LA-UR -80-3494

MASTER

TITLE: SCALING LAWS FOR FRC FORMATION AND PREDICTION OF FEX-C PARAMETERS

AUTHOR(S): Richard E. Siemon
R. Richard Bartsch - CTR-3

SUBMITTED TO: CT Symposium, Los Alamos, NM

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CONTRACT W-7405-ENG. 36

Scaling Laws for FRC Formation and Prediction of FRX-C Parameters

R. E. Siemon and R. R. Bartsch

A semi-emperical method has been developed to extrapolate the experimental results from FRX-B,[1] a field-reversed theta pinch which generates an FRC (Field-Reversed Configuration--a compact toroid with no toroidal field), to the larger size FRX-C.[2] Even though there are many uncertainties about details of the dynamic processes by which an FRC is formed, the scaling exercise has proven useful in identifying limitations in the original FRX-C design and the design has been modified to have a lower voltage and larger capacitance. The goal of FRX-C remains unchanged: to test the confinement scaling of an FRC in a larger device over a wider range of temperatures. Of particular interest is the testing of possible MHD stability limits as the ratio of plasma size to gyro radius increases.

1. Non-adiabatic implosion heating. The high-voltage capacitor bank is approximated as a capacitor (C) initially charged to voltage V connected to the coil of radius r_w and length L through a source inductance L_s. The total temperature ($T_e + T_i$) resulting from the implosion process is

$$\frac{kT}{e} = X_{11} \left(\frac{L_s}{L_s+L_c}\right) \left(\frac{r_t}{r_w}\right)^2 \left(\frac{c/\omega_{pi}}{r_t}\right) V \ , \qquad (1)$$

where X_{11} is a dimensionless emperical constant, $L_c = \mu_0 \pi r_w^2/L$ is the coil inductance, r_t is the discharge tube radius, c/ω_{pi} is a characteristic length that can be used to estimate the sheath thickness at the initial pressure p_0, and e is the electron charge. This scaling is derived by ignoring the bias field and assuming that the energy is initially imparted to the ions by a moving sheath described by the snow plow equation $\rho_0 v^2 = B^2/2\mu_0$ where ρ_0 is the initial mass density, v is the sheath velocity, and B is the externally applied rising magnetic field.[2] The scaling can be justified in the presence of a small bias field or if the bias is adjusted to be proportional to $V^{1/2}$ as discussed below with regard to the Green-Newton limit.[3] From FRX-B data, X_{11} is found to have the value 0.06.

2. Initial equilibrium. Following the implosion, it is assumed that the current continues to rise until a compact toroid of length L equal to the coil length is established in an equilibrium state. The balance between plasma pressure and field line tension in an elongated equilibrium requires[1]

$$\langle\beta\rangle = 1 - \frac{1}{2} x_s^2 \ . \qquad (2)$$

Here $\langle\beta\rangle = (1/\pi r_s^2) \int \beta(r)\ 2\pi r\, dr$ is the volume averaged $\beta = p(r)/(B^2/2\mu_0)$ defined with respect to the external magnetic field, B, and x_s is the ratio of the separatrix radius to r_w. A useful analytic form for $\beta(r)$ is a sharp-boundary profile[1]

$$\beta(r) = \langle\beta\rangle \qquad 0 < r < r_s, \qquad \beta(r) = 0 \qquad r_s < r < r_w \ . \qquad (3)$$

The plasma current flows at the separatrix and at the major radius $R = r_s/\sqrt{2}$.

3. Trapped flux. The preionized plasma contains magnetic flux as a result of the initial bias field, B_1. Preionization, field reversal, implosion, and other formation processes result in a fairly small fraction of the initial flux being trapped in the equilibrium. The fraction X_F, about 0.13 according to the FRX-B experiment, is taken as an emperical constraint on the trapped, poloidal equilibrium flux, ϕ:

$$\phi = X_F B_1 \pi r_t^2 \ . \qquad (4)$$

It was pointed out by Green and Newton[3] that there exists an upper bound, B_{max}, on the bias field because of the finite time for field reversal,

$$B_{max} = E_\theta^{1/2} \left(\mu_o \rho_o\right)^{1/4} \ ,$$

where the electric field at the inner wall of the discharge tube is $E_\theta = (V/2\pi r_w)(r_t/r_w)\, L_c/(L_c+L_s)$. The value of bias field is taken for scaling purposes to be $B_1 = X_B B_{max}$ where X_B is a multiplying factor less than unity. A typical value of X_B used in the FRX-B experiment is 0.75, but $X_B = 1.0$ is assumed for FRX-C.

Given the magnetic profile implied by Eq. (3), the poloidal flux is constrained by these arguments to be

$$\phi = \frac{\pi}{2\sqrt{2}} x_s^3 r_w^2 B = X_F \pi r_t^2 X_B E_\theta^{1/2} (\mu_o \rho_o)^{1/4} \ . \qquad (5)$$

4. Trapped particles. Of the initial particles in the discharge tube, a fraction X_p (0.35 in FRX-B) is observed to be confined in the equilibrium formed. Thus the plasma volume of a compact toroid of length ℓ and radius $x_s r_w$ determines the density

$$n x_s^2 \ell = X_p n_o L (r_t/r_w)^2 \ . \qquad (6)$$

5. Adiabatic compression. For a given device and fill pressure p_o, the values of constants $X_{||}$, X_F, X_B, and X_p with Eqs. (1)-(6) specify the value of $B = B_L$ that is required to form a compact toroid of length $\ell = L$.

To avoid end effects and to insure that the compact toroid is compressed to a length less than the coil length, it is desirable to use enough capacitance that the field increases to a final value B larger than B_L. An adiabatic compression model is used to describe the compression phase despite the fact that the quarter-period rise time in typical experimental arrangements is roughly the same as the thermal transit time of particles along the length of the system. Assuming adiabatic compression pV^γ = const., and the sharp boundary profile of Eq. (3), one finds

$$z_s = (x_s/x_{sL})^{8/5} \left((1 - \tfrac{1}{2} x_{sL}^2)/(1 - \tfrac{1}{2} x_s^2)\right)^{3/5} \ , \qquad (7)$$

where $z_s \equiv \ell/L$, and x_{sL} is the value of x_s when the compact toroid is first formed with length L. The magnitude of current in the coil at the end of the compression phase is approximately given by

$$I = F_{CB} V/\left((L_s+L_c)/C\right)^{1/2} - I_{bias} \ , \qquad (8)$$

where F_{CB} is an inefficiency factor (~0.75) associated primarily with crowbar switches. Eq. (8) is equivalent to assuming that the plasma has no effect on the coil inductance, an approximation that can be justified to within a few percent despite the fact that the final B field between the separatrix and metal coil does depend on the shape of the compact toroid:

$$B = \left(\frac{\mu_o}{L}\right) \frac{I}{\left(1 - x_s^2(1 - z_s)\right)} \, . \qquad (9)$$

6. Maximum values of x_s and p_o. According to both experiment[4] and theory[5] it is desirable to make x_s as large as possible in order to minimize the pressure gradients that tend to result from the equilibrium relationship of Eq. (2). The maximum x_s is x_{sL} (no compression) and is determined by the above equations to be

$$x_{sL} = \frac{\alpha^{1/2}}{2}\left((1 + 16/\alpha)^{1/2} - 1\right)^{1/2}$$

$$\alpha = \frac{2}{\pi}\left(\frac{r_t}{r_w}\right)^2 \frac{X_F^2\, X_B^2}{X_p\, X_H}$$

It is important to note that bank parameters such as V, C, etc. do not affect the result. The small values of X_F so far observed are not fully understood, and it may be possible to increase x_{sL} by increasing X_F in future experiments. The upper limit on x_s corresponding to $\ell = L$ is not directly observable because the non-uniform magnetic fields at the ends of the coil perturb the assumed equilibrium when $\ell \simeq L$.

There also exists an upper limit on the initial fill pressure, $p_o(\max)$, corresponding to $\ell = L$. For a given capacitor bank it is impossible to operate above $p_o(\max)$ because the plasma energy cannot be confined with the available magnetic field. For $p_o(\max)$ in mTorr using MKS units,

$$p_o(\max) = (1/6.6\times10^{19})\,(4\mu_o/m_i)\,(F_{CB}^4/X_B^4 r_t^2)\,(CV/L)^2\,\left(1/(1 + B_L/B_1)\right)^4 ,$$

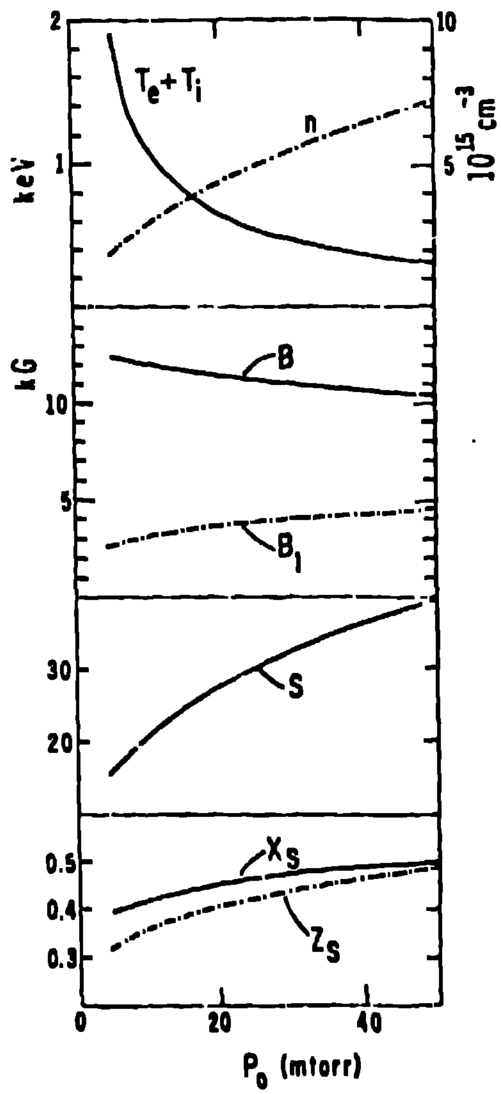
$$B_L/B_1 = X_F(2\sqrt{2}/x_{sL}^3)\,(r_t/r_w)^2 .$$

7. FRX-C parameters. Plotted in Fig. 1 are predictions of FRX-C parameters based on the above equations, assuming the 3/4-bank parameters listed in Table I. The mechanical configuration has two feed slots giving an effective voltage of 110 kV from the individual 2.8-μf, 55-kV capacitors. The x_s value ranges from 0.4 to 0.5 which is the same as that obtained on FRX-B. The scaling parameter $S \equiv R/\rho_i$, where R is the major radius and ρ_i is the ion gyro radius in the external field, is larger than in FRX-B as desired for the scaling

TABLE I. FRX-C Device Parameters

Coil Diameter	0.45 m	
Coil Length	2.0 m	
Equivalent Circuit	Full bank	3/4-bank
C	112 μf	84 μf
L_s	16.8 nH	20.0 nH
V	110 kV	110 kV
$\frac{1}{2}CV^2$	678 kJ	508 kJ

studies. Larger values of x_s and S would be achieved if the trapped flux X_F exceeds 0.13 as assumed in these calculations.



References

1. W. T. Armstrong et al., "Field-Reversed Experiments (FRX) on Compact Toroids," Phys. Fluids, to be published.
2. H. Dreicer et al., "Proposal for FRX-C and Multiple-Cell Compact Torus Experiments," Los Alamos Scientific Laboratory proposal LA-8045-P, 1979.
3. T. S. Green and A. A. Newton, Phys. Fluids 9, 1386 (1966).
4. A. G. Es'kov et al., "Principles of Plasma Heating and Confinement in a Compact Toroid Configuration" (Proc. Plasma Phys. and Contr. Nucl. Fusion Research 1978, Vol. II) IAEA, Innsbruck (1978), p. 187.
5. W. T. Armstrong et al., "Compact Toroids Experiments and Theory," (Proc. Plasma Phys. and Contr. Nucl. Fusion Research 1980) IAEA, Brussels, to be published.