

PROGRESS REPORT

NONLINEAR DYNAMICS AND PLASMA TRANSPORT

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I. INTRODUCTION AND PROGRAM OUTLINE

This progress report details work done on a program in nonlinear dynamical aspects of plasma turbulence and transport funded by DOE since 1989. This program has been in cooperation with laboratories in the USSR [now Russia and the Confederation of Independent States (CIS)]. The purpose of this program has been:

a. To promote the utilization of recent pathbreaking developments in nonlinear science in plasma turbulence and transport.

b. To promote cooperative scientific investigations between the US and CIS in the related areas of nonlinear science and plasma turbulence and transport.

In the work reported in our progress report, we have studied simple models which are motivated by observation on actual fusion devices. The models focus on the important physical processes without incorporating the complexity of the geometry of real devices. This allows for a deeper analysis and understanding of the system both analytically and numerically.

The strong collaboration between the Russian visitors and the US participants has lead to a fruitful and strong research program that taps the complementary analytic and numerical capabilities of the two groups.

Over the years several distinguished Russian visitors have interacted with various members of the group and set up collaborative work which forms a significant part of proposed research. Dr. Galeev, Director of the Space Research Institute of Moscow and Dr. Novakovskii from the Kurchatov Institute are two such ongoing collaborations.

II. PROGRESS REPORT

In this section we summarize the work completed over the last year in three different areas of research.

- a. Shear flow generation in plasmas and fluids.
- b. Nonlinear Dynamics and Visualization of 3D Flows.
- c. Self-Consistent MHD Behavior in the Presence of Chaotic Field Lines.

A. Shear Flow Generation in Plasmas and Fluids (Drake, Finn, Guzdar, Rogalsky)

1. Nonlinear Interaction of Rayleigh-Taylor and Shear Instabilities (Finn)

Results on the nonlinear behavior of the Rayleigh-Taylor or interchange instability with very weak magnetic shear and consequent development of shear flow by the shear or peeling instability¹ were obtained.² It was found that the shear flow is generated at sufficient amplitude to reduce greatly the convective transport. For high viscosity, the time-asymptotic state consists of an equilibrium with shear flow and vortex flow (with islands, or "cat's eyes"), or a relaxation oscillation involving an interplay between the shear instability and the Rayleigh-Taylor instability in the presence of shear. For low viscosity, the dominant new feature we found is a high-frequency nonlinear standing wave consisting of convective vortices (islands) localized near the top and bottom boundaries. The localization of these vortices is due to the smaller shear near the boundary regions. The convective transport is largest around these convective vortices

near the boundary and there is a region of good confinement near the center. For very small viscosity μ and thermal conduction coefficient κ , these vortices or islands are strongly localized near the boundaries. For intermediate μ and κ , these islands were found to overlap, giving chaotic $\mathbf{E} \times \mathbf{B}$ advection and enhanced transport. The possible relevance of this behavior to the H mode and edge-localized modes (ELM's) in the tokamak edge region has been explored. We have suggested that the equilibria with shear may correspond to the H-mode, the low frequency relaxation oscillations may correspond to ELM's and the high frequency oscillations may correspond to "grassy ELM's". This work appeared in Phys. Fluids B 5, 415 (1993).

2. 2D Nonlinear Dynamics of Four Driven Vortices

The interaction of four, alternately driven counter-rotating vortices in a two dimensional box, with impenetrable free-slip boundary conditions in the x direction and periodic boundary conditions in the y direction, has been studied numerically.³ For viscosity above a critical value the nonlinear state consists of four alternately counter-rotating vortices. For a lower value of the viscosity the system evolves to a nonlinear steady state consisting of four vortices and shear flow generated by the "peeling instability" [Drake et al., Phys. Fluids B 4, 447 (1992)]. For a still lower viscosity the steady state nonlinear state undergoes a Hopf bifurcation. The periodic state is caused by a secondary instability associated with vortex pairing. However, the vorticity of the shear flow, though periodic, has a definite sign. With a further decrease in the viscosity, a global bifurcation gives rise to a periodic state during which the vorticity of the shear flow changes sign. At even lower viscosity, there is a

transition to a steady state, involving dominantly shear flow and a two-vortex state. Finally, this state undergoes a bifurcation to a temporally chaotic state, with the further decrease of viscosity. The results were compared to some recent experiments in fluids with driven vortices [P. Tabeling et al, J. Fluid Mech. **215**, 511 (1990)].

B. Nonlinear Dynamics and Visualization of 3D Flows (Chernikov, Finn, Guzdar, Rogalsky, Usikov)

1. Diffusion of Stochastic Webs Near the Percolation Threshold

For a time dependent Hamiltonian which represents a linear oscillator perturbed by periodic kicks, the phase portrait of the oscillator reveals the formation of a stochastic web.⁴ The rate of diffusion in the stochastic web was estimated to be the product of the global separatrix rate and the ratio of the phase space of the web to the total phase space. In the present work, exact results for scaling characteristics of the stochastic webs with quasi-crystal symmetries, near the percolation threshold has been obtained.⁵ This problem has application in plasmas and in hydrodynamics (stochastic heating of particles and passive particle advection or test particle diffusion) and solid state physics (electrical conduction in metal insulator composites).

2. 3D Force-Free MHD Equilibria and their Visualization

Computations of three dimensional force-free MHD equilibria, $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ with $\lambda = \lambda_0$, a constant (Taylor states) were performed. These equilibria were determined by boundary conditions of the normal component of \mathbf{B} on a surface.⁶ This surface cor-

responds to the z-electrodes in a flux core spheromak or, in space applications, to the solar photosphere or the earth's ionosphere. It was found that as λ_0 is increased, i.e. as the helicity is increased, the field lines become kinked, and for sufficiently large λ_0 develop knots. The relationship between the kinking and knotting properties of these equilibria and the presence of a kink instability and related loss of equilibrium was explored. Magnetic reconnection must be involved for an unknotted loop equilibrium to become knotted. It was concluded that there is indeed a loss of equilibrium process associated with the existence of an unstable kink mode, and that this process leads to the creation of a closed hyperbolic field line (X-line) about which this reconnection creating knotted field lines is centered. The field lines were visualized in 3D by AVS (Applications Visualization System) on the DEC-5000 Workstation at Maryland. We developed several modules for use with AVS for tracing field lines from numerical data and for computing Poincare sections.

3. Loss of Equilibrium and Reconnection in Tearing of 2D Equilibria

Two-dimensional tearing-like behavior was studied in reduced resistive magnetohydrodynamics (MHD) with flux conserving boundary conditions on an elongated rectangular box.⁷ This study was begun in order to understand the role of loss of equilibrium in axisymmetric MHD evolution in a tokamak, specifically the spontaneous splitting process in tokamaks with highly elongated cross sections. The tearing-like perturbations do not destroy the symmetries of the initial state, either discrete or continuous. In such cases linear instability is typically not directly observed. However, it was found that there can be a loss of equilibrium (a saddle-node bifurcation)

associated with the existence of a tearing unstable state. The results in a very elongated tokamak, with pinching coils to elongate its flux surfaces, were compared to those in a model for the magnetotail or for solar arcades. The loss of equilibrium was demonstrated by means of a nonlinear energy functional. The importance of the fact that the splitting or tearing process occurs by means of a loss of equilibrium is that a large amount of free energy can be released, in the form of reconnection. Also, with this kind of bifurcation, there is a possibility of hysteresis, suggesting that a tokamak can be heated at the center by repeatedly forcing such a splitting and unsplitting process.

C. Self-Consistent MHD Behavior in the Presence of Chaotic Field Lines (Finn)

For equilibrium or low frequency behavior, chaotic field lines occur only in 3D. However, the beginnings of a systematic study of this general problem have been made in 2D by applying an external force which produces large amplitude Alfvén waves with resonance surfaces near the mode rational surface of a tearing mode. It was found that the tearing mode saturates at a lower level or can be stabilized completely if the Alfvén wave amplitude is large enough. This stabilization is due in part to a quasilinear flattening of the current profile, by overlap of the Alfvén resonances with each other and with the tearing mode. However, another contributing factor in the stabilization is the velocity shear formed by the presence of the large amplitude Alfvén waves in the presence of a sheared background magnetic field. This work was accepted

for an oral presentation at the 1993 Sherwood Conference.

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III. PROPOSED RESEARCH

A. Rayleigh-Taylor Instability with Shear Flow

The earlier work on the Rayleigh-Taylor (RT) instability¹, reported in the progress report was motivated by our 3D simulations on the Resistive Ballooning Modes², for tokamak edge plasmas. The RT model was a simpler 2D model which captured some of the features of the Resistive Ballooning modes without the complications of toroidal geometry. However, the main focus of the study¹ was the change in the nonlinear dynamical behavior of the system as the viscosity was varied. The box size ratio, L_y/L_x was chosen to be 0.8, so that the RT instability would produce tall vortices, which are susceptible to the inviscid “peeling” instability.³⁻⁴ From our 3D simulations we have found that a chain of alternately counter-rotating vortices are generated in the linear phase. Therefore we wish to extend the 2D RT model to allow for multiple pairs of counter-rotating vortices, thereby mimicking the more realistic 3D simulations. This can be readily done by changing the box ratio to 0.4 (as a first step). This would allow the generation of two pairs of counter-rotating vortices. We will then investigate the nonlinear dynamics of this system by changing the viscosity, as was done for the single vortex pair.¹

B. Transition from Marfes to Detached Plasmas

In the edge region of tokamaks it is observed that, with the increase of density, a toroidally symmetric ring of high density, localized poloidally on the inside of the

tokamak, is created.⁵ This region of high density, called a marfe, is a source of significant line radiation from carbon and oxygen. Recently a simple model for thermal transport using an exponential radiation cooling function was studied. It was found that by increasing the impurity concentration the thermal equilibria evolved from a marfe-like state to a detached plasma state.⁶ However what leads to the location of the marfe on the inside of the tokamak as well as the accessibility of the states was not addressed. We will investigate both analytically and numerically a time dependent 2D (radial and poloidal) thermal model for the edge region, which includes the essential feature of poloidally asymmetric energy transport. Our work on the resistive ballooning modes² has clearly demonstrated that large poloidally asymmetric anomalous transport can be generated by these modes, with large transport on the outside of the torus compared to the inside. The ensuing parallel flows can convect particles to the inside where a stagnation point can seed the formation of the marfe. Also the time dependent study will answer the question related to accessibility and stability of the marfe and detached states.

C. The Impact of Self-Consistency on Thermal Transport Driven by Magnetic Stochasticity

Stochastic magnetic fields have been widely invoked to explain cross-field thermal transport during disruptions and during the more quiescent phase of tokamak operation. Electrons streaming along the locally stochastic magnetic field lines produce an effective transport across the “average” magnetic field. Nearly all of the work on

transport in a stochastic magnetic field has not been self-consistent: the transport has simply been calculated in an imposed magnetic field. Because the thermal velocity of electrons in present tokamaks is so large, even rather small magnetic perturbations can cause significant transport.

The large thermal transport in a stochastic magnetic field is, of course, produced by large electron parallel flows driven by the effective parallel temperature gradient. Such electron flows also produce large parallel currents which can strongly alter the magnetic field. It is well known, for example, that to lowest order such parallel temperature gradients simply cause the magnetic fields to rotate at the diamagnetic frequency, producing no thermal transport.⁹ Thus, the thermal transport calculated in a non-self-consistent magnetic field is very likely an extreme over-estimate of the actual transport which is induced by the magnetic perturbations.

The role of self-consistency in magnetic turbulence has recently been investigated analytically in a collisionless plasma.^{7,8} The conclusions of these two papers are diametrically opposed. In Ref. 7 the basic conclusion was that in the collisionless case the magnetic fluctuations could not enhance thermal transport above that caused by electrostatic perturbations. In Ref. 8 the conclusion was that thermal transport was given by the test particle diffusion rate.

We have begun a numerical/analytic study of self-consistent thermal transport induced by magnetic stochasticity. For simplicity, we model the plasma with the Braginskii fluid equations. Thus, we may not be able to directly settle the contradictory results in Refs. 7 and 8. Nevertheless, the fluid calculation is clearly tractable and

will also shed light on the collisionless limit.

We have written a numerical code which advances the electron temperature equation in a 3-D box which is periodic in the y and z directions with an imposed jump in the temperature across the x boundaries. At the present time the magnetic field is simply a specified function $\mathbf{B}(x, y, z)$, i.e., the magnetic field is not yet self-consistent so that $\partial\mathbf{B}/\partial t = 0$. The temperature equation is evolved until it reaches a steady state and the thermal flux is then calculated. When the magnetic field is regular (magnetic islands with only a single helicity) the transport rate is a hybrid of κ_{\parallel} and κ_{\perp} , consistent with theoretical estimates. When the magnetic field is stochastic the transport is insensitive to κ_{\perp} and is very large.

Our next step is to include the self-consistent evolution of the magnetic field using the Braginskii equations. Parallel currents will cause the magnetic field to propagate. The expected reduction in transport will be calculated. Parallel analytic calculations will be completed and compared with the numerical results.

D. Neoclassical Theory of Poloidal and Toroidal Rotation of Tokamak Plasmas

The mechanism of spin-up of tokamaks and their relevance to L-H transitions in tokamaks has been an active area of research. The anomalous Stringer/Winsor spin-up mechanism suggested by Hassam et al.⁹ is a strong candidate for explaining the observed bifurcation. In this work it was assumed that the ions were in the Pfirsch-Schluter regime. The next step is to extend this work into the plateau and

the banana regimes for the ions. This work can have implications for the hotter devices like TFTR and JET where the ions are in the banana regime. Also this work will address what happens whether there are neoclassical spin-up mechanisms in the core region of tokamaks.

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APPENDICES

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