribution to the heat capacity than that estimated above. So all in all, the energy deposited in the metal should be somewhat reduced from that corresponding to the temperature obtained from Eq. (6). The extent of such a reduction is uncertain, although the arguments given above indicate that it should not be as large as a factor of two and is probably less. The reduction in temperature, at least for temperatures < 70 K, is less than the reduction in energy. All in all, then, computations based on Eq. (6) should give roughly correct results. In any case, the results are conservative because they overestimate the temperature and therefore the quench pressure.

Thermal Expulsion

No thermal expulsion can occur until a time l/c has elapsed, because that is the length of time it takes for a signal to propagate from the center of the hydraulic path (where the heated slug is assumed to be located) to the ends. For the WH-LCT coil, this time is 185 ms. After this much time has elapsed, helium may be expelled, but the expulsion rate will be small because of the strong frictional retardation of the helium flow. As is noted in Ref. 1, the helium velocity far from the moving hot-cold interface (the "piston") is given by the asymptotic formula

$$v = \frac{3}{2}\frac{c^2 Dt}{fx^2} \ . \tag{7}$$

If we set $x = l$ in this formula, we obtain an estimate of the expulsion velocity that is valid as long as this quantity is much smaller than the velocity of the hot-cold front itself.

After considerable time has elapsed and the helium flow pattern has become slug flow, the expulsion velocity becomes equal to the velocity of the hot-cold front. Shown in Figs. 1(a), 1(b), and 1(c) are dash-dot curves, which for short times are given by Eq. (7) and for longer times are graphically faired into the curve of hot-cold front velocity. This dash-dot curve represents a rough estimate of the helium expulsion rate. The time at which it enters the curve of hot-cold front velocity gives a rough estimate of the duration of the "piston" phase of helium flow.

Discussion

The model proposed here is based on very broad assumptions that allow us to sidestep the wealth of difficulties outlined in the introduction. Arguments have been adduced that support the validity of the model, but in view of the complexity of the real situation, its verisimilitude can probably be established only by comparison with experiment.

Interestingly, the piston (early time) model and the slug (late time) model do not give very different results for the pressure and the hot-cold front velocity. Nevertheless, the theory cannot be simplified by neglect of the more complicated piston model, because the two models give very different results for the expulsion velocity, the piston model giving much smaller velocities at early times.

The hot-cold front velocity in Fig. 1(a), which refers to a normal zone with an initial length of 40 cm, has a broad minimum around 40 cm/s for times in the interval from 1 to 5 s. This is not too different from what was observed in WH-LCT,[2] but detailed comparison with our model is difficult because of the uncertainty in the initial length of the normal zone. Directly after the inductive heater pulse, the normal zone shrinks because heat transfer to the helium is augmented by the heating-induced flow. This shrinkage usually takes place in several tens of milliseconds. When the normal zone does not

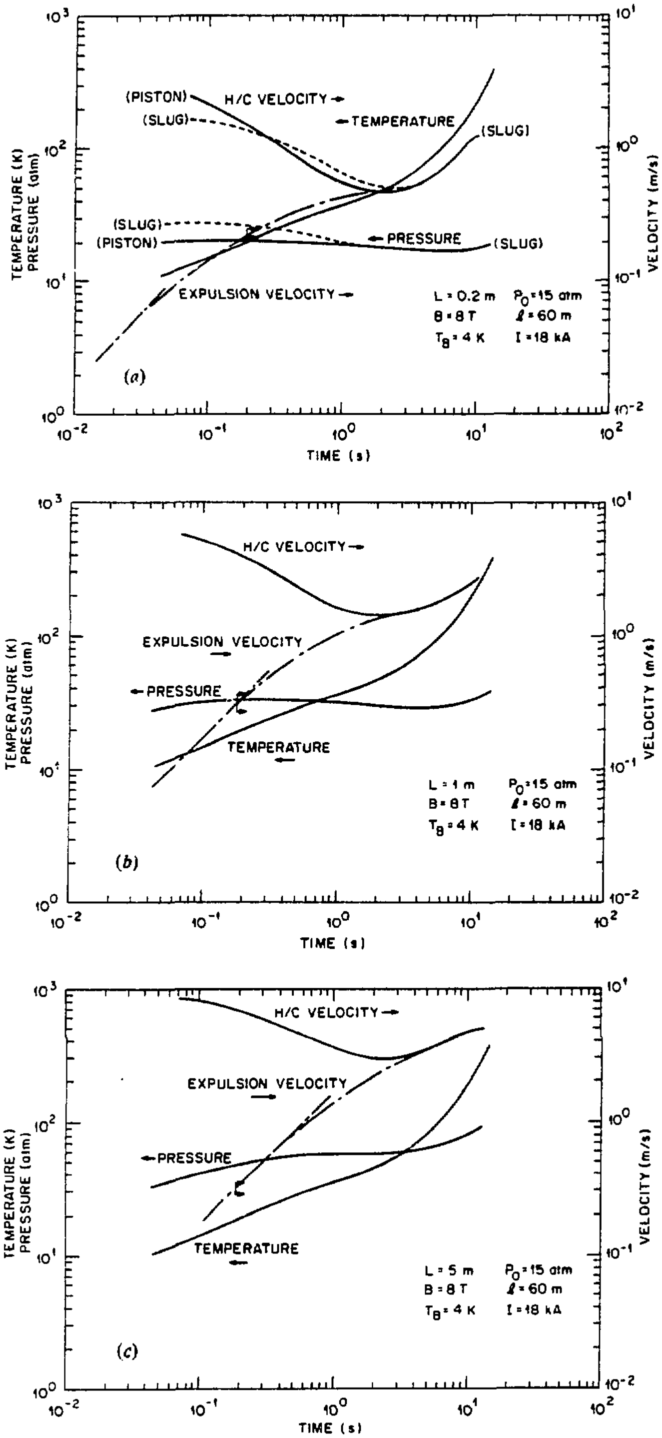

Fig. 1. The temperature, pressure, expansion (hot-cold) velocity, and expulsion velocity as functions of time for a current density J_{Cu} of 15 kA/cm^2 and a field of 8.0 T, with (a) $L = 0.2$ m, (b) $L = 1$ m, (c) $L = 5$ m. In the sketches for $L = 1$ and 5 m, the pressure and expansion velocity have been determined using the piston model for short times and the slug model for long times.

recover, a central portion of unspecified length remains normal, and it is this portion that acts as the initial nucleus of the nonrecovering normal zone. In the experiments cited here, the unshrunk length of the initial normal zone was about 40 cm.

The comparison of the predictions of the model for short initial normal zones with the experiments done on WH-LCT is further clouded by existence of another avenue of axial heat transfer, namely, conduction through the copper matrix. Estimates of the normal-superconducting propagation velocity based on this mechanism (that also take into account the heat capacity of the unexpanded helium) lead to velocities around 50 cm/s. So for very small initial normal zones, the advancing normal-superconducting front may force the hot-cold front along with it. In this case, the hot helium will not always consist of the same atoms as in the initial normal zone. On the other hand, when the initial normal zone is large, and the hot-cold expansion velocity is substantially larger than the normal-superconducting propagation velocity, the advancing hot-cold front will force the normal-superconducting front along with it, and the hot helium will always consist of the same atoms, as we presupposed.

The clear implication of this model is that the severity of pressure protection problems increases with the length of the initial normal zone. So if the conductor is designed to withstand the quench pressure when an entire hydraulic path goes normal all at once, it should be able to withstand the quench pressure arising from a local normal zone. Furthermore, the model calculations indicate that monitoring the rate of thermal expulsion is a feasible nonelectrical means of detecting a nascent quench.[3]

References

[1] Lawrence Dresner, "Protection considerations for force-cooled superconductors," in *Proceedings of the 11th Symposium on Fusion Engineering*, Austin, TX, November 18–22, 1985, IEEE Cat. No. CH2251-7, pp. 1218–1222.

[2] L. Dresner, D. T. Fehling, M. S. Lubell, J. W. Lue, J. N. Luton, T. J. McManamy, S. S. Shen, and C. T. Wilson, "Stability tests of the Westinghouse coil in the International Fusion Superconducting Magnet Test Facility (IFSMTF)," *IEEE Trans. Magn.* vol. MAG-24(2), pp. 779–782, 1988.

[3] Lawrence Dresner, "Quench detection by fluid dynamic means in cable-in-conduit superconductors," *Advances in Cryogenic Engineering*, vol. 33, pp. 167–174, 1988.

List of Symbols

A	dimensionless coefficient in Eq. (1).
c	sonic speed (m/s)
D	hydraulic diameter (m)
f	Fanning friction factor (dimensionless)
J_{Cu}	current density in the copper matrix (A/m^2)
l	half-length of a hydraulic path (m)
L	initial half-length of the heated slug (m)
n	power of the time with which the displacement of the hot-cold front varies
P	pressure (Pa)
P_0	initial pressure (Pa)
R	gas constant of helium per unit mass (J/kg-K)
$S_{eff\,Cu}$	the effective heat capacity of the copper matrix per unit volume (J/m^3-K)
t	time (s)
T	temperature (K)
T_b	initial temperature (K)
X	the instantaneous half-length of the heated slug (m)
$\dot{X}$	dX/dt (m/s)
y	X/L, the expansion ratio (dimensionless)
ρ_{Cu}	the resistivity of copper (Ω-m)
ρ_0	the helium density (kg/m^3)