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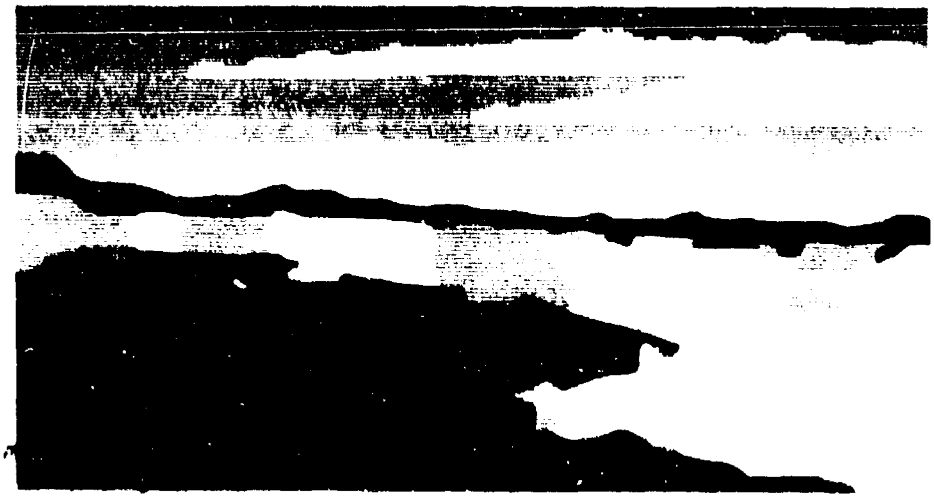

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MEASURING PION BETA DECAY WITH HIGH-ENERGY PION BEAMS

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ABSTRACT

Improved measurements of the pion beta decay rate are possible with an intense high-energy pion beam. The rate for the decay $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ is predicted by the Standard Model (SM) to be $R(\pi^+ \rightarrow \pi^0 e^+ \nu_e) = 0.3999 \pm 0.0005\ \mathrm{s}^{-1}$. The best experimental number, obtained using in-flight decays, is $R(\pi^+ \rightarrow \pi^0 e^+ \nu_e) = 0.394 \pm 0.015\ \mathrm{s}^{-1}$. A precise measurement would test the SM by testing the unitarity of the Cabibbo-Kobayashi-Maskawa matrix, for which one analysis of the nuclear beta decay data has shown a 0.4% discrepancy. Several nuclear correction factors, needed for nuclear decay, are not present for pion beta decay, so that an experiment at the 0.2% level would be a significant one. Detailed study of possible designs will be needed, as well as extensive testing of components. The reduction of systematic errors to the 0.1% level can only be done over a period of years with a highly stable apparatus and beam. At a minimum, three years of occupancy of a beam line, with 800 hours per year, would be required.

I. INTRODUCTION

The proposal to build a "pion accelerator," PILAC, at LAMPF led to discussions of making an improved measurement of the pion beta decay rate. The clean beam—well-defined phase space and low contamination in comparison with previous beams—can greatly reduce systematic uncertainties, and the intensity can provide the large number of events necessary. This note discusses the goal for a new experiment, and whether it can be reached with a realizable apparatus.

The rate for the decay $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ was predicted from the rate for superallowed Fermi nuclear beta decays in 1958, using the conserved vector current hypothesis (CVC).[1] Since that time, the development of the Standard Model (SM) has incorporated CVC and enabled detailed calculations of radiative corrections.[2] Decades of experimental and theoretical effort have resulted in refined measurements and detailed corrections so that the prediction is now one of the most precise in particle physics.

Within the SM, the rate is given by

$$R = \frac{G_\mu^2 |V_{ud}|^2}{30\pi^3} \left[1 - \frac{\Delta}{2m_+}\right]^3 \Delta^5 F(\varepsilon,\Delta)(1+\delta) ,$$

where G_μ is the weak interaction coupling constant for muon decay, V_{ud} is the first element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, Δ is the $\pi^+ - \pi^0$ mass difference, m_+ is the mass of the π^+, $F(\varepsilon, \Delta)$ is a phase space function near unity with $\varepsilon = m_e/\Delta$, and δ is a radiative correction. To calculate the rate we use the most recent values from the 1990 PDG report,[4] with the exception that we use the unitarity of the CKM matrix to give a value of $|V_{ud}|^2$. Details of the calculation are given in Section VI. We find

$$R(\pi^+ \to \pi^0 e^+ \nu_e) = 0.3999 \pm 0.0005 \ \mathrm{s}^{-1}$$

(giving a branching fraction of $B(\pi^+ \to \pi^0 e^+ \nu_e) \sim 1.04 \times 10^{-8}$). The intent of this calculation is not so much to make a definitive prediction of the rate as to illustrate that straightforward application of the theory and data existing at a particular time gives an extremely precise result. New developments in both areas will change the prediction in detail but will not change the fact that a very precise measurement will be a test of fundamental assumptions and a spur to theoretical calculations: the uncertainty in this 0.1% prediction owes as much, if not more, to theoretical uncertainties as to experimental ones. It is worth noting, however, that the prediction has changed by 1.2% in the last few years due to a new measurement of the $\pi^+ - \pi^0$ mass difference, which changed Δ by three standard deviations.[5]

The best experimental number, from a Los Alamos experiment (Exp. 32) using in-flight decays, is[6]

$$R(\pi^+ \to \pi^0 e^+ \nu_e) = 0.394 \pm 0.015 \ \mathrm{s}^{-1} .$$

It is obviously important to improve the experimental measurement to fully test the precise prediction. Indeed there are tentative plans for a new experiment with a goal of 0.5%, using stopping pions at PSI.[7]

Quite apart from the challenge to experimentalists of a precise theoretical prediction, there is a new motivation that has arisen: the question of the unitarity of the CKM matrix, which describes the electroweak interactions. Recent analyses of nuclear beta decay data have found a difference from unitarity, for the CKM matrix: e.g., Ref. 9 finds

$$V_u^2 = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9960 \pm 0.0013 .$$

The value for $|V_{ud}|$ is from nuclear beta decay, the value for $|V_{us}|$ is from kaon and hyperon decays and that for $|V_{ub}|$ from B meson decays. This three-standard-deviation discrepancy has prompted scrutiny of the derivation of each element of the CKM matrix. (A useful summary on this topic is given in Ref. 10.) The term of direct interest to us is $|V_{ud}|$, since it can be determined from $R(\pi^+ \to \pi^0 e^+ \nu_e)$.

Before embarking on an experimental program, it is necessary to look at other ways in which the same information may be obtained. This includes nuclear beta decay, neutron decay and the use of pion decays at rest. These are discussed in the following paragraphs.

The coupling constant for superallowed Fermi nuclear beta decays, $G_{\beta V}$, is also $G_\mu |V_{ud}|$ in the SM. It is related to the Ft value by $G_{\beta V} = \pi^3 \ln 2/(m_e^5 Ft)$. Ft for a given decay is obtained from the raw value ft by the formula

$$Ft = ft \cdot \overline{C(E)}(1 + \delta_1 + \delta_2 + \delta_3 + \alpha/\pi C_{NS}) \cdot (1 + \delta_c) + \delta_t \,,$$

where $\overline{C(E)}$ is a nuclear shape correction factor, δ_1 a radiative correction of $O(\alpha)$, δ_2 one of $O(Z\alpha^2)$, δ_3 one of $O(Z^2\alpha^3)$,[8] $\alpha/\pi C_{NS}$ a nuclear structure correction, δ_c the isospin-breaking correction, and δ_t a combination of small corrections which vary little with Z (see, e.g., Ref. 9). The raw ft values are strongly Z-dependent, and some of the corrections have a Z-dependence other than the explicit one.

Recently, Hardy et al.[11] have reanalyzed the nuclear beta decay data, using up-to-date values for atomic masses, transition energies, and calibration energies. This results in a consistent set of Ft values for the eight decays treated, which in turn gives $V_u^2 = 0.9970 \pm 0.0021$. This new analysis removes the difficulty present with previous analyses[9,12] that the Ft values depended on Z, which would be a violation of CVC. We should note here that there are two calculations of δ_c, which differ on average by 0.16%; Ref. 11 averages the two. A previous approach to the dependence on Z was to parameterize it and extrapolate to $Z = 0$. This was done for a linear dependence[12] and for a quadratic dependence.[13] The results are, respectively, $V_u^2 = 0.9995 \pm 0.0009$ and $V_u^2 = 0.9989 \pm 0.0012$. (Reference 14 has argued that isospin-breaking effects are much larger than previously considered. However, Ref. 15 has contradicted this thesis.)

Despite the new consistency among the Ft values, it is natural to be concerned that systematic effects remain in the theory (there is certainly one in the absolute values of δ_c) and the experimental data. To detect such systematic effects, it is necessary to perform new experiments or experiments of a completely different type, that have different theoretical corrections. Since some of the corrections in nuclear beta decay are explicitly Z-dependent, low-Z nuclei are particularly interesting; an effort to measure the rate for ^{10}C is under way.

Neutron decay is a low-Z decay, which requires a measurement of $|g_A/g_V|$ to extract Ft. The most recent neutron lifetime experiment[16] combined with the best measurement of $|g_A/g_V|$ (Ref. 17) gives a value of Ft that is about 0.9% different from the nuclear value,[18] with about 0.7% uncertainty, mostly from $|g_A/g_V|$.

In the case of pion beta decay, there are fewer corrections, and no corrections that depend on Z, since the Z of the daughter (π^0) is zero. There are some theoretical uncertainties (e.g., π-η mixing) but a precise measurement would spur study of such problems.

How good an experiment has to be done? The calculation of $|V_{ud}|^2$ could have systematic effects of the order of 0.5% from nuclear beta decay. This then is the level at which discrimination is needed: to make an independent measurement of V_{ud} useful for testing the unitarity of the KM matrix, or to detect a discrepancy of 0.5%, we need a 0.2% (or better!) experiment.

To exploit high-energy pion beams to make a measurement of the pion beta decay rate, an in-flight-decay experiment is most appropriate. The pion decay length is $c\tau_\beta = 1.2\times10^8$ m. Thus, at the momentum (1050 MeV $/c$—the peak intensity of PILAC), the probability of decay is 1.1×10^{-9} m^{-1} so that an intense beam, long decay region, and good acceptance are required for a high-statistics experiment. Recent improvements in detector technology give promise of being able to handle the high rates involved while controlling systematic errors, as described below.

Note that a decay-in-flight experiment measures the decay rate directly, with the pion lifetime entering only indirectly. In this there is a small advantage over decay-at-rest experiments, in which the lifetime (currently measured to 0.1%) enters directly. As for other data that are used to calculate the rate, it would be useful to confirm the new measurement[5] of the mass difference, since it is three standard deviations different from the previous value.

II. PREVIOUS EXPERIENCE

To best understand the relative importance of the various factors in designing an experiment, we turn to previous experience with an in-flight decay experiment, LAMPF Exp. 32 (Ref. 6). A sketch of the apparatus is shown in Fig. 1. Pions that beta-decay in flight produce a daughter π^0 in the forward direction with nearly the same beam energy. The π^0 decays into two photons. These photons were converted into e^+ - e^- pairs by three layers of lead glass converters and their energies were measured by total-absorption lead-glass blocks. Scintillation counters behind each converter layer provided the trigger and determination of the position of the conversion. A veto counter in front of each lead-glass array reduced charged-particle triggers. The trigger and event selection was based only on the two photons. In 10% of the cases, the electron was detected; these events were used only as a check of systematic effects. Three cuts were applied in the analysis: a cut requiring time coincidence of the two photons, a cut on the correlation between energies and positions of conversion, and a cut on coplanarity of the photon conversion points and the beam. The signal region was defined as the range of total energy (the sum of the energies of the two photons) from 420 to 800 MeV. Figure 2 shows the spectrum of the total energy for events passing all cuts (except on total energy); it is remarkably clean, with only 1% background in the signal region.

The pion beta decay rate, R, is given by

$$N_\beta = R\cdot N_\pi \cdot P^2 \cdot (\beta\gamma c)^{-1}\int \eta(z)\,dz \cdot \prod_{i=1}^{n} F_i\ ,$$

where N_β is the number of "signal" events, N_π is the number of beam pions entering the apparatus, P^2 is the joint photon conversion efficiency, $(\beta\gamma c)^{-1}$ is the proper time of the pion per unit flight path, $\eta(z)$ is the geometric efficiency of detection as a function of position z along the beam, and the factors F_i are corrections. The combination $(\beta\gamma c)^{-1}\int \eta(z)\,dz = T_\pi$ is the effective proper time spent by a beam pion in the decay region and is determined by a Monte Carlo calculation. For Exp. 32, these quantities are given in Table I, which is adapted from Table IX of Ref. 6, where more detail can be found. The number of signal events is found from

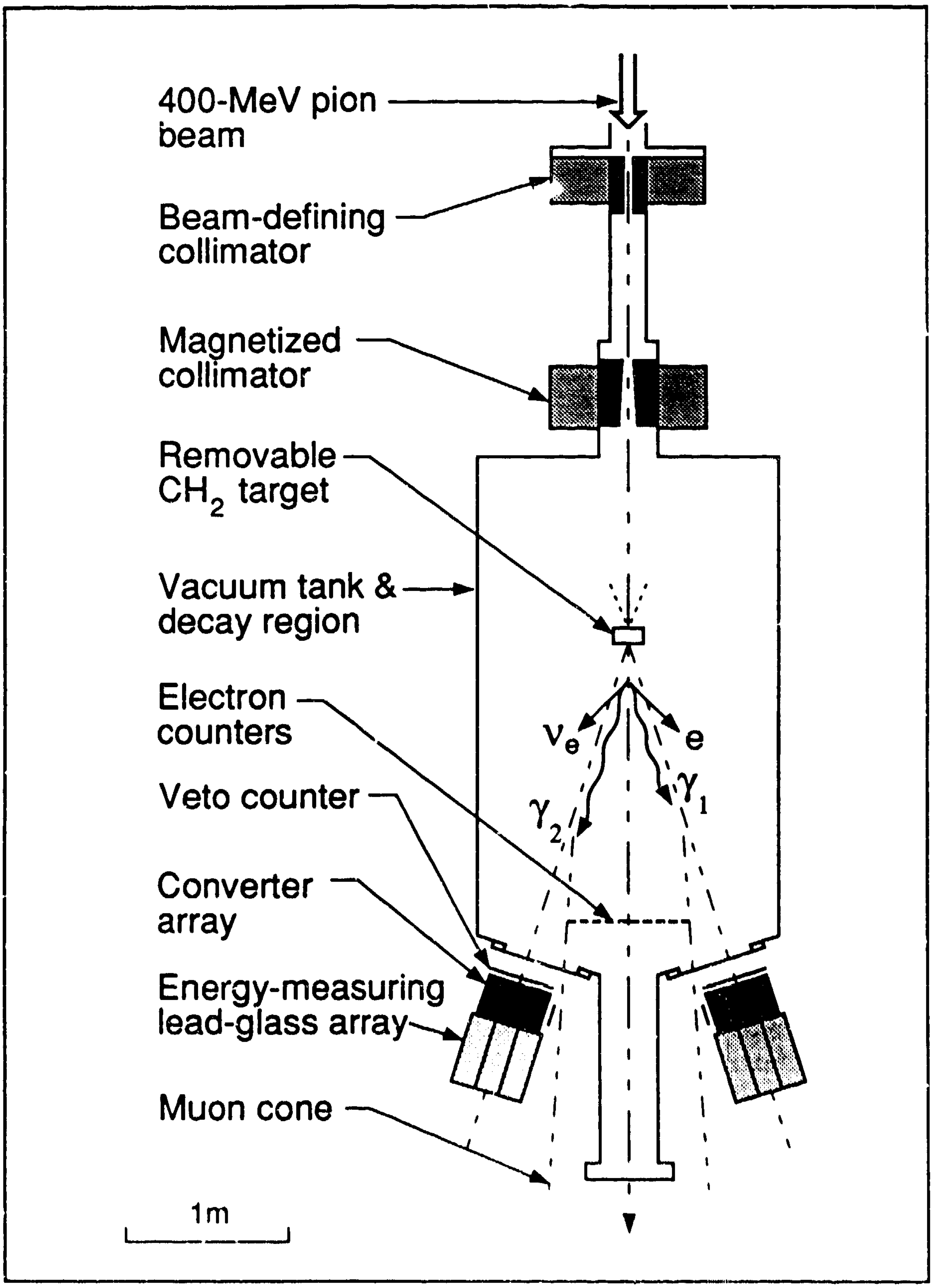

Fig. 1. Sketch of apparatus of LAMPF Exp. 32.

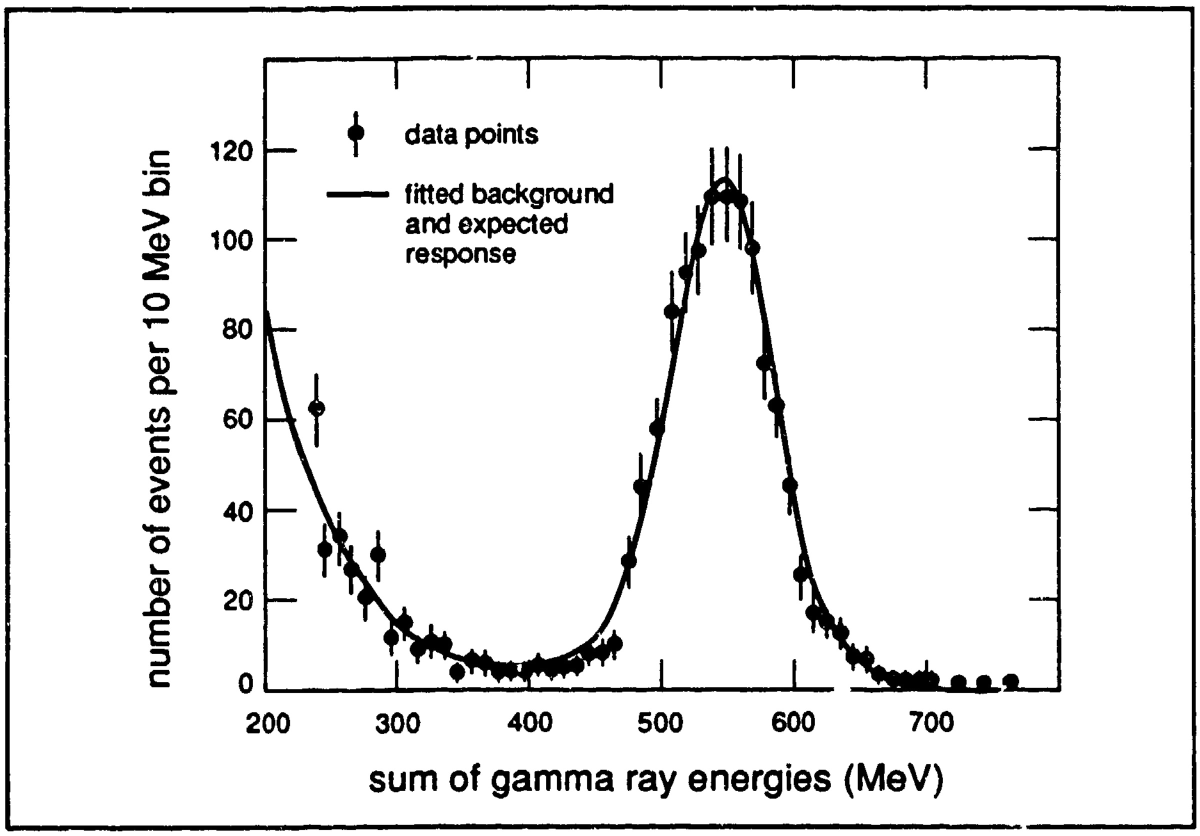

Fig. 2. Distribution of the sum of energy events, from Exp. 32.

Table I. Factors entering into decay-rate calculation, from LAMPF Exp. 32.

Symbol	*Description*	*Value*	*Uncertainty*
N	Number of events	1259	2.8%[a]
N_b	Random background	16.4	0.2%[b]
C	Correction for cuts	1.044	0.33%[b]
W	Weights	0.952	0.38%[b]
$N_{e\nu\gamma}$	Adjustment for $\pi \rightarrow e\nu\gamma$	0.9907	0.54%[b]
N_β	Number of good events	1223.9 ± 36.2	3.0%[a]
N_π	Number of beam pions	2.144×10^{14}	1.0%[b]
P^2	Conversion efficiency	0.5151	1.2%[a]
T	Time in decay region (s)	3.534×10^{-11}	0.88%[b]
F_1	Dalitz, early conversions	0.9430	0.5%[b]
F_2	Trigger efficiency	0.8917	0.9%[b]
F_3	Software efficiency	0.9581	0.5%[b]
F_4	Event-selection efficiency	0.9880	0.7%[b]
R	Rate for $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ (s^{-1})	0.394	3.8%

[a] Error is primarily statistical.

[b] Error is primarily systematic.

$$N_\beta = (N - N_b)\cdot C \cdot W - N_{e\nu\gamma}\ ,$$

where N is the number of events passing all cuts (including that on total measured energy), N_b is the fitted random background, C is the correction for the energy cut, W is the weight for detection efficiency by class of event, and $N_{e\nu\gamma}$ is the adjustment for the background from $\pi \rightarrow e\nu\gamma$.

It should be noted that the fit in Fig. 2 is a *two-parameter* fit. The background spectrum and the expected spectrum from pion beta-decay events were found from calibration data, and the only free parameters were the total amplitudes of each spectrum.

As can be seen, the dominant uncertainty is statistical, depending on the number of events. The background correction is small, and systematics dominate. In the detailed analysis, over 100 measured and calculated quantities were used, contributing uncertainties ranging from 0.01% to 1%. In the following section, we discuss the sources of the major uncertainties in the context of a new experiment at higher energies.

III. AN EXPERIMENT AT HIGHER ENERGIES

The geometry of an in-flight experiment is influenced strongly by the fact that the momentum of the daughter π^0 in the center of mass is very low. The π^0 has essentially the same momentum in the laboratory as the original π^+. Thus, the problem is to detect the two photons from the decay of the π^0 with good efficiency. We consider a detector patterned after the previous one (Fig. 1). A cylindrical detector surrounding the decay volume has some advantages, but requires a larger detector area.

The choice of operating energy is governed by the desire to resolve pion beta decay from background. Since the decay is detected by the presence of two photons with a total energy approximately equal to the beam energy, energy resolution is important. Resolution of photon detectors improves with energy, so the highest energy compatible with good intensity is chosen: 1050 MeV $/c$ at PILAC, for example.

Note that we do not attempt to detect the decay electron for the rate measurement—this would reduce the acceptance by a substantial factor, though it would improve the discrimination against background. It would also increase the systematic errors, since some of the electrons have very low kinetic energies. It should be noted that the calculated rate includes the radiative decays $\pi^+ \rightarrow \pi^0 e^+ \nu_e \gamma$, which are automatically included in our method but would imply a systematic correction if the electron were measured. However, detecting the electron would be helpful in calibration and analysis of apparatus performance.

We chose a length of about 10 m for the decay volume, which is compatible with the decay length in the lab and the scale of a reasonable apparatus. The sensitivity of the apparatus increases approximately linearly with this length, so a longer decay volume (and larger detector) would give even more sensitivity.

To increase acceptance, the photon detector should be annular, rather than consisting of two rectangles. One restriction on the detector is that it must be entirely outside the "muon cone," the volume in which muons from upstream $\pi^+ \rightarrow \mu^+ \nu_\mu$ decay are found. The singles rate inside this cone is prohibitive. This gives an inner dimension for the annulus of 0.4 m. Another restriction is that the apparatus must not detect both photons from π^0's produced by charge exchange in the collimator. This implies an outside dimension of 1.3 m. With these dimensions, the acceptance of the detector for $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ is about 15%.

The detector has the advantage that it is continuous in azimuthal angle so that the acceptance can be more confidently calculated. By using scintillating fibers[19] or straws, it is possible to measure position precisely and distinguish incident charged particles from neutral ones. New technology makes it easy to increase the thickness of the converter, and hence the photon detection efficiency, without losing position resolution or energy resolution. To be more specific, we might use up to four converter sheets, using a lead-scintillating-fiber combination. Position of conversion might be determined by straw tubes or sci-fi layers. For the total-absorption energy measurement either lead-sci-fi or BaF_2 might be used. The veto/electron detection system might be several sci-fi or straw layers. Figure 3 shows a schematic view of the detection apparatus at higher energies.

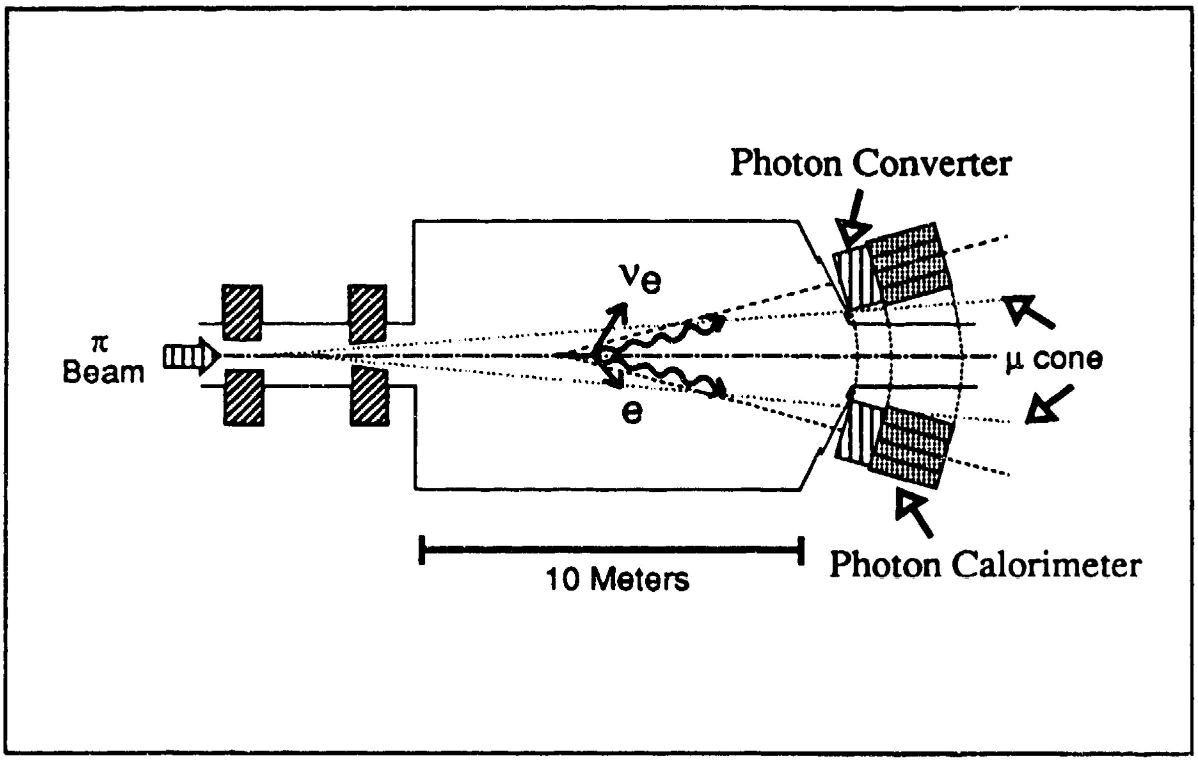

Fig. 3. Sketch of proposed apparatus for a pion beta-decay experiment with a high-energy pion beam.

We compare the parameters of the proposed apparatus at PILAC with that of experiment Exp. 32 (Ref. 6) in Table II. As can be seen, the statistics are adequate. We discuss the factors of Table I and their uncertainties in the new design in the following sections.

Table II. Comparison of proposed apparatus with LAMPF Exp. 32.			
Parameter	*Exp. 32*	*Proposed*	*Factor*
Beam momentum (MeV/c)	522	1050	—
Decay length (m)	1.82	10	5.5
Half-detector area (m^2)	0.24	2.3	9.6
Fraction of detected electrons	0.1	0.3	3.3
Acceptance	4%	15%	3.8
Effective proper time (s)	3.5×10^{-11}	64.3×10^{-11}	18
Beam intensity $(\pi^+ s^{-1})$	2×10^8	10×10^8	5
Joint conversion efficiency	0.52	0.72	1.4
Running time (h)	300	2400	8
Number of events	1259	1.2×10^6	1008

A. Signal and Backgrounds

To achieve statistical significance at the 0.1–0.2% level, about 10^6 events are needed. The parameters of Table II show that this is possible, given sufficient running time.

The trigger rate for Exp. 32 was about 0.5 Hz, consistent with random coincidences of single photons from upstream pion interactions at a probability of 5×10^{-6} per incident pion. Scaling to the new experiment by detector solid angle and beam intensity gives a trigger rate of about 100 Hz, well within data-acquisition capability. Singles rates in scintillators in Exp. 32 were as high as 1 MHz in some counters; increased segmentation should reduce occupancy in a new design. Cleaner beams and better collimation should reduce the random background (N_b) by a large factor. This is important to keep the background subtraction small.

Practical photon detectors have an asymmetric pulse-height response, with a long tail extending to low pulse heights; this gives rise to the factor C. In Exp. 32 this was measured by charge exchange on polyethylene, carbon, and hydrogen. With much greater statistics in the calibration, higher photon energy, and the use of detailed simulation, it seems reasonable to improve the uncertainty on this factor by three, to 0.1%.

The factor W of Table I results from the character of the detector of Exp. 32. The solid angle was defined by the edges of scintillation counters up to 15 cm wide. With an azimuthally continuous detector, which can measure the position of each conversion to 1 mm, this factor will be eliminated.

Background from $\pi \rightarrow e\nu\gamma$ will be reduced by a more efficient veto, which is also position sensitive, e.g., several layers of straws. The major uncertainty in the quantity $N_{e\nu\gamma}$ comes from uncertainty in the veto inefficiency; this can surely be reduced. The process $\mu \rightarrow e\nu\nu\gamma$ will be negligible as a source of background, as will charge-exchange interactions with gas in the decay region.

B. Beam Monitoring

In Exp. 32, the beam was monitored by three ionization chambers and by counter telescopes that detected muons from $\pi^+ \rightarrow \mu^+ \nu_\mu$. It appeared that the muon telescopes were stable to the 0.2% level, while the ion chambers drifted in calibration by up to 1%. This, and the difficulty of extrapolating from low beam intensities where the beam could be directly counted, resulted in the (systematic) uncertainty on N_π. However, in Exp. 32 the devices were calibrated only once. It seems reasonable that frequent calibrations and better telescopes can monitor the pion beam at the 0.1% level.

A major problem in Exp. 32 was contamination of the pion beam with protons (1.5%), electrons (0.5%) and muons ($0.5 \pm 0.1\%$). PILAC will remove protons and electrons, leaving the muons, for which it should be possible to make an improved estimate. An experiment based at a higher energy proton facility could utilize incident π^-, thus eliminating the problem of proton contamination.

C. Conversion Efficiency and Acceptance

The uncertainty on P^2 was largely due to the statistics of the calibration runs. To reduce this uncertainty, much longer and more frequent runs are required, as is detailed understanding of the detector. Reduced detector inefficiency will automatically reduce the uncertainty, so it is important to construct a high-efficiency detector.

The acceptance is given by the effective proper time in the decay region, T_π. This is found by a Monte Carlo calculation using measured beam and apparatus parameters. Long and detailed calibration runs, in combination with methods of simulation that have recently matured (EGS4, GEANT) can surely improve greatly over our previous techniques.

D. Correction Factors

The Dalitz decay and early conversion corrections (F_1) depend on a knowledge of the veto efficiency; as noted, this can be greatly improved.

The uncertainty of the trigger efficiency (F_2) in Exp. 32 was mainly due to the part of the trigger that was sensitive to the pulse-height from the photon detectors. Modern electronics should be able to do better.

A main contribution to the uncertainty in the software efficiency (F_3) came from early TDC stops in the high-occupancy scintillators; a more-segmented detector and multi-hit TDCs should reduce this. At the 0.1% level, a good knowledge of the software efficiency also depends on very careful control of the data-taking and data-analysis conditions and frequent calibration runs.

The event selection (F_4) on energy-position correlation and coplanarity (see Section II) can have a reduced uncertainty with additional calibration, good energy and position resolution and a well-defined beam.

IV. CONCLUSION

It may be possible to do an experiment at the level desired. Detailed study of possible specific designs will be needed, as well as extensive testing of components. It should be emphasized that a reduction of systematic errors to the 0.1% level can only be done over a period of years with a highly stable apparatus and beam. At a minimum, three years of occupancy of a beam line, with 800 hours per year, would be required.

V. ACKNOWLEDGMENTS

The advice of Peter Herczeg in the preparation of this note is gratefully acknowledged.

VI. APPENDIX: CALCULATION OF RATE

In this section we give a sample calculation of the rate, R, for $\pi^+ \to \pi^0 e^+ \nu_e$ decay. Following Sirlin[2] and Källén,[20] we write (using appropriate units)

$$R = \frac{G_\mu^2 |V_{ud}|^2}{30\pi^3 \hbar} \left[1 - \frac{\Delta}{2m_+}\right]^3 \Delta^5 F(\varepsilon, \Delta)(1+\delta) ,$$

where G_μ is the weak interaction coupling constant for muon decay, V_{ud} is the first element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, Δ is the $\pi^+ - \pi^0$ mass difference, m_+ is the mass of the π^+, $F(\varepsilon, \Delta)$ is a phase space function near unity with $\varepsilon = m_e/\Delta$, δ is a radiative correction, and

$$F(\varepsilon, \Delta) = \sqrt{1-\varepsilon}\left(1 - \frac{9\varepsilon}{2} - 4\varepsilon^2\right) + \frac{15}{2}\varepsilon^2 \ln\left(\frac{1+\sqrt{1-\varepsilon}}{\sqrt{\varepsilon}}\right) - \frac{3}{7}\left(\frac{\Delta}{2m_+ - \Delta}\right)^2 .$$

To calculate the rate, we generally use values from the 1990 PDG report,[4] with some exceptions as noted. We use the unitarity of the CKM matrix within the minimal SM to give a value of $|V_{ud}|^2$: $|V_{ud}|^2 = 1 - |V_{us}|^2 - |V_{ub}|^2$. We use $G_\mu = 1.16639(1) \times 10^{-11}$ MeV^{-2},[22] $|V_{us}| =$ 0.2205(18) and $|V_{ub}|/|V_{cb}| = 0.09(4)$, $|V_{cb}| = 0.044(9)$ [yielding $|V_{ud}|^2 = 0.9514(8)$], $\hbar =$ $6.5821220(20) \times 10^{-22}$ MeV s, $\Delta = 4.59364(48)$ MeV/c^2,[5] $m_+ = 139.5675(4)$ MeV/c^2, and $m_e = 0.51099906(15)$ MeV/c^2 .

For the radiative correction, we follow Marciano:[3]

$$1 + \delta = \left\{1 + \frac{\alpha}{2\pi}\left[\ln(m_p/m_A) + 2C\right] + \frac{\alpha(m_p)}{2\pi}\left[\bar{g}(E_m) + A_g\right]\right\} \cdot S(m_p, m_Z)$$

where α is the fine structure constant, $\left(\ln(m_p/m_A)+2C\right)$ is an axial-vector contribution, m_p is the proton mass, m_A is a cutoff usually taken to be of the order of the A_1 mass, C is a correction for the remaining effects, $\alpha(m_p)$ is the fine-structure constant evaluated at m_p, $\bar{g}(E_m)$ is the "Sirlin function,"[21] E_m is the maximum electron energy, A_g is a strong interaction correction, and $S(m_p,m_Z)$ is a QED short-distance enhancement factor dependent on $\alpha(m)$ evaluated at the masses of all elementary fermions with mass $m < m_Z$.

We use $\alpha^{-1} =$ 137.0359895(61), $m_A = 1.2 \pm 0.4$ GeV/c^2, $C=0$, $\alpha^{-1}(m_p) =$ 133.150, $\bar{g}(E_m) = 9.0371$,[23] $A_g = -0.34$,[2] and we calculate $S =$ 1.02246(8) (for the fermion masses we used[4] $m_e = 0.511$ MeV/c^2, $m_d = 9.9(1.1)$ MeV/c^2, $m_u = 5.6(1.1)$ MeV/c^2, $m_\mu = 105.658$ MeV/c^2, $m_s = 0.199(33)$ GeV/c^2, $m_c = 1.35(5)$ GeV/c^2, $m_\tau = 1.784$ GeV/c^2, $m_b = 5.0(5)$ GeV/c^2, $m_t = 140.(50.)$ GeV/c^2, $m_W = 80.6(4)$ GeV/c^2, and $m_Z = 91.161(31)$ GeV/c^2). These values give $1+\delta = 1.0329(5)$.

Taking all the above uncertainties to be standard deviations of normal distributions, we calculate

$$R(\pi^+ \to \pi^0 e^+ \nu_e) = 0.3999 \pm 0.0005 \text{ s}^{-1}$$

The factors contributing the largest uncertainties are: $|V_{ud}|$ (0.00032), Δ (0.00021), and δ (0.00019). In δ, the factor contributing most to the overall uncertainty of 0.0005 is the choice of m_A (0.00044).

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